# More on Single Valued Neutrosophic R-dynamic Vertex Coloring and R-dynamic Edge Coloring of graphs

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## Abstract

The notion of neutrosophic sets facilitates the analysis of values that are unclear or indeterminate. In this paper, we discuss the single valued neutrosophic R-dynamic vertex coloring of Cartesian product of SVNG's and join of SVNG's. Further we have described the concept of single valued neutrosophic R-dynamic edge coloring and provided some examples and theorems.
**Keywords:** Single Valued Neutrosophic Graph; Single Valued Neutrosophic Vertex Coloring; Single Valued Neutrosophic R-dynamic Vertex Coloring; Single Valued Neutrosophic Edge Coloring; Single Valued Neutrosophic R-dynamic Edge Coloring.

## 1 Introduction

Rosenfeld [20], 10 years after Zadeh's seminal study in fuzzy sets [23], was the first to develop fuzzy graphs. Fuzzy graph theory is presently being used in a variety of domains in modern science and technology, including information theory, neural networks, expert systems, cluster analysis, medical diagnostics, control theory, and many others. In 2004, Munoz et al. [15] presented the fuzzy chromatic number, which was further developed in 2006 by C. Eslahchi and B. N. Onagh [11]. In 2009, S. Lavanya and R. Sattanathan [13] proposed the idea of fuzzy total coloring for the first time. Arindam Dey and Anita Pal discussed fuzzy vertex coloring with fuzzy graphs using the $\alpha$-cut in the paper [7] in 2012. Anjaly Kishore, M.S.Sunitha, published a research paper [4] in 2014 on strong chromatic number of fuzzy graphs.
To deal with data on membership and non-membership values, intuitionistic fuzzy sets are utilised. Kassimir T. Atanassov [6] developed the concepts of intuitionistic fuzzy sets and intuitionistic fuzzy graphs in 1986 and 1999, respectively. Ismail and Rifayathali [12] addressed intuitionistic fuzzy graph coloring with $(\alpha, \beta)$ cuts in 2015, while Rifayathali et al. [16] published studies on intuitionistic fuzzy coloring and strong intuitionistic fuzzy coloring in 2017 and 2018.
The principles of membership and non-membership are insufficient to predict the result of all real-time circumstances. In situations when the vagueness or indeterminacy of a decision must be considered, intuitionistic fuzzy logic is unsuitable to provide a solution. F. Smarandache discovered a solution as a result of this condition: "Neutrosophic logic". Smarandache [21] developed the concept of Neutrosophic Sets in 1998, which is a generalised form of intuitionistic fuzzy sets that incorporates three categories of values: truth, indeterminacy, and falsity membership values. Single-valued neutrosophic sets were studied by Wang et al. [22] in 2010. In 2015, Dhavaseelan et al. [10] introduced and discussed Strong Neutrosophic graphs, and in 2016, Akram and Shahzadi [1-3] introduced and discussed Single Valued Neutrosophic graph definitions and operations of neutrosophic graphs. Broumi et al. [8, 9] expanded on their prior work with single-valued neutrosophic graphs. Rohini et al. established the concept of single valued neutrosophic vertex coloring, single valued neutrosophic edge coloring, and single valued neutrosophic total coloring of single valued neutrosophic graph with examples in the research publications [17, 18] published in 2019. In addition, Rohini et al. have expanded on their work on single valued neutrosophic vertex coloring and developed the concept of single valued neutrosophic irregular vertex coloring in [19].

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Bruce Montgomery proposed the concept of r-dynamic coloring in [14]. The r-dynamic coloring of a graph is an appropriate coloring of the graph in which each vertex $u$ receives $min\{r, d(u)\}$ different colors from its neighbours. In [5] we have introduced the concept of single valued neutrosophic R-dynamic vertex coloring and furthermore in this paper, we discuss the single valued neutrosophic R-dynamic vertex coloring of Cartesian product of SVNG's and join of SVNG's. Also we have introduce the concept of single valued neutrosophic R-dynamic edge coloring and provided some examples and theorems.

# 2 Preliminaries

**Definition 2.1.** [21] Consider S as a collection of objects(points). Then the **neutrosophic set** P in S is described by truth membership function $t_P(s)$, an indeterminacy function $i_P(s)$ and a falsity membership (non-membership) function $f_P(s)$. The mappings $t_P(s)$, $i_P(s)$ and $f_P(s)$ are real standard or non-standard subsets of $]0^-, 1^+[$ which means $t_P(s) : S \to ]0^-, 1^+[$, $i_P(s) : S \to ]0^-, 1^+[$ and $f_P(s) : S \to ]0^-, 1^+[$. Also $0^- \leq t_P(s) + i_P(s) + f_P(s) \leq 3^+$.

**Definition 2.2.** [2] A **Single Valued Neutrosophic Graph (SNVG)** $G = (C, D)$ is a pair consisting of $C : N \to [0, 1]$ which is a single valued neutrosophic set on N and $D : N \times N \to [0, 1]$, a single valued neutrosophic relation on N with the following characteristics:
$t_D(uv) \leq min\{t_C(u), t_C(v)\}$
$i_D(uv) \leq min\{i_C(u), i_C(v)\}$
$f_D(uv) \leq max\{f_C(u), f_C(v)\}$
for all $u, v \in N$. The sets C and D are said to be single valued neutrosophic vertex set and edge set of G respectively. The single valued neutrosophic relation D is symmetric if it satisfies $t_D(uv) = t_D(vu)$, $i_D(uv) = i_D(vu)$ and $f_D(uv) = f_D(vu)$ for all $u, v \in N$.

**Definition 2.3.** [3] A **complete neutrosophic graph (CSVNG)** is an SVNG $G = (C, D)$ which complies criteria below:
$t_D(uv) = min\{t_C(u), t_C(v)\}$
$i_D(uv) = min\{i_C(u), i_C(v)\}$
$f_D(uv) = max\{f_C(u), f_C(v)\}$
for all $u, v \in C$.

**Definition 2.4.** [17] The collection $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_p\}$ of SVN fuzzy sets is termed as **p-Single Valued Neutrosophic Vertex Coloring(SVNVC)** of SVNG $G = (C, D)$ if the following criteria hold:
1. $\vee\gamma_s(u) = C, \forall u \in C$
2. $\gamma_s \wedge \gamma_t = 0$
3. For each incident vertices of the edge $uv$ of G, $min\{\gamma_s(t_C(u)), \gamma_s(t_C(v))\} = 0$, $min\{\gamma_s(i_C(u)), \gamma_s(i_C(v))\} = 0$ and $max\{\gamma_s(f_C(u)), \gamma_s(f_C(v))\} = 1, (1 \leq s \leq p)$.
This is indicated as $\chi^v(G)$ and is termed as the SVN chromatic number of the SVNG G.

**Definition 2.5.** [5] A family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_p\}$ of SVN fuzzy sets is termed as **p-Single Valued Neutrosophic R-dynamic Vertex Coloring (SVNRVC)** of a SVNG $G = (C, D)$ if the following criteria hold:
1. $\vee\gamma_s(u) = C, \forall u \in C$
2. $\gamma_s \wedge \gamma_t = 0$
3. For each incident vertices of the edge $uv$ of G, $min\{\gamma_s(t_C(u)), \gamma_s(t_C(v))\} = 0$, $min\{\gamma_s(i_C(u)), \gamma_s(i_C(v))\} = 0$ and $max\{\gamma_s(f_C(u)), \gamma_s(f_C(v))\} = 1, (1 \leq s \leq p)$.
4. Every vertex $u$ with $m$ number of incident edges, the corresponding incident vertices of the vertex $u$ receives atleast $min\{R, m\}$ different members(colors) from the set $\Gamma$.
Here, $1 \leq R \leq M$ where $M$ represents the maximum number of incident edges of the vertices of SVNG G.
The least value of $p$ is the SVNRVC of SVNG G which is denoted as $\chi^v_R(G)$, is called the Single Valued Neutrosophic R-dynamic chromatic number of the SVNG G.
Further we define Single Valued Neutrosophic dynamic chromatic number of the SVNG G, $\chi^v_2(G)$ by replacing $R = 2$ in criterion 4 i.e., (4) becomes every vertex $u$ with $m$ number of incident edges, the corresponding incident vertices of the vertex $u$ receives atleast $min\{2, m\}$ different members(colors) from the set $\Gamma$.

**Definition 2.6.** [8] **Path** $P_m$ in a single valued neutrosophic graph $G = (C, D)$ is an arrangement of distinct vertices $v_1, v_2, \cdots, v_m$ which complies the criteria $t_D(v_{k-1}, v_k) > 0$, $i_D(v_{k-1}, v_k) > 0$ and $f_D(v_{k-1}, v_k) > 0$ for $2 \leq k \leq m$.

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**Definition 2.7.** [8] A **cycle** $C_m$ in a single valued neutrosophic graph $G = (C, D)$ is a sequence of distinct vertices $v_1, v_2, \cdots, v_m, v_1$ which satisfies the condition $t_D(v_{k-1}, v_k) > 0$, $i_D(v_{k-1}, v_k) > 0$ and $f_D(v_{k-1}, v_k) > 0$ for $2 \leq k \leq$, together with $t_D(v_1, v_m) > 0$, $i_D(v_1, v_m) > 0$ and $f_D(v_1, v_m) > 0$.

**Definition 2.8.** [3] Consider $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two single valued neutrosophic graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. Then the cartesian product $G_1 \times G_2$ is defined to be the pair $(C, D)$ such that
1. $t_C(u_1, u_2) = min\{t_{C_1}(u_1), t_{C_2}(u_2)\}$
$i_C(u_1, u_2) = min\{i_{C_1}(u_1), i_{C_2}(u_2)\}$
$f_C(u_1, u_2) = max\{f_{C_1}(u_1), f_{C_2}(u_2)\}$
2. $t_D((u, u_2)(u, v_2)) = min\{t_{C_1}(u), t_{D_2}(u_2v_2)\}$
$i_D((u, u_2)(u, v_2)) = min\{i_{C_1}(u), i_{D_2}(u_2v_2)\}$
$f_D((u, u_2)(u, v_2)) = max\{f_{C_1}(u), f_{D_2}(u_2v_2)\}$ for all $u \in V_1$ and for all $u_2v_2 \in E_2$.
3. $t_D((u_1, v), (v_1, v)) = min\{t_{D_1}(u_1v_1), t_{C_2}(v)\}$
$i_D((u_1, v), (v_1, v)) = min\{i_{D_1}(u_1v_1), i_{C_2}(v)\}$
$f_D((u_1, v), (v_1, v)) = max\{f_{D_1}(u_1v_1), f_{C_2}(v)\}$ for all $v \in V_2$ and for all $u_1v_1 \in E_1$.

**Definition 2.9.** [3] Consider $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two single valued neutrosophic graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. Then the join $G_1 + G_2$ is defined to be the pair $(C, D)$ such that

$$1. t_C(u) = \begin{cases} t_{C_1}(u) & if \quad u \in V_1 \; and \; u \notin V_2 \\ t_{C_2}(u) & if \quad u \in V_2 \; and \; u \notin V_1 \\ max\{t_{C_1}(u), t_{C_2}(u)\} & if \quad u \in V_1 \cap V_2 \end{cases}$$

$$2.\ i_C(u) = \begin{cases} i_{C_1}(u) & if \quad u \in V_1 \; and \; u \notin V_2 \\ i_{C_2}(u) & if \quad u \in V_2 \; and \; u \notin V_1 \\ max\{i_{C_1}(u), i_{C_2}(u)\} & if \quad u \in V_1 \cap V_2 \end{cases}$$

$$3.\ f_C(u) = \begin{cases} f_{C_1}(u) & if \quad u \in V_1 \; and \; u \notin V_2 \\ f_{C_2}(u) & if \quad u \in V_2 \; and \; u \notin V_1 \\ min\{f_{C_1}(u), f_{C_2}(u)\} & if \quad u \in V_1 \cap V_2 \end{cases}$$

$$4.\ t_D(uv) = \begin{cases} t_{D_1}(uv) & if \quad uv \in E_1 \; and \; uv \notin E_2 \\ t_{D_2}(uv) & if \quad uv \in E_2 \; and \; uv \notin E_1 \\ max\{t_{D_1}(uv), t_{D_2}(uv)\} & if \quad uv \in E_1 \cap E_2 \\ min\{t_{C_1}(u), t_{C_2}(v)\} & if \quad uv \in E' \end{cases}$$

$$5.\ i_D(uv) = \begin{cases} i_{D_1}(uv) & if \quad uv \in E_1 \; and \; uv \notin E_2 \\ i_{D_2}(uv) & if \quad uv \in E_2 \; and \; uv \notin E_1 \\ max\{i_{D_1}(uv), i_{D_2}(uv)\} & if \quad uv \in E_1 \cap E_2 \\ min\{i_{C_1}(u), i_{C_2}(v)\} & if \quad uv \in E' \end{cases}$$

$$6.\ f_D(uv) = \begin{cases} f_{D_1}(uv) & if \quad uv \in E_1 \; and \; uv \notin E_2 \\ f_{D_2}(uv) & if \quad uv \in E_2 \; and \; uv \notin E_1 \\ min\{f_{D_1}(uv), f_{D_2}(uv)\} & if \quad uv \in E_1 \cap E_2 \\ max\{f_{C_1}(u), f_{C_2}(v)\} & if \quad uv \in E' \end{cases}$$

# 3 Single Valued Neutrosophic Dynamic Coloring of Graphs

In this section we have determined the Single Valued Neutrosophic Dynamic Chromatic Number of some graphs like Cartesian product of path with path, path with complete graph; join of path and path, path with cycle and path with complete SVNG.

**Theorem 3.1.** *Let $l \geq 2, m \geq 2$, then the single valued neutrosophic dynamic chromatic number of the cartesian product of path $P_l$ with path $P_m$ is $\chi_2^v(P_l \times P_m) = 4$.*

*Proof.* Consider path $P_l = (C_1, D_1)$ with SVN vertex set $C_1 = \{v_1, v_2, \cdots, v_l\}$ and path $P_m = (C_2, D_2)$ with SVN vertex set $C_2 = \{u_1, u_2, \cdots, u_m\}$. Then the cartesian product of path $P_l$ with path $P_m$ is SVNG $P_l \times P_m = (C, D)$ with SVN vertex set $C = \{(v_j, u_k) : 1 \leq j \leq l, 1 \leq k \leq m\}$ and edge set defined as in the definition 2.8.
Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be the collection of fuzzy sets determined on vertices of $P_l \times P_m$ for $R = 2$ as follows:

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Case 1: When $m \equiv 0(mod\,4)$ and $m \equiv 2(mod\,4)$.

$$\gamma_1((v_j,u_k)) = \begin{cases} (t_C((v_j,u_k)), i_C((v_j,u_k)), f_C((v_j,u_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 1(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k \equiv 3(mod\,4), \\ & \quad\ j \equiv 3(mod\,4)\ and\ k \equiv 2(mod\,4,) \\ & \quad\ j \equiv 0(mod\,4)\ and\ k \equiv 0(mod\,4) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2((v_j,u_k)) = \begin{cases} (t_C((v_j,u_k)), i_C((v_j,u_k)), f_C((v_j,u_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 2(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k \equiv 0(mod\,4), \\ & \quad\ j \equiv 3(mod\,4)\ and\ k \equiv 1(mod\,4), \\ & \quad\ j \equiv 0(mod\,4)\ and\ k \equiv 3(mod\,4) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3((v_j,u_k)) = \begin{cases} (t_C((v_j,u_k)), i_C((v_j,u_k)), f_C((v_j,u_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 3(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k \equiv 1(mod\,4), \\ & \quad\ j \equiv 3(mod\,4)\ and\ k \equiv 0(mod\,4), \\ & \quad\ j \equiv 0(mod\,4)\ and\ k \equiv 2(mod\,4) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4((v_j,u_k)) = \begin{cases} (t_C((v_j,u_k)), i_C((v_j,u_k)), f_C((v_j,u_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 0(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k \equiv 2(mod\,4), \\ & \quad\ j \equiv 3(mod\,4)\ and\ k \equiv 3(mod\,4), \\ & \quad\ j \equiv 0(mod\,4)\ and\ k \equiv 1(mod\,4) \\ (0,0,1) & for \quad otherwise \end{cases}$$

Case 2 : When $m \equiv 1(mod\,4)$.

$$\gamma_1((v_j,u_k)) = \begin{cases} (t_C((v_j,u_k)), i_C((v_j,u_k)), f_C((v_j,u_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 1(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k \equiv 3(mod\,4), \\ & \quad\ j \equiv 3(mod\,4)\ and\ k \equiv 2(mod\,4), \\ & \quad\ j \equiv 0(mod\,4)\ and\ k \equiv 0(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k = m \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2((v_j,u_k)) = \begin{cases} (t_C((v_j,u_k)), i_C((v_j,u_k)), f_C((v_j,u_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 2(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k \equiv 0(mod\,4), \\ & \quad\ j \equiv 3(mod\,4)\ and\ k \equiv 1(mod\,4), \\ & \quad\ j \equiv 0(mod\,4)\ and\ k \equiv 3(mod\,4), \\ & \quad\ j \equiv 0(mod\,4)\ and\ k = m \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3((v_j,u_k)) = \begin{cases} (t_C((v_j,u_k)), i_C((v_j,u_k)), f_C((v_j,u_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 3(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k \equiv 1(mod\,4), \\ & \quad\ j \equiv 3(mod\,4)\ and\ k \equiv 0(mod\,4), \\ & \quad\ j \equiv 0(mod\,4)\ and\ k \equiv 2(mod\,4), \\ & \quad\ j \equiv 1(mod\,4)\ and\ k = m \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4((v_j,u_k)) = \begin{cases} (t_C((v_j,u_k)), i_C((v_j,u_k)), f_C((v_j,u_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 0(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k \equiv 2(mod\,4), \\ & \quad\ j \equiv 3(mod\,4)\ and\ k \equiv 3(mod\,4), \\ & \quad\ j \equiv 0(mod\,4)\ and\ k \equiv 1(mod\,4), \\ & \quad\ j \equiv 3(mod\,4)\ and\ k = m \\ (0,0,1) & for \quad otherwise \end{cases}$$

Case 3 : When $m \equiv 3(mod\,4)$.

$$\gamma_1((v_j,u_k)) = \begin{cases} (t_C((v_j,u_k)), i_C((v_j,u_k)), f_C((v_j,u_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 1(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k \equiv 3(mod\,4), \\ & \quad\ j \equiv 3(mod\,4)\ and\ k \equiv 2(mod\,4), \\ & \quad\ j \equiv 0(mod\,4)\ and\ k \equiv 0(mod\,4), \\ & \quad\ j \equiv 2(mod\,4)\ and\ k = m-1, \\ & \quad\ j \equiv 0(mod\,4)\ and\ k = m \\ (0,0,1) & for \quad otherwise \end{cases}$$

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$$\gamma_2((v_j,u_k))=\begin{cases}(t_C((v_j,u_k)),i_C((v_j,u_k)),f_C((v_j,u_k)) & for\quad j\equiv 1(mod\,4)\ and\ k\equiv 2(mod\,4),\\ & \quad j\equiv 2(mod\,4)\ and\ k\equiv 0(mod\,4),\\ & \quad j\equiv 3(mod\,4)\ and\ k\equiv 1(mod\,4),\\ & \quad j\equiv 0(mod\,4)\ and\ k\equiv 3(mod\,4),\\ & \quad j\equiv 0(mod\,4)\ and\ k=m-1,\\ & \quad j\equiv 2(mod\,4)\ and\ k=m\\ (0,0,1) & for\quad otherwise\end{cases}$$

$$\gamma_3((v_j,u_k))=\begin{cases}(t_C((v_j,u_k)),i_C((v_j,u_k)),f_C((v_j,u_k)) & for\quad j\equiv 1(mod\,4)\ and\ k\equiv 3(mod\,4),\\ & \quad j\equiv 2(mod\,4)\ and\ k\equiv 1(mod\,4),\\ & \quad j\equiv 3(mod\,4)\ and\ k\equiv 0(mod\,4),\\ & \quad j\equiv 0(mod\,4)\ and\ k\equiv 2(mod\,4),\\ & \quad j\equiv 1(mod\,4)\ and\ k=m-1,\\ & \quad j\equiv 3(mod\,4)\ and\ k=m\\ (0,0,1) & for\quad otherwise\end{cases}$$

$$\gamma_4((v_j,u_k))=\begin{cases}(t_C((v_j,u_k)),i_C((v_j,u_k)),f_C((v_j,u_k)) & for\quad j\equiv 1(mod\,4)\ and\ k\equiv 0(mod\,4),\\ & \quad j\equiv 2(mod\,4)\ and\ k\equiv 2(mod\,4),\\ & \quad j\equiv 3(mod\,4)\ and\ k\equiv 3(mod\,4),\\ & \quad j\equiv 0(mod\,4)\ and\ k\equiv 1(mod\,4),\\ & \quad j\equiv 3(mod\,4)\ and\ k=m-1,\\ & \quad j\equiv 1(mod\,4)\ and\ k=m\\ (0,0,1) & for\quad otherwise\end{cases}$$

Case 4 : For $P_3 \times P_3$.

$$\gamma_1((v_j,u_k))=\begin{cases}(t_C((v_j,u_k)),i_C((v_j,u_k)),f_C((v_j,u_k)) & for\quad jk=11,13,32\\ (0,0,1) & for\quad otherwise\end{cases}$$

$$\gamma_2((v_j,u_k))=\begin{cases}(t_C((v_j,u_k)),i_C((v_j,u_k)),f_C((v_j,u_k)) & for\quad jk=12,31,33\\ (0,0,1) & for\quad otherwise\end{cases}$$

$$\gamma_3((v_j,u_k))=\begin{cases}(t_C((v_j,u_k)),i_C((v_j,u_k)),f_C((v_j,u_k)) & for\quad jk=21,23\\ (0,0,1) & for\quad otherwise\end{cases}$$

$$\gamma_4((v_j,u_k))=\begin{cases}(t_C((v_j,u_k)),i_C((v_j,u_k)),f_C((v_j,u_k)) & for\quad jk=22\\ (0,0,1) & for\quad otherwise\end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ ensures that SVNRVC requirements are met. Families with fewer than four points did not match our defining criteria. Hence $\chi_2^v(P_l \times P_m) = 4$. □

**Theorem 3.2.** *Let $l \geq 2, m \geq 2$, then the single valued neutrosophic dynamic chromatic number of the cartesian product of path $P_l$ with CSVNG $K_m$ is $\chi_2^v(P_l \times K_m) = m$.*

*Proof.* Consider path $P_l = (C_1, D_1)$ with SVN vertex set $C_1 = \{v_1, v_2, \cdots, v_l\}$ and CSVNG $K_m = (C_2, D_2)$ with SVN vertex set $C_2 = \{u_1, u_2, \cdots, u_m\}$. Then the cartesian product of path $P_l$ with CSVNG $K_m$ is SVNG $P_l \times K_m = (C, D)$ with SVN vertex set $C = \{(v_j, u_k) : 1 \leq j \leq l, 1 \leq k \leq m\}$ and edge set defined as in the definition 2.8.

Let $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_m\}$ be the collection of fuzzy sets determined on vertices of $P_l \times K_m$ for $R = 2$ as follows:

$$\gamma_1((v_j,u_k))=\begin{cases}(t_C((v_j,u_k)),i_C((v_j,u_k)),f_C((v_j,u_k)) & for\quad j\equiv 1(mod\,2)\ and\ k=1,\\ & \quad j\equiv 0(mod\,2)\ and\ k=m\\ (0,0,1) & for\quad otherwise\end{cases}$$

For $2 \leq i \leq m$,

$$\gamma_i((v_j,u_k))=\begin{cases}(t_C((v_j,u_k)),i_C((v_j,u_k)),f_C((v_j,u_k)) & for\quad j\equiv 1(mod\,2)\ and\ k=i,\\ (t_C((v_j,u_k)),i_C((v_j,u_k)),f_C((v_j,u_k)) & \quad j\equiv 0(mod\,2)\ and\ k=i-1\\ (0,0,1) & for\quad otherwise\end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_m\}$ ensures that SVNRVC requirements are met. Families with fewer than $m$ points did not match our defining criteria. Hence $\chi_2^v(P_l \times K_m) = m$. □

**Theorem 3.3.** *Let $l \geq 2, m \geq 2$, then the single valued neutrosophic dynamic chromatic number of the join of path $P_l$ with path $P_m$ is $\chi_2^v(P_l + P_m) = 4$.*

*Proof.* Consider path $P_l = (C_1, D_1)$ with SVN vertex set $C_1 = \{v_1, v_2, \cdots, v_l\}$ and path $P_m = (C_2, D_2)$ with SVN vertex set $C_2 = \{u_1, u_2, \cdots, u_m\}$. Also the vertex set of these SVNG's are disjoint. Then the join of path $P_l$ with path $P_m$ is SVNG $P_l + P_m = (C, D)$ with SVN vertex set $C = \{v_1, v_2, \cdots, v_l, u_1, u_2, \cdots, u_m\}$. Also the membership, indeterminacy and falsity functions of the vertices of $P_l + P_m$ is determined using the definition in 2.9 and edge set is also determined in the same way.

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Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be the collection of fuzzy sets determined on vertices of $P_l + P_m$ for $R = 2$ as follows:

$$\gamma_1(v_j) = \begin{cases} (t_C(v_j), i_C(v_j), f_C(v_j)) & for \quad j \equiv 1(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(v_j) = \begin{cases} (t_C(v_j), i_C(v_j), f_C(v_j)) & for \quad j \equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(u_k) = \begin{cases} (t_C(u_k), i_C(u_k), f_C(u_k)) & for \quad k \equiv 1(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(u_k) = \begin{cases} (t_C(u_k), i_C(u_k), f_C(u_k)) & for \quad k \equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_4\}$ ensures that SVNRVC requirements are met. Families with fewer than four points did not match our defining criteria. Hence $\chi_2^v(P_l + P_m) = 4$. □

**Theorem 3.4.** *Let $l \geq 2, m \geq 3$, then the single valued neutrosophic dynamic chromatic number of the join of path $P_l$ with cycle $C_m$ is* $\chi_2^v(P_l + C_m) = \begin{cases} 4 & when \quad m\ is\ even \\ 5 & when \quad m\ is\ odd \end{cases}$.

*Proof.* Consider path $P_l = (C_1, D_1)$ with SVN vertex set $C_1 = \{v_1, v_2, \cdots, v_l\}$ and cycle $C_m = (C_2, D_2)$ with SVN vertex set $C_2 = \{u_1, u_2, \cdots, u_m\}$. Also the vertex set of these SVNG's are disjoint. Then the join of path $P_l$ with cycle $C_m$ is SVNG $P_l+C_m = (C, D)$ with SVN vertex set $C = \{v_1, v_2, \cdots, v_l, u_1, u_2, \cdots, u_m\}$. Also the membership, indeterminacy and falsity functions of the vertices of $P_l + C_m$ is determined using the definition in 2.9 and edge set is also determined in the same way.

Case 1 : When m is even.

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be the collection of fuzzy sets determined on vertices of $P_l + C_m$ for $R = 2$ as follows:

$$\gamma_1(v_j) = \begin{cases} (t_C(v_j), i_C(v_j), f_C(v_j)) & for \quad j \equiv 1(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(v_j) = \begin{cases} (t_C(v_j), i_C(v_j), f_C(v_j)) & for \quad j \equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(u_k) = \begin{cases} (t_C(u_k), i_C(u_k), f_C(u_k)) & for \quad k \equiv 1(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(u_k) = \begin{cases} (t_C(u_k), i_C(u_k), f_C(u_k)) & for \quad j \equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_4\}$ ensures that SVNRVC requirements are met. Families with fewer than four points did not match our defining criteria.

Case 2 : When m is odd.

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ be the collection of fuzzy sets determined on vertices of $P_l + C_m$ for $R = 2$ as follows:

$$\gamma_1(v_j) = \begin{cases} (t_C(v_j), i_C(v_j), f_C(v_j)) & for \quad j \equiv 1(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(v_j) = \begin{cases} (t_C(v_j), i_C(v_j), f_C(v_j)) & for \quad j \equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(u_k) = \begin{cases} (t_C(u_k), i_C(u_k), f_C(u_k)) & for \quad k \equiv 1(mod\,2)\ and\ k \neq m \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(u_k) = \begin{cases} (t_C(u_k), i_C(u_k), f_C(u_k)) & for \quad k \equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_5(u_k) = \begin{cases} (t_C(u_k), i_C(u_k), f_C(u_k)) & for \quad k = m \\ (0,0,1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_5\}$ ensures that SVNRVC requirements are met. Families with fewer than five points did not match our defining criteria.

Hence $\chi_2^v(P_l + C_m) = \begin{cases} 4 & when \quad m\ is\ even \\ 5 & when \quad m\ is\ odd \end{cases}$. □

**Theorem 3.5.** *Let $l \geq 2, m \geq 2$, then the single valued neutrosophic dynamic chromatic number of the join of path $P_l$ with CSVNG $K_m$ is $\chi_2^v(P_l + K_m) = m + 2$.*

*Proof.* Consider path $P_l = (C_1, D_1)$ with SVN vertex set $C_1 = \{v_1, v_2, \cdots, v_l\}$ and CSVNG $K_m = (C_2, D_2)$ with SVN vertex set $C_2 = \{u_1, u_2, \cdots, u_m\}$. Also the vertex set of these SVNG's are disjoint. Then the join of path $P_l$ with CSVNG $K_m$ is SVNG $P_l + K_m = (C, D)$ with SVN vertex set

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$C = \{v_1, v_2, \cdots, v_l, u_1, u_2, \cdots, u_m\}$. Also the membership, indeterminacy and falsity functions of the vertices of $P_l + K_m$ is determined using the definition in 2.9 and edge set is also determined in the same way. Let $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_{m+2}\}$ be the collection of fuzzy sets determined on vertices of $P_l + K_m$ for $R = 2$ as follows:

$$\gamma_1(v_j) = \begin{cases} (t_C(v_j), i_C(v_j), f_C(v_j)) & for \quad j \equiv 1(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(v_j) = \begin{cases} (t_C(v_j), i_C(v_j), f_C(v_j)) & for \quad j \equiv 0(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

For $3 \leq k \leq m+2$,

$$\gamma_k(u_i) = \begin{cases} (t_C(u_i), i_C(u_i), f_C(u_i)) & for \quad i = k-2 \\ (0,0,1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_{m+2}\}$ ensures that SVNRVC requirements are met. Families with fewer than $m + 2$ points did not match our defining criteria. Hence $\chi_2^v(P_l + K_m) = m + 2$. □

# 4 Single-Valued Neutrosophic Edge Coloring

**Definition 4.1.** [17] The collection $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_p\}$ of SVN fuzzy sets is termed as **p-Single Valued Neutrosophic Edge Coloring(SVNEC)** of SVNG $G = (C, D)$ if the following criteria hold:
1. $\vee\gamma_s(uv) = D, \forall uv \in D$
2. $\gamma_s \wedge \gamma_t = 0$
3. For each strong edge $uv$ of G, $min\{\gamma_s(t_D(uv))\} = 0$, $min\{\gamma_s(i_D(uv))\} = 0$ and $max\{\gamma_s(f_D(uv))\} = 1$, $(1 \leq s \leq p)$.
This is indicated as $\chi^e(G)$ and is termed as the SVN edge chromatic number of the SVNG G.

**Example 4.2.** Consider the following SVNG $G_1 = (C, D)$ with SVN vertex set $C = \{v_1, v_2, v_3, v_4, v_5\}$ and SVN edge $D = \{v_jv_k | jk = 12, 23, 34, 45, 15\}$ with

$$(t_C(v_j), i_C(v_j), f_C(v_j)) = \begin{cases} (0.4, 0.2, 0.5) & j = 1 \\ (0.3, 0.4, 0.6) & j = 2, 4 \\ (0.4, 0.4, 0.7) & j = 3 \\ (0.2, 0.3, 0.8) & j = 5 \end{cases}$$

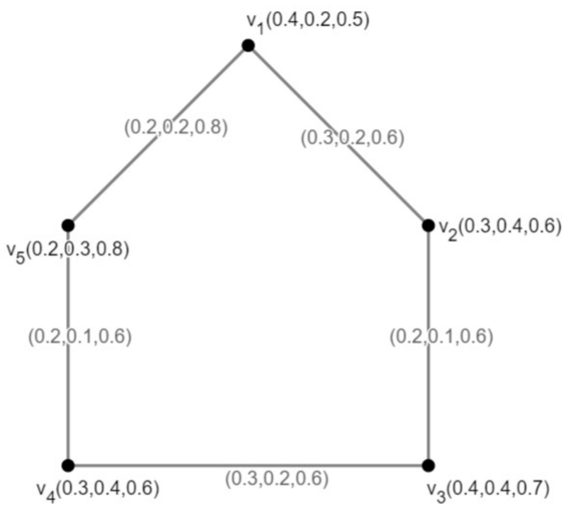


Figure 1: $G_1$

$$(t_D(v_jv_k), i_D(v_jv_k), f_D(v_jv_k)) = \begin{cases} (0.3, 0.2, 0.6) & jk = 12, 34 \\ (0.2, 0.1, 0.6) & jk = 23, 45 \\ (0.2, 0.2, 0.8) & jk = 15 \end{cases}$$

Figure 1 depicts the SVNG $G_1$.
Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be collection of SVN fuzzy sets determined on D as below:

$$\gamma_1(v_jv_k) = \begin{cases} (0.3, 0.2, 0.6) & for \quad jk = 12, 34 \\ (0,0,1) & for \quad otherwise \end{cases}$$

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$$\gamma_2(v_jv_k) = \begin{cases} (0.2, 0.1, 0.6) & for \quad jk = 23, 45 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$
$$\gamma_3(v_jv_k) = \begin{cases} (0.2, 0.2, 0.8) & for \quad j = 15 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

Hence the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ ensures that SVNEC requirements are met. Families with fewer than 3 points did not match our defining criteria. Hence $\chi^e G_1 = 3$.

**Definition 4.3.** The collection $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_p\}$ of SVN fuzzy sets is termed as **p-Single Valued Neutrosophic R-dynamic Edge Coloring(SVNREC)** of SVNG $G = (C, D)$ if the following criteria hold:
1. $\vee\gamma_s(uv) = D, \forall uv \in D$
2. $\gamma_s \wedge \gamma_t = 0$
3. For each strong edge $uv$ of G, $min\{\gamma_s(t_D(uv))\} = 0, min\{\gamma_s(i_D(uv))\} = 0$ and $max\{\gamma_s(f_D(uv))\} = 1$, $(1 \leq s \leq p)$.
4. For each edge $uv$ with at least $m$ number of incident edges, the corresponding edges receives at least $min\{R, m\}$ different members(colors) from the set $\Gamma$.
Here, $1 \leq R \leq M$ where $M$ represents the maximum number of incident edges at the edges of SVNG G.
The least value of $p$ is the SVNREC of SVNG G is denoted as $\chi^e_R(G)$, is called the Single Valued Neutrosophic R-dynamic edge chromatic number of the SVNG G.

**Example 4.4.** Consider the following SVNG $G_2 = (C, D)$ with SVN vertex set $C = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and SVN edge $D = \{v_jv_k | jk = 12, 23, 24, 34, 45, 56, 15, 16\}$ with

$$(t_C(v_j), i_C(v_j), f_C(v_j)) = \begin{cases} (0.5, 0.3, 0.4) & j = 1, 4 \\ (0.7, 0.6, 0.8) & j = 2 \\ (0.8, 0.4, 0.6) & j = 3, 6 \\ (0.6, 0.5, 0.7) & j = 5 \end{cases}$$

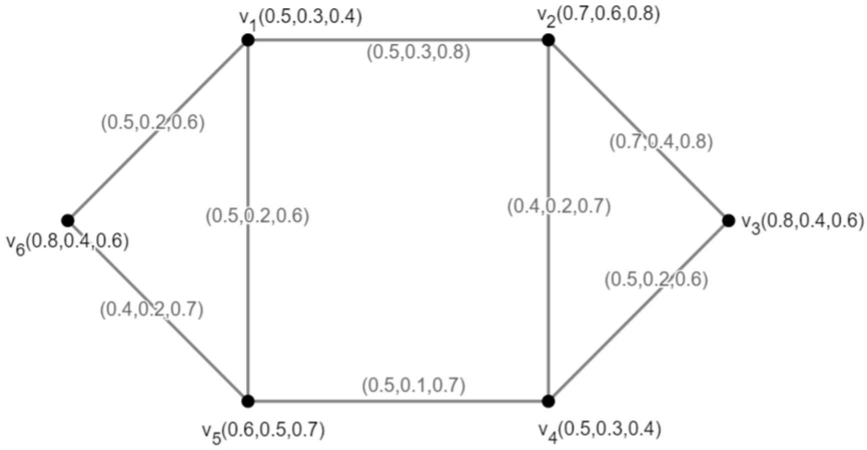


Figure 2: $G_2$

$$(t_D(v_jv_k), i_D(v_jv_k), f_D(v_jv_k)) = \begin{cases} (0.5, 0.3, 0.8) & jk = 12 \\ (0.4, 0.2, 0.7) & jk = 24, 56 \\ (0.5, 0.2, 0.6) & jk = 34, 15, 16 \\ (0.7, 0.4, 0.8) & jk = 23 \\ (0.5, 0.1, 0.7) & jk = 45 \end{cases}$$

Figure 2 depicts the SVNG $G_2$.
Here $M = 4$ so $1 \leq R \leq 4$.
For $1 \leq R \leq 2$ let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be collection of SVN fuzzy sets determined on D as below:

$$\gamma_1(v_jv_k) = \begin{cases} (0.5, 0.3, 0.8) & for \quad jk = 12 \\ (0.4, 0.2, 0.7) & for \quad jk = 56 \\ (0.5, 0.2, 0.6) & for \quad jk = 34 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$
$$\gamma_2(v_jv_k) = \begin{cases} (0.5, 0.2, 0.6) & for \quad jk = 16 \\ (0.7, 0.4, 0.8) & for \quad jk = 23 \\ (0.5, 0.1, 0.7) & for \quad jk = 45 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

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$$\gamma_3(v_jv_k) = \begin{cases} (0.4,0.2,0.7) & for \quad jk=24 \\ (0.5,0.2,0.6) & for \quad jk=15 \\ (0,0,1) & for \quad otherwise \end{cases}$$

Hence the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ ensures that SVNREC requirements are met. Families with fewer than 3 points did not match our defining criteria. Hence $\chi_R^e G_2 = 3$.

For $R = 3$ let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be collection of SVN fuzzy sets determined on D as below:

$$\gamma_1(v_jv_k) = \begin{cases} (0.5,0.2,0.6) & for \quad jk=34,16 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(v_jv_k) = \begin{cases} (0.4,0.2,0.7) & for \quad jk=24,56 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(v_jv_k) = \begin{cases} (0.7,0.4,0.8) & for \quad jk=23 \\ (0.5,0.2,0.6) & for \quad jk=15 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(v_jv_k) = \begin{cases} (0.5,0.3,0.8) & for \quad jk=12 \\ (0.5,0.1,0.7) & for \quad jk=45 \\ (0,0,1) & for \quad otherwise \end{cases}$$

Hence the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ ensures that SVNREC requirements are met. Families with fewer than 4 points did not match our defining criteria. Hence $\chi_3^e G_2 = 4$.

For $R = 4$ let $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_6\}$ be collection of SVN fuzzy sets determined on D as below:

$$\gamma_1(v_jv_k) = \begin{cases} (0.5,0.3,0.8) & for \quad jk=12 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(v_jv_k) = \begin{cases} (0.5,0.2,0.6) & for \quad jk=34,16 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(v_jv_k) = \begin{cases} (0.4,0.2,0.7) & for \quad jk=24 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(v_jv_k) = \begin{cases} (0.5,0.2,0.6) & for \quad jk=15 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_5(v_jv_k) = \begin{cases} (0.7,0.4,0.8) & for \quad jk=23 \\ (0.4,0.2,0.7) & for \quad jk=56 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_6(v_jv_k) = \begin{cases} (0.5,0.1,0.7) & for \quad jk=45 \\ (0,0,1) & for \quad otherwise \end{cases}$$

Hence the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \cdots, \gamma_6\}$ ensures that SVNREC requirements are met. Families with fewer than 6 points did not match our defining criteria. Hence $\chi_4^e G_2 = 6$.

Thus $\chi_R^e(G_2) = \begin{cases} 3 & for \quad 1 \le R \le 2 \\ 4 & for \quad R=3 \\ 6 & for \quad R=4 \end{cases}$

**Theorem 4.5.** *Let $n \ge 3$, $P_l$ be a path graph then* $\chi_R^e(P_l) = \begin{cases} 2 & for \quad R=1 \\ 3 & for \quad R=2 \end{cases}$

*Proof.* Let the SVN vertex set of $P_l$ be $C = \{u_1, u_2, \cdots, u_l\}$ and SVN edge set be $D = \{u_{j-1}u_j : 2 \le j \le l\}$. For the path graph $P_l$, $1 \le R \le 2$.

Let $\Gamma = \{\gamma_1, \gamma_2\}$ be collection of fuzzy sets determined on edges for $R = 1$ as follows:

$$\gamma_1(u_{j-1}u_j) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \text{ is even} \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(u_{j-1}u_j) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \text{ is odd} \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2\}$ ensures that SVNREC requirements are met. Families with fewer than 2 points did not match our defining criteria. Hence $\chi_R^e(P_l) = 2$.

When $R = 2$, let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be collection of fuzzy sets determined on edges as follows:

$$\gamma_1(u_{j-1}u_j) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 2 (mod\, 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(u_{j-1}u_j) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 0 (mod\, 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(u_{j-1}u_j) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 1 (mod\, 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ ensures that SVNREC requirements are met. Families with fewer than 3

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points did not match our defining criteria. Thus $\chi^e_2(P_l) = 3$.

Hence $\chi^e_R(P_l) = \begin{cases} 2 & for \quad R = 1 \\ 3 & for \quad R = 2 \end{cases}$ □

**Theorem 4.6.** *Let $l \geq 3$, $C_l$ be a cycle then* $\chi^e_R(C_l) = \begin{cases} 2 & for \quad R = 1 \quad and\, l\, is\, even \\ 3 & for \quad R = 1 \quad and\, l\, is\, odd \\ 5 & for \quad R = 2 \quad and\, l = 5 \\ 3 & for \quad R = 2 \quad and\, l = 3n \\ 4 & for \quad R = 2 \quad and\, otherwise \end{cases}$

*Proof.* Let the SVN vertex set of $C_l$ be $C = \{u_1, u_2, \cdots, u_l\}$ and SVN edge set be $D = \{u_{j-1}u_j, u_{1l} : 2 \leq j \leq l\}$. For the path graph $C_l$, $1 \leq R \leq 2$.

Case 1 : When $R = 1\; and\, l\, is\, even.$

Let $\Gamma = \{\gamma_1, \gamma_2\}$ be collection of fuzzy sets determined on edges for $R = 1$ as follows:

$$\gamma_1(u_ku_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j\; is\; even \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(u_ku_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j\; is\; odd \\ (t_D(u_ku_h), i_D(u_ku_h), f_D(u_ku_h)) & for \quad kh = 1l \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2\}$ ensures that SVNREC requirements are met. Families with fewer than 2 points did not match our defining criteria. Hence $\chi^e_1(C_l) = 2$.

Case 2 : When $R = 1\; and\, l\, is\, odd.$

Let $\Gamma = \{\gamma_1, \gamma_2\}$ be collection of fuzzy sets determined on edges for $R = 1$ as follows:

$$\gamma_1(u_ku_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j\; is\; even \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(u_ku_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j\; is\; odd \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(u_ku_h) = \begin{cases} (t_D(u_ku_h), i_D(u_ku_h), f_D(u_ku_h)) & for \quad kh = 1l \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ ensures that SVNREC requirements are met. Families with fewer than 3 points did not match our defining criteria. Hence $\chi^e_1(C_l) = 3$.

Case 3 : When $R = 2\; and\; l = 5$.

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ be a family of fuzzy sets determined on edges for $R = 2$ as follows:

$$\gamma_1(u_ku_h) = \begin{cases} (t_D(u_ku_h), i_D(u_ku_h), f_D(u_ku_h)) & for \quad kh = 12 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(u_ku_h) = \begin{cases} (t_D(u_ku_h), i_D(u_ku_h), f_D(u_ku_h)) & for \quad kh = 23 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(u_ku_h) = \begin{cases} (t_D(u_ku_h), i_D(u_ku_h), f_D(u_ku_h)) & for \quad kh = 34 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(u_ku_h) = \begin{cases} (t_D(u_ku_h), i_D(u_ku_h), f_D(u_ku_h)) & for \quad kh = 45 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_5(u_ku_h) = \begin{cases} (t_D(u_ku_h), i_D(u_ku_h), f_D(u_ku_h)) & for \quad kh = 15 \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ ensures that SVNREC requirements are met. Families with fewer than 5 points did not match our defining criteria. Hence $\chi^e_2(C_l) = 5$.

Case 4 : When $R = 2$ and $l = 3n, n = 1, 2, \cdots$.

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be a family of fuzzy sets determined on edges as follows:

$$\gamma_1(u_ku_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 2(mod\, 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(u_ku_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 0(mod\, 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(u_ku_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 1(mod\, 3) \\ (t_D(u_ku_h), i_D(u_ku_h), f_D(u_ku_h)) & for \quad kh = 1l \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ ensures that SVNREC requirements are met. Families with fewer than 3 points did not match our defining criteria. Hence $\chi^e_2(C_l) = 3$ for $l = 3n$.

Case 5 : When $R = 2$ and otherwise.

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be a family of fuzzy sets determined on edges as follows:

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When $l = 3n+1, n = 1, 2, \cdots$.

$$\gamma_1(u_k u_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 2(mod\, 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(u_k u_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 0(mod\, 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(u_k u_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 1(mod\, 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(u_k u_h) = \begin{cases} (t_D(u_k u_h), i_D(u_k u_h), f_D(u_k u_h)) & for \quad kh = 1l \\ (0,0,1) & for \quad otherwise \end{cases}$$

When $l = 3n+2, n = 1, 2, \cdots$

$$\gamma_1(u_k u_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 2(mod\, 3) \\ (t_D(u_k u_h), i_D(u_k u_h), f_D(u_k u_h)) & for \quad kh = (l-2)(l-1) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(u_k u_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 0(mod\, 3) \\ (t_D(u_k u_h), i_D(u_k u_h), f_D(u_k u_h)) & for \quad kh = (l-3)(l-2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(u_k u_h) = \begin{cases} (t_D(u_{j-1}u_j), i_D(u_{j-1}u_j), f_D(u_{j-1}u_j)) & for \quad j \equiv 1(mod\, 3) \\ (t_D(u_k u_h), i_D(u_k u_h), f_D(u_k u_h)) & for \quad kh = 1l \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(u_k u_h) = \begin{cases} (t_D(u_k u_h), i_D(u_k u_h), f_D(u_k u_h)) & for \quad kh = (l-4)(l-3), (l-1)l \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ ensures that SVNREC requirements are met. Families with fewer than $4$ points did not match our defining criteria. Thus $\chi_2^v(C_l) = 4$ when otherwise.

□

# 5 Conclusions

In this paper we have analysed about the single valued neutrosophic R-dynamic vertex coloring of Cartesian product of SVNG's and join of SVNG's. Also we have amalgamated the idea of Single Valued Neutrosophic edge Coloring and r-dynamic coloring to introduce a new concept Single Valued Neutrosophic R-dynamic Edge Coloring (SVNREC). We have defined the new coloring and provided some examples.

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