Multi Criteria Decision Making Algorithm Via Complex Neutrosophic Nano Topological Spaces

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Abstract

The scope of this paper is to instigate the contemporary notion of complex neutrosophic nano topological spaces and explore some of its properties. We also illustrate the properties with numerical quantities. Furthermore, an algorithm for multi-criteria decision-making (MCDM) in medical diagnosis under uncertainty by using complex neutrosophic nano topological spaces are developed (CNNTSs). A comparison table is presented to show the difference between novel concept and the existing methods.

Keywords. complex neutrosophic topology, complex neutrosophic nano topological spaces, complex neutrosophic nano-closed sets, complex neutrosophic interior and closure.

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1 Introduction

Multi-criteria decision-making (MCDM) is a method that specifically considers the best possible alternatives. In medieval times, decisions were made without coping with data uncertainties, which could lead to a potential outcome. Inadequate outcomes for real-life organizational conditions. The results would be ambivalent, undefined, or wrong if we deduce the result of obtained data without hesitation. MCDM played an important role in real life problem such as management, diagnose the diseases, economics and industries. Each time hundreds of decisions are taken by each decision maker to execute the major part his/her work but it should be a logical judgment. Medical diagnosis with MCDM provides options for doctors to recognize signs of disease and severity of illness. MCDM is used to solve complex and complex problems with various parameters for this. In MCDM, the issue must be identified by defining potential alternatives , evaluating each alternative on the basis of the criteria set by the decision-maker or community of decision-makers and, ultimately, choosing the best alternative. A variety of valuable mathematical methods like this as fuzzy sets, neutrosophic sets and soft sets have been developed to tackle the complexities and complexity of multi-criteria decision-making problems.

Fuzzy set theory was introduced by Zadeh's [25]. MCDM algorithm via rough fuzzy information was introduced and developed by Zafer et al. [26]. Among different generalized FSs, in sight of IFSs introduced by Attanasov [3] lacking a logical scheme to effectively process inconsistent and indeterminate knowledge embedded in practical situations, Smarandache [22, 23] introduced the structure of neutrosophic logic, and NSs, which could be viewed as a generalized form of fuzzy logic and intuitionistic fuzzy logic and FSs, IFSs. Compared with IFSs, through incorporating an indeterminacy membership function which is focused on separately, NSs are able to effectively express the informations of inconsistency, incompleteness, and indeterminacy. NS has been applied in science and engineering field such as Algebra, topology [8, 9, 10, 11, 12, 13, 15, 16, 17], Graph theory and Image processing. Ali and Smarandache [1] developed novel complex neutrosophic sets which consists of both amplitude values and phase values and it is applicable for the problems of uncertainty and periodicity. simultaneously, Complex Neutrosophic set has been applied in science and engineering field. Neutrosophic topology which is a generalization of general topology, FT, IFT and its applications [2, 8, 10, 11, 12, 14, 15, 16] in various fields gained attention in the literature. Lellis Thivagar et al. [7] developed concept of NNT to the literature. It gained the attention of researchers to develop the theory of nano topology and via neutrosophic sets [8, 10, 14].

In this era, several mathematicians have been focusing on correlation coefficients, similarity measures, aggregation operators, topological spaces, and applications for decision-making. These structures provide different formula for different sets and have better solution to decision-making problems. It has various applications in different fields like medical diagnosis, pattern recognition, social sciences, artificial intelligence, business, and decision making problems with multi-attributes.

1.1 Motivation and Objective

The expanded and hybrid motivation and goal work is given in the entire manuscript, step by step. We make sure other hybrid systems of Fuzzy Sets are special CNS cases, under some necessary circumstances. The robustness, durability, simplicity and superiority of our proposed model and algorithms are discussed. This model is the most common type and is used to collect large-scale data in medical, engineering and artificial applications. Intelligence, forestry, and other issues of everyday life. Similar research can be easily replicated in the future with other methods and different forms of hybrid structure.

The scheme of this manuscript is organized as follows. Section 2, gives the preliminary definitions of the literature in NS are discussed. In Section 3, we introduced a novel idea of CNNTSs and established some of its operations like interior and closure with the help of illustrations, and defined a score function. In Section 4, We proposed an algorithm and flowchart for MCDM problem. In Section 5, As a numerical example, we established a method for the solution of MCDM problem based on medical diagnosis using CNNTS. We also presented the efficiency, advantage, consistency and validity of the algorithms proposed. With some current methodologies we provided a brief overview and comparative review of our proposed approach. The conclusion of this work is essentially summed up and future scope of research are presented in Section 6.

2 Preliminaries

The definitions from [1, 6, 7] are used in sequel.

Definition 2.1. [1] An object $\mathfrak{S}$ defined on a universe of discourse $\mathfrak{U}$ is called CNS, if it can be expressed as $\mathfrak{S} = \{(\zeta, \langle \mathfrak{M}(\zeta), \mathfrak{I}(\zeta), \mathfrak{F}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$. The values $\mathfrak{M}(\zeta), \mathfrak{I}(\zeta), \mathfrak{F}(\zeta)$ and their number can be in the complex plane all inside the unit circle, and so is in the following form, $\mathfrak{M}(\zeta) = p(\zeta)e^{j\mu(\zeta)}, \mathfrak{I}(\zeta) = q(\zeta)e^{j\nu(\zeta)}, \mathfrak{F}(\zeta) = r(\zeta)e^{j\omega(\zeta)}$ where $p(\zeta), q(\zeta), r(\zeta)$ and $\mu(\zeta), \nu(\zeta), \omega(\zeta)$ are respectively the amplitude terms and the phase terms, $\mu(\zeta), \nu(\zeta), \omega(\zeta) \in [0, 1]$ such that $^{-}0 \leq p(\zeta) + q(\zeta) + r(\zeta) \leq 3^{+}$ and $\mu(\zeta), \nu(\zeta), \omega(\zeta)$ are real valued with $j = \sqrt{-1}$. The scaling factors μ, ν and $\omega \in [0, 2\pi]$.

Definition 2.2. [6] Let $\mathfrak{C}$ be a family of CNS on $U \neq \emptyset$. Then $(X, \mathfrak{C})$ is called a neutrosophic complex topological space if it satisfies the following:

- 0_C and 1_C are member of $\mathfrak{C}$.
- Arbitrary union of neutrosophic complex set C in $\mathfrak{C}$ if each C in $\mathfrak{C}$
- Finite intersection of neutrosophic complex set C in $\mathfrak{C}$ if each C in $\mathfrak{C}$

Definition 2.3. [7] Let $\mathfrak{R}$ and $\mathfrak{S}$ be the equivalence relation and neutrosophic set, respectively defined on universe of discourse $\mathfrak{U}$. The membership $\mathfrak{M}_S$,the indeterminacy $\mathfrak{I}_S$ and nonmembership $\mathfrak{F}_S$ are the components of $\mathfrak{S}$. The approximation space $(\mathfrak{U}, \mathfrak{R})$ has three components namely, lower $\mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{S})$,upper $\mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{S})$ and boundary approximation $\mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{S})$ where

(i) The upper approximation of S with respect to R is denoted by $\mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{S})$. That is, $\mathfrak{CNU})_{\mathfrak{R}}(\mathfrak{S}) = \{\langle \zeta, \mathfrak{M}_{\overline{\mathfrak{R}}\mathfrak{S}}(\zeta), \mathfrak{I}_{\overline{\mathfrak{R}}\mathfrak{S}}(\zeta), \mathfrak{F}_{\overline{\mathfrak{R}}\mathfrak{S}}(\zeta)\rangle \, |\xi \in [\zeta]_R, \zeta \in \mathfrak{U}\}$

(ii) The lower approximation of S with respect to R is the set is denoted by $\mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{S})$. That is, $\mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{S}) = \{\langle \zeta, \mathfrak{M}_{\underline{\mathfrak{R}}\mathfrak{S}}(\zeta), \mathfrak{I}_{\underline{\mathfrak{R}}\mathfrak{S}}(\zeta), \mathfrak{F}_{\underline{\mathfrak{R}}\mathfrak{S}}(\zeta)\rangle \, |\xi \in [\zeta]_R, \zeta \in \mathfrak{U}\}$

(iii) The boundary region of S with respect to R is the set of all objects which can be classified neither as S nor as not S with respect to R and is denoted by $\mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{S})$. $\mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{S}) = \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{S}) - \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{S})$.

where, $\mathfrak{M}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \wedge_{\xi\in[\zeta]_{\mathfrak{R}}} \mathfrak{M}_{(\mathfrak{A})}(\xi)$, $\mathfrak{M}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \vee_{\xi\in[\zeta]_{\mathfrak{R}}} \mathfrak{M}_{(\mathfrak{A})}(\xi)$
$\mathfrak{I}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \vee_{\xi\in[\zeta]_{\mathfrak{R}}} \mathfrak{I}_{(\mathfrak{A})}(\xi)$, $\mathfrak{I}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \wedge_{\xi\in[\zeta]_{\mathfrak{R}}} \mathfrak{I}_{(\mathfrak{A})}(\xi)$
$\mathfrak{F}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \vee_{\xi\in[\zeta]_{\mathfrak{R}}} \mathfrak{F}_{(\mathfrak{A})}(\xi)$, $\mathfrak{F}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \wedge_{\xi\in[\zeta]_{\mathfrak{R}}} \mathfrak{F}_{(\mathfrak{A})}(\xi)$.

3 Complex Neutrosophic Nano Topological Spaces

Definition 3.1. Two objects $\mathfrak{S}_1 = \{(\zeta, \langle \mathfrak{M}_{\mathfrak{S}_1}(\zeta), \mathfrak{I}_{\mathfrak{S}_1}(\zeta), \mathfrak{F}_{\mathfrak{S}_1}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$ and $\mathfrak{S}_2 = \{(\zeta, \langle \mathfrak{M}_{\mathfrak{S}_2}(\zeta), \mathfrak{I}_{\mathfrak{S}_2}(\zeta), \mathfrak{F}_{\mathfrak{S}_2}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$ are defined on $\mathfrak{U}$, the universe of discourse, and their union and intersection are defined and denoted as follows

1. The union of $\mathfrak{S}_1$ and $\mathfrak{S}_2$ is $\mathfrak{S}_1 \cup \mathfrak{S}_2 = \{(\zeta, \langle \mathfrak{M}_{\mathfrak{S}_1\cup\mathfrak{S}_2}(\zeta), \mathfrak{I}_{\mathfrak{S}_1\cup\mathfrak{S}_2}(\zeta), \mathfrak{F}_{\mathfrak{S}_1\cup\mathfrak{S}_2}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$, where

$$\mathfrak{M}_{\mathfrak{S}_1\cup\mathfrak{S}_2}(\zeta) = [p_{\mathfrak{S}_1}(\zeta) \vee p_{\mathfrak{S}_2}(\zeta)]e^{j[\mu_{\mathfrak{S}_1}(\zeta)\vee\mu_{\mathfrak{S}_2}(\zeta)]}$$
$$\mathfrak{I}_{\mathfrak{S}_1\cup\mathfrak{S}_2}(\zeta) = [q_{\mathfrak{S}_1}(\zeta) \wedge q_{\mathfrak{S}_2}(\zeta)]e^{j[\nu_{\mathfrak{S}_1}(\zeta)\wedge\nu_{\mathfrak{S}_2}(\zeta)]}$$
$$\mathfrak{F}_{\mathfrak{S}_1\cup\mathfrak{S}_2}(\zeta) = [r_{\mathfrak{S}_1}(\zeta) \wedge r_{\mathfrak{S}_2}(\zeta)]e^{j[\omega_{\mathfrak{S}_1}(\zeta)\wedge\omega_{\mathfrak{S}_2}(\zeta)]}$$

2. The intersection of $\mathfrak{S}_1$ and $\mathfrak{S}_2$ is $\mathfrak{S}_1 \cap \mathfrak{S}_2 = \{(\zeta, \langle \mathfrak{M}_{\mathfrak{S}_1\cap\mathfrak{S}_2}(\zeta), \mathfrak{I}_{\mathfrak{S}_1\cap\mathfrak{S}_2}(\zeta), \mathfrak{F}_{\mathfrak{S}_1\cap\mathfrak{S}_2}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$, where

$$\mathfrak{M}_{\mathfrak{S}_1\cap\mathfrak{S}_2}(\zeta) = [p_{\mathfrak{S}_1}(\zeta) \wedge p_{\mathfrak{S}_2}(\zeta)]e^{j[\mu_{\mathfrak{S}_1}(\zeta)\wedge\mu_{\mathfrak{S}_2}(\zeta)]}$$
$$\mathfrak{I}_{\mathfrak{S}_1\cap\mathfrak{S}_2}(\zeta) = [q_{\mathfrak{S}_1}(\zeta) \vee q_{\mathfrak{S}_2}(\zeta)]e^{j[\nu_{\mathfrak{S}_1}(\zeta)\vee\nu_{\mathfrak{S}_2}(\zeta)]}$$
$$\mathfrak{F}_{\mathfrak{S}_1\cap\mathfrak{S}_2}(\zeta) = [r_{\mathfrak{S}_1}(\zeta) \vee r_{\mathfrak{S}_2}(\zeta)]e^{j[\omega_{\mathfrak{S}_1}(\zeta)\vee\omega_{\mathfrak{S}_2}(\zeta)]}$$

3. The symmetric difference of $\mathfrak{S}_1$ and $\mathfrak{S}_2$ is $\mathfrak{S}_1 - \mathfrak{S}_2 = \{(\zeta, \langle \mathfrak{M}_{\mathfrak{S}_1-\mathfrak{S}_2}(\zeta), \mathfrak{I}_{\mathfrak{S}_1-\mathfrak{S}_2}(\zeta), \mathfrak{F}_{\mathfrak{S}_1-\mathfrak{S}_2}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$, where

$$\mathfrak{M}_{\mathfrak{S}_1-\mathfrak{S}_2}(\zeta) = [p_{\mathfrak{S}_1}(\zeta) \wedge r_{\mathfrak{S}_2}(\zeta)]e^{j[\mu_{\mathfrak{S}_1}(\zeta)\wedge\omega_{\mathfrak{S}_2}(\zeta)]}$$
$$\mathfrak{I}_{\mathfrak{S}_1-\mathfrak{S}_2}(\zeta) = [q_{\mathfrak{S}_1}(\zeta) \vee (1 - q_{\mathfrak{S}_2}(\zeta))]e^{j[\nu_{\mathfrak{S}_1}(\zeta)\vee(2\pi-\nu_{\mathfrak{S}_2}(\zeta))]}$$
$$\mathfrak{F}_{\mathfrak{S}_1-\mathfrak{S}_2}(\zeta) = [r_{\mathfrak{S}_1}(\zeta) \vee p_{\mathfrak{S}_2}(\zeta)]e^{j[\omega_{\mathfrak{S}_1}(\zeta)\vee\mu_{\mathfrak{S}_2}(\zeta)]}$$

Definition 3.2. Let $\mathfrak{S}_1 = \{(\zeta, \langle \mathfrak{M}_{\mathfrak{S}_1}(\zeta), \mathfrak{I}_{\mathfrak{S}_1}(\zeta), \mathfrak{F}_{\mathfrak{S}_1}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$ and $\mathfrak{S}_2 = \{(\zeta, \langle \mathfrak{M}_{\mathfrak{S}_2}(\zeta), \mathfrak{I}_{\mathfrak{S}_2}(\zeta), \mathfrak{F}_{\mathfrak{S}_2}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$ are the two objects defined on a universe of discourse $\mathfrak{U}$, then

1. $\mathfrak{S}_1 \subseteq \mathfrak{S}_2$ if and only if $\mathfrak{M}_{\mathfrak{S}_1} \leq \mathfrak{M}_{\mathfrak{S}_2}$, $\mathfrak{I}_{\mathfrak{S}_1} \geq \mathfrak{I}_{\mathfrak{S}_2}$ and $\mathfrak{F}_{\mathfrak{S}_1} \geq \mathfrak{F}_{\mathfrak{S}_2}$ such that

$$\mathfrak{M}_{\mathfrak{S}_1} \leq \mathfrak{M}_{\mathfrak{S}_2} = [p_{\mathfrak{S}_1}(\zeta) \leq p_{\mathfrak{S}_2}(\zeta)]e^{j[\mu_{\mathfrak{S}_1}(\zeta)\leq\mu_{\mathfrak{S}_2}(\zeta)]}$$
$$\mathfrak{I}_{\mathfrak{S}_1} \geq \mathfrak{I}_{\mathfrak{S}_2} = [q_{\mathfrak{S}_1}(\zeta) \geq q_{\mathfrak{S}_2}(\zeta)]e^{j[\nu_{\mathfrak{S}_1}(\zeta)\geq\nu_{\mathfrak{S}_2}(\zeta)]}$$
$$\mathfrak{F}_{\mathfrak{S}_1} \geq \mathfrak{F}_{\mathfrak{S}_2} = [r_{\mathfrak{S}_1}(\zeta) \geq r_{\mathfrak{S}_2}(\zeta)]e^{j[\omega_{\mathfrak{S}_1}(\zeta)\geq\omega_{\mathfrak{S}_2}(\zeta)]}$$

Definition 3.3. Let $\mathfrak{S} = \{(\zeta, \langle \mathfrak{M}(\zeta), \mathfrak{I}(\zeta), \mathfrak{F}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$ be a non-void set and $\mathfrak{R}$ an equivalence relation on $\mathfrak{S}$. Let $\mathfrak{A}$ be a CNS in with membership $\mathfrak{M}_{\mathfrak{A}}$, indeterminacy $\mathfrak{I}_{\mathfrak{A}}$ and non-membership $\mathfrak{F}_{\mathfrak{A}}$. The complex neutrosophic nano minor (lower) approximation, complex neutrosophic nano major (upper) approximation and complex neutrosophic nano border (boundary) of $\mathfrak{A}$ in the approximation space $(\mathfrak{S}, \mathfrak{R})$ denoted by $\mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A})$, $\mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A})$ and $\mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A})$ are respectively defined as:

(i) $\mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \zeta, \mathfrak{M}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta), \mathfrak{I}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta), \mathfrak{F}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta)\rangle : \xi \in [\zeta]_{\mathfrak{R}}, \zeta \in \mathfrak{U}\}$

(ii) $\mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \zeta, \mathfrak{M}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta), \mathfrak{I}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta), \mathfrak{F}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta)\rangle : \xi \in [\zeta]_{\mathfrak{R}}, \zeta \in \mathfrak{U}\}$

(iii) $\mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}) - \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A})$ where

$$\mathfrak{M}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \wedge_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{M}_{(\mathfrak{A})}(\xi),\ \mathfrak{M}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \vee_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{M}_{(\mathfrak{A})}(\xi)$$
$$\mathfrak{I}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \vee_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{I}_{(\mathfrak{A})}(\xi),\ \mathfrak{I}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \wedge_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{I}_{(\mathfrak{A})}(\xi)$$
$$\mathfrak{F}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \vee_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{F}_{(\mathfrak{A})}(\xi),\ \mathfrak{F}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \wedge_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{F}_{(\mathfrak{A})}(\xi).$$

The triplet $(\mathfrak{CNL}_{\mathfrak{R}}, \mathfrak{CNU}_{\mathfrak{R}}, \mathfrak{CNB}_{\mathfrak{R}})$ is said to be complex neutrosophic approximation space.

Definition 3.4. Let the Universe be $\mathfrak{U}$, equivalence relation on the non-void set $\mathfrak{S} \subseteq \mathfrak{U}$ be $\mathfrak{R}$ and if $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_\sim, 1_\sim, \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A})\}$, where $\mathfrak{A} \subseteq \mathfrak{S}$ and $\tau_{\mathfrak{R}}$ satisfies the following axioms:

1. $0_\sim, 1_\sim \in \tau_{\mathfrak{R}}$
2. If $\mathfrak{A}_{\mathfrak{i}} \in \tau_{\mathfrak{R}}(\mathfrak{A})$, for $\mathfrak{i} = 1, 2, 3, ..,$ then
$$\bigcup_{\mathfrak{i}=1}^{\infty} \mathfrak{A}_{\mathfrak{i}} \in \tau_{\mathfrak{R}}(\mathfrak{A})$$
3. If $\mathfrak{A}_{\mathfrak{i}} \in \tau_{\mathfrak{R}}(\mathfrak{A})$, for $\mathfrak{i} = 1, 2, 3, ..n$, then
$$\bigcap_{\mathfrak{i}=1}^{n} \mathfrak{A}_{\mathfrak{i}} \in \tau_{\mathfrak{R}}(\mathfrak{A})$$

then $\tau_{\mathfrak{R}}(\mathfrak{A})$ is termed as CNNTS on $\mathfrak{S}$ with respect to $\mathfrak{A}$. where the neutrosophic complex sets $1_\sim$ and $0_\sim$ are defined by $1_\sim = \{(\zeta, \langle 1e^{j0}, 0e^{j1}, 0e^{j1}\rangle) : \zeta \in \mathfrak{U}\}$ and $0_\sim = \{(\zeta, \langle 0e^{j1}, 1e^{j0}, 1e^{j0}\rangle) : \zeta \in \mathfrak{U}\}$ respectively. Whereas, we call $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{A}))$ as CNNTS. The components of $\tau_{\mathfrak{R}}(\mathfrak{A})$ are called CNNOS.

The complement $\mathfrak{A}^{\mathfrak{c}}$ of a CNNOS $\mathfrak{A}$ in a CNNTS. $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{A}))$ is called a CNNCS in $\mathfrak{S}$.

Example 3.1. Suppose we have a factory that includes a car part. The factory has three workers in this section. Each employee in this plant gets 10 car components, to be polished every day. The quality assurance unit at the factory maintains that though three employees are polishing correctly / successfully. The car parts, some of the staff are doing a job higher output than the rest. The amplitude (number of jobs done) and phase (attribute) (quality of the job done) of CNS and their major, minor and border approximations are given below: Let $\mathfrak{S} = \{\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3\}$ be the universe of discourse. Let $\mathfrak{S}/\mathfrak{R} = \{\{\mathfrak{a}_1, \mathfrak{a}_2\}, \{\mathfrak{a}_3\}\}$ be an equivalence relation on $\mathfrak{S}$ and $\mathfrak{A} = \{\langle \mathfrak{a}_1, (0.8e^{j\pi 0.7}, 0.5e^{j\pi 0.4}, 0.6e^{j\pi 0.2})\rangle, \langle \mathfrak{a}_2, (0.3e^{j\pi 0.4}, 0.4e^{j\pi 0.3}, 0.1e^{j\pi 0.5})\rangle,$ $\langle \mathfrak{a}_3, (0.1e^{j\pi 0.3} 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6})\rangle\}$ be a neutrosophic set on $\mathfrak{S}$, then

$\mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \mathfrak{a}_1, (0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.4}, 0.6e^{j\pi 0.5})\rangle, \langle \mathfrak{a}_2, (0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.4}, 0.6e^{j\pi 0.5})\rangle,$
$\langle \mathfrak{a}_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6})\rangle\}$,
$\mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \mathfrak{a}_1, (0.8e^{j\pi 0.7}, 0.4e^{j\pi 0.3}, 0.1e^{j\pi 0.2})\rangle, \langle \mathfrak{a}_2, (0.8e^{j\pi 0.7}, 0.4e^{j\pi 0.3}, 0.1e^{j\pi 0.2})\rangle,$
$\langle \mathfrak{a}_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6})\rangle\}$ and
$\mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \mathfrak{a}_1, (0.1e^{j\pi 0.2}, 0.6e^{j\pi 0.7}, 0.8e^{j\pi 0.7})\rangle, \langle \mathfrak{a}_2, (0.1e^{j\pi 0.2}, 0.6e^{j\pi 0.7}, 0.8e^{j\pi 0.7})\rangle,$
$\langle \mathfrak{a}_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6})\rangle\}$.
$\mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}) \cup \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \mathfrak{a}_1, (0.8e^{j\pi 0.7}, 0.4e^{j\pi 0.3}, 0.1e^{j\pi 0.2})\rangle, \langle \mathfrak{a}_2, (0.8e^{j\pi 0.7}, 0.4e^{j\pi 0.3}, 0.1e^{j\pi 0.2})\rangle,$
$\langle \mathfrak{a}_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6})\rangle\} = \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A})$
$\mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}) \cap \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \mathfrak{a}_1, (0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.4}, 0.6e^{j\pi 0.5})\rangle, \langle \mathfrak{a}_2, (0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.4}, 0.6e^{j\pi 0.5})\rangle,$
$\langle \mathfrak{a}_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6})\rangle\} = \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A})$
$0_\sim \cap \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}) = 0_\sim$, $0_\sim \cap \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}) = 0_\sim$, $0_\sim \cap \mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A}) = 0_\sim$,
$0_\sim \cup \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{CNU}_{\mathfrak{R}}$, $0_\sim \cup \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{CNL}_{\mathfrak{R}}$, $0_\sim \cup \mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{CNB}_{\mathfrak{R}}$,
$1_\sim \cap \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{CNU}_{\mathfrak{R}}$, $1_\sim \cap \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{CNL}_{\mathfrak{R}}$, $1_\sim \cap \mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{CNB}_{\mathfrak{R}}$,
$1_\sim \cup \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}) = 1_\sim$, $1_\sim \cup \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}) = 1_\sim$, $1_\sim \cup \mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A}) = 1_\sim$,
Therefore, $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_\sim, 1_\sim, \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A})\}$ forms a topology.

Remark 3.1. In CNNTS, the complex neutrosophic nano border will be a non-void set. Since the symmetric difference between complex neutrosophic nano major and complex neutrosophic nano minor approximations is defined here as the maximum and minimum of the values in the complex neutrosophic sets.

Proposition 3.1. Let $\mathfrak{U}$ be a non-void universe and $\mathfrak{A}$ be a complex neutrosophic set on $\mathfrak{U}$. Then the following statements hold:

1. The collection $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_\sim, 1_\sim\}$, is the in-discrete complex neutrosophic nano topology on $\mathfrak{U}$.

2. If $\mathfrak{CNL}_{\mathfrak{R}} = \mathfrak{CNU}_{\mathfrak{R}} = \mathfrak{CN}_{\mathfrak{R}}$, then the complex neutrosophic nano topology is $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\sim}, 1_{\sim}, \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A})\}$.

3. If $\mathfrak{CNL}_{\mathfrak{R}} = \mathfrak{CNB}_{\mathfrak{R}}$, then $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\sim}, 1_{\sim}, \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{CNU}_{\mathfrak{R}}(\mathfrak{A})\}$ is a complex neutrosophic nano topology.

4. If $\mathfrak{CNU}_{\mathfrak{R}} = \mathfrak{CNB}_{\mathfrak{R}}$, then the complex neutrosophic nano topology is $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\sim}, 1_{\sim}, \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A})\}$

Definition 3.5. Let $(\mathfrak{U}; \tau_{\mathfrak{R}})$ be any CNNTS with respect to complex neutrosophic subset of $\mathfrak{U}$ and let $\mathfrak{A}$ be a complex neutrosophic nano set in $\mathfrak{S}$. Then the complex neutrosophic nano interior and complex neutrosophic nano closure of $\mathfrak{A}$ are defined as follows:

1. $\mathfrak{A}^{\mathfrak{o}} = \cup\{\mathfrak{G} : \mathfrak{G}$ is a CNNOS in $\mathfrak{S}$ and $\mathfrak{G} \subseteq \mathfrak{A}\}$,

2. $\mathfrak{A}^{-} = \cap\{\mathfrak{G} : \mathfrak{G}$ is a CNNCS in $\mathfrak{S}$ and $\mathfrak{G} \supseteq \mathfrak{A}\}$.

Remark 3.2. For any complex neutrosophic nano set $\mathfrak{A}$ in $(\mathfrak{U}; \tau_{\mathfrak{R}})$, we have

1. $[\mathfrak{A}^c]^- = [\mathfrak{A}^{\mathfrak{o}}]^c$.

2. $[\mathfrak{A}^c]^{\mathfrak{o}} = [\mathfrak{A}^-]^c$.

3. $\mathfrak{A}$ is a CNNCS if and only if $\mathfrak{A}^- = \mathfrak{A}$.

4. $\mathfrak{A}$ is a CNNOS if and only if $\mathfrak{A}^{\mathfrak{o}} = \mathfrak{A}$.

5. $\mathfrak{A}^-$ is a CNNCS in $\mathfrak{U}$.

6. $\mathfrak{A}^{\mathfrak{o}}$ is a CNNOS in $\mathfrak{U}$.

Theorem 3.1. Let $(\mathfrak{U}; \tau_{\mathfrak{R}})(\mathfrak{S})$ be a complex neutrosophic nano topological space with respect to $\mathfrak{S}$ where $\mathfrak{S}$ is a complex neutrosophic subset of $\mathfrak{U}$. Let $\mathfrak{A}_1$ and $\mathfrak{A}_2$ be complex neutrosophic subsets of $\mathfrak{U}$. Then the following statements hold:

1. $\mathfrak{A} \subseteq \mathfrak{A}^-$.

2. $\mathfrak{A}$ is complex neutrosophic nano closed if and only if $\mathfrak{A}^- = \mathfrak{A}$.

3. $0_{\sim}^- = 0_{\sim}$ and $1_{\sim}^- = 1_{\sim}$.

4. $\mathfrak{A}_1 \subseteq \mathfrak{A}_2 \Rightarrow {\mathfrak{A}_1}^- \subseteq {\mathfrak{A}_2}^-$.

5. $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^- = {\mathfrak{A}_1}^- \cup {\mathfrak{A}_2}^-$.

6. $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^- = {\mathfrak{A}_1}^- \cap {\mathfrak{A}_2}^-$.

7. $(\mathfrak{A}^-)^- = \mathfrak{A}^-$.

Proof.

1. By definition of complex neutrosophic nano closure, $\mathfrak{A} \subseteq \mathfrak{A}^-$

2. If $\mathfrak{A}$ is a complex neutrosophic nano closed set, then $\mathfrak{A}$ is the smallest complex neutrosophic nano closed set containing itself and hence $\mathfrak{A}^- = \mathfrak{A}$. Conversely, if $\mathfrak{A}^- = \mathfrak{A}$, then $\mathfrak{A}$ is the smallest complex neutrosophic nano closed set containing itself and hence $\mathfrak{A}$ is a complex neutrosophic nano closed set.

3. Since $0_\sim$ and $1_\sim$ are complex neutrosophic nano closed sets in $(\mathfrak{U}; \tau_{\mathfrak{R}})(\mathfrak{S})$, $0_\sim^- = 0_\sim$ and $1_\sim^- = 1_\sim$.

4. If CNN set $\mathfrak{A}_1$ is a subset of CNN set $\mathfrak{A}_2$, since CNN set $\mathfrak{A}_2$ is a subset of ${\mathfrak{A}_2}^-$, then CNN set $\mathfrak{A}_1$ is a subset of ${\mathfrak{A}_2}^-$. That is, ${\mathfrak{A}_2}^-$ is a CNNCS containing $\mathfrak{A}_1$. But ${\mathfrak{A}_1}^-$ is the smallest CNNCS containing $\mathfrak{A}_1$. Therefore, ${\mathfrak{A}_1}^- \subseteq {\mathfrak{A}_2}^-$

5. Since CNN set $\mathfrak{A}_1$ is a subset of union of two CNN sets$\mathfrak{A}_1$ and $\mathfrak{A}_2$ and CNN set $\mathfrak{A}_2$ is a subset of union of two CNN sets $\mathfrak{A}_1$ and $\mathfrak{A}_2$, ${\mathfrak{A}_1}^- \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^-$. Then closure of CNN set $\mathfrak{A}_1$ is a subset of closure of union of two CNN sets$\mathfrak{A}_1$ and $\mathfrak{A}_2$ and closure of CNN set $\mathfrak{A}_2$ is a subset of closure of union of two CNN sets $\mathfrak{A}_1$ and $\mathfrak{A}_2$. Therefore, union of closure of CNN sets ${\mathfrak{A}_1}^-$, ${\mathfrak{A}_2}^-$ is a subset of closure of union of $({\mathfrak{A}_1}^-, \mathfrak{A}_2)^-$. By the fact that $\mathfrak{A}_1 \cup \mathfrak{A}_2 \subseteq {\mathfrak{A}_1}^- \cup {\mathfrak{A}_2}^-$, and since $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^-$ is the smallest complex neutrosophic nano closed set containing $\mathfrak{A}_1 \cup \mathfrak{A}_2$, so $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^- \subseteq {\mathfrak{A}_1}^- \cup {\mathfrak{A}_2}^-$. Thus, $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^- = {\mathfrak{A}_1}^- \cup {\mathfrak{A}_2}^-$.

6. Since $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_1$ and $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_2$, $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^- \subseteq {\mathfrak{A}_1}^- \cap {\mathfrak{A}_2}^-$.

7. Since $\mathfrak{A}^-$ is a complex neutrosophic nano closed set, then $(\mathfrak{A}^-)^- = \mathfrak{A}^-$.

Theorem 3.2. $(\mathfrak{U}; \tau_{\mathfrak{R}})(\mathfrak{S})$ be a complex neutrosophic nano topological space with respect to $\mathfrak{S}$ where $\mathfrak{S}$ is a complex neutrosophic subset of $\mathfrak{U}$. Let $\mathfrak{A}$ be a complex neutrosophic subset of $\mathfrak{U}$. Then

1. $1_\sim - \mathfrak{A}^o = (1_\sim - \mathfrak{A})^-$.

2. $1_\sim - \mathfrak{A}^- = (1_\sim - \mathfrak{A})^o$.

Theorem 3.3. Let $(\mathfrak{U}; \tau_{\mathfrak{R}})(\mathfrak{S})$ be a complex neutrosophic nano topological space with respect to $\mathfrak{S}$ where $\mathfrak{S}$ is a complex neutrosophic subset of $\mathfrak{U}$. Let $\mathfrak{A}_1$ and $\mathfrak{A}_2$ be complex neutrosophic subsets of $\mathfrak{U}$. Then the following statements hold:

1. $\mathfrak{A}$ is complex neutrosophic nano open if and only if $\mathfrak{A}^o = \mathfrak{A}$.

2. $0_\sim^o = 0_\sim$ and $1_\sim^o = 1_\sim$.

3. $\mathfrak{A}_1 \subseteq \mathfrak{A}_2 \Rightarrow \mathfrak{A}_1^o \subseteq \mathfrak{A}_2^o$.

4. $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^o = \mathfrak{A}_1^o \cup \mathfrak{A}_2^o$.

5. $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^o = \mathfrak{A}_1^o \cap \mathfrak{A}_2^o$.

6. $(\mathfrak{A}^o)^o = \mathfrak{A}^o$.

Proof.

1. $\mathfrak{A}$ is a complex neutrosophic nano open set if and only if $1_\sim - \mathfrak{A}$ is a complex neutrosophic nano closed set, if and only if $(1_\sim - \mathfrak{A})^- = 1_\sim - \mathfrak{A}$, if and only if $1_\sim - (1_\sim - \mathfrak{A})^- = \mathfrak{A}$ if and only if $\mathfrak{A}^o = \mathfrak{A}$.

2. Since $0_\sim$ and $1_\sim$ are complex neutrosophic nano open sets in $(\mathfrak{U}; \tau_{\mathfrak{R}})(\mathfrak{S})$, $0_\sim^o = 0_\sim$ and $1_\sim^o = 1_\sim$.

3. If $\mathfrak{A}_1 \subseteq \mathfrak{A}_2$, since $\mathfrak{A}_2 \supseteq \mathfrak{A}_2^o$, then $\mathfrak{A}_1 \supseteq \mathfrak{A}_2^o$. That is, $\mathfrak{A}_2^o$ is a complex neutrosophic nano open set containing $\mathfrak{A}_1$. But $\mathfrak{A}_1^o$ is the largest complex neutrosophic nano open set contained in $\mathfrak{A}_1$. Therefore, ${\mathfrak{A}_1}^o \subseteq {\mathfrak{A}_2}^o$

4. Since $\mathfrak{A}_1 \subseteq \mathfrak{A}_1 \cup \mathfrak{A}_2$ and $\mathfrak{A}_2 \subseteq \mathfrak{A}_1 \cup \mathfrak{A}_2$, $\mathfrak{A}_1^o \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^o$ and $\mathfrak{A}_2^o \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^o$. Therefore, $\mathfrak{A}_1^o \cup \mathfrak{A}_2^o \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^o$. By the fact that $\mathfrak{A}_1 \cup \mathfrak{A}_2 \subseteq \mathfrak{A}_1^o \cup \mathfrak{A}_2^o$, and since $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^o$ is the largest complex neutrosophic nano open set containing $\mathfrak{A}_1 \cup \mathfrak{A}_2$, so $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^o \subseteq \mathfrak{A}_1^o \cup \mathfrak{A}_2^o$. Thus, $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^o = \mathfrak{A}_1^o \cup \mathfrak{A}_2^o$.

5. Since $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_1$ and $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_2$, $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^o \subseteq \mathfrak{A}_1^o \cap \mathfrak{A}_2^o$.

6. Since $\mathfrak{A}^o$ is a complex neutrosophic nano open set, then $(\mathfrak{A}^o)^o = \mathfrak{A}^o$.

Definition 3.6. Let $\mathfrak{A} = \{\mathfrak{M}, \mathfrak{I}, \mathfrak{F}\}$ be a CNS, a score function $\mathfrak{S}_{\mathfrak{cr}}(.)$, based on the truth-membership degree ($\mathfrak{M}$), indeterminacy-membership degree ($\mathfrak{I}$), and falsity membership degree ($\mathfrak{F}$) which is defined as

$$\mathfrak{S}_{\mathfrak{cr}}(CNN) = \frac{1}{6m}\sum_{i=1}^{m}\{[p_i(\zeta) + (2 - (q_i(\zeta) + r_i(\zeta))] + \frac{1}{2\pi}[\mu_i(\zeta) + (4\pi - (\nu_i(\zeta) + \omega_i(\zeta))]\}$$

The score function of a complex neutrosophic number measures the accuracy of the number CNN in favor of truth membership degree.

Clearly, if in some cases CNS has $\mathfrak{M} + \mathfrak{F} = 1$ then $\mathfrak{S}_{\mathfrak{cr}}(.)$ reduces to $\mathfrak{K}_{\mathfrak{cr}}(.)$. Based on it, a prioritized comparison method for any two CNSs $\mathfrak{A}_1$ and $\mathfrak{A}_2$ is defined as

1. if $\mathfrak{K}_{\mathfrak{cr}}(\mathfrak{A}_1) < \mathfrak{K}_{\mathfrak{cr}}(\mathfrak{A}_2)$, then $\mathfrak{A}_1 \prec \mathfrak{A}_2$
2. if $\mathfrak{K}_{\mathfrak{cr}}(\mathfrak{A}_1) = \mathfrak{K}_{\mathfrak{cr}}(\mathfrak{A}_2)$, then $\mathfrak{A}_1 \sim \mathfrak{A}_2$
3. if $\mathfrak{S}_{\mathfrak{cr}}(\mathfrak{A}_1) < \mathfrak{S}_{\mathfrak{cr}}(\mathfrak{A}_2)$, then $\mathfrak{A}_1 \prec \mathfrak{A}_2$
4. if $\mathfrak{S}_{\mathfrak{cr}}(\mathfrak{A}_1) > \mathfrak{S}_{\mathfrak{cr}}(\mathfrak{A}_2)$, then $\mathfrak{A}_1 \succ \mathfrak{A}_2$
5. if $\mathfrak{S}_{\mathfrak{cr}}(\mathfrak{A}_1) = \mathfrak{S}_{\mathfrak{cr}}(\mathfrak{A}_2)$, then $\mathfrak{A}_1 \sim \mathfrak{A}_2$

4 Complex neutrosophic nano topology in multi-criteria decision-making

MCDM is a procedure for seeking a best solution that has the highest degree of satisfaction from a set of possible alternative solutions. These types of MCDM problems arise in a many real-time situations, and they are characterized by multiple criteria.

A novel complex neutrosophic nano topological approach are presented in this section for decision-making problems with complex neutrosophic information. A methodological procedure for selecting the right alternatives and attributes in the decision-making environment is proposed as the following necessary steps.

4.1 Proposed Algorithm and Flowchart

Algorithm: (Ideal decision making with CNNTSs)

Input part:

Step-1: Consider the universe of discourse (set of objects) $\mathfrak{V}$, the set of alternatives $\mathfrak{E}$, the set of decision attributes $\mathfrak{D}$. Consider an in-discernibility relation $\mathfrak{R}$ on $\mathfrak{V}$. Construct a complex neutrosophic set in matrix form related to the attributes. The objects and attributes are represented as columns and rows respectively and the table entries indicate the values of the attributes.

Calculation part:

Step-2: Frame the in-discernibility relation $\mathfrak{R}$.

Step-3: Construct the complex neutrosophic nano topologies $\mathfrak{C}^*_{\tau_1\eta}$ and $\mathfrak{C}^*_{\tau_2\xi}$.

Step-4: Find the score value of each of the entries of the CNN topological spaces

$$\mathfrak{S}_{\mathfrak{cr}}(CNN) = \frac{1}{6m}\sum_{i=1}^{m}\{[p_i(\zeta) + (2-(q_i(\zeta)+r_i(\zeta))] + \frac{1}{2\pi}[\mu_i(\zeta) + (4\pi - (\nu_i(\zeta)+\omega_i(\zeta))]\}$$

The accuracy of the CNN number measure provides a support the truth membership degree.

Conclusion Part:

Step-5: Final Decision

Organize the complex neutrosopic score values of the alternatives $\mathfrak{G}_1 \leq \mathfrak{G}_2 \leq .. \leq \mathfrak{G}_\beta$ and the attributes $\mathfrak{H}_1 \leq \mathfrak{H}_2 \leq .. \leq \mathfrak{H}_\gamma$. Choose the attribute $\mathfrak{H}_\gamma$ for the alternative $\mathfrak{G}_1$ and $\mathfrak{H}_\gamma - 1$ for the alternative $\mathfrak{G}_2$ etc. If $\beta \leq \gamma$, then ignore $\mathfrak{H}_\xi$, where $\xi = 1, 2, \beta - \gamma$.

The flow chart of proposed Algorithm for MCDM is given in the Figure 1.

5 Numerical Example

For example of the proposed solution we find a medical diagnosis issue. Medicine Diagnosis entails uncertainty and an growing amount of knowledge available to doctors Fresh tools for pharmacy. Therefore, all the collected knowledge can be in a dynamic neutrosophic type. The three components of a complex neutrosophic set are real-evaluated combinations Truth of matter amplitude term in combination with phase term, indeterminate amplitude term actual evaluated with phase term, and with real-evaluated, phase term false amplitude. And in a neutrosophical dynamic medical diagnosis environment, it is given to deal with further indeterminacy circumstances.

The method of classifying various sets of symptoms under a single disease name is very critical and complicated In certain realistic circumstances, each dimension has the possibility within a periodic form of the neutrosophic sets. So, further indeterminacy is involved in

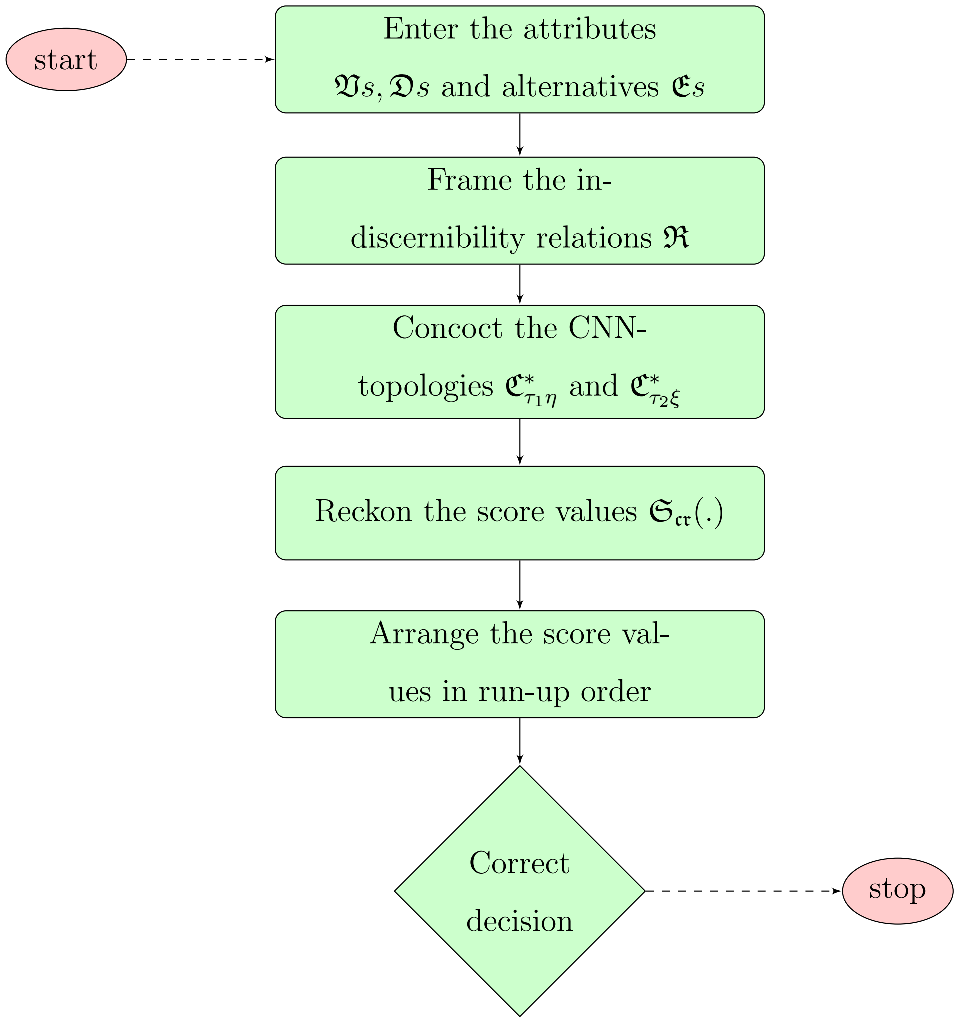

Figure 1: Flow chart of Algorithm

the medical diagnosis. Complicated situations are addressed by complex neutrosophic nano topologies. This strategy is generally more versatile, when it comes to less places of indeterminacy, and easier to use.With a score function between patients versus symptoms and symptoms versus diseases, the proposed algorithm of complex neutrosophic nano topological spaces has the right medical diagnosis in complex neutrosophic milieu.

The key feature of this suggested method is that it appraises complex factual participation, specific indeterminate and misrepresentation of each dimension taking periodic form in neutrosophic set.

Let $\mathfrak{P} = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3, \mathfrak{p}_4\}$ be the set of patients, $\mathfrak{D} = \{\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4\}$ be the set of diseases and $\mathfrak{S} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_4, \mathfrak{s}_5\}$ be the set of symptoms. The symptoms, practitioner decided to include, looked something like this: Temperature, Headache, Stomach pain, cough, Chest pain. Normal representation showed following

are the diseases to be the most indicated ones: Viral Fever, Malaria, Stomach problem, Chest problem.

Our work is to analyze the patient and decide the patient's illness in a complex neutrosophical environment.

Table 1: The complex neutrosophic system of relationship between Patients and Symptoms

	$\mathfrak{p}_1$	$\mathfrak{p}_2$	$\mathfrak{p}_3$	$\mathfrak{p}_4$
$\mathfrak{s}_1$	$\left\langle 0.7e^{j\pi 0.9}, 0.4e^{j\pi 0.8}, 0.3e^{j\pi 0.7} \right\rangle$	$\left\langle 0.5e^{j\pi 0.6}, 0.5e^{j\pi 0.9}, 0.2e^{j\pi 0.8} \right\rangle$	$\left\langle 0.7e^{j\pi 0.7}, 0.4e^{j\pi 0.8}, 0.6e^{j\pi 0.9} \right\rangle$	$\left\langle 0.5e^{j\pi 0.9}, 0.5e^{j\pi 0.7}, 0.5e^{j\pi 0.6} \right\rangle$
$\mathfrak{s}_2$	$\left\langle 0.4e^{j\pi 0.2}, 0.4e^{j\pi 0.7}, 0.3e^{j\pi 0.3} \right\rangle$	$\left\langle 0.4e^{j\pi 0.5}, 0.6e^{j\pi 0.4}, 0.4e^{j\pi 0.6} \right\rangle$	$\left\langle 0.5e^{j\pi 0.1}, 0.4e^{j\pi 0.7}, 0.7e^{j\pi 0.9} \right\rangle$	$\left\langle 0.8e^{j\pi 0.7}, 0.0e^{j\pi 0.2}, 0.7e^{j\pi 0.5} \right\rangle$
$\mathfrak{s}_3$	$\left\langle 0.8e^{j\pi 0.7}, 0.1e^{j\pi 0.8}, 0.2e^{j\pi 0.2} \right\rangle$	$\left\langle 0.2e^{j\pi 0.7}, 0.1e^{j\pi 0.2}, 0.6e^{j\pi 0.7} \right\rangle$	$\left\langle 0.3e^{j\pi 0.7}, 0.1e^{j\pi 0.9}, 0.6e^{j\pi 0.2} \right\rangle$	$\left\langle 0.1e^{j\pi 0.5}, 0.2e^{j\pi 0.1}, 0.7e^{j\pi 0.6} \right\rangle$
$\mathfrak{s}_4$	$\left\langle 0.3e^{j\pi 0.4}, 0.2e^{j\pi 0.6}, 0.5e^{j\pi 0.6} \right\rangle$	$\left\langle 0.9e^{j\pi 0.6}, 0.3e^{j\pi 0.5}, 0.2e^{j\pi 0.9} \right\rangle$	$\left\langle 0.4e^{j\pi 0.2}, 0.3e^{j\pi 0.4}, 0.2e^{j\pi 0.4} \right\rangle$	$\left\langle 0.9e^{j\pi 0.2}, 0.1e^{j\pi 0.6}, 0.3e^{j\pi 0.8} \right\rangle$
$\mathfrak{s}_5$	$\left\langle 0.2e^{j\pi 0.4}, 0.1e^{j\pi 0.6}, 0.7e^{j\pi 0.9} \right\rangle$	$\left\langle 0.3e^{j\pi 0.4}, 0.4e^{j\pi 0.6}, 0.7e^{j\pi 0.9} \right\rangle$	$\left\langle 0.3e^{j\pi 0.6}, 0.1e^{j\pi 0.5}, 0.6e^{j\pi 0.9} \right\rangle$	$\left\langle 0.4e^{j\pi 0.6}, 0.2e^{j\pi 0.4}, 0.2e^{j\pi 0.8} \right\rangle$

Table 2: The complex neutrosophic system of relationship between Symptoms and Diseases

	$\mathfrak{s}_1$	$\mathfrak{s}_2$	$\mathfrak{s}_3$	$\mathfrak{s}_4$	$\mathfrak{s}_5$
$\mathfrak{d}_1$	$\left\langle 0.4e^{j\pi 0.8}, 0.4e^{j\pi 0.6}, 0.3e^{j\pi 0.3} \right\rangle$	$\left\langle 0.7e^{j\pi 0.5}, 0.4e^{j\pi 0.7}, 0.3e^{j\pi 0.4} \right\rangle$	$\left\langle 0.6e^{j\pi 0.2}, 0.3e^{j\pi 0.7}, 0.7e^{j\pi 0.8} \right\rangle$	$\left\langle 0.6e^{j\pi 0.7}, 0.4e^{j\pi 0.8}, 0.6e^{j\pi 0.5} \right\rangle$	$\left\langle 0.7e^{j\pi 0.3}, 0.4e^{j\pi 0.7}, 0.3e^{j\pi 0.7} \right\rangle$
$\mathfrak{d}_2$	$\left\langle 0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.6}, 0.4e^{j\pi 0.5} \right\rangle$	$\left\langle 0.4e^{j\pi 0.4}, 0.5e^{j\pi 0.5}, 0.6e^{j\pi 0.7} \right\rangle$	$\left\langle 0.6e^{j\pi 0.2}, 0.5e^{j\pi 0.8}, 0.9e^{j\pi 0.5} \right\rangle$	$\left\langle 0.6e^{j\pi 0.8}, 0.2e^{j\pi 0.3}, 0.5e^{j\pi 0.6} \right\rangle$	$\left\langle 0.4e^{j\pi 0.7}, 0.6e^{j\pi 0.8}, 0.3e^{j\pi 0.6} \right\rangle$
$\mathfrak{d}_3$	$\left\langle 0.9e^{j\pi 0.6}, 0.3e^{j\pi 0.6}, 0.3e^{j\pi 0.4} \right\rangle$	$\left\langle 0.3e^{j\pi 0.6}, 0.2e^{j\pi 0.3}, 0.7e^{j\pi 0.6} \right\rangle$	$\left\langle 0.4e^{j\pi 0.6}, 0.2e^{j\pi 0.8}, 0.7e^{j\pi 0.5} \right\rangle$	$\left\langle 0.2e^{j\pi 0.6}, 0.1e^{j\pi 0.5}, 0.5e^{j\pi 0.7} \right\rangle$	$\left\langle 0.8e^{j\pi 0.4}, 0.6e^{j\pi 0.5}, 0.3e^{j\pi 0.9} \right\rangle$
$\mathfrak{d}_4$	$\left\langle 0.4e^{j\pi 0.5}, 0.3e^{j\pi 0.7}, 0.6e^{j\pi 0.4} \right\rangle$	$\left\langle 0.8e^{j\pi 0.7}, 0.2e^{j\pi 0.8}, 0.3e^{j\pi 0.6} \right\rangle$	$\left\langle 0.6e^{j\pi 0.3}, 0.4e^{j\pi 0.5}, 0.1e^{j\pi 0.5} \right\rangle$	$\left\langle 0.7e^{j\pi 0.4}, 0.2e^{j\pi 0.7}, 0.4e^{j\pi 0.6} \right\rangle$	$\left\langle 0.4e^{j\pi 0.2}, 0.3e^{j\pi 0.1}, 0.4e^{j\pi 0.7} \right\rangle$

Step-2: Frame the in-discernibility relation for the correlation between the symptoms is given as $\mathfrak{R} = \{\{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_4\}, \{\mathfrak{s}_3, \mathfrak{s}_5\}\}$.

Step-3: Construct the complex neutrosophic nano topological spaces for each patient and each disease with respect to the symptoms as follows:

Complex neutrosophic nano topologies for patients are $\mathfrak{C}^*_{\tau_1 \eta}$

1. $\mathfrak{C}^*_{\tau_1}(\mathfrak{p}_1) = \{1_\sim, 0_\sim, \langle 0.7e^{j\pi 0.9}, 0.2e^{j\pi 0.6}, 0.3e^{j\pi 0.3}\rangle, \langle 0.8e^{j\pi 0.7}, 0.1e^{j\pi 0.6}, 0.2e^{j\pi 0.2}\rangle, \langle 0.3e^{j\pi 0.2}, 0.4e^{j\pi 0.8}, 0.5e^{j\pi 0.7}\rangle, \langle 0.2e^{j\pi 0.4}, 0.1e^{j\pi 0.8}, 0.7e^{j\pi 0.9}\rangle, \langle 0.3e^{j\pi 0.2}, 0.6e^{j\pi 1.2}, 0.7e^{j\pi 0.9}\rangle, \langle 0.2e^{j\pi 0.2}, 0.9e^{j\pi 1.2}, 0.8e^{j\pi 0.9}\rangle\}$

2. $\mathfrak{C}^*_{\tau_1}(\mathfrak{p}_2) = \{1_\sim, 0_\sim, \langle 0.9e^{j\pi 0.6}, 0.3e^{j\pi 0.4}, 0.2e^{j\pi 0.6}\rangle, \langle 0.3e^{j\pi 0.7}, 0.1e^{j\pi 0.2}, 0.6e^{j\pi 0.7}\rangle, \langle 0.4e^{j\pi 0.5}, 0.6e^{j\pi 0.9}, 0.4e^{j\pi 0.9}\rangle, \langle 0.2e^{j\pi 0.4}, 0.4e^{j\pi 0.6}, 0.7e^{j\pi 0.9}\rangle, \langle 0.2e^{j\pi 0.5}, 0.4e^{j\pi 1.1}, 0.9e^{j\pi 0.9}\rangle, \langle 0.2e^{j\pi 0.4}, 0.6e^{j\pi 1.4}, 0.7e^{j\pi 0.9}\rangle\}$

3. $\mathfrak{C}^*_{\tau_1}(\mathfrak{p}_3) = \{1_\sim, 0_\sim, \langle 0.7e^{j\pi 0.7}, 0.3e^{j\pi 0.4}, 0.2e^{j\pi 0.4}\rangle, \langle 0.3e^{j\pi 0.7}, 0.1e^{j\pi 0.5}, 0.6e^{j\pi 0.2}\rangle, \langle 0.4e^{j\pi 0.1}, 0.4e^{j\pi 0.8}, 0.7e^{j\pi 0.9}\rangle, \langle 0.3e^{j\pi 0.6}, 0.1e^{j\pi 0.9}, 0.6e^{j\pi 0.9}\rangle, \langle 0.2e^{j\pi 0.1}, 0.6e^{j\pi 1.2}, 0.7e^{j\pi 0.9}\rangle, \langle 0.3e^{j\pi 0.2}, 0.9e^{j\pi 1.1}, 0.6e^{j\pi 0.9}\rangle\}$

4. $\mathfrak{C}^*_{\tau_1}(\mathfrak{p}_4) = \{1_\sim, 0_\sim, \langle 0.9e^{j\pi 0.9}, 0e^{j\pi 0.2}, 0.3e^{j\pi 0.5}\rangle, \langle 0.4e^{j\pi 0.6}, 0.2e^{j\pi 0.1}, 0.2e^{j\pi 0.6}\rangle, \langle 0.5e^{j\pi 0.2}, 0.5e^{j\pi 0.7}, 0.7e^{j\pi 0.8}\rangle, \langle 0.1e^{j\pi 0.5}, 0.4e^{j\pi 0.4}, 0.7e^{j\pi 0.8}\rangle, \langle 0.3e^{j\pi 0.2}, 0.5e^{j\pi 1.3}, 0.9e^{j\pi 0.9}\rangle, \langle 0.1e^{j\pi 0.5}, 0.6e^{j\pi 1.6}, 0.7e^{j\pi 0.8}\rangle\}$

Complex neutrosophic nano topologies for diseases are $\mathfrak{C}^*_{\tau_2 \xi}$

1. $\mathfrak{C}^*_{\tau_2}(\mathfrak{d}_1) = \{1_\sim, 0_\sim, \langle 0.7e^{j\pi 0.8}, 0.4e^{j\pi 0.6}, 0.3e^{j\pi 0.3}\rangle, \langle 0.7e^{j\pi 0.3}, 0.3e^{j\pi 0.7}, 0.3e^{j\pi 0.7}\rangle, \langle 0.4e^{j\pi 0.5}, 0.4e^{j\pi 0.8}, 0.6e^{j\pi 0.5}\rangle, \langle 0.6e^{j\pi 0.2}, 0.4e^{j\pi 0.7}, 0.7e^{j\pi 0.8}\rangle, \langle 0.3e^{j\pi 0.3}, 0.6e^{j\pi 1.2}, 0.7e^{j\pi 0.8}\rangle, \langle 0.3e^{j\pi 0.2}, 0.6e^{j\pi 1.3}, 0.7e^{j\pi 0.8}\rangle\}$

2. $\mathfrak{C}^*_{\tau_2}(\mathfrak{d}_2) = \{1_\sim, 0_\sim, \langle 0.6e^{j\pi 0.8}, 0.2^{j\pi 0.3}, 0.4e^{j\pi 0.5}\rangle, \langle 0.6e^{j\pi 0.7}, 0.5e^{j\pi 0.8}, 0.3e^{j\pi 0.5}\rangle, \langle 0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.6}, 0.6e^{j\pi 0.7}\rangle, \langle 0.4e^{j\pi 0.2}, 0.6e^{j\pi 0.8}, 0.9e^{j\pi 0.6}\rangle, \langle 0.3e^{j\pi 0.4}, 0.5e^{j\pi 1.4}, 0.6e^{j\pi 0.8}\rangle, \langle 0.3e^{j\pi 0.2}, 0.5e^{j\pi 1.2}, 0.9e^{j\pi 0.7}\rangle\}$

3. $\mathfrak{C}^*_{\tau_2}(\mathfrak{d}_3) = \{1_\sim, 0_\sim, \langle 0.9e^{j\pi 0.6}, 0.1e^{j\pi 0.3}, 0.3e^{j\pi 0.4}\rangle, \langle 0.8e^{j\pi 0.6}, 0.2e^{j\pi 0.5}, 0.3e^{j\pi 0.5}\rangle, \langle 0.2e^{j\pi 0.6}, 0.3e^{j\pi 0.6}, 0.7e^{j\pi 0.7}\rangle, \langle 0.4e^{j\pi 0.4}, 0.6e^{j\pi 0.8}, 0.7e^{j\pi 0.9}\rangle, \langle 0.2e^{j\pi 0.4}, 0.7e^{j\pi 1.4}, 0.9e^{j\pi 0.7}\rangle, \langle 0.3e^{j\pi 0.4}, 0.4e^{j\pi 1.2}, 0.8e^{j\pi 0.9}\rangle\}$

4. $\mathfrak{C}^*_{\tau_2}(\mathfrak{d}_4) = \{1_\sim, 0_\sim, \langle 0.8e^{j\pi 0.7}, 0.2e^{j\pi 0.7}, 0.3e^{j\pi 0.4}\rangle, \langle 0.6e^{j\pi 0.3}, 0.3e^{j\pi 0.1}, 0.1e^{j\pi 0.5}\rangle, \langle 0.4e^{j\pi 0.4}, 0.3e^{j\pi 0.8}, 0.6e^{j\pi 0.6}\rangle, \langle 0.4e^{j\pi 0.2}, 0.4e^{j\pi 0.5}, 0.4e^{j\pi 0.7}\rangle, \langle 0.3e^{j\pi 0.4}, 0.7e^{j\pi 1.2}, 0.8e^{j\pi 0.7}\rangle, \langle 0.1e^{j\pi 0.2}, 0.6e^{j\pi 1.5}, 0.6e^{j\pi 0.7}\rangle\}$

Step-4: Computation of complex neutrosophic score functions for the patients and diseases are done as in step-4 f the algorithm are as follows

Score values for the patients are

$\mathfrak{S}_{cr}(\mathfrak{p}_1) = 0.5052, \mathfrak{S}_{cr}(\mathfrak{p}_2) = 0.4917, \mathfrak{S}_{cr}(\mathfrak{p}_3) = 0.4906, \mathfrak{S}_{cr}(\mathfrak{p}_4) = 0.5042$

Score values for the diseases are

$\mathfrak{S}_{cr}(\mathfrak{d}_1) = 0.5010, \mathfrak{S}_{cr}(\mathfrak{d}_2) = 0.4875, \mathfrak{S}_{cr}(\mathfrak{d}_3) = 0.5073, \mathfrak{S}_{cr}(\mathfrak{d}_4) = 0.5146$

Step-5: Arrange the complex neutrosophic score values for the alternatives $\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3, \mathfrak{p}_4$ and the attributes $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4$ in run-up order. We consider the sequences below $\mathfrak{p}_3 \leq \mathfrak{p}_2 \leq \mathfrak{p}_4 \leq \mathfrak{p}_1$ and $\mathfrak{d}_2 \leq \mathfrak{d}_1 \leq \mathfrak{d}_3 \leq \mathfrak{d}_4$. Thus the patient $\mathfrak{p}_3$ suffers with disease $\mathfrak{d}_4$= chest problem, Thus the patient $\mathfrak{p}_2$ suffers with disease $\mathfrak{d}_3$= stomach problem, Thus the patient $\mathfrak{p}_4$ suffers with disease $\mathfrak{d}_1$ = viral fever and Thus the patient $\mathfrak{p}_1$ suffers with disease $\mathfrak{d}_2$= Malaria.

The comparison table show the difference between novel complex neutrosophic nano topological space with existing work.

Sets	Uncertainty	Truth Value of an element	False value of an element	Indeterminacy of an element	Roughness & boundary of a set	unit complex plane
GT	-	-	-	-	-	-
FT	✓	✓	-	-	-	-
IFT	✓	✓	✓	-	-	-
NT	✓	✓	✓	✓	-	-
NNT	✓	✓	✓	✓	✓	-
CNNT	✓	✓	✓	✓	✓	✓

6 Conclusion

It is the opinion that complex neutrosophic information can be best dealt with by unclear, vague, indeterminate, contradictory and incomplete periodic / redundant information werte. This paper aimed at bringing out the complex neutrosophic nano topology which is more versatile and adaptable to real-time issues than rest of the types of general fluffy sets. Definitions of complex nano topology in neutrosophy has been identified, followed by the closure and interior operations. A new form of MCDM technique in the complex neutrosophic set has been introduced and applied to a medical diagnostic issue. To show the advantages and applicability, a comparison was made between the proposed method and the existing methods. The results are critical in enriching the complex neutrosophical awareness provided for the decision making applications. Future research plans to use the MCDM technique for more practical applications and advance the practical interval valued complex neutro nano topological logic method for prediction of forecasting problems.

Abbreviation:

MCDM	- Multi-criteria Decision Making
NNT	- Neutrosophic nano Topological spaces
CNNCS	- Complex neutrosophic nano closed set
CNN	- Complex neutrosophic nano
CNNOS	- Complex neutrosophic nano open set
CNNTS	- Complex neutrosophic nano topological space
NS	- Neutrosophic set
IFS	- intuitionistic fuzzy sets
NNt	- neutrosophic nano topology
CNS	- complex neutrosophic set

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References

[1] M. Ali, F. Smarandache. Complex neutrosophic set. Neural Computing and Applications, 12, (2015). DOI:10.1007/s00521-015-2154-y.

[2] I. Arokiarani, R. Dhavaseelan, S. Jafari, M. Parimala. On some new notions and functions in neutrosophic topological spaces, Neutrosophic Sets Syst., 16, (2017), 16-19.

[3] K. T. Atanassov. Intuitionstic fuzzy sets, Fuzzy sets and systems, 20, (1986), 87-96.

[4] D. Coker. An introduction to fuzzy topological spaces, Fuzzy sets and systems, 88, (1997), 81-89.

[5] K. Kuratowski.Topology Vol. II (transl.), Academic Press, New York, (1966).

[6] M. Karthika, M. Parimala, S. Jafari, F. Smarandache, Mohammad Alshumrani, Cenap Ozel, R. Udhayakumar. Neutrosophic complex $\alpha\psi$ connectedness in neutrosophic complex topological spaces. Neutrosophic sets and systems, 29,(2019), 158-164.

[7] M. Lellis Thivagar, S. Jafari, V. Sutha Devi,V. Antonysamy, A novel approach to nano topology via neutrosophic sets, Neutrosophic sets and systems, 20(2018), 86-94.

[8] M. Parimala, S. Jafari, and S. Murali, Nano Ideal Generalized Closed Sets in Nano Ideal Topological Spaces, Annales Univ. Sci. Budapest., 60, (2017), 3-11.

[9] M. Parimala, R. Jeevitha, S. Jafari, F. Smarandache and R. Udhayakumar. Neutrosophic $\alpha\psi$-Homeomorphism in Neutrosophic Topological Spaces, *Information*,9,187,(2018), 1-10. ; doi:10.3390/info9080187.

[10] M.Parimala, R.Jeevitha, A.Selvakumar. A New Type of Weakly Closed Set in Ideal Topological Spaces, International Journal of Mathematics and its Applications, 5(4-C), (2017), 301-312.

[11] M. Parimala, M. Karthika, R. Dhavaseelan, S. Jafari. On neutrosophic supra precontinuous functions in neutrosophic topological spaces, New Trends in Neutrosophic Theory and Applications , 2, (2018), 371-383.

[12] M. Parimala, M. Karthika, S. Jafari, F. Smarandache , R. Udhayakumar. Neutrosophic nano ideal topological structures, Neutrosophic sets and systems, 24(2019), 70-77.

[13] M. Parimala, M. Karthika, S. Jafari, F. Smarandache, R. Udhayakumar. Decision-Making via Neutrosophic Support Soft Topological Spaces. Symmetry, 10(2018), 1-10. doi:10.3390/sym10060217

[14] M. Parimala and R. Perumal, Weaker form of open sets in nano ideal topological spaces, Global Journal of Pure and Applied Mathematics, 12(1), (2016), 302-305.

[15] M. Parimala, F. Smarandache, S. Jafari, R. Udhayakumar, On Neutrosophic $\alpha\psi$ -Closed Sets, Information, 9, (2018), 103, 1-7.

[16] M. Riaz, F. Smarandache, F. Karaaslan, M. R. Hashmi, I. Nawaz. Neutrosophic Soft Topology and its Applications to Multi-Criteria Decision Making. Neutrosophic sets and systems,35,(2020), 198-219.

[17] M. Riaz, Naeem, K.; Zareef, I.; Afzal, D. Neutrosophic N-Soft Sets with TOPSIS method for Multiple Attribute Decision Making. Neutrosophic sets and systems, 32 (2020), 1-24

[18] M. Riaz, B. Davvaz, A. Fakhar, A. Firdous. Hesitant fuzzy soft topology and its applications to multi-attribute group decision-making. Soft Computing, 2020,1-21,https://doi.org/10.1007/s00500-020-04938-0.

[19] M. Riaz, S.T. Tehrim. Cubic bipolar fuzzy set with application to multi-criteria group decision making using geometric aggregation operators. Soft Computing, 2020, https://doi.org/10.1007/s00500-020-04927-3.

[20] A. A. Salama, S. A. Alblowi. Neutrosophic Set and Neutrosophic Topological Spaces, IOSR J. Math., 3, (2012), 31-35.

[21] A. A. Salama. Florentin Smarandache, Valeri Kromov. Neutrosophic Closed Set and Neutrosophic Continuous Functions, Neutrosophic Sets and Systems, 4, (2014),4-8.

[22] F. Smarandache. Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic Set, Probability and Statistics; University of New Mexico, Gallup, NM, USA, (2002).

[23] F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Research Press, Rehoboth, NM, USA, (1999).

[24] F. Smarandache. Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra, Neutrosophic Sets and Systems, 33,(2020), 290-296.

[25] L. A. Zadeh. Fuzzy Sets, Information and Control, 18, (1965), 338-353.

[26] F. Zafer, M. Akram, A novel decision-making method based on rough fuzzy information. Int. J. Fuzzy Syst. 2017, doi:10.1007/s40815-017-0368-0.