



H-Max Single-Valued Neutrosophic Distance Measure in Medical Diagnosis

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Abstract

As an instance of a neutrosophic set, a single-valued neutrosophic set (SVNS) can be used to handle uncertainty, inaccuracy, indeterminacy, and inconsistency. In this paper, a new not-so-sophisticated distance measure between two SVNSs is defined by considering the cross-assessment between characteristic functions for the forward and backward differences. Furthermore, a single-valued neutrosophic similarity measure, a single-valued neutrosophic entropy measure, and their basic properties are presented and proven. In addition, an application to medical diagnosis is shown to illustrate the effective applicability of the proposals.

Keywords: Single-Valued Neutrosophic Distance, similarity measure, entropy measure, medical diagnosis

1.Introduction

In order to approach uncertainty, in 1965, the fuzzy set (FS) theory was introduced by Zadeh [1] and has achieved a great success in many real applications. Zadeh defined the degree of membership of a FS A defined on a universe X by one real value $\mu_A(x) \in [0,1]$. However, in many real-life systems, it may be very difficult to assign the membership value for a FS to handle imprecise, incomplete, and inconsistent information. In 1983, Atanassov [2] proposed a generalization of FS, named the intuitionistic fuzzy set (IFS) on a universe X , where the truth membership, $\mu_A(x) \in [0,1]$, is considered as well as the falsity membership, $\nu_A(x) \in [0,1]$, for

description of an object. They are tied together by condition $\mu_A(x) + \nu_A(x) \in [0, 1]$ for each $x \in X$. In a belief system, the inconsistent and indeterminate information is not handled satisfactorily by IFS. In 1999, the concept of neutrosophic set (NS) was originally introduced by Smarandache [3]. This is a branch of philosophy and is a mathematical tool for studying the neutralities. A NS is represented independently by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+]$. Some of the recent developments on how to use neutrosophic data can be mentioned as: in 2018 a mathematical model for clustering the neutrosophic big data sets was determined [4]; in 2019 a Python tool for bipolar neutrosophic matrices is presented in [5]; in 2020 a novel social choice theory based multi-criteria decision making method under neutrosophic environment was introduced [6]; and a smart agriculture mechanism model equipped with neutrosophy theory was studied [7]. In order to apply NS in real scientific and engineering areas, Wang et al [8] introduced a single-valued neutrosophic set (SVNS) and it has received considerable attention for various researchers. Some applications of SVNSs in decision-making can be found in [9-16].

A distance measure is an effective tool in decision-making problems, such as, intuitionistic fuzzy and picture fuzzy distance measures have been studied for use in medical diagnosis and Controlling Network Power Consumption [17-21]. From 2013 to present, several distance measures of SVNSs were introduced such as Hausdorff distance [22], Cosine similarity [23] measures, and the distance measures of Ye [24], Aydoğdu [25], and Huang [9]. In 2016, the measure of Huang [9] has solved the limitation of Ye's measure [24] in some cases where it is necessary to distinguish the differences between several objects, but it is quite complicated in calculation. Regarding the need to distinguish the differences, in an intuitionistic fuzzy environment, the H-max distance measure of Ngan et al. [18] has solved well by considering the cross-assessment between characteristic functions. From a comprehensive consideration of the problem, this paper focuses on proposing a new distance measure of SVNSs and its application in medical diagnosis for the purpose of increasing accuracy and reducing computation time. This research based on some of the following main motivations.

- H-max distance measure of intuitionistic fuzzy sets in decision making [18] and H-max distance measure of the picture fuzzy sets [21].
- The single-valued neutrosophic weighted distance measure, the single-valued neutrosophic weighted similarity measure, and the single-valued neutrosophic entropy measure of Huang [9].
- The entropy measure is used to calculate the weights of attributes in decision-making problem [26].

The main contribution of the proposal is to extend the H-max distance measure to SVNSs, thereby building the formula of entropy measure of SVNSs and they are included in the new diagnostic method.

In Section 2, some basic concepts are reviewed. In Section 3, the notions of the distance measure, the similarity measure, and the entropy measure are introduced. In Section 4, an application to medical diagnosis given to show the effectiveness of the new distance measure applied in medical diagnosis. A conclusion is set in section 5.

2. Preliminaries

Definition 1. [7] Let X be a universe set and $x \in X$. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard of nonstandard subsets of $]0, 1^+[$, and there is no restriction on the sum of them. Hence

$$T_A(x): x \rightarrow]0, 1^+[, \quad I_A(x): x \rightarrow]0, 1^+[, \quad F_A(x): x \rightarrow]0, 1^+[, \text{ and}$$

$$0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+.$$

To make it easier to access science and engineering problems, a subclass of a NS which is the concepts of SVN S was proposed by Wang et al. [8].

Definition 2. [8] Let X be a universe set and $x \in X$. A SVN S A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, and can be denoted by $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \}$. For each point x in X , $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

The inclusion, equality, and complement relations are defined by Wang et al. [8] for SVN Ss A, B as follows:

$$A \subseteq B \text{ if and only if } T_A(x) \leq T_B(x), \quad I_A(x) \geq I_B(x) \text{ and } F_A(x) \geq F_B(x) \text{ for any } x \text{ in } X.$$

$$A = B \text{ if and only if } T_A(x) = T_B(x), \quad I_A(x) = I_B(x) \text{ and } F_A(x) = F_B(x) \text{ for any } x \text{ in } X.$$

$$A^c = \{ \langle x, F_A(x), 1 - I_A(x), T_A(x) \rangle \mid x \in X \} \text{ where } A^c \text{ denotes the complement of } A.$$

Definition 3. [9] A mapping, $E : \text{SVN Ss} \rightarrow [0, 1]$, is an entropy measure if the conditions (e1)-(e4) are satisfied.

$$(e1) \quad E(A) = 0 \text{ if and only if } A \text{ or } A^c \text{ is a crisp set;}$$

$$(e2) \quad E(A) = 1 \text{ if and only if } A = A^c, \text{ i.e., } T_A(x_i) = F_A(x_i) \text{ and } I_A(x_i) = 0.5 \text{ for all } x_i \in X.$$

$$(e3) \quad E(A) \leq E(B) \text{ if } A \text{ is less fuzzy than } B, \text{ i.e., if}$$

$$T_A(x_i) \leq T_B(x_i), F_A(x_i) \geq F_B(x_i) \text{ for } T_B(x_i) \leq F_B(x_i) \text{ and } I_A(x_i) = I_B(x_i) = 0.5, \text{ or}$$

$$T_A(x_i) \geq T_B(x_i), F_A(x_i) \leq F_B(x_i) \text{ for } T_B(x_i) \geq F_B(x_i) \text{ and } I_A(x_i) = I_B(x_i) = 0.5;$$

$$(e4) \quad E(A) = E(A^c).$$

In 2016, Huang [9] introduced the weighted distance measure on SVNShs as follows.

$$d_\lambda(A, B) = \left[\sum_{j=1}^n \omega_j \left(\sum_{i=1}^4 \beta_i \varphi_i(x_j) \right)^\lambda \right]^{1/\lambda}$$

where $\lambda > 0$, $\beta_i \in [0, 1]$ and $\sum_{i=1}^4 \beta_i = 1$, $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$,

$$\varphi_1(x_j) = \frac{|T_A(x_j) - T_B(x_j)|}{3} + \frac{|I_A(x_j) - I_B(x_j)|}{3} + \frac{|F_A(x_j) - F_B(x_j)|}{3},$$

$$\varphi_2(x_j) = \max \left\{ \frac{2 + T_A(x_j) - I_A(x_j) - F_A(x_j)}{3}, \frac{2 + T_B(x_j) - I_B(x_j) - F_B(x_j)}{3} \right\} \\ - \min \left\{ \frac{2 + T_A(x_j) - I_A(x_j) - F_A(x_j)}{3}, \frac{2 + T_B(x_j) - I_B(x_j) - F_B(x_j)}{3} \right\},$$

$$\varphi_3(x_j) = \frac{|T_A(x_j) - T_B(x_j) + I_B(x_j) - I_A(x_j)|}{2}, \varphi_4(x_j) = \frac{|T_A(x_j) - T_B(x_j) + F_B(x_j) - F_A(x_j)|}{2}.$$

In 2018, by reasoning about the need for the cross-evaluation, Ngan et al. [18] defined the H-max distance measure on IFSs by

$$d_{Hm}(A, B) = \frac{1}{3m} \sum_{i=1}^m (|\mu_1(x_i) - \mu_2(x_i)| + |\nu_1(x_i) - \nu_2(x_i)| + \max\{\mu_1(x_i), \nu_2(x_i)\} - \max\{\mu_2(x_i), \nu_1(x_i)\}).$$

In 2020, Smarandache and Ngan et al. [21] introduced the picture fuzzy distance measure based on $d_{Hm}(A, B)$ as

$$D_0(x, y) = \frac{1}{3} (|x_1 - y_1| + |x_2 - y_2| + |x_3 - y_3| + |\max\{x_1, y_3\} - \max\{x_3, y_1\}|), \forall x, y \in P^*,$$

where $P^* = \{x = (x_1, x_2, x_3) \mid x_1, x_2, x_3, \sum_{i=1}^3 x_i \in [0, 1]\}$.

Note that a picture fuzzy set, proposed by Cuong [27], is considered as a special case of neutrosophic sets when when all three components, $T_A(x)$, $I_A(x)$, and $F_A(x)$, are dependent, i.e., $T_A(x) + I_A(x) + F_A(x) \in [0, 1]$.

Remark 1. We agree with the argument of Huang [9], that is, some existing measures give an equal rating that is not tight. Suppose in a match, there are three players whose performances are represented by three SVNSSs $A = \langle 0.8, 0, 0.8 \rangle$, $B = \langle 0.7, 0, 0.7 \rangle$, and $C = \langle 0.7, 0, 0.9 \rangle$ on $X = \{x\}$. Obviously, the distance between A and B is more closer than the distance between A and C , such as, by the distance of Huang [9], we obtain that $d(A, B) = 0.06 < d(A, C) = 0.14$. However, this distance measure is quite cumbersome. Furthermore, by the distance and similarity measures introduced in [22-25], we have $d(A, B) = d(A, C)$, $s(A, B) = s(A, C)$, and $s(A, B) < s(A, C)$, which are not consistent with our intuition (see Table 1).

Table 1. The values of the distance and similarity measures between A, B , and C .

[25]	[23]	[24]	[22]	[9]
$s(A, B) = 0.93$	$s(A, B) = 0.53$	$d(A, B) = 0.08$	$d(A, B) = 0.1$	$d(A, B) = 0.06$
$=$	$<$	$=$	$=$	$<$
$s(A, C) = 0.93$	$s(A, C) = 0.56$	$d(A, C) = 0.08$	$d(A, C) = 0.1$	$d(A, C) = 0.14$

3. H-max single-valued neutrosophic distance measure

Definition 4. Let A and B be two SVNSSs in a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i) \rangle \mid x_i \in X\}$ and $B = \{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i) \rangle \mid x_i \in X\}$. The H-max single-valued neutrosophic weighted distance measure is defined by

$$d_{H-N}(A, B) = \sum_{i=1}^n \chi_i \left(\omega_1 |T_A(x_i) - T_B(x_i)| + \omega_2 |I_A(x_i) - I_B(x_i)| + \omega_3 |F_A(x_i) - F_B(x_i)| \right. \\ \left. + \omega_4 \left| \max\{T_A(x_i), I_B(x_i)\} - \max\{I_A(x_i), T_B(x_i)\} \right| \right. \\ \left. + \omega_5 \left| \max\{T_A(x_i), F_B(x_i)\} - \max\{F_A(x_i), T_B(x_i)\} \right| \right)$$

where $\omega_j \in (0, 1)$ for $j = 1, 2, 3, 4, 5$, and $\sum_{j=1}^5 \omega_j = 1$, $\chi_i \in [0, 1]$ for $i = 1, 2, \dots, n$ and $\sum_{i=1}^n \chi_i = 1$.

Remark 2. Revisiting Remark 1, we have $d_{H-N}(A, B) < d_{H-N}(A, C)$, where $A = \langle 0.8, 0, 0.8 \rangle$, $B = \langle 0.7, 0, 0.7 \rangle$, and $C = \langle 0.7, 0, 0.9 \rangle$. Specifically, when $\omega_j = \frac{1}{5}$, $j = 1, 2, 3, 4, 5$, we have $d_{H-N}(A, B) = 0.06$ and $d_{H-N}(A, C) = 0.08$. Furthermore, the proposed distance measure is less cumbersome than that of Huang [9]. However, its complexity is intentional. Considering the case

Huang [9] mentioned, we have $d_{H-N}(A, B) = d_{H-N}(A, C)$, where $A = \langle 0.9, 0, 0.3 \rangle$, $B = \langle 0.7, 0, 0.1 \rangle$, and $C = \langle 0.7, 0, 0.5 \rangle$. This means, in this case, our proposed distance measure is not suitable to distinguish the differences between three players.

Proposition 1. The distance measure $d_{H-N}(A, B)$ satisfies the following properties:

- (d1) $0 \leq d_{H-N}(A, B) \leq 1$;
- (d2) $d_{H-N}(A, B) = 0$ if and only if $A = B$;
- (d3) $d_{H-N}(A, B) = d_{H-N}(B, A)$;
- (d4) If $A \subseteq B \subseteq C$, C is an SVN in X , then $d_{H-N}(A, B) \leq d_{H-N}(A, C)$ and $d_{H-N}(B, C) \leq d_{H-N}(A, C)$.

Proof.

(d1) For $i = 1, 2, \dots, n$, since $|T_A(x_i) - T_B(x_i)|$, $|I_A(x_i) - I_B(x_i)|$, $|F_A(x_i) - F_B(x_i)| \in [0, 1]$, and $|\max\{T_A(x_i), I_B(x_i)\} - \max\{I_A(x_i), T_B(x_i)\}|$, $|\max\{T_A(x_i), F_B(x_i)\} - \max\{F_A(x_i), T_B(x_i)\}| \in [0, 1]$, hence $0 \leq d_{H-N}(A, B) \leq 1$.

$$(d2) \quad d_{H-N}(A, B) = 0 \Leftrightarrow \begin{cases} T_A(x_i) = T_B(x_i) \\ I_A(x_i) = I_B(x_i), \forall i = 1, 2, \dots, n \\ F_A(x_i) = F_B(x_i) \end{cases} \Leftrightarrow A = B.$$

(d3) The symmetry of measure $d_{H-N}(A, B)$ with respect to their argument is obvious.

(d4) Let $A \subseteq B \subseteq C$, we have $T_A(x_i) \leq T_B(x_i) \leq T_C(x_i)$, $I_A(x_i) \geq I_B(x_i) \geq I_C(x_i)$ and $F_A(x_i) \geq F_B(x_i) \geq F_C(x_i)$ for $i = 1, 2, \dots, n$. Then,

$$\begin{aligned} |T_A(x_i) - T_B(x_i)| &\leq |T_A(x_i) - T_C(x_i)|, \\ |I_A(x_i) - I_B(x_i)| &\leq |I_A(x_i) - I_C(x_i)|, \\ |F_A(x_i) - F_B(x_i)| &\leq |F_A(x_i) - F_C(x_i)|. \end{aligned}$$

Moreover, we have

$$\begin{aligned} \max\{T_C(x_i), I_A(x_i)\} &\geq \max\{T_B(x_i), I_A(x_i)\} \geq \max\{T_A(x_i), I_B(x_i)\} \geq \max\{T_A(x_i), I_C(x_i)\}, \\ \max\{T_C(x_i), F_A(x_i)\} &\geq \max\{T_B(x_i), F_A(x_i)\} \geq \max\{T_A(x_i), F_B(x_i)\} \geq \max\{T_A(x_i), F_C(x_i)\}, \end{aligned}$$

hence

$$\left| \max \{T_B(x_i), I_A(x_i)\} - \max \{T_A(x_i), I_B(x_i)\} \right| \leq \left| \max \{T_C(x_i), I_A(x_i)\} - \max \{T_A(x_i), I_C(x_i)\} \right|,$$

$$\left| \max \{T_B(x_i), F_A(x_i)\} - \max \{T_A(x_i), F_B(x_i)\} \right| \leq \left| \max \{T_C(x_i), F_A(x_i)\} - \max \{T_A(x_i), F_C(x_i)\} \right|.$$

Thus, $d_{H-N}(A, B) \leq d_{H-N}(A, C)$. Similarly, we also have $d_{H-N}(B, C) \leq d_{H-N}(A, C)$.

Definition 5. Let A and B be two SVNNS in a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, $A = \{ \langle x_i, T_A(x_i), I_A(x_i), F_A(x_i) \rangle \mid x_i \in X \}$ and $B = \{ \langle x_i, T_B(x_i), I_B(x_i), F_B(x_i) \rangle \mid x_i \in X \}$. The H-max single-valued neutrosophic weighted similarity measure is defined by

$$s_{H-N}(A, B) = 1 - d_{H-N}(A, B).$$

Proposition 2. The similarity measure $s_{H-N}(A, B)$ satisfies the following properties:

$$(s1) \quad 0 \leq s_{H-N}(A, B) \leq 1;$$

$$(s2) \quad s_{H-N}(A, B) = 1 \text{ if and only if } A = B;$$

$$(s3) \quad s_{H-N}(A, B) = s_{H-N}(B, A);$$

$$(s4) \quad \text{If } A \subseteq B \subseteq C, \text{ } C \text{ is an SVNNS in } X, \text{ then } s_{H-N}(A, B) \geq s_{H-N}(A, C) \text{ and } s_{H-N}(B, C) \geq s_{H-N}(A, C).$$

Proposition 3. $s_{H-N}(A, A^c)$ is an entropy measure for a SVNNS A .

Proof.

(e1) Let A be a crisp set, i.e., $T_A(x_i) = 1, I_A(x_i) = F_A(x_i) = 0$ or $F_A(x_i) = 1, T_A(x_i) = I_A(x_i) = 0$ for all $x_i \in X$. No matter in which cases (A or A^c is a crisp set), then $s_{H-N}(A, A^c) = 0$. On the other hand, let $s_{H-N}(A, A^c) = 0$, we can easily get A or A^c is a crisp set.

(e2) From (s2) in Proposition 2, we obtain that $s_{H-N}(A, A^c) = 1$ if and only if $A = A^c$.

(e3) Let A is less fuzzy than B , assume that $T_A(x_i) \leq T_B(x_i)$, $F_A(x_i) \geq F_B(x_i)$ for $T_B(x_i) \leq F_B(x_i)$ and $I_A(x_i) = I_B(x_i) = 0.5$, then

$$T_A(x_i) \leq T_B(x_i) \leq F_B(x_i) \leq F_A(x_i),$$

$$\max \{T_A(x_i), 0.5\} \leq \max \{T_B(x_i), 0.5\} \leq \max \{F_B(x_i), 0.5\} \leq \max \{F_A(x_i), 0.5\}, \text{ and}$$

$$d_{H-N}(A, A^c) = \sum_{i=1}^n \chi_i \left((\omega_1 + \omega_3 + \omega_5) |T_A(x_i) - F_A(x_i)| + \omega_4 |\max \{T_A(x_i), 0.5\} - \max \{0.5, F_A(x_i)\}| \right)$$

$$d_{H-N}(B, B^c) = \sum_{i=1}^n \chi_i \left((\omega_1 + \omega_3 + \omega_5) |T_B(x_i) - F_B(x_i)| + \omega_4 |\max \{T_B(x_i), 0.5\} - \max \{0.5, F_B(x_i)\}| \right)$$

We obtain that $d_{H-N}(A, A^c) \geq d_{H-N}(B, B^c)$ and then $s_{H-N}(A, A^c) \leq s_{H-N}(B, B^c)$. The other case can be also proved by the same way.

(e4) Since (s3) in Proposition 2, we have $E(A) = E(A^c)$.

4. An application of the H-Max Single-Valued Neutrosophic Distance Measure to medical diagnosis

4.1. The proposed method named Two- d_{H-N} .

The considered problem: Consider a medical dataset with m records of m corresponding patients P_i , $i = 1, 2, \dots, m$, and with n attributes S_j , $j = 1, 2, \dots, n$, of a disease D with k disease classes labeled C_t , $t = 1, 2, \dots, k$, of D . Find the disease label (diagnosis) for each patient P_i .

The method Two- d_{H-N} :

- **Step 1.** For each attribute, choose three suitable fuzzification functions (based on experts) representing the truth membership function T , the indeterminacy-membership function I , and the falsity-membership function F .

From the original data and the fuzzification functions, built two single-valued neutrosophic matrices (the elements of the matrices are the single-valued neutrosophic values):

(M1): Matrix of the attributes on patients (P_i and S_j are the i^{th} row and the j^{th} column of (M1), respectively, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$).

(M2): Matrix of the classification results. (M2) is a $k \times n$ matrix (C_t is the t^{th} row of (M2), $t = 1, 2, \dots, k$).

- **Step 2.** Calculate the entropy of the attributes $E(S_j)$.

- **Step 3.** Calculate the similarity $s_{H-N}(P_i, C_t)$ between the attributes of each patient (P_i) with all of the disease classes C_t , $t=1,2,\dots,k$, where the weight of each attribute is corresponding to its entropy.
- **Step 4.** For the i^{th} patient, find the highest similarity value $\hat{s}_{H-N}(P_i, C_{t_0}) = s_{H-N}(P_i, C_{t_0})$, $t_0 \in [1, k]$. Conclude that the classification label of the i^{th} patient is t_0 .

The time complexity:

- The time complexity of Step 1: $O(m.n) + O(k.n)$
- The time complexity of Step 2: $O(n)$
- The time complexity of Step 3: $O(m.k)$
- The time complexity of Step 4: $O(m)$

Thus, the time complexity of the method Two- $\mathcal{D}_{H-N}$ is $O(m.n + k.n + n + m.k + m)$.

4.2. Numeric example

In this section, a numeric example for the method Two- $\mathcal{D}_{H-N}$ made on 5 patients, which are the 36th patient, the 57th patient, the 77th patient, the 97th patient, and the 525th patient, randomly selected of the ILPD (Indian Liver Patient Dataset) Data Set taken from UCI.

Table 2 shows information of the dataset, where there are 10 considered attributes: Age, Gender, Total Bilirubin (TB), Direct Bilirubin (DB), Alkaline Phosphatase (ALP), Alamine Aminotransferase (ALT), Aspartate Aminotransferase (AST), Total proteins (TP), Albumin (ALB), and Albumin and Globulin Ratio (A/G). There are 2 classification labels: 1 (liver patient) and 2 (non-liver patient).

Table 2. Records of the 36th patient, the 57th patient, the 77th patient, the 97th patient, and the 525th patient of the ILPD dataset

No	Age	Gender	TB	DB	ALP	ALT	AST	TP	ALB	A/G	Label
36	30	Male	1.3	0.4	482	102	80	6.9	3.3	0.9	1
57	33	Male	0.8	0.2	198	26	23	8	4	1	2
77	31	Male	0.9	0.2	518	189	17	5.3	2.3	0.7	1
97	39	Male	3.8	1.5	298	102	630	7.1	3.3	0.8	1
525	29	Male	0.8	0.2	156	12	15	6.8	3.7	1.1	2

The method Two- d'_{H-N} includes the following steps:

- **Step 1:** In this step, the data is reviewed and evaluated before the fuzzification process.

Table 3. Data preprocessing

	S_1 (TB) (normal: 0.8-1.2)	S_2 (DB) (normal: 0.2-0.4)	S_3 (ALP) (normal: 20-140)	S_4 (ALT) (normal: 29-33)	S_5 (AST) (normal: 23-33)	S_6 (ALB) (normal: 3.5-4.8)	S_7 [A/G= ALB/(TP- ALB)] (normal: 1-1.5)
P_1	1.3	0.4	482	102	80	3.3	0.9
P_2	0.8	0.2	198	26	23	4	1
P_3	0.9	0.2	518	189	17	2.3	0.7
P_4	3.8	1.5	298	102	630	3.3	0.8
P_5	0.8	0.2	156	12	15	3.7	1.1
	Max=32.6 Max(2)=5.3	Max=17.1 Max(2)=2.3	Max=2110 Max(2)=486	Max=2000 Max(2)=119	Max=2946 Max(2)=178	Min=1.7 Min(2)=2.2	Min=0.3 Min(2)=0.5

In Table 3:

- Select 7 required attributes out of the original 10 attributes.
- Given the normal threshold of each attribute (obtained from expert knowledge).
- Differentiate the correlation between attributes with disease classes. Specifically, the orange background indicates that if a patient has liver disease, this attribute will increase; the green background indicates that if a patient has liver disease, this attribute is reduced.
- The last row: Specify the maximum values (Max) and the minimum values (Min) of attributes; and the maximum value (Max(2)) and the minimum values (Min(2)) corresponding to label classification 2 (non-liver patient) from the ILPD dataset.

Fuzzification process: Two fuzzification functions are used, which are Right-function and Left-function.

$$R_{a,b}(x) = \begin{cases} 0 & : x \leq a \\ \frac{x-a}{b-a} & : a < x \leq b \\ 1 & : b < x \end{cases}, \quad L_{a',b'}(x) = \begin{cases} 1 & : x \leq a' \\ \frac{x-b'}{a'-b'} & : a' < x \leq b' \\ 0 & : b' < x \end{cases}$$

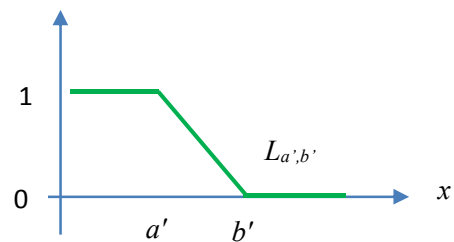
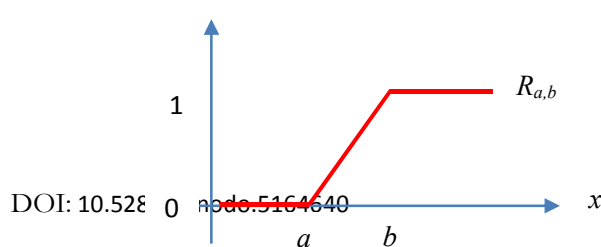


Figure 1. Graphs of the fuzzification functions

The attributes on patients:

$$S_1 = \langle T_1(x), I_1(x), F_1(x) \rangle = \langle R_{1,2,5,3}(x), L_{0,2,3}(x), L_{0,6,4}(x) \rangle$$

$$S_2 = \langle T_2(x), I_2(x), F_2(x) \rangle = \langle R_{0,4,2,3}(x), L_{0,1,1}(x), L_{0,15,1,5}(x) \rangle$$

$$S_3 = \langle T_3(x), I_3(x), F_3(x) \rangle = \langle R_{140,486}(x), L_{80,250}(x), L_{100,400}(x) \rangle$$

$$S_4 = \langle T_4(x), I_4(x), F_4(x) \rangle = \langle R_{33,119}(x), L_{5,60}(x), L_{30,100}(x) \rangle$$

$$S_5 = \langle T_5(x), I_5(x), F_5(x) \rangle = \langle R_{33,178}(x), L_{10,100}(x), L_{23,150}(x) \rangle$$

$$S_6 = \langle T_6(x), I_6(x), F_6(x) \rangle = \langle L_{2,2,3,5}(x), R_{2,4}(x), R_{3,5}(x) \rangle$$

$$S_7 = \langle T_7(x), I_7(x), F_7(x) \rangle = \langle L_{0,5,1}(x), R_{0,3,1,5}(x), R_{0,8,2,5}(x) \rangle$$

Two single-valued neutrosophic matrices (M1) and (M2): (see Tables 4 and 5)

Table 4. Matrix of the attributes on patients

(M1)	S_1	S_2	S_3	S_4	S_5	S_6	S_7
P_1	$\langle 0.02, 0.6, 0.7 \rangle$	$\langle 0, 0.6, 0.8 \rangle$	$\langle 0.9, 0, 0 \rangle$	$\langle 0.8, 0, 0 \rangle$	$\langle 0.3, 0.2, 0.5 \rangle$	$\langle 0.1, 0.6, 0.1 \rangle$	$\langle 0.2, 0.5, 0.08 \rangle$
P_2	$\langle 0, 0.7, 0.9 \rangle$	$\langle 0, 0.8, 0.9 \rangle$	$\langle 0.1, 0.3, 0.6 \rangle$	$\langle 0, 0.6, 1 \rangle$	$\langle 0, 0.8, 1 \rangle$	$\langle 0, 1, 0.5 \rangle$	$\langle 0, 0.5, 0.1 \rangle$
P_3	$\langle 0, 0.7, 0.9 \rangle$	$\langle 0, 0.8, 0.9 \rangle$	$\langle 1, 0, 0 \rangle$	$\langle 1, 0, 0 \rangle$	$\langle 0, 0.9, 1 \rangle$	$\langle 0.9, 0.1, 0 \rangle$	$\langle 0.6, 0.3, 0 \rangle$
P_4	$\langle 0.6, 0, 0.05 \rangle$	$\langle 0.5, 0, 0 \rangle$	$\langle 0.4, 0, 0.3 \rangle$	$\langle 0.8, 0, 0 \rangle$	$\langle 1, 0, 0 \rangle$	$\langle 0.1, 0.6, 0.1 \rangle$	$\langle 0.4, 0.4, 0 \rangle$
P_5	$\langle 0, 0.7, 0.9 \rangle$	$\langle 0, 0.8, 0.9 \rangle$	$\langle 0.04, 0.5, 0.8 \rangle$	$\langle 0, 0.8, 1 \rangle$	$\langle 0, 0.9, 1 \rangle$	$\langle 0, 0.8, 0.3 \rangle$	$\langle 0, 0.6, 0.2 \rangle$

Table 5. Matrix of the classification results

(M2)	$\hat{S}_1$	$\hat{S}_2$	$\hat{S}_3$	$\hat{S}_4$	$\hat{S}_5$	$\hat{S}_6$	$\hat{S}_7$
La := 1	$\langle 1, 0, 0 \rangle$	$\langle 1, 0, 0 \rangle$	$\langle 1, 0, 0 \rangle$	$\langle 1, 0, 0 \rangle$	$\langle 1, 0, 0 \rangle$	$\langle 1, 0, 0 \rangle$	$\langle 1, 0, 0 \rangle$
La := 2	$\langle 0, 1, 1 \rangle$	$\langle 0, 1, 1 \rangle$	$\langle 0, 1, 1 \rangle$	$\langle 0, 1, 1 \rangle$	$\langle 0, 1, 1 \rangle$	$\langle 0, 1, 1 \rangle$	$\langle 0, 1, 1 \rangle$

where $\hat{S}_j$, $j = 1, 2, 3, 4, 5$, are the attribute characteristics of the disease classes.

- **Step 2:** Calculating the entropy of the attributes:

$$E(S_1) = s_{H-N}(S_1, S_1^c) = 0.31; \quad E(S_2) = s_{H-N}(S_2, S_2^c) = 0.26;$$

$$E(S_3) = s_{H-N}(S_3, S_3^c) = 0.35; \quad E(S_4) = s_{H-N}(S_4, S_4^c) = 0.12;$$

$$E(S_5) = s_{H-N}(S_5, S_5^c) = 0.18; \quad E(S_6) = s_{H-N}(S_6, S_6^c) = 0.57$$

$$E(S_7) = s_{H-N}(S_7, S_7^c) = 0.76$$

where $\chi_i = \frac{1}{5}, i = 1, 2, 3, 4, 5$ and $\omega_j = \frac{1}{5}, j = 1, 2, 3, 4, 5$.

- **Step 3:** Calculating the similarity $s_{H-N}(P_i, (La := 1))$ and $s_{H-N}(P_i, (La := 2))$, $i = 1, 2, 3, 4, 5$,

where $\omega_j = \frac{1}{5}, j = 1, 2, 3, 4, 5$ and $\chi_i = \frac{E(S_i)}{\sum_{i=1}^5 E(S_i)}$, i.e.,

$$\chi_1 = 0.12, \chi_2 = 0.1, \chi_3 = 0.14, \chi_4 = 0.05, \chi_5 = 0.07, \chi_6 = 0.22, \chi_7 = 0.3.$$

$$s_{H-N}(P_1, (La := 1)) = 0.43, \quad s_{H-N}(P_1, (La := 2)) = 0.415,$$

$$s_{H-N}(P_2, (La := 1)) = 0.17, \quad s_{H-N}(P_2, (La := 2)) = 0.59,$$

$$s_{H-N}(P_3, (La := 1)) = 0.62, \quad s_{H-N}(P_3, (La := 2)) = 0.33,$$

$$s_{H-N}(P_4, (La := 1)) = 0.59, \quad s_{H-N}(P_4, (La := 2)) = 0.266,$$

$$s_{H-N}(P_5, (La := 1)) = 0.15, \quad s_{H-N}(P_5, (La := 2)) = 0.7,$$

- **Step 4.** Making decisions:

- Since $s_{H-N}(P_1, (La := 1)) > s_{H-N}(P_1, (La := 2))$, we obtain that the classification label of 1st patient is 1. That means P_1 is a liver patient.
- Since $s_{H-N}(P_2, (La := 1)) < s_{H-N}(P_2, (La := 2))$, we obtain that the classification label of 2nd patient is 2. That means P_2 is a non-liver patient.
- Since $s_{H-N}(P_3, (La := 1)) > s_{H-N}(P_3, (La := 2))$, we obtain that the classification label of 3th patient is 1. That means P_3 is a liver patient.

- Since $s_{H-N}(P_4, (La := 1)) > s_{H-N}(P_4, (La := 2))$, we obtain that the classification label of 4th patient is 1. That means P_4 is a liver patient.
- Since $s_{H-N}(P_5, (La := 1)) < s_{H-N}(P_5, (La := 2))$, we obtain that the classification label of 5th patient is 2. That means P_2 is a non-liver patient.

These conclusions and the classification results of Table 2 are the same. Also on the dataset ILPD, Ngan et al. [19] tested 14 diagnostic methods based on the considered intuitionistic fuzzy distance measures. The best mean absolute error obtained from their experiment is 0.28.

5. Conclusions

In this paper, based on the H-max distance measure on IFSs, a new distance measure on SVNss is proposed. Further, a single-valued neutrosophic similarity measure, a single-valued neutrosophic entropy measure, and their basic properties are presented and proven. In addition, an application to medical diagnosis is shown to illustrate the effective applicability of the proposals. There, the proposed diagnostic method named Two- d'_{H-N} and a numerical example are clarified in detail. The diagnostic results according to the proposed method and the actual results are the same. Furthermore, the diagnostic method is relatively simple to perform. In the future, we will test the proposed diagnostic method on real datasets and compare the performance with related methods. Furthermore, we will develop the distance measure for neutrosophic sets.

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