GENERALIZED POSSIBILITY NEUTROSOPHIC SOFT SET AND ITS APPLICATION

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Abstract

Today, experts are emphasizing to establish innovative ideas to cope with the complexity, imprecision and ambiguity that exist in practical problems, together with suitable examples to elucidate their hypotheses. Neutrosophic set and its hybridisations are broadly adopted in many decision making challenges. More researchers are working to discuss the validity of neutrosophy and its combinations in decision making issues. In this work we develop a new hybridized structure of neutrosophic soft set named Generalized Possibility Neutrosophic Soft Set (GPNSS) and discuss its basic properties. We define set theoretical operations between two possibility neutrosophic soft sets and study some of their features. We also present the GPNS decision making approach, which is based on the AND-product of GPNSS. Finally, we provide a numerical example to demonstrate how the technique may be effectively applied to the circumstances investigated.

Keywords: Neutrosophic set, soft set, generalised possibility neutrosophic soft set, decision making.

1. Introduction

Lotfi L. A. Zadeh [12] introduced fuzzy set as an extension of crisp set or non-fuzzy set in 1965. A fuzzy set is a class of objects with a continuum of grade of membership and such a set is described by a membership ranging between zero and one. The intuitionistic fuzzy set are sets whose elements have degrees of membership and non-membership. Intuitionistic fuzzy sets was imported by Krassimir Atanassov [[9], [10]] as an extension of fuzzy set, which itself develops the classical notation of a set. The intuitionistic fuzzy sets can only operate the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. It does not operate the indeterminate and incompatible information which remain in belief system.

Smarandache [[4], [5], [6]] introduced the approach of neutrosophic set which is a mathematical tool for handling problems contains imprecise, indeterminate and incompatible data. The neutrosophic factors T, I, F which represents the membership, indeterminacy, and non-membership values respectively, where $]^{-}0, 1^{+}[$ is the non-standard unit

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interval, and thus defines the neutrosophic set. Nowadays the theory of neutrosophy is combained with existing theories and widely applied in many decision problems [2, [13], 25, [26].

In 1999, Molodtsov [[3]] introduced the notion of soft set theory for dealing with complicated problems and different types of uncertainties and the approach has been applied in diverse practical problems . Several researchers have incorporated various mathematical hybrid structures by generalizing and expanding classical soft set theory [[11], [14], [19]]. Maji combined the notion of soft sets and neutrosophic set together by introducing a new notion called neutrosophic soft set [[17]] and gave application of neutrosophic soft set in decision making problem [17]. Broumi studied generalized neutrosophic soft sets [7, 8, 20, 22] and intuitionistic neutrosophic soft set [[20], [21]] and discussed their basic definitions and operations and implemented its applications in decision making problems. The properties and applications on the neutrosophic soft set and its extensions are studied increasingly [1, 7]. Since then, many scholars did researches for MCDM problems with neutrosophic information and its combinations [15, 18, 23, 24, 26].

Motivated by the above said considerations, this study is organized as follows:

- A new notion of Generalized Possibility Neutrosophic Soft Set (GPNSS) is introduced considering the fact that each element of the universal set is represented with a possibility degree of belongingness and the parameters are im[lemented with a weight.
- Various algebraic properties such as generalized possibility neutrosophic soft subset, generalized neutrosophic soft null set, and generalized neutrosophic soft universal set are defined.
- By employing the principles of n-norm and n-conorm, the set theoretical operations of generalized possible neutrosophic soft sets such as union, intersection, and complement are discussed and specific algebraic properties of these operations are analysed
- Further, a new decision making method using AND-product of generalized possibility neutrosophic soft sets called Generalized Possibility Neutrosophic Soft Decision Making Method (GPNSDM) is proposed. Finally a numerical example that shows the applicability of proposed technique is presented.

1. Preliminaries

DEFINITION 1.1

If X is a collection of objects denoted generically by x, then a fuzzy set Ã in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

$\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in Ã that maps X to the membership space M (When M contains only the two points 0 and 1, Ã is non fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a nonfuzzy set). The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

DEFINITION 1.2

An intuitionistic fuzzy set A in U is given by

$A = \{\langle u, \mu_A(u), \vartheta_A(u)\rangle \mid u \in U\}$

Where $\mu_A : U \rightarrow [0,1]$, $\vartheta_A : U \rightarrow [0,1]$

And $0 \leq \mu_A(u) + \vartheta_A(u) \leq 1 \; \forall u \in U$

For each u, the numbers $\mu_A(u)$ and $\vartheta_A(u)$ are the degree of membership and degree of non membership of u to A, respectively.

DEFINITION 1.3

A neutrosophic set A on the universe of discourse X is defined as A = $\{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}$, where T, I, F: X → $]-0, 1+[$ and $-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3+$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]-0, 1+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]-0, 1+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0,1].

DEFINITION 1.4

A neutrosophic set A is contained in another neutrosophic set B i.e. A ⊆ B if $\forall x \in X$, $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

DEFINITION 1.6

Let U be an initial universe set and E be a set of parameters or attributes with respect to U. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U).

DEFINITION 1.7

Let U be an initial universe set and E be a set of parameters. Consider A ⊂ E. Let P(U) denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F: A → P(U).

DEFINITION 1.8

The class of all value sets of a neutrosophic soft set (F, E) is called value-class of the neutrosophic soft set and is denoted by $C_{(F,E)}$. For the above Example, $C_{(F,E)} = \{v_1, v_2, \cdots v_{10}\}$. Clearly, $C_{(F,E)} \subset P(U)$.

DEFINITION 1.9

Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. (F, A) is said to be neutrosophic soft subset of (G, B) if A ⊂ B and $T_{F(e)}(x) \leq T_{G(e)}(x)$, $I_{F(e)}(x) \leq I_{G(e)}(x)$, $F_{F(e)}(x) \geq F_{G(e)}(x)$, $\forall e \in A, x \in U$. We denote it by (F, A) ⊆ (G, B). (F, A) is said to be neutrosophic soft super set of (G, B) if (G, B)is a neutrosophic soft subset of (F, A). We denote it by (F, A) ⊇ (G, B).

DEFINITION 1.10

Equality of two neutrosophic soft sets. Two NSSs (F, A) and (G, B) over the common universe U are said to be equal if (F, A) is neutrosophic soft subset of (G, B) and (G, B) is neutrosophic soft subset of (F, A). We denote it by (F, A) = (G, B).

DEFINITION 1.11

NOT set of a set of parameters. Let E =$\{e_1, e_2, \cdots e_n\}$ be a set of parameters. The NOT set of E is denoted by $\rceil E$ is defined by ⌉ E = { $\rceil e_1, \rceil e_2, \cdots \rceil e_n$ } where $\rceil e_i$ = not e_i, $\forall i$ (it may be noted that ⌉ and ⌉ are different operators).

DEFINITION 1.12

Complement of a neutrosophic soft set. The complement of a neutrosophic soft set (F, A) denoted by $(F, A)^c$ and is defined as $(F, A)^c$ = (F^c, ⌉A); where F^c : ⌉A → P(U) is a mapping given by $F^c(\alpha)$ = neutrosophic soft complement with $T_{F^c(x)} = F_{F(x)}$, $I_{F^c(x)} = I_{F(x)}$ and $F_{F^c(x)} = T_{F(x)}$.

DEFINITION 1.13

Empty or Null neutrosophic soft set with respect to a parameter. A neutrosophic soft set (H, A) over the universe U is termed to be empty or null neutrosophic soft set with respect to the parameter A if $T_{H(e)}(m) = 0$, $F_{H(e)}(m) = 0$ and $I_{H(e)}(m) = 0$, $\forall m \in U$, $\forall e \in A$ In this case the null neutrosophic soft set (NNSS) is denoted by Φ_A.

2. GENERALIZED POSSIBILITY NEUTROSOPHIC SOFT SETS

In this section, we introduce a new type of set called Generalized Possibility Neutrosophic Soft Set (GPNSS) and study a Generalized Possibility Neutrosophic Soft Decision Making Method (GPNSDMM).

DEFINITION 2.1

Let U be an initial universe, E be a parameter set, N(U) be the collection of all neutrosophic sets of U and F(U) is collection of all fuzzy subset of U. A generalized possibility neutrosophic soft set (GPNS-set) $P_{[\alpha,\beta]}$ over U is a set of triple defined by

$$P_{[\alpha,\beta]} = \left\{ \left(e_k, \left\{ \left(\frac{u_j}{P(e_k)(u_j)}, \alpha(e_k)(u_j) \right) \right\}, \beta(P(e_k)) \right) : e_k \in E \right\}$$

or a mapping defined by $P_{[\alpha,\beta]} : E \rightarrow$ N(U) × F(U) × N(U), where, i ∈ $\wedge_1$ and k ∈ $\wedge_2$, P is a mapping given by P : E → N(U) and $\alpha(e_k)$ is a fuzzy set such that α : E → F(U).

For each parameter

$$e_k \in \text{E, P}(e_k) = \{\langle u_j, T_{P(e_k)}(u_j), I_{P(e_k)}(u_j), F_{P(e_k)}(u_j) \rangle : u_j \in U\}$$

indicates neutrosophic value set of parameter e_k and where T, I, F are the truth, indeterminacy and falsity values respectively of the element $u_i \in$ U. For each $u_i \in$ U and $e_k \in$ E, $0 \leq T_{P(e_k)}(u_j) + I_{P(e_k)}(u_j) + F_{P(e_k)}(u_j) \leq 3$. Also $\alpha\,(e_k)$, degrees of priority given for the belongingness of elements of U in P(e_k) and $\beta(e_k)$ the possibility degree and weightage given to the parameters by the experts. So we can write

$$P_{[\alpha,\beta]}\,(e_k) = \left\{\left(\frac{u_1}{P(e_k)(u_1)}\,,\alpha(e_k)(u_1)\right)\left(\frac{u_2}{P(e_k)(u_2)}\,,\alpha(e_k)(u_2)\right),\ldots,\left(\frac{u_n}{P(e_k)(u_n)}\,,\alpha(e_k)(u_n)\right),\beta(e_k)\right\}.$$

EXAMPLE 2.2

Let U = $\{u_1, u_2, u_3\}$ be a set of three restaurants. Let E =$\{e_1, e_2, e_3\}$ be a set of qualities where e_1=Taste, e_2 = Variety, e_3 =Service and let α : E →F(U) and β : E →N(U). We can define a function $P_{[\alpha,\beta]} : E \rightarrow$ N(U) × F(U) × N(U) as follow:

$$P_{[\alpha,\beta]} = \begin{cases} P_{[\alpha,\beta]}(e_1) = \left\{\left[\left(\frac{u_1}{(0.4,0.1,0.5)},0.7\right),\left(\frac{u_2}{(0.6,0.2,0.4)},0.3\right),\left(\frac{u_3}{(0.3,0.4,0.7)},0.6\right)\right],(0.3,0.5,0.7)\right\} \\ P_{[\alpha,\beta]}(e_2) = \left\{\left[\left(\frac{u_1}{(0.7,0.3,0.4)},0.5\right),\left(\frac{u_2}{(0.4,0.6,0.1)},0.7\right),\left(\frac{u_3}{(0.6,0.2,0.8)},0.3\right)\right],(0.6,0.8,0.2)\right\} \\ P_{[\alpha,\beta]}(e_3) = \left\{\left[\left(\frac{u_1}{(0.5,0.6,0.7)},0.2\right),\left(\frac{u_2}{(0.4,0.2,0.6)},0.5\right),\left(\frac{u_3}{(0.5,0.4,0.3)},0.4\right)\right],(0.5,0.3,0.8)\right\} \end{cases}$$

For the purpose of storing a GPNS in a computer, we can use matrix notation of generalized possibility neutrosophic soft set $P_{[\alpha,\beta]}$. For Example, matrix notation of GPNS $P_{[\alpha,\beta]}$ can be written as follows: for m, n $\in \wedge$,

$$P_{[\alpha,\beta]} = \begin{pmatrix} (\langle 0.4, 0.1, 0.5\rangle, 0.7)(\langle 0.6, 0.2, 0.4\rangle, 0.3)(\langle 0.3, 0.4, 0.7\rangle, 0.6)(0.3,0.5,0.7) \\ (\langle 0.7, 0.3, 0.4\rangle, 0.5)(\langle 0.4, 0.6, 0.1\rangle, 0.7)(\langle 0.6, 0.2, 0.8\rangle, 0.3)(0.6,0.8,0.2) \\ (\langle 0.5, 0.5, 0.4\rangle, 0.1)(\langle 0.4, 0.2, 0.6\rangle, 0.5)(\langle 0.5, 0.4, 0.3\rangle, 0.4)(0.5,0.3,0.8) \end{pmatrix}$$

where the m-th row vector shows P(e_m) and n-th column vector shows u_n

DEFINITION 2.3

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]}$ $\in$ GPN (U, E). Then, $P_{[\alpha,\beta]}$ is said to be a generalized possibility neutrosophic soft subset (GPNS-subset) of $Q_{[\gamma,\delta]}$, and denoted by $P_{[\alpha,\beta]} \subseteq Q_{[\gamma,\delta]}$ if

1. α(e) and β(e) are a fuzzy subset of γ(e) and δ(e), for all e ∈ E
2. P is a neutrosophic subset of Q.

EXAMPLE 2.4

Let U = $\{u_1, u_2, u_3\}$ be a set of three broadband service, and let E =$\{e_1, e_2, e_3\}$ be a set of qualities where e_1= Stability, e_2= Security, e_3 = price. Let $P_{[\alpha,\beta]}$ be a GPNS-set define as follows:

$$P_{[\alpha,\beta]} = \begin{cases} P_{[\alpha,\beta]}(e_1) = \left\{\left[\left(\frac{u_1}{(0.4,0.7,0.6)},0.8\right),\left(\frac{u_2}{(0.8,0.6,0.8)},0.3\right),\left(\frac{u_3}{(0.4,0.5,0.8)},0.7\right)\right],(0.3,0.5,0.7)\right\} \\ P_{[\alpha,\beta]}(e_2) = \left\{\left[\left(\frac{u_1}{(0.7,0.4,0.6)},0.6\right),\left(\frac{u_2}{(0.5,0.7,0.5)},0.7\right),\left(\frac{u_3}{(0.7,0.4,0.8)},0.5\right)\right],(0.6,0.8,0.2)\right\} \\ P_{[\alpha,\beta]}(e_3) = \left\{\left[\left(\frac{u_1}{(0.5,0.6,0.7)},0.2\right),\left(\frac{u_2}{(0.6,0.5,0.8)},0.5\right),\left(\frac{u_3}{(0.6,0.5,0.7)},0.4\right)\right],(0.5,0.3,0.8)\right\} \end{cases}$$

Also we can define a function $Q_{[\gamma,\delta]}$: E → N(U) × F(U)× N(U) as follows:

$$Q_{[\gamma,\delta]} \begin{cases} Q_{[\gamma,\delta]}(e_1) = \left\{\left[\left(\frac{u_1}{(0.4,0.7,0.6)},0.8\right),\left(\frac{u_2}{(0.8,0.6,0.8)},0.3\right),\left(\frac{u_3}{(0.4,0.5,0.8)},0.7\right)\right],(0.3,0.5,0.7)\right\} \\ Q_{[\gamma,\delta]}(e_2) = \left\{\left[\left(\frac{u_1}{(0.8,0.6,0.5)},0.7\right),\left(\frac{u_2}{(0.6,0.8,0.3)},0.8\right),\left(\frac{u_3}{(0.8,0.5,0.6)},0.5\right)\right],(0.6,0.8,0.2)\right\} \\ Q_{[\gamma,\delta]}(e_3) = \left\{\left[\left(\frac{u_1}{(0.6,0.7,0.5)},0.4\right),\left(\frac{u_2}{(0.7,0.6,0.7)},0.7\right),\left(\frac{u_3}{(0.8,0.6,0.4)},0.6\right)\right],(0.5,0.3,0.8)\right\} \end{cases}$$

It is clear that $P_{[\alpha,\beta]}$ is GPNS –subset of $Q_{[\gamma,\delta]}$.

DEFINITION 2.5

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]}$ ∈ GPN (U, E). Then, $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ are called generalized possibility neutrosophic soft equal set and denote by $P_{[\alpha,\beta]} = Q_{[\gamma,\delta]}$ if $P_{[\alpha,\beta]} \subseteq Q_{[\gamma,\delta]}$ and $P_{[\alpha,\beta]} \supseteq Q_{[\gamma,\delta]}$.

DEFINITION 2.6

Let $P_{[\alpha,\beta]}$ ∈ GPN (U, E). Then, $P_{[\alpha,\beta]}$ is said to be generalized possibility neutrosophic soft null set, denoted by $\phi_{[\alpha,\beta]}$, if $\forall e \in$, $\phi_{[\alpha,\beta]} : E \to$ N(U) × F(U) × N(U) such that $\phi_{[\alpha,\beta]}(e) = \left\{\left[\frac{u}{\phi(e)(u)}, \alpha(e)(u) : u \in U\right] \beta(P(e))\right\}$ where ϕ(e) ={⟨u , 0 , 0 , 1⟩: $u \in U$} and $\alpha(e) = \{(u, 0): u \in U\}$ and $\beta(e) = \{(u, 0,0,1): u \in U\}$.

DEFINITION 2.7

Let $P_{[\alpha,\beta]}$ ∈ GPN(U,E). Then, $P_{[\alpha,\beta]}$ is said to be generalized possibility neutrosophic soft universal set, denoted by $U_{[\alpha,\beta]}$, if $\forall\ e \in E$, $U_{[\alpha,\beta]} : E \to$ N(U) × F(U) × N(U) such that $U_{[\alpha,\beta]}(e) = \left\{\left[\frac{u}{U(e)(u)}, \alpha(e)(u) : u \in U\right], \beta(e)\right\}$ where U(e) = {⟨u , 1 , 1 , 0⟩: $u \in U$} and $\alpha(e) = \{(u, 1): u \in U\}$ and $\beta(e) = \{(u, 1,1,0): u \in U\}$.

PROPOSITION 2.8

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]}$ and $H_{[\theta,\phi]}$ ∈ GPN (U, E). Then,

1. $\phi_{[\alpha,\beta]} \subseteq P_{[\alpha,\beta]}$
2. $P_{[\alpha,\beta]} \subseteq U_{[\alpha,\beta]}$
3. $P_{[\alpha,\beta]} \subseteq \phi_{[\alpha,\beta]}$ and $\phi_{[\alpha,\beta]} \subseteq H_{[\theta,\phi]}$ implies $P_{[\alpha,\beta]} \subseteq H_{[\theta,\phi]}$

***Proof*.** The proof follows from the Definitions (2.5) - (2.7)

DEFINITION 2.9

Let $P_{[\alpha,\beta]} \in$ GPN(U,E), where

$P_{[\alpha,\beta]}(e_k) = \{([P(e_k)(u_i), \alpha(e_k)(u_i)], \beta(\mathrm{P}(e_k))): e_k \in E, u_i \in U\}$

$P(e_k) = \{\langle u, T_{P(e_k)}(u_i), I_{P(e_k)}(u_i), F_{P(e_k)}(u_i)\rangle\}\ \forall e_k \in E, u \in U.$

Thenfore $e_k \in E$ and $u_i \in U$, and

1. $P^T_{[\alpha,\beta]}$ is said to be truth- membership part of $P_{[\alpha,\beta]}$

   $P^T_{[\alpha,\beta]} = \{(P^T_{kj}(e_k), \alpha_{kj}(e_k))\}$ and $P^T_{kj}(e_k) = \{(u_j, T_{P(e_k)}(u_j))\}$, $\alpha_{kj}(e_k) = \{(u_j, \alpha(e_k)(u_j))\}$

2. $P^I_{[\alpha,\beta]}$ is said to be indeterminacy - membership part of $P_{[\alpha,\beta]}$

   $P^I_{[\alpha,\beta]} = \{(P^I_{kj}(e_k), \alpha_{kj}(e_k))\}$ and $P^I_{kj}(e_k) = \{(u_j, I_{P(e_k)}(u_j))\}$, $\alpha_{kj}(e_k) = \{(u_j, \alpha(e_k)(u_j))\}$

3. $P^F_{[\alpha,\beta]}$ is said to be falsity- membership part of $P_{[\alpha,\beta]}$

   $P^F_{[\alpha,\beta]} = \{(P^F_{kj}(e_k), \alpha_{kj}(e_k))\}$ and $P^F_{kj}(e_k) = \{(u_j, F_{P(e_k)}(u_j))\}$, $\alpha_{kj}(e_k) = \{(u_j, \alpha(e_k)(u_j))\}$

We can write a GPNS in form $P_{[\alpha,\beta]} = \left(P^T_{[\alpha,\beta]}, P^I_{[\alpha,\beta]}, P^F_{[\alpha,\beta]}\right)$.

A GPNS can be expressed in matrix form.

Let us consider generalized possibility neutrosophic soft set $P_{[\alpha,\beta]}$ given in Example 2.4. Then generalized possibility neutrosophic soft set $P_{[\alpha,\beta]}$ can be expressed in

matrix form as follows :

$$P^T_{[\alpha,\beta]} = \begin{pmatrix} (0.4, 0.8)\ (0.8, 0.3)(0.4, 0.7)(0.3) \\ (0.7, 0.6)(0.5, 0.7)(0.7, 0.5)(0.6) \\ (0.5, 0.2)(0.6, 0.5)(0.6, 0.4)(0.5) \end{pmatrix}$$

$$P^I_{[\alpha,\beta]} = \begin{pmatrix} (0.7, 0.8)(0.6, 0.3)(0.5, 0.7)(0.5) \\ (0.4, 0.6)(0.7, 0.7)(0.4, 0.5)(0.8) \\ (0.6, 0.2)(0.5, 0.5)(0.5, 0.4)(0.3) \end{pmatrix}$$

$$P^F_{[\alpha,\beta]} = \begin{pmatrix} (0.6, 0.8)(0.8, 0.3)(0.8, 0.7)(0.7) \\ (0.6, 0.6)(0.5, 0.7)(0.8, 0.5)(0.2) \\ (0.7, 0.2)(0.8, 0.5)(0.7, 0.4)(0.8) \end{pmatrix}$$

DEFINITION 2.10

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]} \in$ GPN (U, E). The union of two GPNSs $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ over U, denote by $P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}$ is defined by

$$P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]} = \left\{ \left(e_k, \left\{ \left(\rho, \alpha_{kj}(e_k) \oplus \gamma_{kj}(e_k) \right) \right\}, \beta_{kj}(P(e_k)) \oplus \delta_{kj}(e_k) \right) : e_k \in E \right\}$$

Where

$$\rho = \frac{u_j}{\left(P^T_{kj}(e_k) \oplus Q^T_{kj}(e_k), P^I_{kj}(e_k) \oplus Q^I_{kj}(e_k), P^F_{kj}(e_k) \otimes Q^F_{kj}(e_k) \right)}$$

and $\oplus$ represents n-conorm and $\otimes$ represents n-norm functions respectively.

DEFINITION 2.11

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]} \in$ GPN (U, E). The intersection of two GPNSs

$P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ over U , denote by $P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}$ is defined by

$$P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]} = \left\{ \left(e_k, \left\{ \left(\theta, \alpha_{kj}(e_k) \otimes \gamma_{kj}(e_k) \right) \right\}, \beta_{kj}(P(e_k)) \otimes \delta_{kj}(e_k) \right) : e_k \in E \right\}$$

Where

$$\theta = \frac{u_j}{\left(P^T_{kj}(e_k) \otimes Q^T_{kj}(e_k),\ P^I_{kj}(e_k) \otimes Q^I_{kj}(e_k),\ P^F_{kj}(e_k) \oplus Q^F_{kj}(e_k) \right)}$$

and $\oplus$ represents n-conorm and $\otimes$ represents n-norm functions respectively.

EXAMPLE 2.12

Let us consider the GPNSs $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ considered in Example 2.4. Let us suppose that n-norm is defined by a $\oplus$ b = $min\{a, b\}$ and the n-conorm is defined by a$\otimes$b = $max\{a, b\}$ f(0.3,0.5,0.7)or a, b ∈ [0, 1]

$P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]} =$

$$\begin{cases} \left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)(e_1) = \left\{ \left[\left(\frac{u_1}{(0.4,0.7,0.6)}, 0.8 \right), \left(\frac{u_2}{(0.8,0.6,0.8)}, 0.3 \right), \left(\frac{u_3}{(0.4,0.5,0.8)}, 0.7 \right) \right], (0.3,0.5,0.7) \right\} \\ \left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)(e_2) = \left\{ \left[\left(\frac{u_1}{(0.7,0.4,0.5)}, 0.6 \right), \left(\frac{u_2}{(0.5,0.7,0.3)}, 0.7 \right), \left(\frac{u_3}{(0.8,0.5,0.6)}, 0.5 \right) \right], (0.6,0.8,0.2) \right\} \\ \left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)(e_3) = \left\{ \left[\left(\frac{u_1}{(0.6,0.7,0.5)}, 0.4 \right), \left(\frac{u_2}{(0.7,0.6,0.7)}, 0.7 \right), \left(\frac{u_3}{(0.8,0.6,0.4)}, 0.6 \right) \right], (0.5,0.3,0.8) \right\} \end{cases}$$

And

$P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]} =$

$$\begin{cases} \left(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}\right)(e_1) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.4,0.7,0.6})}, 0.8\right), \left(\frac{u_2}{(\mathbf{0.8,0.6,0.8})}, 0.3\right), \left(\frac{u_3}{(\mathbf{0.4,0.5,0.8})}, 0.7\right)\right], (0.3,0.5,0.7)\right\} \\ \left(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}\right)(e_2) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.8,0.6,0.5})}, 0.7\right), \left(\frac{u_2}{(\mathbf{0.6,0.8,0.3})}, 0.8\right), \left(\frac{u_3}{(\mathbf{0.8,0.5,0.6})}, 0.5\right)\right], (0.6,0.8,0.2)\right\} \\ \left(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}\right)(e_3) = \left\{\left[\left(\frac{u_1}{(\mathbf{0.6,0.7,0.5})}, 0.4\right), \left(\frac{u_2}{(\mathbf{0.7,0.6,0.7})}, 0.7\right), \left(\frac{u_3}{(\mathbf{0.8,0.6,0.4})}, 0.6\right)\right], (0.5,0.3,0.8)\right\} \end{cases}$$

PROPOSITION 2.13

Let $P_{[\alpha,\beta]}$, $Q_{[\gamma,\delta]}$ and $H_{[\theta,\phi]} \in$ GPN (U, E). Then,

1. $P_{[\alpha,\beta]} \cap \phi_\alpha = \phi_\alpha$ and $P_{[\alpha,\beta]} \cap U_\alpha = P_{[\alpha,\beta]}$.
2. $P_{[\alpha,\beta]} \cup \phi = P_{[\alpha,\beta]}$ and $P_{[\alpha,\beta]} \cup U_\alpha = U_\alpha$.
3. $P_{[\alpha,\beta]} \cap (Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}) = (P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}$ and
4. $P_{[\alpha,\beta]} \cup (Q_{[\gamma,\delta]} \cup H_{[\theta,\phi]}) = (P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}) \cap H_{[\theta,\phi]}$.
5. $P_{[\alpha,\beta]} \cap (Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}) = (P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}) \cup (P_{[\alpha,\beta]} \cap H_{[\theta,\phi]})$ and
6. $P_{[\alpha,\beta]} \cup (Q_{[\gamma,\delta]} \cap H_{[\theta,\phi]}) = (P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}) \cap (P_{[\alpha,\beta]} \cup H_{[\theta,\phi]})$.

$proof.$ The proof can be obtained from Definitions 2.10 and 2.11

DEFINITION 2.14

Let $P_{[\alpha,\beta]} \in$ GPN (U, E). Complement of GPNS $P_{[\alpha,\beta]}$, denoted by $P^c_{[\alpha,\beta]}$, is defined by

$$P^c_{[\alpha,\beta]} = \left\{\left(e, \left\{\left(\frac{u_j}{\sim(P(e_k))}, \sim\left(\alpha_{kj}(e_k)(u_j)\right)\right), \sim(\beta_{kj} P(e_k))\right\}\right), e \in E\right\}$$

Where, $\left(\sim(P_{kj})(e_k)\right) = \left(\sim(P^T_{kj}(e_k)), \sim(P^I_{kj}(e_k)), \sim(P^F_{kj}(e_k))\right), \forall\ k \in \wedge_1, j \in \wedge_2$.

EXAMPLE 2.15

Let us consider the GPNS $P_{[\alpha,\beta]}$ taken in Example 2.4. Suppose that the negation is defined by $\left(\sim P^T_{kj}(e_k)\right) = P^F_{kj}(e_k)$, $\sim(P^F_{kj}(e_k)) = P^T_{kj}(e_k)$, $\sim (P^I_{kj}(e_k)) = 1-P^I_{kj}(e_k)$ and $\sim\left(\alpha_{ij}(e_k)\right) = 1- \alpha_{kj}(e_k)$ and $\sim(\beta^T_{ij}(e_k)) = \beta^F_{ij}(e_k)$ $\sim(\beta^F_{ij}(e_k)) = \beta^T_{ij}(e_k)$, $\sim(\beta^I_{ij}(e_k)) = 1-\beta^I_{ij}(e_k)$ respectively.

Then, $\sim P_{[\alpha,\beta]}$ is defined as follow:

$\sim P_{[\alpha,\beta]}$

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$$= \begin{cases} \sim P_{[\alpha,\beta]}(e_1) = \left\{\left[\left(\frac{u_1}{(0.6,0.3,0.4)},0.2\right),\left(\frac{u_2}{(0.8,0.6,0.8)},0.7\right),\left(\frac{u_3}{(0.8,0.5,0.4)},0.2\right)\right],(0.7,0.5,0.3)\right\} \\ \sim P_{[\alpha,\beta]}(e_2) = \left\{\left[\left(\frac{u_1}{(0.6,0.6,0.7)},0.4\right),\left(\frac{u_2}{(0.5,0.3,0.5)},0.3\right),\left(\frac{u_3}{(0.8,0.6,0.7)},0.3\right)\right],(0.2,0.2,0.6)\right\} \\ \sim P_{[\alpha,\beta]}(e_3) = \left\{\left[\left(\frac{u_1}{(0.7,0.4,0.5)},0.8\right),\left(\frac{u_2}{(0.8,0.6,0.6)},0.5\right),\left(\frac{u_3}{(0.7,0.5,0.6)},0.4\right)\right],(0.8,0.7,0.5)\right\} \end{cases}$$

PROPOSITION 2.16

Let $P_{[\alpha,\beta]} \in$ GPN (U, E). Then

1. $\sim\phi_{[\alpha,\beta]} = U_{[\alpha,\beta]}$
2. $\sim U_{[\alpha,\beta]} = \phi_{[\alpha,\beta]}$
3. $\sim(\sim P_{[\alpha,\beta]}) = P_{[\alpha,\beta]}$.

Proof. It is clear from Definition 2.14.

PROPOSITION 2.17

Let $P_{[\alpha,\beta]}, Q_{[\gamma,\delta]} \in$ GPN(U,E). Then De Morgans law is valid.

1. $\sim(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}) = \sim P_{[\alpha,\beta]} \cap \sim Q_{[\gamma,\delta]}$.
2. $\sim(P_{[\alpha,\beta]} \cap Q_{[\gamma,\delta]}) = \sim P_{[\alpha,\beta]} \cup \sim Q_{[\gamma,\delta]}$.

Proof:

1. Let i, j ϵ A, $\sim\left(P_{[\alpha,\beta]} \cup Q_{[\gamma,\delta]}\right)$

$$= \sim\left\{\left(e_k,\left\{\left(\left(\frac{u_j}{\left(P_{kj}^T(e_k) \oplus Q_{kj}^T(e_k),\ P_{kj}^I(e_k) \oplus Q_{kj}^I(e_k),\ P_{kj}^F(e_k) \otimes Q_{kj}^F(e_k)\right)}',\ \alpha_{kj}(e_k) \oplus \gamma_{kj}(e_k)\right), \beta_{kj}(e_k) \oplus \delta_{kj}(e_k)\right\}\right) : u_j \in U, e_k \in E\right\}$$

$$= \left\{\left(e_k,\left\{\left(\left(\frac{u_j}{\left(P_{kj}^F(e_k) \otimes Q_{kj}^F(e_k),\ \sim\left(P_{kj}^I(e_k) \oplus Q_{kj}^I(e_k)\right),\ P_{kj}^T(e_k) \oplus Q_{kj}^T(e_k)\right)}',\ \sim(\alpha_{kj}(e_k) \oplus \gamma_{kj}(e_k))\right), \sim\left((\beta_{kj}(e_k) \oplus \delta_{kj}(e_k)\right)\right\}\right) : u_j \in U, e_k \in E\right\}$$

$$= \left\{\left(e_k, \left\{\left(\left(\frac{u_j}{\left(P^F_{kj}(e_k) \otimes Q^F_{kj}(e_k),\ \sim\left(P^I_{kj}(e_k)\right) \oplus \sim\left(Q^I_{kj}(e_k)\right), P^T_{kj}(e_k) \oplus Q^T_{kj}(e_k)\right)}, \sim(\alpha_{kj}(e_k)) \otimes \sim(\gamma_{kj}(e_k))\right), \sim\left((\beta_{kj}(e_k)\right) \otimes \sim\left(\delta_{kj}(e_k)\right)\right\}\right) : u_j \in U, e_k \in E\right\}$$

$$= \left\{\left(e_k, \left\{\left(\frac{u_j}{\left(P^F_{kj}(e_k),\ \sim\left(P^I_{kj}(e_k)\right),\ P^T_{kj}(e_k)\right)}, \sim(\alpha_{kj}(e_k))\right), \sim\left(\beta_{kj}(e_k)\right)\right\}\right) : u_j \in U, e_k \in E\right\}$$

$$= \left\{\left(e_k, \left\{\left(\frac{u_j}{\left(P^T_{kj}(e_k), \left(P^I_{kj}(e_k)\right), P^F_{kj}(e_k)\right)}, (\alpha_{kj}(e_k))\right), \left(\beta_{kj}(e_k)\right)\right\}\right) : u_j \in U, e_k \in E\right\}^c$$

$$\cap \left\{\left(e_k, \left\{\left(\frac{u_j}{Q^T_{kj}(e_k),\ Q^I_{kj}(e_k),\ Q^F_{kj}(e_k)}, \gamma_{kj}(e_k)\right), \left(\delta_{kj}(e_k)\right)\right\}\right) : u_j \in U, e_k \in E\right\}^c$$

$$= \sim P^c_{[\alpha,\beta]} \cap \sim Q^c_{[\gamma,\delta]}$$

The techniques used to prove (2) are similar to those used for (1), therefore we skip this proof.

DEFINITION 2.18

Let $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]} \in GPN(U, E)$. Then 'AND' product of GPNS set $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ denoted by $P_{[\alpha,\beta]} \wedge Q_{[\gamma,\delta]}$ is defined as follows:

$$P_{[\alpha,\beta]} \wedge Q_{[\gamma,\delta\ ]} = \left\{\left((e_k, e_1), P^T_{kj}(e_k) \wedge g^T_{lj}(e_1), P^I_{kj}(e_k) \wedge g^I_{lj}(e_1), P^F_{kj}(e_k) \vee g^F_{lj}(e_1)), \alpha_{kj}(e_k) \wedge \gamma_{lj}(e_1)), \beta_{kj}(e_k) \wedge \delta_{lj}(e_1)\right) : (e_k, e_1) \in E \times E, j, k, l \in \wedge\right\}$$

DEFINITION 2.19

Let $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]} \in GPN (U, E)$. Then 'OR' product of GPNS set $P_{[\alpha,\beta]}$ and $Q_{[\gamma,\delta]}$ denoted by $P_{[\alpha,\beta]} \vee Q_{[\gamma,\delta]}$ is defined as follows:

$$P_{[\alpha,\beta]} \wedge Q_{[\gamma,\delta]} = \left\{\left((e_k, e_1), P^T_{kj}(e_k) \vee Q^T_{lj}(e_1), P^I_{kj}(e_k) \vee Q^I_{lj}(e_1), P^F_{kj}(e_k) \wedge Q^F_{lj}(e_1)), \alpha_{kj}(e_k) \vee \gamma_{lj}(e_1)), \beta_{kj}(e_k) \vee \delta_{lj}(e_1)\right) : (e_k, e_1) \in E \times E, j, k, l \in \wedge\right\}$$

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2. **GENERALIZED POSSIBILITY NEUTROSOPHIC SOFT SET IN DECISION MAKING**

DEFINITION 3.1

Let $Q_{[\gamma,\delta]}$, $L_{[\theta,\phi]}\epsilon$ GPN (U, E), $P_{[\alpha,\beta]}=Q_{[\gamma,\delta]}\wedge L_{[\theta,\phi]}$ and $P^T_{[\alpha,\beta]}$, $P^I_{[\alpha,\beta]}$ and $P^F_{[\alpha,\beta]}$ be the truth, indeterminacy and falsity matrices of $\wedge$ −product matrix, respectively. Then, weighted matrices of $\left(P_{[\alpha,\beta]}\right)^T$, $\left(P_{[\alpha,\beta]}\right)^I$ and $\left(P_{[\alpha,\beta]}\right)^F$ denoted $\wedge^{\mathrm{T}}$, $\wedge^{\mathrm{I}}$and $\wedge^{\mathrm{F}}$ are defined as follows:

- $\wedge^{\mathrm{T}}(e_{kj},u_r)=T_{Q_{[\gamma,\delta]}\wedge L_{[\theta,\phi]}(e_{kj})}(u_r)+(\gamma_{kr}(e_k)\wedge\theta_{jr}(e_j))-T_{Q_{[\gamma,\delta]}\wedge L_{[\theta,\phi]}(e_{kj})}(u_r)\times(\alpha_{\mathrm{kr}}(e_k)\wedge(\gamma_{jr}(e_j)),\beta(P(e_k))$
- $\wedge^{\mathrm{I}}(e_{kj},u_r)=I_{Q_{[\gamma,\delta]}\wedge L_{[\theta,\phi]}(e_{kj})}(u_r)+(\gamma_{kr}(e_k)\wedge\theta_{jr}(e_j))-I_{Q_{[\gamma,\delta]}\wedge L_{[\theta,\phi]}(e_{kj})}(u_r)\times(\alpha_{\mathrm{kr}}(e_k)\wedge(\gamma_{jr}(e_j)),\beta(P(e_k))$
- $\wedge^{\mathrm{F}}(e_{kj},u_r)=P_{Q_{[\gamma,\delta]}\wedge L_{[\theta,\phi]}(e_{kj})}(u_r)\times(\gamma_{kr}(e_k)\wedge\theta_{jr}(e_j)),\beta(\mathrm{P}(e_k))$

DEFINITION 3.2

Let$Q_{[\gamma,\delta]}$, $L_{[\theta,\phi]}\epsilon$ GPN(U, E), $P_{[\alpha,\beta]}=Q_{[\gamma,\delta]}\wedge L_{[\theta,\phi]}$ and let $\wedge^{\mathrm{T}}$, $\wedge^{\mathrm{I}}$and $\wedge^{\mathrm{F}}$ be the weighted matrices of $P_{[\alpha,\beta]})^T$, $P_{[\alpha,\beta]})^I$and $P_{[\alpha,\beta]})^F$ respectively. Then, in the weighted matrices $\wedge^{\mathrm{T}}$, $\wedge^{\mathrm{I}}$and $\wedge^{\mathrm{F}}$scores of $u_n\ \epsilon$ U denoted by $s^T(u_n)$, $s^I(u_n)$ and $s^F(u_n)$ are defined as follows:

$s^T(u_n)=\sum_{\mathrm{k,j}\,\epsilon\wedge}(\pi^T_{kj}(u_n)\times\beta(\mathrm{P}(e_k)))$

$s^I(u_n)=\sum_{\mathrm{k,j}\,\epsilon\wedge}(\pi^I_{kj}(u_n)\times\beta(P(e_k)))$

$s^F(u_n)=\sum_{\mathrm{k,j}\,\epsilon\wedge}(\pi^F_{kj}(u_n)\times\beta(P(e_k)))$

Where $\pi^T_{kj}(u_m)=\begin{cases}\wedge^{\mathrm{T}}(e_{kJ},u_n), & \wedge^{\mathrm{T}}(e_{kJ},u_n)=\max\{\wedge^{\mathrm{T}}(e_{kJ},u_m):u_m\epsilon\ \mathrm{U}\}\\ 0, & otherwise\end{cases}$

$\pi^I_{kj}(u_m)=\begin{cases}\wedge^{\mathrm{I}}(e_{kJ},u_n), & \wedge^{\mathrm{I}}(e_{kJ},u_n)=\max\{\wedge^{\mathrm{I}}(e_{kJ},u_m):u_m\epsilon\ \mathrm{U}\}\\ 0, & otherwise\end{cases}$

$\pi^F_{kj}(u_m)=\begin{cases}\vee^{\boldsymbol{F}}(e_{kJ},u_n), & \vee^{\boldsymbol{F}}(e_{kJ},u_n)=\max\{\vee^{\boldsymbol{F}}(e_{kJ},u_m):u_m\epsilon\ \mathrm{U}\}\\ 0, & otherwise\end{cases}$

DEFINITION 3.3

Let $s^T(u_n)$, $s^I(u_n)$ and $s^F(u_n)$be scores of $u_n\ \epsilon$ U in the weighted matrices $\wedge^{\mathrm{T}}$, $\wedge^{\mathrm{I}}$ and $\wedge^{\mathrm{F}}$.Then, decision score of $u_n\ \epsilon$ U, denoted by ds(u_n), is defined by ds(u_n) = $s^T(u_n)+s^I(u_n)$ - $s^F(u_n)$. Now, we construct a GPNS – decision making method by the following algorithm:

ALGORITHM

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Step 1: Input the GPNSs,

Step 2: Construct the ∧-product matrix,

Step3: Construct the truth, indeterminacy and falsity matrices of the ∧-product matrix,

Step4: Construct the weighted matrices $\wedge^T$, $\wedge^I$ and $\wedge^F$.

Step5: Compute score of $u_t \in U$, for each of the weighted matrices,

Step6: Compute decision score, for all $u_t \in U$,

Step7: The optimal decision is to select $u_t = maxds(u_i)$

Example

Assume that U = $\{C_1, C_2, C_3\}$ is a set of three colleges and E = $\{e_1, e_2, e_3\}$ = {Academics, Sports, Fine Arts} is a set of parameters which is best college among the three, α-represents priority given to the parameters by the college and β-represents the weightage given to the parameters by the experts.

Suppose that a committee wants to award a particular college.

Step 1: Based on the choice parameters, GPNSs $Q_{[\gamma,\delta]}$ and $L_{[\theta,\phi]}$, constructed by two experts are as follows:

$$Q_{[\gamma,\delta]} = \left\{\begin{matrix} Q_{[\gamma,\delta]}(e_1) = \left\{\left(\frac{u_1}{(0.6,0.5,0.6)},0.7\right),\left(\frac{u_2}{(0.7,0.3,0.5)},0.3\right),\left(\frac{u_3}{(0.8,0.7,0.6)},0.5\right),(0.3,0.5,0.4)\right\} \\ Q_{[\gamma,\delta]}(e_2) = \left\{\left(\frac{u_1}{(0.4,0.3,0.6)},0.5\right),\left(\frac{u_2}{(0.8,0.9,0.4)},0.6\right),\left(\frac{u_3}{(0.3,0.5,0.5)},0.7\right),(0.7,0.3,0.6)\right\} \\ Q_{[\gamma,\delta]}(e_3) = \left\{\left(\frac{u_1}{(0.8,0.7,0.6)},0.5\right),\left(\frac{u_2}{(0.5,0.6,0.3)},0.4\right),\left(\frac{u_3}{(0.6,0.4,0.6)},0.3\right),(0.9,0.2,0.3)\right\} \end{matrix}\right\}$$

also we can define a function $Q_{[\gamma,\delta]}$: E → N(U) × F(U) × F(U) as follows:

$$L_{[\theta,\phi]} = \left\{\begin{matrix} L_{[\theta,\phi]}(e_1) = \left\{\left(\frac{u_1}{(0.4,0.5,0.6)},0.7\right),\left(\frac{u_2}{(0.8,0.4,0.5)},0.3\right),\left(\frac{u_3}{(0.5,0.6,0.3)},0.5\right),(0.3,0.6,0.2)\right\} \\ L_{[\theta,\phi]}(e_2) = \left\{\left(\frac{u_1}{(0.5,0.7,0.6)},0.5\right),\left(\frac{u_2}{(0.3,0.6,0.4)},0.8\right),\left(\frac{u_3}{(0.5,0.7,0.3)},0.9\right),(o.8,0.1,0.5)\right\} \\ L_{[\theta,\phi]}(e_3) = \left\{\left(\frac{u_1}{(0.3,0.2,0.7)},0.8\right),\left(\frac{u_2}{(0.9,0.5,0.6)},0.4\right),\left(\frac{u_3}{(0.7,0.5,0.4)},0.5\right),(0.4,0.3,0.9)\right\} \end{matrix}\right\}$$

Step 2: Let us consider GPNS ∧-product $P_{[\alpha,\beta]} = Q_{[\gamma,\delta]} \wedge L_{[\theta,\phi]}$ which is the mapping ∧ **:** E × E → N(U) × F(U) × N(U) given as follows:

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$\wedge$	u_1, α	u_2, α	u_3, α	β
e_{11}	$(\langle 0.4,0.5,0.6\rangle, 0.7)$	$(\langle 0.7,0.3,0.5\rangle, 0.3)$	$(\langle 0.5,0.6,0.6\rangle, 0.5)$	$(0.3,0.5,0.4)$
e_{12}	$(\langle 0.5,0.5,0.6\rangle, 0.5)$	$(\langle 0.3,0.3,0.5\rangle, 0.3)$	$(\langle 0.5,0.7,0.6\rangle, 0.5)$	$(0.3,0.1,0.5)$
e_{13}	$(\langle 0.3,0.2,0.7\rangle, 0.6)$	$(\langle 0.7,0.3,0.6\rangle, 0.2)$	$(\langle 0.7,0.5,0.6\rangle, 0.6)$	$(0.3,0.3,0.9)$
e_{21}	$(\langle 0.4,0.3,0.6\rangle, 0.5)$	$(\langle 0.8,0.4,0.5\rangle, 0.3)$	$(\langle 0.3,0.5,0.5\rangle, 0.5)$	$(0.3,0.3,0.6)$
e_{22}	$(\langle 0.4,0.3,0.6\rangle, 0.5)$	$(\langle 0.3,0.6,0.4\rangle, 0.6)$	$(\langle 0.3,0.5,0.5\rangle, 0.7)$	$(0.7,0.1,0.6)$
e_{23}	$(\langle 0.3,0.2,0.7\rangle, 0.5)$	$(\langle 0.8,0.5,0.6\rangle, 0.5)$	$(\langle 0.3,0.5,0.5\rangle, 0.5)$	$(0.4,0.3,0.9)$
e_{31}	$(\langle 0.4,0.3,0.6\rangle, 0.5)$	$(\langle 0.5,0.4,0.5\rangle, 0.3)$	$(\langle 0.5,0.4,0.6\rangle, 0.3)$	$(0.3,0.2,0.5)$
e_{32}	$(\langle 0.5,0.3,0.6\rangle, 0.5)$	$(\langle 0.3,0.6,0.4\rangle, 0.4)$	$(\langle 0.5,0.4,0.6\rangle, 0.3)$	$(0.8,0.1,0.5)$
e_{33}	$(\langle 0.3,0.2,0.6\rangle, 0.5)$	$(\langle 0.5,0.5,0.6\rangle, 0.4)$	$(\langle 0.6,0.4,0.6\rangle, 0.3)$	$(0.4,0.3,0.9)$

Step 3: We construct matrices $P^T_{[\alpha,\beta]}$, $P^I_{[\alpha,\beta]}$ and $P^F_{[\alpha,\beta]}$ as follows:

$\wedge$	u_1, α	u_2, α	u_3, α	β^T
e_{11}	(0.4,0.7)	(0.7,0.3)	(0.5,0.5)	(0.3)
e_{12}	(0.5,0.5)	(0.3,0.3)	(0.5,0.5)	(0.3)
e_{13}	(0.3,0.7)	(0.7,0.3)	(0.7,0.6)	(0.3)
e_{21}	(0.4,0.5)	(0.8,0.3)	(0.3,0.5)	(0.3)
e_{22}	(0.4,0.5)	(0.3,0.6)	(0.3,0.7)	(0.7)
e_{23}	(0.3,0.5)	(0.8,0.5)	(0.3,0.5)	(0.4)
e_{31}	(0.4,0.5)	(0.5,0.3)	(0.5,0.3)	(0.3)
e_{32}	(0.5,0.5)	(0.4,0.4)	(0.5,0.3)	(0.8)
e_{33}	(0.3,0.5)	(0.5,0.4)	(0.6,0.3)	(0.4)

$\wedge$	u_1, α	u_2, α	u_3, α	β^I
e_{11}	(0.4,0.7)	(0.3,0.3)	(0.6,0.5)	(0.5)
e_{12}	(0.4,0.5)	(0.3,0.3)	(0.7,0.5)	(0.1)
e_{13}	(0.2,0.7)	(0.3,0.3)	(0.5,0.6)	(0.3)
e_{21}	(0.3,0.5)	(0.4,0.3)	(0.5,0.5)	(0.3)
e_{22}	(0.3,0.5)	(0.6,0.6)	(0.5,0.7)	(0.1)
e_{23}	(0.2,0.5)	(0.5,0.5)	(0.5,0.5)	(0.3)
e_{31}	(0.3,0.5)	(0.4,0.3)	(0.4,0.3)	(0.2)
e_{32}	(0.3,0.5)	(0.6,0.4)	(0.4,0.3)	(0.1)
e_{33}	(0.2,0.5)	(0.5,0.4)	(0.4,0.3)	(0.3)

$\wedge$	u_1, α	u_2, α	u_3, α	β^F
e_{11}	(0.6,0.7)	(0.5,0.3)	(0.6,0.5)	(0.4)
e_{12}	(0.6,0.5)	(0.5,0.3)	(0.6,0.5)	(0.5)
e_{13}	(0.7,0.7)	(0.6,0.3)	(0.6,0.6)	(0.9)
e_{21}	(0.6,0.5)	(0.5,0.3)	(0.5,0.5)	(0.6)
e_{22}	(0.6,0.5)	(0.4,0.6)	(0.5,0.7)	(0.6)
e_{23}	(0.7,0.5)	(0.6,0.5)	(0.5,0.5)	(0.9)
e_{31}	(0.6,0.5)	(0.5,0.3)	(0.6,0.3)	(0.5)
e_{32}	(0.6,0.5)	(0.4,0.4)	(0.6,0.3)	(0.5)
e_{33}	(0.7,0.5)	(0.6,0.4)	(0.6,0.3)	(0.9)

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Matrix $P^F_{[\alpha,\beta]}$ of ∧ -product

Step 4: We obtain weighted matrices $\wedge^T$, $\wedge^I$ and $\wedge^F$ as follows:

$\wedge^T$	u_1,α	u_2,α	u_3,α	β^T
e_{11}	0.82	0.79	0.75	0.3
e_{12}	0.75	0.51	0.70	0.3
e_{13}	0.79	0.79	0.88	0.3
e_{21}	0.70	0.86	0.65	0.3
e_{22}	0.70	0.72	0.79	0.7
e_{23}	0.65	0.90	0.65	0.4
e_{31}	0.70	0.65	0.65	0.3
e_{32}	0.75	0.58	0.65	0.8
e_{33}	0.65	0.7	0.72	0.4

$\wedge^I$	u_1,α	u_2,α	u_3,α	β^I
e_{11}	0.85	0.51	0.80	0.5
e_{12}	0.70	0.51	0.85	0.1
e_{13}	0.76	0.51	0.80	0.3
e_{21}	0.65	0.58	0.75	0.3
e_{22}	0.65	0.84	0.85	0.1
e_{23}	0.60	0.75	0.75	0.3
e_{31}	0.65	0.58	058	0.2
e_{32}	0.65	0.76	0.58	0.1
e_{33}	0.60	0.70	0.58	0.3

$\wedge^F$	u_1,α	u_2,α	u_3,α	β^F
e_{11}	0.42	0.15	0.15	0.4
e_{12}	0.15	0.12	0.15	0.5
e_{13}	0.49	0.18	0.24	0.9
e_{21}	0.30	0.12	0.15	0.6
e_{22}	0.15	0.24	0.21	0.6
e_{23}	0.35	0.20	0.20	0.9
e_{31}	0.30	0.09	0.09	0.5
e_{32}	0.15	0.12	0.09	0.5
e_{33}	0.30	0.12	0.12	0.9

Above are the matrices $P^T_{[\alpha,\beta]}$, $P^I_{[\alpha,\beta]}$ and $P^F_{[\alpha,\beta]}$ from left to right, respectively

Step 5: For all $u \in U$, we find the scores.

$s^T(u_1)$ = 1.281, $s^T(u_2)$ = 0.618, $s^T(u_3)$ = 1.105

$s^I(u_1)$ = 0.555, $s^I(u_2)$ = 0.286, $s^I(u_3)$ = 0.86

$s^F(u_1)$ = 1.599, $s^F(u_3)$ = 0.144, $s^F(u_3)$ = 0.075

Step 6: For each $u \in U$, we find a decision scores.

$ds(u_1)$ = 1.281 + 0.555 – 1.599 = 0.237

$ds(u_2)$ = 0.618 + 0.286 – 0.44 = 0.464

$ds(u_3)$ = 1.105 + 0.86 – 0.075 =1.89

Step 7: Then the optimal selection of the committee is the college C_3.

Conclusion

In this work, we devised the concept of Generalisd Possibility Neutrosophic Soft Set (GPNSS) and studied generalized possibility neutrosophic soft set operations and discussed some properties related with defined operations. We constructed a MCDM based on generalized possibility neutrosophic soft set and a numerical example is presented. The proposed method can be applied in many different fields to solve the related problems.

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