# n-Cyclic Refined Neutrosophic Groups

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**Abstract**

This paper introduces for the first time the concept of n-cyclic refined neutrosophic group as a direct application of the concept of n-cyclic refined neutrosophic set. Also, it discusses some of its elementary properties such as AH-subgroups, kernels, and direct products.

**Keywords:** n-cyclic refined neutrosophic group , AH-subgroup, AH-homomorphism

## 1.Introduction

Neutrosophy is a philosophical concept founded by Smarandache to generalize the fuzzy logic [1,32] . Neutrosophic sets were very applicative in many areas of science such as topology [2,27,60,61,63], decision making [21,35], applied mathematics [3,4,20,25,28,57,64], and pure mathematics [9,10,19,39,43,44,47,50,51,52].

Neutrosophic sets were used in the study of algebraic structures such as modules [6], rings [12], groups [13], and matrices [6,53,62]. In the literature, we find three interesting generalizations of neutrosophic sets, where refined neutrosophic sets [13] n-refined neutrosophic sets [15,50], and n-cyclic refined neutrosophic sets [31]. These kinds lead us to many interesting generalizations of classical algebraic structures such as n-refined neutrosophic modules [5], n-refined spaces and matrices [8,46,54], refined neutrosophic ring [23,48,49], and n-cyclic refined neutrosophic modules [31].

In this work, we use the concept of n-cyclic refined neutrosophic set to defined n-cyclic refined neutrosophic groups. These groups will be studied carefully, and many elementary properties will be discussed through this paper, especially AH-subgroups, kernels, and homomorphisms.

## 2. Preliminaries

**Definition 2.1**: [31]

Let (R,+,×) be a ring and $I_k; 1 \leq k \leq n$ be n indeterminacies. We define $R_n$(I)={$a_0 + a_1 I + \cdots + a_n I_n$ ; $a_i \in R$} to be n-cyclic refined neutrosophic ring.

Operations on $R_n$(I) are defined as:

$$\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n} (x_i + y_i) I_i \,, \sum_{i=0}^{n} x_i I_i \times \sum_{i=0}^{n} y_i I_i = \sum_{i,j=0}^{n} (x_i \times y_j) I_i I_j = \sum_{i,j=0}^{n} (x_i \times y_j) I_{(i+j\ mod n)}$$

Where × is the multiplication on the ring R.

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It is obvious that $R_n$(I) is a ring in the algebraic ordinary concept.

**Definition 2.2: [31]**

Let $R_n$(I) be an n-cyclic refined neutrosophic ring, it is called commutative if $xy = yx$ for all x , y ∈ $R_n$(I). If there is 1∈ $R_n$(I) such that $1.x = x.1 = x$, then it is called an n-cyclic refined neutrosophic ring with unity.

**Definition 2.3: [31]**

Let $R_n$(I) be a commutative n-cyclic refined neutrosophic ring and $P: R_n(\mathrm{I}) \to R_n(\mathrm{I})$ is a function defined as $P(x) = \sum_{i=0}^{m} a_i x^i$ such that $a_i \in R_n$(I), we call P an n-cyclic refined neutrosophic polynomial on $R_n$(I).

We denote by $R_n$(I)[x] to be the ring of n-cyclic refined neutrosophic polynomials over $R_n$(I).

Since $R_n$(I) is a classical ring, then $R_n$(I)[x] is a classical ring.

**Definition 2.4 : [31]**

Let (M,+,.) be a module over the ring R, we say that $M_n(I) = M + MI_1 + \cdots + MI_n = \{x_0 + x_1 I_1 + \cdots + x_n I_n;\ x_i \in M\}$ is a weak n-cyclic refined neutrosophic module over the ring R. Elements of $M_n(I)$ are called n-cyclic refined neutrosophic vectors, elements of R are called scalars.

If we take scalars from the n- cyclic refined neutrosophic ring $R_n(I)$, we say that $M_n(I)$ is a strong n-cyclic refined neutrosophic module over the n-cyclic refined neutrosophic ring $R_n(I)$. Elements of $M_n(I)$ are called n-refined neutrosophic scalars.

**Remark 2.5: [31]**

Addition on $M_n(I)$ is defined as:

$\sum_{i=0}^{n} a_i I_i + \sum_{i=0}^{n} b_i I_i = \sum_{i=0}^{n} (a_i + b_i) I_i$.

Multiplication by a scalar $m \in R$ is defined as:

$m.\sum_{i=0}^{n} a_i I_i = \sum_{i=0}^{n} (m.a_i) I_i$.

Multiplication by an n-cyclic refined neutrosophic scalar $m = \sum_{i=0}^{n} m_i I_i \in R_n(I)$ is defined as:

$\sum_{i=0}^{n} m_i I_i\ .\sum_{i=0}^{n} a_i I_i = \sum_{i=0}^{n} (m_i.a_i) I_i I_j$.

Where $\mathrm{a_i} \in \mathrm{M}, \mathrm{m_i} \in \mathrm{R}, \mathrm{I_i I_j} = \mathrm{I_{(i+j \bmod n)}}$.

**3. Main Concepts and discussion**

**Definition 3.1:**

Let I be the neutrosophic element, which refers to indeterminacy, we define n-cyclic refining system of I by the set L={$I^1 = I, I^2, \dots, I^{n-1}$}, where $I^i \neq I^j\ for\ all\ i \neq j\ and\ i,j < n$.

We define a binary operation on L as follows:

$I^j.I^i = I^{i+j(modn)}$. It is clear that L has a structure of the subset $(Z_n/\{0\}, +)$, where $Z_n$ is the additive group of integers modulo n.

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**Definition 3.2**:

Let (G,*) be a group, and I is the neutrosophic element with property $I^n = I\ ; n \geq 2$ , with

$I^i \neq I^j\ for\ all\ i \neq j\ and\ i,j < n$ . We call M(G)=$G \cup GI \cup GI^2 \cup \ldots \cup GI^{n-1}$ an n-cyclic refined neutrosophic group.

It is easy to see that 2-cyclic refined neutrosophic group is the classical neutrosophic group.

**Remark 3.3:**

The sets $GI^k$ are groups under the binary operation (x$I^k$)(y$I^k$)=(xy$I^k$) ; $k < n$ with identity $I^k$ and each one of them must be isomorphic to G.

**Definition 3.4:**

Let M(G) be an n-cyclic refined neutrosophic group, H be a subset of M(G). We call H an AH-subgroup if H=$H_0 \cup H_1 I \cup H_2 I^2 \cup \ldots \cup H_{n-1} I^{n-1}$,where $H_i$ is a subgroup of G for all i.

We call H an AHS-subgroup if $H_0 = H_1 = \cdots = H_{n-1}$.

We call H an AH-normal if $H_i$ is normal subgroup of G for all i.

We call H an AHS-normal if it is AHS-subgroup and AH-normal.

**Definition 3.5 :**

Let M(G) , M(H) be two n , m-cyclic refined neutrosophic groups respectively, $f: M(G) \rightarrow M(H)$ be a map , we say that $f$ is an AH- homomorphism if it is a homomrphism between G , H i.e. $f(xy) = f(x)f(y) \forall\ x, y \in G\ and\ f(I^k) = (I')^k$ such $I'$ is the neutrosophic element of H.

We define $AH - Ker(f)\ = Kerf_G \cup Kerf_G I \cup \ldots \cup Kerf_G I^{n-1}$ , we regard that $AH - Ker(f)$ is an AHS-normal subgroup of M(G)

We say that $f$ is an isomorphism if it is a correspondence one-to-one homomorphism.

If H = $H_0 \cup H_1 I \cup \ldots \cup H_{n-1} I^{n-1}\ and\ K = K_0 \cup K_1 I \cup \ldots \cup K_{n-1} I^{n-1}$ are two AH-subgroups of M(G). We say that they are isomorphic if $H_i \cong K_i$ for all i .

**Definition 3.6** :

Let H , K be two AH-subgroups of M(G). We define

$HK = H_0 K_0 \cup H_1 K_1 I^1 \cup \ldots \cup H_{n-1} K_{n-1} I^{n-1}$.

**Definition 3.7 :**

Let M(G) be an n-cyclic refined neutrosophic group, $H = H_0 \cup H_1 I \cup H_2 I^2 \cup \ldots \cup H_{n-1} I^{n-1}$ be an AH-normal subgroup of M(G). We define the corresponding AH-factor as M(G)/H = $(G/H_0) \cup (G/H_1)I \cup \ldots \cup (G/H_{n-1})I^{n-1}$.

**Definition 3.8 :**

Let M(G) be an n-cyclic refined neutrosophic group. We define the AH-center of M(G) by

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$Z(M(G)) = Z(G) \cup Z(G)I \cup \ldots \cup Z(G)I^{n-1}$.

It is easy to see that $Z(M(G))$ is an AHS-normal subgroup of M(G).

**Definition 3.9 :**

Let M(G) be an n-cyclic refined neutrosophic group. We say that M(G) is abelian if G is abelian, i.e. M(G)=Z(M(G)).

M(G) is said to be cyclic if G is cyclic.

**Theorem 3.10 :**

Let M(G) be an n-cyclic refined neutrosophic group, then

(a) If H is an AH-normal subgroup and M(G) is abelian, then M(G)/H is abelian.

(b) If M(G) is finite and H is an AHS-subgroup, then O(H) divides O(M(G))= n O(G).

(c) If H is an AH-normal subgroup and M(G) is cyclic then M(G)/H is cyclic.

Proof:

(a) Since M(G)/H = $(G/H_0) \cup (G/H_1)I \cup \ldots \cup (G/H_{n-1})I^{n-1}$ and $G/H_i$ is abelian for all i, then M(G)/H is abelian.

(b) We have that O(H)= n O($H_0$) and O($H_0$) divides the order of G then O(H) divides O(M(G))= n O(G).

(c) Since $G/H_i$ is cyclic for all i then M(G)/H is cyclic.

**Theorem 3.11** :

Let M(G) be an n-cyclic refined neutrosophic group and H , K be two AH-subgroups, then

(a) $H \cap K$ is an AH-subgroup.

$(b)$ If H , K are AHS-subgroups, then $H \cap K$ is an AHS-subgroup.

(c) If H , K are AH-normal subgroups, then $H \cap K$ and HK are AH-normal subgroups.

(d) If H, K are AHS-normal subgroups, then $H \cap K$ and HK are AHS-normal subgroups.

Proof : Suppose that H = $H_0 \cup H_1 I \cup \ldots \cup H_{n-1}I^{n-1}$ $and$ $K = K_0 \cup K_1 I \cup \ldots \cup K_{n-1}I^{n-1}$ then $H \cap K = (H_0 \cap K_0) \cup (H_1 \cap K_1)I \cup \ldots \cup (H_{n-1} \cap K_{n-1})I^{n-1}$ by this argument we can easily find that the proof holds.

**Theorem 3.12 :**

Let M(G) , M(H) be two n , m-cyclic refined neutrosophic groups respectively, and $f: M(G) \to M(H)$ be a homomorphism, then

(a) $n \geq m$.

(b) If K is an AH-subgroup of M(G) then f(K) is an AH-subgroup of M(H).

(c) If K is an AHS-subgroup of M(G) then f(K) is an AHS-subgroup of M(H).

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(d) If K is an AH-normal subgroup of M(G) then f(K) is an AH-normal subgroup of f(M(G)).

(e) If K is an AHS-normal subgroup of M(G) then f(K) is an AHS-normal subgroup of f(M(G)).

(f) M(G)/Kerf ≅ f (M(G)).

Proof :

(a) Suppose that $n < m$ then $I' = f(I) = f(I^n) = (I')^n$ and this is a contradiction thus $n \geq m$.

(b) Suppose that $K = K_0 \cup K_1 I \cup \ldots \cup K_{n-1} I^{n-1}$, then $f(K) = f(K_0) \cup f(K_1)I \cup \ldots \cup f(K_{n-1})I^{n-1}$ with subgroups f$(K_i)$ $for\ all\ i$ of M(H), so that $f(K)$ is an AH-subgroup of $M(H)$.

(c) It is obvious that if $K_i = K_j, then\ f(K_i) \cong f(K_j)$, thus $f(K)$ is an AHS-subgroup of M(H).

(d) , (e) hold directly from (b) and (c) and from the fact that if $K_i$ is normal, then $f(K_i)$ is normal.

(f) From the definition, we find $M(G)/Kerf = (G/Kerf_G) \cup (G/Kerf_G)I \cup \ldots \cup (G/Kerf_G)I^{n-1}$; but $G/Kerf_G \cong f(G)$, thus $M(G)/Kerf \cong f(G) \cup f(G)I \cup \ldots \cup f(G)I^{n-1} = f(M(G))$.

**Theorem 3.13 :**

Let M(G) be an n-cyclic refined neutrosophic group and H , K be two AH-normal subgroups with K$\leq H$, then $(M(G)/K)/(H/K) \cong M(G)/H$.

Proof :

Suppose that = $H_0 \cup H_1 I \cup \ldots \cup H_{n-1} I^{n-1}\ and\ K = K_0 \cup K_1 I \cup \ldots \cup K_{n-1} I^{n-1}$ with K$\leq H$, then $(M(G)/K)/(H/K) = \ ((G/K_0) \cup \ldots \cup (G/K_{n-1})I^{n-1})/((H_0/K_0) \cup \ldots \cup (H_{n-1}/K_{n-1})I^{n-1}) \cong$ $G/K_0/(H_0/K_0) \cup \ldots \cup (G/K_{n-1})/(H_{n-1}/K_{n-1})I^{n-1} \cong G/H_0 \cup (G/H_1)I \cup \ldots \cup (G/H_{n-1})I^{n-1} = M(G)/H$.

**Theorem 3.14** :

Let M(G) be an n-cyclic refined neutrosophic group, and H is an AH-normal subgroup, then for each AH-subgroup T of M(G)/H there is an AH-subgroup of M(G) contains H.

Proof : It can be proved as the classical case.

**Definition 3.15** :

Let M(G) , M(H) be two n-cyclic refined neutrosophic groups,

we define $M(G) \times M(H) = (G \times H) \cup (G \times H)II' \cup \ldots \cup (G \times H)I^n(I')^n$ with $(II')^k = I^k(I')^k$ for all k , it is clear that $M(G) \times M(H)$ is an n-generalized neutrosophic group with neutrosophic element $II'$.

**Theorem 3.16** :

Let M(G) , M(H) be two n-cyclic refined neutrosophic groups, then

(a) If M(G) , M(H) are abelian then $M(G) \times M(H)$ is abelian.

(b) If T,S are two AH-subgroups of M(G) , M(H) respectively, then $T \times S$ is an AH-subgroup of $M(G) \times M(H)$.

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(c) If T,S are two AH-normal subgroups of M(G) , M(H) respectively, then $T \times S$ is an AH-normal subgroup of $M(G) \times M(H)$.

(d) If T,S are two AHS-subgroups of M(G) , M(H) respectively, then $T \times S$ is an AHS-subgroup of $M(G) \times M(H)$.

(e) If T,S are two AHS-normal subgroups of M(G) , M(H) respectively then $T \times S$ is an AHS-normal subgroup of $M(G) \times M(H)$.

Proof :

(a) It is clear since $G \times H$ is abelian.

(b) Assume that $T = T_0 \cup \dots \cup T_{n-1}I^{n-1}$ and $S = S_0 \cup \dots \cup S_{n-1}(I')^{n-1}$, then

$T \times S = (T_0 \times S_0) \cup \dots \cup (T_{n-1} \times S_{n-1})(II')^{n-1}$ , we can regard that $T_i \times S_i$ is a subgroup of $G \times H$, so $T \times S$ is an AH-subgroup.

(c) It holds directly from (b).

(d) If $S_j \cong T_i$ $and$ $S_j \cong S_i$ then $T_i \times S_i \cong T_j \times S_j$ and then $T \times S$ is an AHS-subgroup.

(e) It holds directly from (d) and (c).

**Theorem 3.17** :

Let M(G) , M(H) be two n-cyclic refined neutrosophic groups and T,S be two AH-normal subgroups of M(G) , M(H) respectively, then

$M(G) \times M(H)/T \times S \cong M(G)/T \times M(H)/S.$

Proof :

Suppose that $T = T_0 \cup \dots \cup T_{n-1}I^{n-1}$ and $S = S_0 \cup \dots \cup S_{n-1}(I')^{n-1}$, then

$M(G)/T \times M(H)/S = (G/T_0 \times H/S_0) \cup (G/T_1 \times H/S_1)II' \cup \dots \cup (G/T_{n-1} \times H/S_{n-1})(II')^{n-1}$

$\cong \mathrm{G} \times \mathrm{H}/\mathrm{T}_0 \times \mathrm{S}_0 \times \dots \times (G \times H/T_{n-1} \times S_{n-1})(II')^{n-1} = M(G) \times M(H)/T \times S.$

**Example 3.18:**

Consider the additive group $(R^*,.)$ and the integer $n = 3$. The corresponding 3-cyclic refined neutrosophic group is

$M(R) = \{a, bI, cI^2; a, b, c \in R^*\}$.

1) We know that $(Q^*,.)$ is a subgroup of $(R^*,.)$, hence L=$Q^* \cup Q^*I \cup Q^*I^2$ =$\{x, yI, zI^2; x, y, z \in Q^*\}$ is a 3-cyclic refined AHS-subgroup.

2) The corresponding AH-factor is $M(R)/_L = R^*/_{Q^*} \cup R^*/_{Q^*} I \cup R^*/_{Q^*} I^2$.

**Example 3.19:**

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Consider the following two groups $G=(Z,+), H=(2Z,+)$, and the integer $n=4$, the corresponding 4-cyclic refined neutrosophic groups are $M(G)=\{a,bI,cI^2,dI^3;a,b,c,d\in Z\}, M(H)=\{2a,2bI,2cI^2,2dI^3;a,b,c,d\in Z\}$,

1) $g:G\to H; g(x)=4x\ is\ a\ group\ homomorphism.\ Ker(g)=\{0\}$.

2) The corresponding AH-homomorphism is $f:M(G)\to M(H); f(x)=4x, and\ f(xI^l)=4xI^l; 1\le l\le 3$.

3) The corresponding AH-kernel is $AH-Ker(f)=Ker(g)\cup Ker(g)I\cup Ker(g)I^2\cup Ker(g)I^3=\{0,0+I,0+I^2,0+I^3\}$

**Conclusion**

In this paper, we have defined for the first time the concept of n-cyclic refined neutrosophic group as a new application of n-cyclic refined neutrosophic sets. We have discussed some of their elementary properties such as AH-subgroups, AH-kernels and direct products.

As a future research directions, we aim to study the n-cyclic refined neutrosophic semi groups and loops.

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