Neutrosophic Bi-topological Group

Riad K. Al-Hamido

Department of Mathematics, College of Science, Al-Furat University, Deir-ez-Zor, Syria.
riad-hamido1983@hotmail.com

Abstract

Neutrosophic topological groups are neutrosophic groups in algebraic sense together with neutrosophic continuous group operations. In this article, we presented neutrosophic bi-topological groups with illustrative examples. Also, We defined eight new patterns of neutrosophic bi-topological groups. Neutrosophic bi-topological group which depends on two neutrosophic topology is a more general than neutrosophic topological groups. Finally, Some basic properties of neutrosophic bi-topological groups are investigated.

Keywords: Neutrosophic topological groups, neutrosophic bi-topological groups, neutrosophic grups.

1. Introduction

The concept of neutrosophic set was first given by F. Smarandache [2,3]. As a generalization of fuzzy set which defined by Zadeh [1], and intuitionistic fuzzy set,via this new logic[4], A.A. Salama and S.A. Alblowi [5] presented neutrosophic topological space. At recent years, the theory of neutrosophic theory becomes very widespread among scientists around the world.

Agboola et al. presented the concept of neutrosophic group and neutrosophic ring in[6,7]. Also, In 2015, he presented the concept of refined neutrosophic algebraic structures in[8], and introduced refined neutrosophic groups. Recently. In 2020, Adeleke et al. in [9,10] studied several refind concepts as refined neutrosophic rings and introduced their basic properties, refined neutrosophic ideals and refined neutrosophic homomorphisms in details.

Resently, In 2020, Q. Imran, et al. in [11] studied several types of neutrosophic topological groups and introduced their basic properties. Also, Sumathi et al. in [12] introduced the concept of neutrosophic topological groups.

This paper is devoted to the study of neutrosophic bi-topological groups, and investigate its basic properties. Also, we provide many new definitions, important Results, some theories and examples.

2. Definition

In this section, NTS (NTG) means neutrosophic topological space (grups). NOS (NCS) means neutrosophic open (closed) sets. Now, we recall some basic definitions as neutrosophic group, and neutrosophic ring,…etc. which are useful in sequel.

Definition 2.1. [10]

Let (G, *) be any group, the neutrosophic group is generated by I and G under * denoted by N(G)={<G∪I>,*}.

Definition 2.2. [12]
A neutrosophic topological group (NTG) is a set G which carries a group structure and a neutrosophic topology with the following two postulates:
(i) The operation function ϕ:G×G→G, given as ϕ(g,h)= g.h is a N-continuous.
(ii) The inversion function ψ:G→G, given as $\psi(g)=g^{-1}$ is a N-continuous.

Definition 2.3.[14]
Let f : $(X,T_1) \to (Y,T_2)$ be a function on neutrosophic topological spaces. Then f is neutrosophic contionuous (N-continuous) if the preimage of each open neutrosophic set in Y is open neutrosophic set in X.

Definition 2.4.
Let f : $(X,T_1) \to (Y,T_2)$ be a function on NTS, then f is called:
(i) Neutrosophic α-continuous (N-α-continuous) iff for each A NOS in Y, then $f^{-1}(A)$ is a NαOS in X [14].
(ii) Neutrosophic α-irresolute (N-α-irresolute) iff for each A NαOS in Y, then $f^{-1}(A)$ is a NαOS in X.

Proposition 2.5. [15]
Every N-continuous function is a N-α-continuous, but the opposite is not valid in general.

Definition 2.6.[11]
Let G be a set that equips with a group structure and a neutrosophic topology. Then G is called:
(i) NTG of pattern (1) iff the operation function ϕ:G × G → G and the inversion function ψ: G → G are both N-α-continuous.
(ii) NTG of pattern (2) iff the operation function ϕ:G × G → G and the inversion function ψ: G → G are both N-α-irresolute.
(iii) NTG of pattern (3) iff the operation function ϕ:G × G→G is N-α-continuous and the inversion function ψ:G →G is N-continuous.
(iv) NTG of pattern (4) iff the operation function ϕ:G × G → G is N-α-irresolute and the inversion function ψ:G → G is N-continuous.
(v) NTG of pattern (5) iff the operation function ϕ:G × G → G is N-α-irresolute and the inversion function ψ: G →G is N-α-continuous.
(vi) NTG of pattern (6) iff the operation function ϕ:G × G→G is N-α-continuous and the inversion function ψ:G → G is N-α-irresolute.
(vii) NTG of pattern (7) iff the operation function ϕ:G × G →G is N-continuous, and the inversion function ψ:G → G is N-α-continuous.
(viii) NTG of pattern (8) iff the operation function ϕ:G × G →G is N-continuous, and the inversion function ψ:G →G is N-α-irresolute.

Remark 2.7.[11]
(i) Every NTG is a NTG of pattern (R), where R=1,3,7.
(ii) Every NTG of pattern (2) is a NTG of pattern (5).
(iii) Every NTG of pattern (2) is a NTG of pattern (6).
(iv) Every NTG of pattern (4) is a NTG of pattern (3).
(v) Every NTG of pattern (4) is a NTG of pattern (5).
(vi) Every NTG of pattern (R) is a NTG of pattern (1), where R=2,3,…,8.

Definition 2.8. [11]
A function f : $(X,T_1) \to (Y,T_2)$ is said to be M-function iff $f^{-1}(Nint(Ncl(A)))\subseteq Nint(Ncl(f^{-1}(A)))$, for every NαOS A of Y.

Remark 2.9.[11]
(i) A NTG of pattern (3) with M-function operation(M-FO) μ is a NTG of pattern (4).
(ii) A NTG of pattern (1) with M-FO ψ and M-FO ϕ is a NTG of pattern (2).
(iii) A NTG of pattern (1) with M-FO ϕ is a NTG of pattern (5).
(iv) A NTG of pattern (1) with M-FO ψ is a NTG of pattern (6).
(v) A NTG of pattern (5) with M-FO ψ is a NTG of pattern (2).
(vi) A NTG of pattern (6) with M-FO ϕ is a NTG of pattern (2).
(vii) A NTG of pattern (7) with M-FO ψ is a NTG of pattern (8).

3. Neutrosophic Bi-Topological Group

In this section, we study neutrosophic bi-topological groups, also, we define new types of neutrosophic bi-topological groups and investigate its basic properties.

Definition 3.1.

Let G be a neutrosophic group. Let T_1, T_2 be two neutrosophic topologies on G. A neutrosophic group G, with two neutrosophic topologies is called a neutrosophic bi-topological group (G, T_1, T_2) (NBi-TG) if and only if the mappings

(1) $\phi : (G, T_i) \times (G, T_i) \to (G, T_i)$ defined by $\phi(x, y) = x.y$, and

(2) ψ: $(G, T_i) \to (G, T_i)$ defined by $\psi(x) = x^{-1}$ for x ∈ G are both T_i −continuous; $for\ i = 1,2$.

An equivalent condition to (i) and (ii) is that, the function $h : (G, T_i) \times (G, T_i) \to (G, T_i)$ defined by $\phi(x, y) = x.y^{-1}$, for x, y ∈ G is T_i −N-continuous; $for\ i = 1,2$.

Example 3.2.

Clearly, every neutrosophic group is a neutrosophic bi-topological group with the discrete neutrosophic topology and indiscrete neutrosophic topology.

Theorem 3.3.

Let G be a neutrosophic group. Let T_1, T_2 be two neutrosophic topologies on G, then :

(G, T_1, T_2) is a neutrosophic bi-topological group (NBi-TG) iff both (G, T_1) , (G, T_2) are neutrosophic topological groups.

Proof.

⇒ : Since (G, T_1, T_2) is a neutrosophic bi-topological group (NBi-TG)**,** then the mappings

(1) $\phi : (G, T_i) \times (G, T_i) \to (G, T_i)$ defined by $\phi(x, y) = x.y$, and

(2) ψ: $(G, T_i) \to (G, T_i)$ defined by $\psi(x) = x^{-1}$ for x ∈ G are both T_i −continuous; $for\ i = 1,2$.

Therefore both (G, T_1) , (G, T_2) are neutrosophic topological groups.

⇐ : Since (G, T_1) is a neutrosophic topological groups, then the mappings

(1) $\phi : (G, T_1) \times (G, T_1) \to (G, T_1)$ defined by $\phi(x, y) = x.y$, and

(2) ψ: $(G, T_1) \to (G, T_1)$ defined by $\psi(x) = x^{-1}$ for x ∈ G are both T_1 −continuous.

Since (G, T_2) is a neutrosophic topological groups, then the mappings

(1) $\phi : (G, T_2) \times (G, T_2) \to (G, T_2)$ defined by $\phi(x, y) = x.y$, and

(2) ψ: $(G, T_2) \to (G, T_2)$ defined by $\psi(x) = x^{-1}$ for x ∈ G are both T_2 −continuous.

Therefore (G, T_1, T_2) is a neutrosophic bi-topological group (NBi-TG).

Definition 3.4.
Let G be a set that equips with a group structure and a neutrosophic topology.Then G is called:
(i) NTG of pattern (1) (NBi-TG(1)) iff the operation function ϕ: $(G, T_i) \times (G, T_i) \to (G, T_i)$ and the inversion function ψ: $(G, T_i) \to (G, T_i)$ are both T_i −N-α-continuous, $for\ i = 1,2$.

(ii) NTG of pattern (2) (NBi-TG(2)) iff the operation function ϕ: $(G, T_i) \times (G, T_i) \rightarrow (G, T_i)$ and the inversion function ψ: $(G, T_i) \rightarrow (G, T_i)$ are both T_i −N-α-irresolute, $for\ i = 1,2$.

(iii) NTG of pattern (3) (NBi-TG(3)) iff the operation function ϕ: $(G, T_i) \times (G, T_i) \rightarrow (G, T_i)$ is T_i −N-α-continuous and the inversion function ψ: $(G, T_i) \rightarrow (G, T_i)$ is T_i −N-continuous, $for\ i = 1,2$.
(iv) NTG of pattern (4) (NBi-TG(4))iff the operation function ϕ: $(G, T_i) \times (G, T_i) \rightarrow (G, T_i)$ is T_i −N-α-irresolute and the inversion function ψ: $(G, T_i) \rightarrow (G, T_i)$is T_i −N-continuous, $for\ i = 1,2$.
(v) NTG of pattern (5) (NBi-TG(5)) iff the operation function ϕ: $(G, T_i) \times (G, T_i) \rightarrow (G, T_i)$ is T_i −N-α-irresolute and the inversion function ψ: $(G, T_i) \rightarrow (G, T_i)$is T_i −N-α-continuous, $for\ i = 1,2$.
(vi) NTG of pattern (6) (NBi-TG(6)) iff the operation function ϕ: $(G, T_i) \times (G, T_i) \rightarrow (G, T_i)$ is T_i −N-α-continuous and the inversion function ψ: $(G, T_i) \rightarrow (G, T_i)$is T_i −N-α-irresolute, $for\ i = 1,2$.
(vii) NTG of pattern (7) (NBi-TG(7)) iff the operation function ϕ: $(G, T_i) \times (G, T_i) \rightarrow (G, T_i)$ is T_i −N-continuous, and the inversion function ψ: $(G, T_i) \rightarrow (G, T_i)$ is T_i −N-α-continuous, $for\ i = 1,2$.
(viii) NTG of pattern (8) (NBi-TG(8)) iff the operation function ϕ: $(G, T_i) \times (G, T_i) \rightarrow (G, T_i)$ is T_i −N-continuous, and the inversion function ψ: $(G, T_i) \rightarrow (G, T_i)$ is T_i −N-α-irresolute, $for\ i = 1,2$.

Theorem 3.5.

Let G be a set that equips with a neutrosophic group structure and two neutrosophic topologies T_1, T_2. Then (G, T_1, T_2) is a NBi-TG of pattern (i) iff both (G, T_1) , (G, T_2) are neutrosophic topological groups of pattern (i), i=1,2,3,…,8.

Proof.

Proof follows from Definition 3.4.

Theorem 3.6.

(i) NBi-TG ⇒ NBi-TG of pattern (i), where i=1,3,7.
(ii) NBi-TG of pattern (2) ⇒ NBi-TG of pattern (5).
(iii) NBi-TG of pattern (2) ⇒ NBi-TG of pattern (6).
(iv) NBi-TG of pattern (4) ⇒ NBi-TG of pattern (3).
(v) NBi-TG of pattern (4) ⇒ NBi-TG of pattern (5).
(vi) NBi-TG of pattern (i) ⇒ NBi-TG of pattern (1), where i=2,3,…,8.

Proof.

(i) Let G be a NBi-TG, then both (G, T_1) , (G, T_2) are neutrosophic topological groups. By Remark 2.7, we have that (G, T_1) and (G, T_2) are neutrosophic topological groups are both NTG of pattern (i), where i=1,3,7.
Hence, G is a NBi-TG of pattern (i), where i=1,3,7.
(ii) Let G be a NBi-TG of pattern (2), then both (G, T_1) , (G, T_2) are NTG of pattern (2). By Remark 2.7, we have that (G, T_1) and (G, T_2) are both NTG of pattern (5).
Hence, G is a NBi-TG of pattern (5).
(iii) Let G be a NBi-TG of pattern (2), then both (G, T_1) , (G, T_2) are NTG of pattern (2). By Remark 2.7, we have that (G, T_1) and (G, T_2) are both NTG of pattern (6).
Hence, G is a NBi-TG of pattern (6).
(iv) Let G be a NBi-TG of pattern (4), then both (G, T_1) , (G, T_2) are NTG of pattern (4). By Remark 2.7, we have that (G, T_1) and (G, T_2) are both NTG of pattern (3).
Hence, G is a NBi-TG of pattern (3).

(v) Let G be a NBi-TG of pattern (4), then both (G, T_1) , (G, T_2) are NTG of pattern (4). By Remark 2.7, we have that (G, T_1) and (G, T_2) are both NTG of pattern (5).
Hence, G is a NBi-TG of pattern (5).
(vi) Let G be a NBi-TG of pattern (i), then both (G, T_1) , (G, T_2) are NTG of pattern (i), i=2,3,…,8.. By Remark 2.7, we have that (G, T_1) and (G, T_2) are both NTG of pattern (1).
Hence, G is a NBi-TG of pattern (1).

Theorem 3.7.

(i) A NBi-TG of pattern (3) with M-FO ϕ is a NBi-TG of pattern (4).
(ii) A NBi-TG of pattern (1) with M-FO ψ and $\mathcal{M}$-FO ϕ is a NBi-TG of pattern (2).
(iii) A NBi-TG of pattern (1) with M-FO ϕ is a NBi-TG of pattern (5).

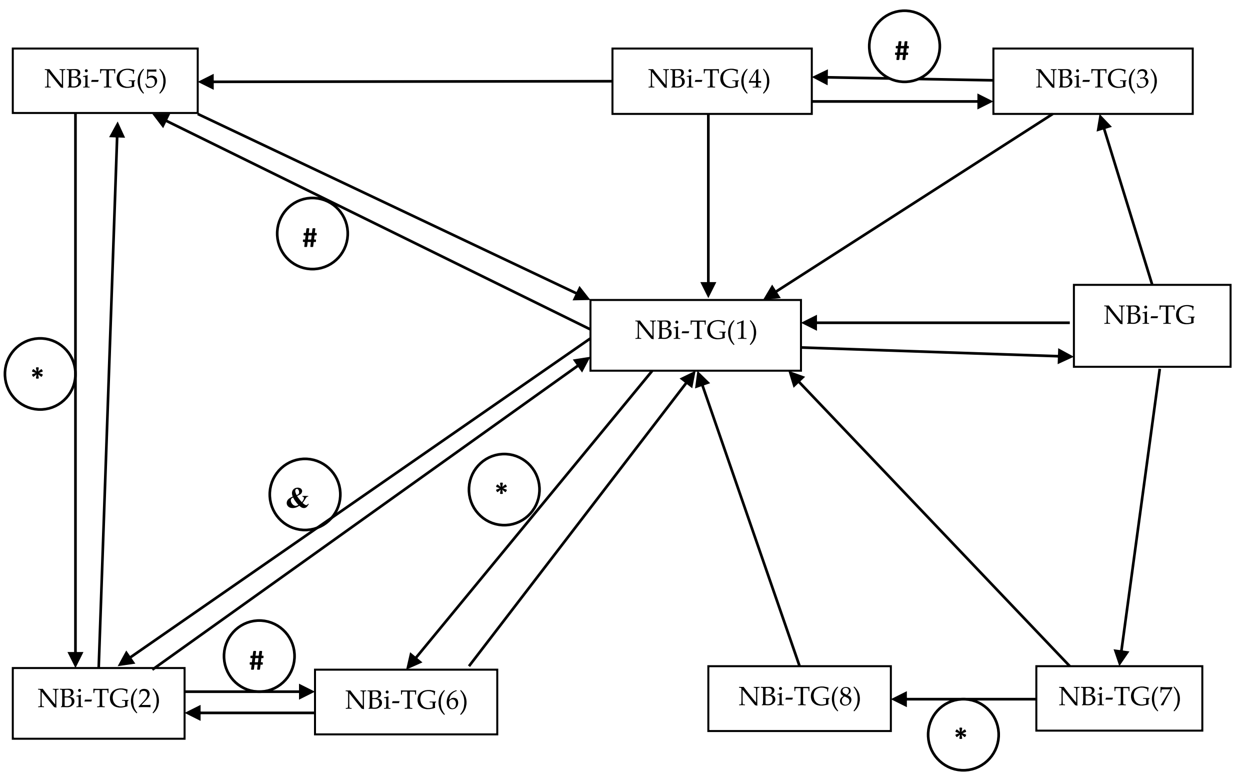
(iv) A NBi-TG of pattern (1) with M-FO ψ is a NBi-TG of pattern (6).
(v) A NBi-TG of pattern (5) with M-FO ψ is a NBi-TG of pattern (2).
(vi) A NBi-TG of pattern (6) with M-FO ϕ is a NBi-TG of pattern (2).
(vii) A NBi-TG of pattern (7) with M-FO ψ is a NBi-TG of pattern (8).

Proof.

(i) Let G be a NBi-TG of pattern (3), then both (G, T_1) , (G, T_2) are NTG of pattern (3). Since μ is M-FO. So by Theorem 2.9, we get that (G, T_1) , (G, T_2) are NTG of pattern (4). Hence NBi-TG of pattern (4).
(ii) Let G be a NBi-TG of pattern (1), then both (G, T_1) , (G, T_2) are NTG of pattern (1). Since μ is M-FO and ψ is M-FO. So by Theorem 2.9, we get that (G, T_1) , (G, T_2) are NTG of pattern (2). Hence NBi-TG of pattern (2).

Remark 3.8.

In the following diagram we show the relationships between NBi-TG of pattern (i) and NBi-TG, i=1,2,…,8.



$*$ means ψ is M –function, # means ϕ is M –function , & means ϕ, ψ are M –function.

Remark 3.9.

Let A be an neutrosophic open set in a NTG (G, T_1). Then $A^{-1}$is neutrosophic open set in (G, T_1).

Proof.

Let f : $(G, T_1) \rightarrow (G, T_1)$ be a mapping defined by f(x)= x, for each x∈G.

Since f is neutrosophic homeomorphism, therefore $f(A) = A^{-1}$ is neutrosophic open set in (G, T_1).

Remark 3.10.

Let A be an neutrosophic open set in a NTG (G, T_2). Then $A^{-1}$is neutrosophic open set in (G, T_2).

Proof.

Let f : $(G, T_2) \rightarrow (G, T_2)$ be a mapping defined by f(x)= x, for each x∈G.

Since f is neutrosophic homeomorphism, therefore $f(A) = A^{-1}$ is neutrosophic open set in (G, T_2).

Conclusion

In this work, we have introduced neutrosophic bi-topological groups and new types of neutrosophic bi-topological groups. Finally, This paper is just a beginning of a new structure and we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application.

References

[1] F. Smarandache; "A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability". American Research Press, Rehoboth, NM, (1999).

[2] F. Smarandache; "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics" University of New Mexico, Gallup, NM 87301, USA (2002).

[3] L. A. Zadeh; "Fuzzy Sets". Inform. Control,Vol. 8, 338-353, (1965).

[4] K. Atanassov; "intuitionistic fuzzy sets". fuzzy sets and systems, Vol. 20, 87-96, (1986).

[5] A. A. Salma; S.A. Alblowi; "Neutrosophic set and neutrosophic topological spaces". IOSR J. Math., Vol. 3(4), 31–35. (2012).

[6] A. A. A. Agboola; A.D. Akinola and O.Y. Oyebola; "Neutrosophic Rings I", Int. J. of Math. Comb., vol. 4, 1-14, (2011).

[7] A. A. A. Agboola; Akwu A.O. and Y.T. Oyebo; "Neutrosophic Groups and Neutrosopic Subgroups", Int. J.of Math. Comb., vol. 3, 1-9, (2012).

[8] A. A. A. Agboola; "On Refined Neutrosophic Algebraic Structures", Neutrosophic Sets and Systems, vol.10, 99-101, (2015).

[9] E. O. Adeleke; A.A.A. Agboola and F. Smarandache; "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science (IJNS), vol. 2(2), pp. 77-81, (2020). (DOI:10.5281/zenodo.3728222)

[10] E.O. Adeleke; A.A.A. Agboola and F. Smarandache; "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science (IJNS), vol. 2(2), pp. 89-94, (2020). (DOI:10.5281/zenodo.3728235.

[11] Q. H. Imran; A. H. Al-Obaidi; F. Smarandache; "On Some Types of Neutrosophic Topological Groups with Respect to Neutrosophic Alpha Open Sets". Neutrosophic Sets Syst, Vol. 33, 426–433, (2020).

[12] R. Sumathi; I. Arockiarani; "Topological Group Structure of Neutrosophic set". Journal of Advanced Studies in Topology, Vol. 7, 12-20, (2016).

[13] W. B. Vasantha Kandasamy; and F. Smarandache; "Neutrosophic Rings", Neutrosophic Sets and Systems, (2006).

[14] A. A. Salama, F. Smarandache and V. Kroumov, Neutrosophic Closed Set and Neutrosophic Continuous Functions. Neutrosophic Sets and Systems, 4(2014), 2-8.

[15] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, On Some New Notions and Functions in Neutrosophic Topological Spaces, Neutrosophic Sets and Systems, Vol.16, 2017, pp.16-19.