Neutrosophic δ-Open Maps and Neutrosophic δ-Closed Maps

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Abstract

The neutrosophic δ-open set is one of the stronger form then neutrosophic topological spaces. In this article, we introduce the concept of neutrosophic δ-open maps and neutrosophic δ-closed maps and investigate their neighbour maps such as $\delta\mathcal{S}$, $\delta\mathcal{P}$ & e^* open maps and closed maps of neutrosophic topological spaces. Also, we analyse some of their related properties and extended to neutrosophic e^*-homeomorphism and neutrosophic $e^*T_{\frac{1}{2}}$-space in neutrosophic topological spaces. Furthermore, these kinds of δ-open functions have strong application in the area of neural network and image processing theory.

Keywords: neutrosophic δ-open map, neutrosophic δ-closed map, neutrosophic e^*-open map, neutrosophic e^*-closed map, netrosophic $e^*T_{\frac{1}{2}}$-space, neutrosophic e^*-homeomorphism.

1 Introduction

In 1965, the idea of fuzzy set (briefly, fs) gives a degree of membership function was first introduced by Zadeh.[24] In 1968, the concept of fuzzy topological space (briefly, fts) was introduced by Chang.[9] In 1983, the next stage of fuzzy set was developed by Atanassov[6–8] which gives a degree of membership and a degree of non-membership functions named as intutionistic fuzzy set (briefly, Ifs). In 1997, Coker[10] introduced the concept of intutionistic fuzzy topological space (briefly, $Ifts$) in intutionistic fuzzy set. In 2005, the concept of neutrosophic crisp set and neutrosophic set (briefly, N_ss) was investigated by Smaradache.[14,19,20] After the introduction of neutrosophic set, there are many fields of mathematics and various applications.[1,11,13,18] In 2012, Salama and Alblowi[15] defined neutrosophic topological space (briefly, N_sts) and many of its applications in.[16,17] The neutrosophic closed sets and neutrosophic continuous functions were introduced by Salama et al.[17] in 2014. Saha[22] defined δ-open sets in topological spaces. Vadivel et al. in[23] introduced δ-open sets in a neutrosophic topological space. The generalization of open and closed functions in topological spaces have been introduced and investigated over the course of years. The open and closed functions stand among the most important and most researched points in the whole of mathematical science. Its importance is significant in various areas of mathematics and related sciences.

In this article, we introduce the idea of neutrosophic δ-open maps and neutrosophic δ-closed maps and relate with their neighbour maps in neutrosophic topological spaces. Futhermore, the work is extended to neutrosophic e^*-homeomorphism and neutrosophic $e^*T_{\frac{1}{2}}$-space in neutrosophic topological spaces and we obtain some of its basic properties.

2 Preliminaries

Definition 2.1. [15] Let Y be a non-empty set. A neutrosophic set (briefly, N_ss) L is an object having the form $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$ where $\mu_L \to [0, 1]$ denote the degree of membership function, $\sigma_L \to [0, 1]$ denote the degree of indeterminacy function and $\nu_L \to [0, 1]$ denote the degree of non-membership function respectively of each element $y \in Y$ to the set L and $0 \le \mu_L(y) + \sigma_L(y) + \nu_L(y) \le 3$ for each $y \in Y$.

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Remark 2.2. [15] A $N_s s$ $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$ can be identified to an ordered triple $\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle$ in $[0,1]$ on Y.

Definition 2.3. [15] Let Y be a non-empty set and the $N_s s$'s L and M in the form $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$, $M = \{\langle y, \mu_M(y), \sigma_M(y), \nu_M(y)\rangle : y \in Y\}$, then

(i) $0_N = \langle y, 0, 0, 1\rangle$ and $1_N = \langle y, 1, 1, 0\rangle$,

(ii) $L \subseteq M$ iff $\mu_L(y) \leq \mu_M(y), \sigma_L(y) \leq \sigma_M(y)$ & $\nu_L(y) \geq \nu_M(y) : y \in Y$,

(iii) $L = M$ iff $L \subseteq M$ and $M \subseteq L$,

(iv) $1_N - L = \{\langle y, \nu_L(y), 1 - \sigma_L(y), \mu_L(y)\rangle : y \in Y\} = L^c$,

(v) $L \cup M = \{\langle y, \max(\mu_L(y), \mu_M(y)), \max(\sigma_L(y), \sigma_M(y)), \min(\nu_L(y), \nu_M(y))\rangle : y \in Y\}$,

(vi) $L \cap M = \{\langle y, \min(\mu_L(y), \mu_M(y)), \min(\sigma_L(y), \sigma_M(y)), \max(\nu_L(y), \nu_M(y))\rangle : y \in Y\}$.

Definition 2.4. [15] A neutrosophic topology (briefly, $N_s t$) on a non-empty set Y is a family Ψ_N of neutrosophic subsets of Y satisfying

(i) $0_N, 1_N \in \Psi_N$.

(ii) $L_1 \cap L_2 \in \Psi_N$ for any $L_1, L_2 \in \Psi_N$.

(iii) $\bigcup L_x \in \Psi_N, \forall\ L_x : x \in X \subseteq \Psi_N$.

Then (Y, Ψ_N) is called a neutrosophic topological space (briefly, $N_s ts$) in Y. The Ψ_N elements are called neutrosophic open sets (briefly, $N_s os$) in Y. A $N_s s$ C is called a neutrosophic closed sets (briefly, $N_s cs$) iff its complement C^c is $N_s os$.

Definition 2.5. [15] Let (Y, Ψ_N) be $N_s ts$ on Y and L be an $N_s s$ on Y, then the neutrosophic interior of L (briefly, $N_s int(L)$) and the neutrosophic closure of L (briefly, $N_s cl(L)$) are defined as

$$N_s int(L) = \bigcup\{I : I \subseteq L \text{ and } I \text{ is a } N_s os \text{ in } Y\}$$

$$N_s cl(L) = \bigcap\{J : L \subseteq J \text{ and } J \text{ is a } N_s cs \text{ in } Y\}.$$

Definition 2.6. [5] Let (Y, Ψ_N) be $N_s ts$ on Y and L be an $N_s s$ on Y. Then L is said to be a neutrosophic regular open set (briefly, $N_s ros$) if $L = N_s int(N_s cl(L))$.

The complement of a $N_s ros$ is called a neutrosophic regular closed set (briefly, $N_s rcs$) in Y.

Definition 2.7. [23] A set K is said to be a neutrosophic

(i) δ interior of G (briefly, $N_s \delta int(K)$) is defined by $N_s \delta int(K) = \bigcup\{B : B \subseteq K$ and B is a $N_s ros$ in $Y\}$.

(ii) δ closure of K (briefly, $N_s \delta cl(K)$) is defined by $N_s \delta cl(K) = \bigcap\{J : K \subseteq J$ and J is a $N_s rcs$ in $Y\}$.

Definition 2.8. [23] A set L is said to be a neutrosophic

(i) δ-open set (briefly, $N_s \delta os$) if $L = N_s \delta int(L)$.

(ii) δ-pre open set (briefly, $N_s \delta \mathcal{P} os$) if $L \subseteq N_s int(N_s \delta cl(L))$.

(iii) δ-semi open set (briefly, $N_s \delta \mathcal{S} os$) if $L \subseteq N_s cl(N_s \delta int(L))$.

(iv) δ-α-open set (briefly, $N_s \delta \alpha os$) if $L \subseteq N_s int(N_s cl(N_s \delta int(L)))$.

(v) e^*-open set (briefly, $N_s e^* os$) if $L \subseteq N_s cl(N_s int(N_s \delta cl(L)))$.

The complement of an $N_s \delta os$ (resp. $N_s \delta \mathcal{P} os$, $N_s \delta \mathcal{S} os$, $N_s \delta \alpha os$ & $N_s e^* os$) is called a neutrosophic δ (resp. δ-pre, δ-semi, $\delta\alpha$ & e^*) closed set (briefly, $N_s \delta cs$ (resp. $N_s \delta \mathcal{P} cs$, $N_s \delta \mathcal{S} cs$, $N_s \delta \alpha cs$ & $N_s e^* cs$)) in Y.

Definition 2.9. [23] Let (X, τ_N) and (Y, σ_N) be any two Nts's. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is said to be neutrosophic e^* continuous (briefly, $N_s e^* Cts$) if the inverse image of every $N_s os$ in (Y, σ_N) is an $N_s e^* os$ in (X, τ_N).

Definition 2.10. [12] Let (X, τ_N) and (Y, σ_N) be any two $N_s ts$'s. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is said to be neutrosophic homeomorphism (briefly, $N_s Hom$) if h and h^{-1} are $N_s Cts$ maps.

3 Neutrosophic δ-open map

Definition 3.1. Let (X, τ_N) and (Y, σ_N) be any two Nts's. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is said to be neutrosophic (resp. δ, $\delta\mathcal{S}$, $\delta\mathcal{P}$ and e^*) open map (briefly, N_sO (resp. $N_s\delta O$, $N_s\delta\mathcal{S}O$, $N_s\delta\mathcal{P}O$ and N_se^*O)) if the image of every N_sos in (X, τ_N) is an N_sos (resp. $N_s\delta os$, $N_s\delta\mathcal{S}os$, $N_s\delta\mathcal{P}os$ and N_se^*os) in (Y, σ_N).

Theorem 3.2. The following statements hold:

(i) Every $N_s\delta O$ map is a N_sO map.

(ii) Every N_sO map is an $N_s\delta\mathcal{S}O$ map.

(iii) Every N_sO map is an $N_s\delta\mathcal{P}O$ map.

(iv) Every $N_s\delta\mathcal{S}O$ map is an N_se^*O map.

(v) Every $N_s\delta\mathcal{P}O$ map is an N_se^*O map.

(vi) Every $N_s\delta\alpha O$ map is an $N_s\delta\mathcal{S}O$ map.

(vii) Every $N_s\delta\alpha O$ map is an $N_s\delta\mathcal{P}O$ map.

Proof. (i) Let λ be an $N_s\delta os$ in X. Since h is $N_s\delta O$ map, $h(\lambda)$ is an $N_s\delta os$ in Y. Since every $N_s\delta os$ is an N_sos,[23] $h(\lambda)$ is an N_sos in Y. Hence h is an N_sO map.

The others are similar. □

$$\begin{tikzcd}
 & \boxed{N_s\delta O \text{ map}} \arrow[d] & \\
 & \boxed{N_sO \text{ map}} \arrow[dl] \arrow[dr] & \\
\boxed{N_s\delta\mathcal{S}O \text{ map}} \arrow[r] & \boxed{N_se^*O \text{ map}} & \boxed{N_s\delta\mathcal{P}O \text{ map}} \arrow[l] \\
 & \boxed{N_s\delta\alpha O \text{ map}} \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Figure 1: $N_s\delta O$ map's in Nts .

Example 3.3. Let $X = \{l\} = Y$ and define N_ss's X_1 in X and Y_1 and Y_2 in Y by

$$X_1 = \langle X, (\frac{\mu_l}{0.2}, \frac{\sigma_l}{0.5}, \frac{\nu_l}{0.8})\rangle,\ Y_1 = \langle Y, (\frac{\mu_l}{0.2}, \frac{\sigma_l}{0.5}, \frac{\nu_l}{0.8})\rangle,\ Y_2 = \langle Y, (\frac{\mu_l}{0.5}, \frac{\sigma_l}{0.5}, \frac{\nu_l}{0.5})\rangle.$$

Then we have $\tau_N = \{0_N, X_1, 1_N\}$ and $\sigma_N = \{0_N, Y_1, Y_2, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be an identity map, then h is N_sO map but not $N_s\delta O$ map.

Example 3.4. Let $X = \{l, m, n\} = Y$ and define N_ss's X_1 in X and Y_1, Y_2 and Y_3 in Y by

$$\begin{aligned}
X_1 &= \langle X, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.4}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.6}, \frac{\nu_n}{0.6})\rangle,\\
Y_1 &= \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle,\\
Y_2 &= \langle Y, (\frac{\mu_l}{0.1}, \frac{\mu_m}{0.1}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.9}, \frac{\nu_m}{0.9}, \frac{\nu_n}{0.6})\rangle,\\
Y_3 &= \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.4}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.6}, \frac{\nu_n}{0.6})\rangle.
\end{aligned}$$

Then we have $\tau_N = \{0_N, X_1, 1_N\}$ and $\sigma_N = \{0_N, Y_1, Y_2, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be an identity map, then h is an

(i) $N_s\delta\mathcal{S}O$ map but not N_sO map (resp. $N_s\delta\alpha O$ map).

(ii) N_se^*O map but not $N_s\delta\mathcal{P}O$ map.

Example 3.5. Let $X = \{l, m, n\} = Y$ and define N_ss's X_1 in X and Y_1, Y_2, Y_3 and Y_4 in Y by

$$\begin{aligned}
X_1 &= \langle X, (\frac{\mu_l}{0.3}, \frac{\mu_m}{0.5}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.7}, \frac{\nu_m}{0.5}, \frac{\nu_n}{0.6})\rangle,\\
Y_1 &= \langle Y, (\frac{\mu_l}{0.3}, \frac{\mu_m}{0.5}, \frac{\mu_n}{0.5}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.7}, \frac{\nu_m}{0.5}, \frac{\nu_n}{0.5})\rangle,\\
Y_2 &= \langle Y, (\frac{\mu_l}{0.4}, \frac{\mu_m}{0.2}, \frac{\mu_n}{0.6}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.6}, \frac{\nu_m}{0.8}, \frac{\nu_n}{0.4})\rangle,\\
Y_3 &= \langle Y, (\frac{\mu_l}{0.4}, \frac{\mu_m}{0.5}, \frac{\mu_n}{0.6}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.6}, \frac{\nu_m}{0.5}, \frac{\nu_n}{0.4})\rangle,\\
Y_4 &= \langle Y, (\frac{\mu_l}{0.3}, \frac{\mu_m}{0.5}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.7}, \frac{\nu_m}{0.5}, \frac{\nu_n}{0.6})\rangle.
\end{aligned}$$

Then we have $\tau_N = \{0_N, X_1, 1_N\}$ and $\sigma_N = \{0_N, Y_1, Y_2, Y_3, Y_1 \cap Y_2, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be an identity map, then h is an

(i) $N_s\delta\mathcal{P}O$ map but not N_sO map (resp. $N_s\delta\alpha O$ map).

(ii) N_se^*O map but not $N_s\delta\mathcal{S}O$ map.

Theorem 3.6. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is N_se^*O iff for every N_ss λ of (X, τ_N), $h(N_sint(\lambda)) \subseteq N_se^*int(h(\lambda))$.

Proof. Necessity: Let h be a N_se^*O map and λ be a N_sos in (X, τ_N). Now, $N_sint(\lambda) \subseteq \lambda$ implies $h(N_sint(\lambda)) \subseteq h(\lambda)$. Since h is a N_se^*O map, $h(N_sint(\lambda))$ is N_se^*os in (Y, σ_N) such that $h(N_sint(\lambda)) \subseteq h(\lambda)$ therefore $h(N_sint(\lambda)) \subseteq N_se^*int(h(\lambda))$.

Sufficiency: Assume λ is a N_sos of (X, τ_N). Then $h(\lambda) = h(N_sint(\lambda)) \subseteq N_se^*int(h(\lambda))$. But N_se^*int $(h(\lambda)) \subseteq h(\lambda)$. So $h(\lambda) = N_se^*int(\lambda)$ which implies $h(\lambda)$ is a N_se^*os of (Y, σ_N) and hence h is a N_se^*O. □

Theorem 3.7. If $h : (X, \tau_N) \to (Y, \sigma_N)$ is a N_se^*O map then $N_sint(h^{-1}(\lambda)) \subseteq h^{-1}(N_se^*int(\lambda))$ for every N_ss λ of (Y, σ_N).

Proof. Let λ be a N_ss of (Y, σ_N). Then $N_sint(h^{-1}(\lambda))$ is a N_sos in (X, τ_N). Since h is N_se^*O, $h(N_sint$ $(h^{-1}(\lambda))$ is N_se^*o in (Y, σ_N) and hence $h(N_sint(h^{-1}(\lambda))) \subseteq N_se^*int(h(h^{-1}(\lambda))) \subseteq N_se^*int(\lambda)$. Thus $N_sint(h^{-1}(\lambda)) \subseteq h^{-1}(N_se^*int(\lambda))$. □

Theorem 3.8. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is N_se^*O iff for each N_ss μ of (Y, σ_N) and for each N_scs λ of (X, τ_N) containing $h^{-1}(\mu)$ there is an N_se^*cs ψ of (Y, σ_N) such that $\mu \subseteq \lambda$ and $h^{-1}(\psi) \subseteq \lambda$.

Proof. Necessity: Assume h is a N_se^*O map. Let μ be the N_scs of (Y, σ_N) and λ is a N_scs of (X, τ_N) such that $h^{-1}(\mu) \subseteq \lambda$. Then $\psi = (h^{-1}(\lambda^c))^c$ is N_se^*cs of (Y, σ_N) such that $h^{-1}(\psi) \subseteq \lambda$.

Sufficiency: Assume ω is a N_sos of (X, τ_N). Then $h^{-1}((h(\omega))^c \subseteq \omega^c$ and ω^c is N_scs in (X, τ_N). By hypothesis there is a N_se^*cs ψ of (Y, σ_N) such that $(h(\omega))^c \subseteq \psi$ and $h^{-1}(\psi) \subseteq \omega^c$. Therefore $\omega \subseteq (h^{-1}(\psi))^c$. Hence $\psi^c \subseteq h(\omega) \subseteq h((h^{-1}(\psi))^c) \subseteq \psi^c$ which implies $h(\omega) = \psi^c$. Since ψ^c is N_se^*os of (Y, σ_N). Hence $h(\omega)$ is N_se^*o in (Y, σ_N) and thus h is N_se^*O map. □

Theorem 3.9. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is N_se^*O iff $h^{-1}(N_se^*cl(\lambda) \subseteq N_scl(h^{-1}(\lambda))$ for every N_ss λ of (Y, σ_N).

Proof. Necessity: Assume h is a N_se^*O map. For any N_ss λ of (Y, σ_N), $h^{-1}(\lambda) \subseteq N_scl(h^{-1}(\lambda))$. Therefore by Theorem 3.8 there exists a N_se^*cs μ in (Y, σ_N) such that $\lambda \subseteq \mu$ and $h^{-1}(\mu) \subseteq N_scl(h^{-1}(\lambda))$. Therefore we obtain that $h^{-1}(N_se^*cl(\lambda)) \subseteq h^{-1}(\mu) \subseteq N_scl(h^{-1}(\lambda))$.

Sufficiency: Assume λ is a N_ss of (Y, σ_N) and μ is a N_scs of (X, τ_N) containing $h^{-1}(\lambda)$. Put $\zeta = cl(\lambda)$, then $\lambda \subseteq \zeta$ and ζ is N_se^*c and $h^{-1}(\zeta) \subsetneq cl(h^{-1}(\lambda)) \subseteq \mu$. Then by Theorem 3.8, h is N_se^*O map. □

Theorem 3.10. If $h : (X, \tau_N) \to (Y, \sigma_N)$ and $g : (Y, \sigma_N) \to (Z, \rho_N)$ are two neutrosophic maps and $g \circ h : (X, \tau_N) \to (Z, \rho_N)$ is N_se^*O. If $g : (Y, \sigma_N) \to (Z, \rho_N)$ is N_se^*Irr then $h : (X, \tau_N) \to (Y, \sigma_N)$ is N_se^*O map.

Proof. Let ψ be a N_sos in (X, τ_N). Then $g \circ h(\psi)$ is N_se^*os of (Z, ρ_N) because $g \circ h$ is N_se^*O map. Since g is N_se^*Irr and $g \circ h(\psi)$ is N_se^*os of (Z, ρ_N), $g^{-1}(g \circ h(\psi)) = h(\psi)$ is N_se^*os in (Y, σ_N). Hence h is N_se^*O map. □

Theorem 3.11. If $h : (X, \tau_N) \to (Y, \sigma_N)$ is N_sO and $g : (Y, \sigma_N) \to (Z, \rho_N)$ is N_se^*O maps then $g \circ h : (X, \tau_N) \to (Z, \rho_N)$ is N_se^*O.

Proof. Let ψ be a N_sos in (X, τ_N). Then $h(\psi)$ is a N_sos of (Y, σ_N) because h is a N_sO map. Since g is N_se^*O, $g(h(\psi)) = (g \circ h)(\psi)$ is N_se^*os of (Z, ρ_N). Hence $g \circ h$ is N_se^*O map. □

4 Neutrosophic e-closed map

Definition 4.1. Let (X, τ_N) and (Y, σ_N) be any two Nts's. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is said to be neutrosophic (resp. δ, $\delta\mathcal{S}$, $\delta\mathcal{P}$ and e^*) closed map (briefly, N_sC (resp. $N_s\delta C$, $N_s\delta\mathcal{S}C$, $N_s\delta\mathcal{P}C$ and N_se^*C)) if the image of every N_scs in (X, τ_N) is a N_scs (resp. $N_s\delta cs$, $N_s\delta\mathcal{S}cs$, $N_s\delta\mathcal{P}cs$ and N_se^*cs) in (Y, σ_N).

Theorem 4.2. The following statements are hold:

(i) Every $N_s\delta C$ map is a N_sC map.

(ii) Every N_sC map is a $N_s\delta\mathcal{S}C$ map.

(iii) Every N_sC map is a $N_s\delta\mathcal{P}C$ map.

(iv) Every $N_s\delta\mathcal{S}C$ map is a N_se^*C map.

(v) Every $N_s\delta\mathcal{P}C$ map is a N_se^*C map.

(vi) Every $N_s\delta\alpha C$ map is a $N_s\delta\mathcal{S}C$ map.

(vii) Every $N_s\delta\alpha C$ map is a $N_s\delta\mathcal{P}C$ map.

Proof. (i) Let λ be a $N_s\delta cs$ in X. Since h is $N_s\delta C$ map, $h(\lambda)$ is a $N_s\delta cs$ in Y. Since every $N_s\delta cs$ is a N_scs,[23] $h(\lambda)$ is a N_scs in Y. Hence h is a N_sC map.
The others are similar. □

$$\begin{tikzcd}
& N_s\delta C \text{ map} \arrow[d] & \\
& N_sC \text{ map} \arrow[dl] \arrow[dr] & \\
N_s\delta\mathcal{S}C \text{ map} \arrow[r] & N_se^*C \text{ map} & N_s\delta\mathcal{P}C \text{ map} \arrow[l] \\
& N_s\delta\alpha C \text{ map} \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Figure 2: $N_s\delta C$ map's in Nts .

Example 4.3. In Example 3.3, h is a N_sC map but not $N_s\delta C$ map.

Example 4.4. In Example 3.4, h is a

(i) $N_s\delta\mathcal{S}C$ map but not N_sC map (resp. $N_s\delta\alpha C$ map).

(ii) N_se^*C map but not $N_s\delta\mathcal{P}C$ map.

Example 4.5. In Example 3.5, h is a

(i) $N_s\delta\mathcal{P}C$ map but not N_sC map (resp. $N_s\delta\alpha C$ map).

(ii) N_se^*C map but not $N_s\delta\mathcal{S}C$ map.

Theorem 4.6. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is N_se^*C iff for each N_ss μ of (Y, σ_N) and for each N_sos λ of (X, τ_N) containing $h^{-1}(\mu)$ there is an N_se^*os ψ of (Y, σ_N) such that $\mu \subseteq \psi$ and $h^{-1}(\psi) \subseteq \lambda$.

Proof. Necessity: Assume h is a N_se^*C map. Let μ be the N_scs of (Y, σ_N) and λ is a N_sos of (X, τ_N) such that $h^{-1}(\mu) \subseteq \lambda$. Then $\psi = Y - h^{-1}(\lambda^c)$ is N_se^*os of (Y, σ_N) such that $h^{-1}(\psi) \subseteq \lambda$.

Sufficiency: Assume ψ is a N_scs of (X, τ_N). Then $(h(\psi))^c$ is a N_ss of (Y, σ_N) and ψ^c is N_sos in (X, τ_N) such that $h^{-1}((h(\psi))^c) \subseteq \psi^c$. By hypothesis there is a N_se^*os ψ of (Y, σ_N) such that $(h(\psi))^c \subseteq \psi$ and $h^{-1}(\psi) \subseteq \psi^c$. Therefore $\psi \subseteq (h^{-1}(\psi))^c$. Hence $\psi^c \subseteq h(\psi) \subseteq h((h^{-1}(\psi))^c) \subseteq \psi^c$ which implies $h(\psi) = \psi^c$. Since ψ^c is N_se^*cs of (Y, σ_N). Hence $h(\psi)$ is N_se^*c in (Y, σ_N) and thus h is N_se^*C map. □

Theorem 4.7. If $h : (X, \tau_N) \to (Y, \sigma_N)$ is N_sC and $g : (Y, \sigma_N) \to (Z, \rho_N)$ is N_se^*C, then $g \circ h : (X, \tau_N) \to (Z, \rho_N)$ is N_se^*C.

Proof. Let ψ be a N_scs in (X, τ_N). Then $h(\psi)$ is N_scs of (Y, σ_N) because h is N_sC map. Now $(g \circ h)(\psi) = g(h(\psi))$ is N_se^*cs in (Z, ρ_N) because g is N_se^*C map. Thus $g \circ h$ is N_se^*C map. □

Theorem 4.8. If $h : (X, \tau_N) \to (Y, \sigma_N)$ is N_se^*C map, then $N_se^*cl(h(\psi)) \subsetneq h(N_scl(\psi))$.

Proof. Obvious. □

Theorem 4.9. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ and $g : (Y, \sigma_N) \to (Z, \rho_N)$ be N_se^*C maps. If every N_se^*cs of (Y, σ_N) is N_sc then, $g \circ h : (X, \tau_N) \to (Z, \rho_N)$ is N_se^*C.

Proof. Let ψ be a N_scs in (X, τ_N). Then $h(\psi)$ is N_se^*cs of (Y, σ_N) because h is N_se^*C map. By hypothesis $h(\psi)$ is N_scs of (Y, σ_N). Now $g(h(\psi)) = (g \circ h)(\psi)$ is N_se^*cs in (Z, ρ_N) because g is N_se^*C map. Thus $g \circ h$ is N_se^*C map. □

Theorem 4.10. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be an objective map, then the following statements are equivalent:

(i) h is a N_se^*O map.

(ii) h is a N_se^*C map.

(iii) h^{-1} is N_se^*Cts map.

Proof. (i) ⇒ (ii): Let us assume that h is a N_se^*O map. By definition, ψ is a N_sos in (X, τ_N), then $h(\psi)$ is a N_se^*os in (Y, σ_N). Here, ψ is N_scs in (X, τ_N), then $X - \psi$ is a N_sos in (X, τ_N). By assumption, $h(X - \psi)$ is a N_se^*os in (Y, σ_N). Hence, $Y - h(X - \psi)$ is a N_se^*cs in (Y, σ_N). Therefore, h is a N_se^*C map.

(ii) ⇒ (iii): Let ψ be a N_scs in (X, τ_N) By (ii), $h(\psi)$ is a N_se^*cs in (Y, σ_N). Hence, $h(\psi) = (h^{-1})^{-1}(\psi)$, so h^{-1} is a N_se^*cs in (Y, σ_N). Hence, h^{-1} is N_se^*Cts.

(iii) ⇒ (i): Let ψ be a N_sos in (X, τ_N) By (iii), $(h^{-1})^{-1}(\psi) = h(\psi)$ is a N_se^*O map. □

5 Neutrosophic e^*-homeomorphism

Definition 5.1. A bijection $h : (X, \tau_N) \to (Y, \sigma_N)$ is called a N_se^*-homeomorphism (briefly N_se^*Hom) if h and h^{-1} are N_se^*Cts.

Theorem 5.2. Each N_sHom is a N_se^*Hom.

Proof. Let h be N_sHom, then h and h^{-1} are N_sCts. But every N_sCts function is N_se^*Cts. Hence, h and h^{-1} are N_se^*Cts. Therefore, h is a N_se^*Hom. □

Example 5.3. Let $X = \{l, m, n\} = Y$ and define N_ss's X_1, X_2 and X_3 in X and Y_1 in Y by

$$\begin{aligned}
X_1 &= \langle X, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6}) \rangle, \\
X_2 &= \langle X, (\frac{\mu_l}{0.1}, \frac{\mu_m}{0.1}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.9}, \frac{\nu_m}{0.9}, \frac{\nu_n}{0.6}) \rangle, \\
X_3 &= \langle X, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.4}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.6}, \frac{\nu_n}{0.6}) \rangle, \\
Y_1 &= \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.4}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.6}, \frac{\nu_n}{0.6}) \rangle.
\end{aligned}$$

Then we have $\tau_N = \{0_N, X_1, X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ be an identity map, then h is $N_s e^* Hom$ but not $N_s Hom$.

Theorem 5.4. Let $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ be a bijective map. If h is $N_s e^* Cts$, then the following are statements are equivalent:

(i) h is a $N_s e^* C$ map.

(ii) h is a $N_s e^* O$ map.

(iii) h^{-1} is a $N_s e^* Hom$.

Proof. (i) $\Rightarrow$ (ii) : Assume that h is a bijective map and a $N_s e^* C$ map. Hence, h^{-1} is a $N_s e^* Cts$ map. We know that each $N_s os$ in (X, τ_N) is a $N_s e^* os$ in (Y, σ_N). Hence, h is a $N_s e^* O$ map.

(ii) $\Rightarrow$ (iii) : Let h be a bijective and $N_s O$ map. Further, h^{-1} is a $N_s e^* Cts$ map. Hence, h and h^{-1} are $N_s e^* Cts$. Therefore, h is a $N_s e^* Hom$.

(iii) $\Rightarrow$ (i): Let h be a $N_s e^* Hom$, then h and h^{-1} are $N_s e^* Cts$. Since each $N_s cs$ in (X, τ_N) is a $N_s e^* cs$ in (Y, σ_N), h is a $N_s e^* C$ map. $\square$

Definition 5.5. A $N_s ts$ (X, τ_N) is said to be a neutrosophic $e^* T_{\frac{1}{2}}$ (briefly, $N_s e^* T_{\frac{1}{2}}$)-space if every $N_s e^* cs$ is $N_s c$ in (X, τ_N).

Theorem 5.6. Let $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ be a $N_s e^* Hom$, then h is a $N_s Hom$ if (X, τ_N) and (Y, σ_N) are $N_s e^* T_{\frac{1}{2}}$-space.

Proof. Assume that ψ is a $N_s cs$ in (Y, σ_N), then $h^{-1}(\psi)$ is a $N_s e^* cs$ in (X, τ_N). Since (X, τ_N) is an $N_s e^* T_{\frac{1}{2}}$-space, $h^{-1}(\psi)$ is a $N_s cs$ in (X, τ_N). Therefore, h is $N_s Cts$. By hypothesis, h^{-1} is $N_s e^* Cts$. Let ζ be a $N_s cs$ in (X, τ_N). Then, $(h^{-1})^{-1}(\zeta) = h(\zeta)$ is a $N_s cs$ in (Y, σ_N), by presumption. Since (Y, σ_N) is a $N_s e^* T_{\frac{1}{2}}$-space, $h(\zeta)$ is a $N_s cs$ in (Y, σ_N). Hence, h^{-1} is $N_s Cts$. Hence, h is a $N_s Hom$. $\square$

Theorem 5.7. Let $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ be a $N_s ts$, then the following are equivalent if (Y, σ_N) is a $N_s e^* T_{\frac{1}{2}}$-space.

(i) h is $N_s e^* C$ map.

(ii) If ψ is a $N_s os$ in (X, τ_N), then $h(\psi)$ is $N_s e^* os$ in (Y, σ_N).

(iii) $h(N_s int(\psi)) \subseteq N_s cl(N_s int(h(\psi)))$ for every $N_s s$ ψ in (X, τ_N).

Proof. (i) $\Rightarrow$ (ii): Obvious.

(ii) $\Rightarrow$ (iii): Let ψ be a $N_s s$ in (X, τ_N). Then, $N_s int(\psi)$ is a $N_s os$ in (X, τ_N). Then, $h(N_s int(\psi))$ is a $N_s e^* os$ in (Y, σ_N). Since (Y, σ_N) is a $N_s e^* T_{\frac{1}{2}}$-space, $h(N_s int(\psi))$ is a $N_s os$ in (Y, σ_N). Therefore, $h(N_s int(\psi)) = N_s int(h(N_s int(\psi))) \subseteq N_s cl(N_s int(h(\psi)))$.

(iii) $\Rightarrow$ (i): Let ψ be a $N_s cs$ in (X, τ_N). Then, ψ^c is a $N_s os$ in (X, τ_N). From, $h(N_s int(\psi^c)) \subseteq N_s cl(N_s int(h(\psi^c)))$. Hence, $h(\psi^c) \subseteq N_s cl(N_s int(h(\psi^c)))$. Therefore, $h(\psi^c)$ is $N_s e^* os$ in (Y, σ_N). Therefore, $h(\psi)$ is a $N_s e^* cs$ in (X, τ_N). Hence, h is a $N_s C$ map. $\square$

Theorem 5.8. Let $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ and $g : (Y, \sigma_N) \rightarrow (Z, \rho_N)$ be $N_s e^* C$, where (X, τ_N) and (Z, ρ_N) are two $N_s ts$'s and (Y, σ_N) a $N_s e^* T_{\frac{1}{2}}$-space, then the composition $g \circ h$ is $N_s e^* C$ map.

Proof. Let ψ be a $N_s cs$ in (X, τ_N). Since h is $N_s e^* c$ and $h(\psi)$ is a $N_s e^* cs$ in (Y, σ_N), by assumption, $h(\psi)$ is a $N_s cs$ in (Y, σ_N). Since g is $N_s e^* c$, $g(h(\psi))$ is $N_s e^* c$ in (Z, ρ_N) and $g(h(\psi)) = (g \circ h)(\psi)$. Therefore, $g \circ h$ is $N_s e^* C$ map. $\square$

Theorem 5.9. Let $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ and $g : (Y, \sigma_N) \rightarrow (Z, \rho_N)$ be two $N_s ts$'s, then the following hold:

(i) If $g \circ h$ is $N_s e^* O$ and h is $N_s Cts$, then g is $N_s e^* O$ map.

(ii) If $g \circ h$ is $N_s O$ and g is $N_s e^* Cts$, then h is $N_s e^* O$ map.

Proof. Obvious. $\square$

6 Conclusions

The open and closed sets play a very a prominent role in general Topology and its applications. Indeed a significant theme in General Topology, Real analysis and many other branches of mathematics concerns the variously modified forms of continuity, separation axioms etc., by utilizing generalized open and closed sets. One of the well-known notions and strong form in topology and their applications is the notion of δ-open sets. The importance of general topological spaces rapidly increases in both the pure and applied directions. In this paper we introduced and investigated the notions of new classes of functions in neutrosophic δ-open set which may have very important applications in mathematics. The new idea of neutrosophic δ-open maps and neutrosophic δ-closed maps and relate with their neighbour maps in neutrosophic topological spaces were discussed. Furthermore, the work was extended as neutrosophic e^*-homeomorphism and neutrosophic $e^*T_{\frac{1}{2}}$-space in N_sts with their related properties. In future, the further research can be carried out on somewhat neutrosophic δ continuous maps and somewhat δ irresolute maps in a neutrosophic topological spaces.

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