

## On $N_{g^*b}$ -closed set in Neutrosophic Topological Spaces]

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### Abstract

The focus of this paper is to introduce the concept of  $N_{g^*b}$ -closed set in Neutrosophic Topological Spaces. Also we analyze their characterization and investigate their properties.

**Keywords:**  $N_{g^*b}$ -closed sets, Neutrosophic Topological Spaces

### 1 Introduction and Preliminaries

The neutrosophic set was introduced by Smarandache<sup>1</sup> and explained, Neutrosophic set is a generalization of intuitionistic fuzzy set. Salama and Alblowi<sup>2</sup> introduced the new concept of neutrosophic topological space in 2012, which had been investigated recently. In the neutrosophic set, all the elements have the degree of membership, indeterminacy and degree of non-membership values. Smarandache neutrosophic system have wide range of real time applications for the fields of Computer Science , Information Systems, Applied Mathematics , Artificial Intelligence, Mechanics, decision making. Medicine, Electrical and Electronic, and Management Science etc. Neutrosophic topological spaces (N-T-S) introduced by Salama et al. R.Dhavaseelan, Saied Jafari<sup>4</sup> are introduced Neutrosophic generalized closed sets. Neutrosophic b closed sets and Neutrosophic generalized b closed sets are introduced C. Maheswari<sup>10</sup> et al. The focus of this article is to introduce the idea of  $N_{g^*b}$ -closed sets and obtain some of its basic properties.

**Definition 1.1.** <sup>2</sup> Let X be a non-empty fixed set. A neutrosophic set A is an object having the form  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ , which represent the degree of membership function (namely  $\mu_A(x)$ ), the degree of indeterminacy (namely  $\sigma_A(x)$ ) and the degree of non-membership function (namely  $\gamma_A(x)$ ) respectively of each element  $x \in X$  to the set A.

**Definition 1.2.** <sup>2</sup>

Neutrosophic Empty set  $0_N$  may be defined as

$$(0_1)0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$$

$$(0_2)0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$$

$$(0_3)0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$$

$$(0_4)0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$$

Neutrosophic Whole set  $1_N$  may be defined as

$$(1_1)1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$$

$$(1_2)1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$$

$$(1_3)1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$$

$$(1_4)1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}.$$

**Definition 1.3.** <sup>2</sup> Let  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$  a NS on X , then the complement of the set ( $\bar{A}$  for short) may be defined as three kinds of complements.

- $\bar{A} = \{ \langle x, 1 - \mu_A(x), \sigma_A(x), 1 - \gamma_A(x) \rangle : x \in X \}.$
- $\bar{A} = \{ \langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}.$
- $\bar{A} = \{ \langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}.$

**Definition 1.4.** <sup>3</sup> Let  $X$  be a non-empty set, and Neutrosophic sets  $A$  and  $B$  in the form  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ ,  $B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$ , then we may consider two possible definitions for subsets ( $A \subseteq B$ ) may be defined as:

- $A \subseteq B \iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x)$  and  $\gamma_A(x) \geq \gamma_B(x)$ , for all  $x \in X$ .
- $A \subseteq B \iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x)$  and  $\gamma_A(x) \geq \gamma_B(x)$ , for all  $x \in X$ .

**Definition 1.5.** <sup>3</sup> Let  $X$  be a non-empty set, and Neutrosophic sets  $A$  and  $B$  in the form  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ ,  $B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$ , then

1.  $A \cap B$  may be defined as:

- $A \cap B = \{ \langle x, \mu_A(x) \cdot \mu_B(x), \sigma_A(x) \cdot \sigma_B(x), \gamma_A(x) \cdot \gamma_B(x) \rangle \}$ .
- $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle \}$ .
- $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle \}$ .

2.  $A \cup B$  may be defined as:

- $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle \}$ .
- $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle \}$ .

**Definition 1.6.** <sup>6</sup> A neutrosophic topology on a non-empty set  $X$  is a family  $\tau$  of neutrosophic sets in  $X$  satisfying the following axioms:

- $0_N, 1_N \in \tau$ ,
- $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$ ,
- $\cup G_i \in \tau$  for arbitrary family  $\{G_i | i \in J\} \subseteq \tau$ .

**Definition 1.7.** <sup>6</sup> Let  $A$  be a neutrosophic set in a neutrosophic topological space  $X$ . Then

- $Nint(A) = \bigcup \{G : G \text{ is a neutrosophic open set in } X \text{ and } G \subseteq A\}$  is called the neutrosophic interior of  $A$ .
- $Ncl(A) = \bigcap \{G : G \text{ is a neutrosophic closed set in } X \text{ and } G \supseteq A\}$  is called the neutrosophic closure of  $A$ .

**Definition 1.8.** <sup>10</sup> A neutrosophic set  $A$  in a neutrosophic topological space  $(X, \tau)$  is called,

1. A neutrosophic semi-closed set if  $Nint(Ncl(A)) \subseteq A$ .
2. A neutrosophic  $\alpha$ -closed set if  $Ncl(Nint(Ncl(A))) \subseteq A$ .
3. A neutrosophic pre-closed set if  $Ncl(Nint(A)) \subseteq A$ .
4. A neutrosophic b-open set if  $A \subseteq Ncl(Nint(A)) \cup Nint(Ncl(A))$ .
5. A neutrosophic b-closed set if  $Ncl(Nint(A)) \cup Nint(Ncl(A)) \subseteq A$ .

**Definition 1.9.** <sup>10</sup> A subset  $\mathcal{A}$  of a space  $(X, \tau)$  is called,

1. A neutrosophic generalized closed (briefly  $\mathcal{N}_g$ -closed) set if  $Ncl(\mathcal{A}) \subseteq U$  whenever  $\mathcal{A} \subseteq U$  and  $U$  is neutrosophic open in  $(X, \tau)$ . The complement of a  $\mathcal{N}_g$ -closed set is called a  $\mathcal{N}_g$ -open set,
2. A neutrosophic generalized  $\alpha$ -closed (briefly  $\mathcal{N}_{g\alpha}$ -closed) set if  $N\alpha cl(\mathcal{A}) \subseteq U$  whenever  $\mathcal{A} \subseteq U$  and  $U$  is neutrosophic  $\alpha$ -open in  $(X, \tau)$ ,
3. A neutrosophic generalized b-closed (briefly  $\mathcal{N}_{gb}$ -closed) set if  $Nbcl(\mathcal{A}) \subseteq U$  whenever  $\mathcal{A} \subseteq U$  and  $U$  is neutrosophic open in  $(X, \tau)$ ,
4. A neutrosophic generalised \* closed (briefly  $\mathcal{N}_{g^*}$ -closed) set if  $Ncl(\mathcal{A}) \subseteq U$  whenever  $\mathcal{A} \subseteq U$  and  $U$  is neutrosophic g-open in  $(X, \tau)$ .
5. A neutrosophic generalized semi-preclosed (briefly  $\mathcal{N}_{gsp}$ -closed) set if  $Nspcl(\mathcal{A}) \subseteq U$  whenever  $\mathcal{A} \subseteq U$  and  $U$  is neutrosophic open in  $(X, \tau)$ .

## 2 Basic properties of $N_{g^*b}$ -closed sets

In this section, we introduce the idea of  $N_{g^*b}$ -closed sets and some of its properties.

**Definition 2.1.** A subset  $A$  of  $(X, \tau)$  is called a  $N_{g^*b}$ -closed set if  $Nbcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $N_g$ -open in  $(X, \tau)$ .

**Theorem 2.2.** Every neutrosophic closed set is  $N_{g^*b}$ -closed set.

*Proof.* Let  $D$  be a neutrosophic closed set in  $X$  such that  $D \subseteq U$ , where  $U$  is neutrosophic  $g$ -open. Since  $D$  is closed,  $Ncl(D) = D$ . Since  $Nbcl(D) \subseteq Ncl(D) = D$ . Therefore  $Nbcl(D) \subseteq U$ . Hence  $D$  is a  $N_{g^*b}$ -closed set in  $X$ .  $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 2.3.** Let  $X = \{x\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.5, 0.5, 0.33) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.3, 0.6, 0.55) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.55, 0.4, 0.3) \rangle \forall x \in X\}.$$

Then  $D$  is a  $N_g$ -closed set and  $N_{g^*b}$ -closed set but it is not a neutrosophic closed set of a neutrosophic topological space  $(X, \tau)$ . Since  $Ncl(D) = A^C$ , which is not equal to a neutrosophic set  $D$ .

**Theorem 2.4.** Every  $N_b$  - closed set is  $N_{g^*b}$  - closed set.

*Proof.* Let  $D$  be a neutrosophic  $b$ -closed set in  $X$  such that  $D \subseteq U$ , where  $U$  is neutrosophic  $g$ -open. Since  $D$  is  $b$ -closed,  $Nbcl(D) = D$ . Therefore  $Nbcl(D) \subseteq U$ . Hence  $D$  is a  $N_{g^*b}$ -closed set in  $X$ .  $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 2.5.** Let  $X = \{x\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.3, 0.5, 0.3) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.3, 0.6, 0.4) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.4, 0.4, 0.3) \rangle \forall x \in X\}.$$

Then  $D$  is a  $N_g$ -closed set and  $N_{g^*b}$ -closed set but it is not a neutrosophic  $b$ -closed set of a neutrosophic topological space  $(X, \tau)$ . Since  $Ncl(Nint(D)) \cup Nint(Ncl(D)) = A$ , which is not equal to a neutrosophic set  $D$ .

**Theorem 2.6.** Every  $N_{g^*b}$  - closed set is  $N_{gb}$  - closed set.

*Proof.* Let  $D$  be a neutrosophic  $N_{g^*b}$ -closed set in  $X$  such that  $D \subseteq U$ , where  $U$  is neutrosophic open. Since every neutrosophic open set is  $N_g$ -open, then  $U$  is  $N_g$ -open. since  $A$  is  $N_{g^*b}$ -closed set, then  $Nbcl(D) \subseteq U$ . Hence  $D$  is  $N_{gb}$ -closed.  $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 2.7.** Let  $X = \{x\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.4, 0.55, 0.6) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

$$E = \{\langle x, (0.9, 0.1, 0.9) \rangle \forall x \in X\}.$$

Then  $D$  is a  $N_{gb}$ -closed set,  $E$  is  $N_g$  - open but it is not a  $N_{g^*b}$ -closed set of a neutrosophic topological space  $(X, \tau)$ . Since  $Ncl(E) = 1_N$ , which is not equal to a neutrosophic set  $D$ .

**Theorem 2.8.** Every  $N_\alpha$ -closed set is  $N_{g^*b}$ -closed set.

*Proof.* Let  $D$  be a  $N_\alpha$ -closed set in  $X$  such that  $D \subseteq U$ , where  $U$  is neutrosophic g-open. Since  $D$  is  $N_\alpha$ -closed,  $N_{\alpha cl}(D) \subseteq U$ . Since  $N_{bcl}(D) \subseteq N_{\alpha cl}(D)$ , then  $N_{bcl}(D) \subseteq U$ . Hence  $D$  is a  $N_{g^*b}$ -closed set in  $X$ .  $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 2.9.** Let  $X = \{x\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.5, 0.5, 0.33) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.3, 0.6, 0.55) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.55, 0.4, 0.3) \rangle \forall x \in X\}.$$

Then  $D$  is a  $N_g$ -closed set and  $N_{g^*b}$ -closed set but it is not a  $N_\alpha$ -closed set of a neutrosophic topological space  $(X, \tau)$ . Since  $N_{\alpha cl}(D) = 1_N$ , which is not equal to a neutrosophic set  $D$ .

**Theorem 2.10.** Every Neutrosophic semi-closed set is  $N_{g^*b}$ -closed set.

*Proof.* Let  $D$  be a neutrosophic semi-closed set in  $X$  such that  $D \subseteq U$ , where  $U$  is neutrosophic g-open. Since  $D$  is neutrosophic semi-closed,  $N_{scl}(D) = D \subseteq U$ . But  $N_{bcl}(D) \subseteq N_{scl}(D)$ , then  $N_{bcl}(D) \subseteq U$ . Hence  $D$  is a  $N_{g^*b}$ -closed set in  $X$ .  $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 2.11.** Let  $X = \{x\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.3, 0.5, 0.3) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.3, 0.6, 0.4) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.4, 0.4, 0.3) \rangle \forall x \in X\}.$$

Then  $D$  is a  $N_g$ -closed set and  $N_{g^*b}$ -closed set but it is not a neutrosophic semi-closed set of a neutrosophic topological space  $(X, \tau)$ . Since  $N_{scl}(D) = A$ , which is not equal to a neutrosophic set  $D$ .

**Theorem 2.12.** Every Neutrosophic preclosed set is  $N_{g^*b}$ -closed set.

*Proof.* Let  $D$  be a neutrosophic preclosed set in  $X$  such that  $D \subseteq U$ , where  $U$  is neutrosophic g-open. Since  $D$  is neutrosophic preclosed,  $N_{pcl}(D) = D \subseteq U$ . But  $N_{bcl}(D) \subseteq N_{pcl}(D)$ , then  $N_{bcl}(D) \subseteq U$ . Hence  $D$  is a  $N_{g^*b}$ -closed set in  $X$ .  $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 2.13.** Let  $X = \{x\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.4, 0.55, 0.6) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

$$E = \{\langle x, (0.9, 0.1, 0.9) \rangle \forall x \in X\}.$$

Then  $D$  is a  $N_{g^*b}$ -closed set,  $E$  is  $N_g$ -open but it is not a neutrosophic preclosed set of a neutrosophic topological space  $(X, \tau)$ . Since  $N_{pcl}(E) = 1_N$ , which is not equal to a neutrosophic set  $D$ .

**Theorem 2.14.** Every  $N_g$ -closed set is  $N_{g^*b}$ -closed set.

*Proof.* Let  $D$  be a  $N_g$ -closed set in  $X$  such that  $D \subseteq U$ , where  $U$  is neutrosophic g-open. Since  $D$  is  $N_g$ -closed,  $N_{cl}(D) \subseteq U$ . But  $N_{bcl}(D) \subseteq N_{cl}(D)$ , then  $N_{bcl}(D) \subseteq U$ . Hence  $D$  is a  $N_{g^*b}$ -closed set in  $X$ .  $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 2.15.** Let  $X = \{x\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.4, 0.55, 0.6) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

$$E = \{\langle x, (0.9, 0.1, 0.9) \rangle \forall x \in X\}.$$

Then  $D$  is a  $N_{g^*b}$ -closed set,  $E$  is neutrosophic closed set but it is not a  $N_g$ -closed set of a neutrosophic topological space  $(X, \tau)$ . Since  $Ncl(E) = 1_N$ , which is not equal to a neutrosophic set  $D$ .

**Theorem 2.16.** Every  $N_{g^*}$ -closed set is  $N_{g^*b}$ -closed set.

*Proof.* Let  $D$  be a  $N_{g^*}$ -closed set in  $X$  such that  $D \subseteq U$ , where  $U$  is neutrosophic g-open. Since  $D$  is  $N_{g^*}$ -closed,  $Ncl(D) \subseteq U$ . But  $Nbcl(D) \subseteq Ncl(D)$ , then  $Nbcl(D) \subseteq U$ . Hence  $D$  is a  $N_{g^*b}$ -closed set in  $X$ .  $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 2.17.** Let  $X = \{x\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.4, 0.55, 0.6) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

$$E = \{\langle x, (0.9, 0.1, 0.9) \rangle \forall x \in X\}.$$

Then  $D$  is a  $N_{g^*b}$ -closed set,  $E$  is  $N_g$ -open but it is not a  $N_{g^*}$ -closed set of a neutrosophic topological space  $(X, \tau)$ . Since  $Ncl(E) = 1_N$ , which is not equal to a neutrosophic set  $D$ .

**Theorem 2.18.** Every  $N_{g^*b}$ -closed set is  $N_{gsp}$ -closed set

*Proof.* Let  $D$  be a neutrosophic  $N_{g^*b}$ -closed set in  $X$  such that  $D \subseteq U$ , where  $U$  is neutrosophic open. Since every neutrosophic open set is  $N_g$ -open then  $U$  is  $N_g$ -open set. Since  $D$  is  $N_{g^*b}$ -closed,  $Nbcl(D) \subseteq U$ . But  $Nspcl(D) \subseteq Nbcl(D)$ , then  $Nspcl(D) \subseteq U$ . Hence  $D$  is  $N_{gsp}$ -closed.  $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 2.19.** Let  $X = \{x\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.4, 0.55, 0.6) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

$$E = \{\langle x, (0.9, 0.1, 0.9) \rangle \forall x \in X\}.$$

Then  $D$  is a  $N_{gsp}$ -closed set,  $E$  is  $N_g$ -open but it is not a  $N_{g^*b}$ -closed set of a neutrosophic topological space  $(X, \tau)$ . Since  $Ncl(E) = 1_N$ , which is not equal to a neutrosophic set  $D$ .

**Theorem 2.20.** If  $A$  is both  $N_g$ -open and  $N_{g^*b}$ -closed sets in  $X$ , then  $A$  is  $N_b$ -closed.

*Proof.* Since  $A$  is  $N_g$ -open and  $N_{g^*b}$ -closed sets in  $X$ ,  $Nbcl(A) \subseteq U$ . But always  $A \subseteq Nbcl(A)$ . Therefore  $A = Nbcl(A)$ . Hence  $A$  is  $N_b$ -closed.  $\square$

**Example 2.21.** Let  $X = \{x\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.5, 0.5, 0.33) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.3, 0.6, 0.55) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.55, 0.4, 0.3) \rangle \forall x \in X\}.$$

Then  $D$  is a  $N_g$ -open and  $N_{g^*b}$ -closed set in  $X$ ,  $Nbcl(D) \subseteq U$ . Since  $D \subseteq Nbcl(D)$ . Therefore  $D = Nbcl(D)$ . Hence  $D$  is  $N_b$ -closed.

**Corollary 2.22.** *The intersection of a  $N_{g^*b}$ -closed set and a neutrosophic closed set is a  $N_{g^*b}$ -closed set.*

**Remark 2.23.** If  $A$  and  $B$  are  $N_{g^*b}$  - closed set, then their union need not be  $N_{g^*b}$  - closed set.

**Example 2.24.** Let  $X = \{a\}$  and the neutrosophic set  $A$  are defined as,

$$A = \{\langle x, (0.3, 0.5, 0.3) \rangle \forall x \in X\},$$

$$B = \{\langle x, (0.4, 0.5, 0.4) \rangle \forall x \in X\},$$

Then the neutrosophic topology  $\tau = \{0_N, A, B, 1_N\}$ , which are neutrosophic open sets in the neutrosophic topological space  $(X, \tau)$ . If

$$D = \{\langle x, (0.4, 0.55, 0.6) \rangle \forall x \in X\},$$

$$E = \{\langle x, (0.3, 0.6, 0.4) \rangle \forall x \in X\},$$

$$U = \{\langle x, (0.6, 0.5, 0.4) \rangle \forall x \in X\}.$$

Then  $D$  is  $N_{g^*b}$  - closed,  $Nbcl(D) \subseteq U$ , whenever  $D \subseteq U$ ,  $U$  is  $N_g$ -open set in  $X$  and  $E$  is  $N_{g^*b}$  - closed,  $Nbcl(E) \subseteq U$ , whenever  $E \subseteq U$ ,  $U$  is  $N_g$ -open set in  $X$ . But  $D \cup E$  is not  $N_{g^*b}$  - closed sets in  $X$ . Since  $cl(D \cup E) = 1_N$ , which is not contained in the neutrosophic set  $U$ .

### 3 Conclusions

In this paper the authors introduced the concept of  $N_{g^*b}$ -closed set in Neutrosophic Topological Spaces and established the relationship between the newly defined set with some kind of existing sets. Also we analyzed their characterization and investigate their properties.

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