



Pairwise Pythagorean Neutrosophic Strongly Irresolvable Spaces (with dependent neutrosophic components between T and F)

¹R.Jansi, ² K.Mohana

¹Research Scholar, Nirmala College for Women, Coimbatore.

²Assistant Professor, Nirmala College for Women, Coimbatore.

Email: mathematicsgasc@gmail.com¹, riyaraju1116@gmail.com²

Abstract

In this paper, we have define the pairwise Pythagorean Neutrosophic (for short, pairwise PN) bitopological spaces (with dependent neutrosophic components between T and F). We also study the Pairwise PN strongly irresolvable spaces .The conditions under which pairwise PN strongly irresolvable spaces become pairwise PN first category spaces and pairwise PN Baire spaces are also investigated.

Keywords: PN bitopological, PN strongly irresolvable spaces, PN Baire.

1 Introduction

Fuzzy sets were introduced by Zadeh [18] and he discussed only membership function. The fuzzy topology concept was first introduced by C.L.Chang [6] in 1968. After the extensions of fuzzy set theory Atanassov [4] generalized this concept and introduced a new concept called intuitionistic fuzzy set (IFS). Yager [16] familiarized the model of Pythagorean fuzzy set.

IFS was failed to deal with indeterminate and inconsistent information which exist in beliefs system, therefore, Smarandache [13] in 1995 introduced new concept known as neutrosophic set(NS) which generalizes fuzzy sets and intuitionistic fuzzy sets and so on. A neutrosophic set includes truth membership, falsity membership and indeterminacy membership. In 2006, F.Smarandache introduced, for the first time, the degree of dependence (and consequently the degree of independence) between the components of the fuzzy set, and also between the components of the neutrosophic set. In 2016, the refined neutrosophic set was generalized to the degree of dependence or independence of subcomponents [14]. A.Kandil [9] introduced the concept of fuzzy bitopological spaces as a generalization of fuzzy topological spaces. Thagaraj and Balasubramanian [15] introduced the concept of fuzzy resolvable and irresolvable spaces. Jansi, Mohana and Florentin Smarandache [8] were firstly studied the concept of Pythagorean Neutrosophic sets with T and F as dependent neutrosophic components.

In this paper we study the pairwise PN strongly irresolvable spaces .Also we studied the conditions under which pairwise PN bitopological strongly irresolvable spaces spaces become pairwise PN first category spaces and pairwise PN Baire spaces are investigated.

2 Preliminaries

Definition 2.1 (16). (**Pythagorean Fuzzy Set**) Let X be a non-empty set and I the unit interval $[0, 1]$. A PF set P is an object having the form $P = \{(x, \mu_P(x), \nu_P(x)) : x \in X\}$ where the function $\mu_P : X \rightarrow [0, 1]$ and $\nu_P : X \rightarrow [0, 1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P , and $0 \leq (\mu_P(x))^2 + (\nu_P(x))^2 \leq 1$ for each $x \in X$.

Definition 2.2 (11). Let X be a non-empty set (universe). A neutrosophic set A on X is an object of the form: $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$, Where $T_A(x), I_A(x), F_A(x) \in [0, 1], 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 2$, for all x in X . $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non-membership. Here $T_A(x)$ and $F_A(x)$ are dependent components and $I_A(x)$ is an independent components.

Definition 2.3 (6). (Pythagorean neutrosophic sets [PN-sets](with T and F are dependent neutrosophic components)) Let X be a non-empty set. A Pythagorean neutrosophic sets with T and F are dependent neutrosophic components (PN) $A = \{(X, T_A(x), I_A(x), F_A(x) : x \in X\}$ where $T_A : X \rightarrow [0, 1]$, $I_A : X \rightarrow [0, 1]$ and $F_A : X \rightarrow [0, 1]$ are the mappings such that $0 \leq T_A^2(x) + I_A^2(x) + F_A^2(x) \leq 2$ and $T_A(x)$ denote the membership degree, $I_A(x)$ denote the Indeterminacy and $F_A(x)$ denote the non-membership degree. Here T and F are dependent neutrosophic components and I is an independent components.

Definition 2.4 (6). Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be two PNs, then their operations are defined as follows:

- (1) $A \subseteq B$ if and if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$
- (2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- (3) $A \cup B = \{(x, \max(T_A, T_B), \min(I_A, I_B), \min(F_A, F_B) : x \in X\}$
- (4) $A \cap B = \{(x, \min(T_A, T_B), \max(I_A, I_B), \max(F_A, F_B) : x \in X\}$
- (5) $A^C = \{(x, F_A, I_A, T_A) : x \in X\}$.

3 PN Bitopological Spaces

Definition 3.1. A Pythagorean neutrosophic (with T and F are dependent neutrosophic components) topology (PNT in Short) on X is a family ρ of PN-sets in X satisfying the following axioms:

- (1) $0_X, 1_X \in \rho$
- (2) $G_1 \cap G_2 \in \rho$, for any $G_1, G_2 \in \rho$
- (3) $\cup G_i \in \rho$ for any family $\{G_i/i \in J\} \subseteq \rho$. Note that $0_X = (0, 1, 1)$ and $1_X = (1, 0, 0)$.

In this case the pair (X, ρ) is called a Pythagorean neutrosophic sets with T and F are dependent neutrosophic components topological space (PNTS in Short) and any PNTS in ρ is known as a Pythagorean neutrosophic sets with T and F are dependent neutrosophic components open set (PNOS in Short) in X .

The Complement A^c of a PNOS A in a PNTS (X, ρ) is called a Pythagorean neutrosophic sets with T and F are dependent neutrosophic components closed set (PNCS in Short) in X .

Definition 3.2. Let (X, ρ) be a PNTS and be a PN in X . Then the PN interior and closure of a PN closure are defined by

$$PNint(A) = \cup \{G/G \text{ is a PNOS in } X \text{ and } G \subseteq A\}$$

$$PNcl(A) = \cap \{K/K \text{ is a PNCS in } X \text{ and } A \subseteq K\}.$$

Note that for any PN A in (X, ρ) , we have $(PNcl(A))^c = PNint(A^c)$ and $(PNint(A))^c = PNcl(A^c)$. Also, note that A is a PN closed set iff $PNcl(A) = A$ and A is a PN open set iff $PNint(A) = A$.

Definition 3.3. A set X on which are defined two (arbitrary)PN topologies ρ_1 and ρ_2 is called PN bitopological spaces and denoted by (X, ρ_1, ρ_2) .

We shall write $PNint_{\rho_i}(A)$ and $PNcl_{\rho_i}(A)$ to mean respectively the PN interior and PN closure of PN set A with respect to the ρ_i in (X, ρ_1, ρ_2) .

Definition 3.4. A PN-set A in a PN bitopological space (X, ρ_1, ρ_2) is called a pairwise PN open set if $A \in \rho_i (i = 1, 2)$. The complement of pairwise PN open set in (X, ρ_1, ρ_2) is called a pairwise PN closed set.

Definition 3.5. A PN-set A in a PN bitopological space (X, ρ_1, ρ_2) is called pairwise PN semi open set if $A \subseteq PNint_{\rho_i} PNcl_{\rho_i}(A) (i = 1, 2)$ and PN semi closed set if $PNint_{\rho_i}(PNcl_{\rho_i}(A)) \subseteq A (i = 1, 2)$.

Definition 3.6. A PN-set A in a PN bitopological space (X, ρ_1, ρ_2) is called a pairwise PN dense set if $PNcl_{\rho_1} PNcl_{\rho_2}(A) = PNcl_{\rho_2} PNcl_{\rho_1}(A) = 1_X$ in (X, ρ_1, ρ_2) .

Definition 3.7. A PN-set A in a PN bitopological space (X, ρ_1, ρ_2) is called pairwise PN nowhere dense if $PNint_{\rho_1} PNcl_{\rho_2}(A) = PNint_{\rho_2} PNcl_{\rho_1}(A) = 0_X$ in (X, ρ_1, ρ_2) .

Definition 3.8. Let (X, ρ_1, ρ_2) be a PN bitopological space. A PN-set A in (X, ρ_1, ρ_2) is called a pairwise PN first category set if $A = \bigcup_{i=1}^{\infty} (A_i)$, where (A_i) 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) . Any other PN set in (X, ρ_1, ρ_2) is said to be a pairwise PN second category set in (X, ρ_1, ρ_2) .

Definition 3.9. If A is a pairwise PN first category set in a PN bitopological space (X, ρ_1, ρ_2) , then the PN A^c is called a pairwise PN residual set in (X, ρ_1, ρ_2) .

Definition 3.10. A PN-set A in a PN bitopological space (X, ρ_1, ρ_2) is called pairwise PN F_{σ} -set in (X, ρ_1, ρ_2) if $A = \bigcup_{i=1}^{\infty} (A_i)$ where $(A_i)^c \in \rho_i$.

Definition 3.11. A PN-set A in a PN bitopological space (X, ρ_1, ρ_2) is called pairwise PN G_{δ} -set in (X, ρ_1, ρ_2) if $A = \bigcap_{i=1}^{\infty} (A_i)$ where $A_i \in \rho_i$.

Definition 3.12. A PN bitopological space (X, ρ_1, ρ_2) is called pairwise PN first category space if the PN set 1_X is a pairwise PN first category set in (X, ρ_1, ρ_2) . That is, $1_X = \bigcup_{i=1}^{\infty} (A_i)$, where A_i 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) .

Otherwise (X, ρ_1, ρ_2) will be called a pairwise PN second category space.

Theorem 3.13. Let (X, ρ_1, ρ_2) be a pairwise PN strongly irresolvable space. Then, A is a pairwise PN dense set in (X, ρ_1, ρ_2) , if and only if (A^c) is a pairwise PN nowhere dense set in (X, ρ_1, ρ_2) .

Proof. Let A be a pairwise PN dense set in (X, ρ_1, ρ_2) . Since (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space $PNcl_{\rho_1}PNint_{\rho_2}(A) = 1_X = PNcl_{\rho_2}PNint_{\rho_1}(A)$.

This implies that $(PNcl_{\rho_1}PNint_{\rho_2}(A))^c = 0_X = (PNcl_{\rho_2}PNint_{\rho_1}(A))^c$.

Therefore $PNint_{\rho_1}PNcl_{\rho_2}(A^c) = 0_X = PNint_{\rho_2}PNcl_{\rho_1}(A^c)$ and hence A^c is a pairwise PN nowhere dense set. \square

Theorem 3.14. Let (X, ρ_1, ρ_2) be a pairwise PN dense and pairwise PN G_{δ} -set in a PN bitopological space (X, ρ_1, ρ_2) , then (A^c) is a pairwise PN σ -nowhere dense set in (X, ρ_1, ρ_2) .

Proof. Let (X, ρ_1, ρ_2) be a pairwise PN dense and pairwise PN G_{δ} -set in a PN bitopological space (X, ρ_1, ρ_2) . Then $PNcl_{\rho_1}PNcl_{\rho_2}(A) = PNcl_{\rho_2}PNcl_{\rho_1}(A) = 1_X$.

This implies that

$$(PNcl_{\rho_1}PNcl_{\rho_2}(A))^c = (PNcl_{\rho_2}PNcl_{\rho_1}(A))^c = 0_X$$

and hence we have $PNint_{\rho_1}PNint_{\rho_2}(A^c) = 0_X = PNint_{\rho_2}PNint_{\rho_1}(A^c)$.

Also, since A is a pairwise PN G_{δ} -set, A^c is a pairwise PN F_{σ} -set in (X, ρ_1, ρ_2) . Hence A^c is a pairwise PN F_{σ} -set in (X, ρ_1, ρ_2) such that $PNint_{\rho_1}PNint_{\rho_2}(A^c) = 0_X = PNint_{\rho_2}PNint_{\rho_1}(A^c)$.

Thus (A^c) is a pairwise PN σ -nowhere dense set in (X, ρ_1, ρ_2) . \square

4 Pairwise PN Strongly Irresolvable Spaces

Definition 4.1. A PN bitopological space (X, ρ_1, ρ_2) is said to be a pairwise PN strongly irresolvable space if $PNcl_{\rho_1}PNint_{\rho_2}(A) = 1_X = PNcl_{\rho_2}PNint_{\rho_1}(A)$, for each pairwise PN dense set A in (X, ρ_1, ρ_2) . That is, (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space if

$$PNcl_{\rho_1}PNcl_{\rho_2}(A) = 1_X = PNcl_{\rho_2}PNcl_{\rho_1}(A) \text{ for a PN-set } A \text{ in } (X, \rho_1, \rho_2), \text{ then}$$

$$PNcl_{\rho_1}PNint_{\rho_2}(A) = 1_X = PNcl_{\rho_2}PNint_{\rho_1}(A) \text{ in } (X, \rho_1, \rho_2).$$

Theorem 4.2. Let (X, ρ_1, ρ_2) be a pairwise PN strongly irresolvable space. Then, A is a pairwise PN dense set in (X, ρ_1, ρ_2) , if and only if (A^c) is a pairwise PN nowhere dense set in (X, ρ_1, ρ_2) .

Proof. Let A be a pairwise PN dense set in (X, ρ_1, ρ_2) .

Since (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space

$$PNcl_{\rho_1}PNint_{\rho_2}(A) = 1_X = PNcl_{\rho_2}PNint_{\rho_1}(A).$$

This implies that $(PNcl_{\rho_1}PNint_{\rho_2}(A))^c = 0_X = (PNcl_{\rho_2}PNint_{\rho_1}(A))^c$.

Therefore $PNint_{\rho_1}PNcl_{\rho_2}(A^c) = 0_X = PNint_{\rho_2}PNcl_{\rho_1}(A^c)$ and hence A^c is a pairwise PN nowhere dense set. \square

Theorem 4.3. Let (X, ρ_1, ρ_2) be a pairwise PN dense and pairwise PN G_{δ} -set in a PN bitopological space (X, ρ_1, ρ_2) , then (A^c) is a pairwise PN σ -nowhere dense set in (X, ρ_1, ρ_2) .

Proof. Let (X, ρ_1, ρ_2) be a pairwise PN dense and pairwise PN G_δ -set in a PN bitopological space (X, ρ_1, ρ_2) .

Then $PNcl_{\rho_1}PNcl_{\rho_2}(A) = PNcl_{\rho_2}PNcl_{\rho_1}(A) = 1_X$.

This implies that

$(PNcl_{\rho_1}PNcl_{\rho_2}(A))^c = (PNcl_{\rho_2}PNcl_{\rho_1}(A))^c = 0_X$ and

hence we have $PNint_{\rho_1}PNint_{\rho_2}(A^c) = 0_X = PNint_{\rho_2}PNint_{\rho_1}(A^c)$.

Also, since A is a pairwise PN G_δ -set, A^c is a pairwise PN F_σ -set in (X, ρ_1, ρ_2) .

Hence A^c is a pairwise PN F_σ -set in (X, ρ_1, ρ_2) such that

$PNint_{\rho_1}PNint_{\rho_2}(A^c) = 0_X = PNint_{\rho_2}PNint_{\rho_1}(A^c)$.

Thus (A^c) is a pairwise PN σ -nowhere dense set in (X, ρ_1, ρ_2) . □

Proposition 4.4. If $PNint_{\rho_i}(B) = 0_X (i = 1, 2)$ for a PN-set B in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) , then B is a pairwise PN nowhere dense set in (X, ρ_1, ρ_2) .

Proof. Let B be a PN-set in (X, ρ_1, ρ_2) such that $PNint_{\rho_i}(B) = 0_X (i = 1, 2)$.

Then, $PNint_{\rho_1}PNint_{\rho_2}(B) = PNint_{\rho_1}(0_X) = 0_X$ and

$PNint_{\rho_2}PNint_{\rho_1}(B) = PNint_{\rho_2}(0_X) = 0_X$ in (X, ρ_1, ρ_2) .

Then, we have $(PNint_{\rho_1}PNint_{\rho_2}(B))^c = 1_X$ and

$(PNint_{\rho_2}PNint_{\rho_1}(B))^c = 1_X$ and

hence $PNcl_{\rho_1}PNcl_{\rho_2}(B^c) = 1_X$ and $PNcl_{\rho_2}PNcl_{\rho_1}(B^c) = 1_X$.

That is, B^c is a pairwise PN dense set in (X, ρ_1, ρ_2) .

Since (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space,

$PNcl_{\rho_1}PNint_{\rho_2}(B^c) = 1_X = PNcl_{\rho_2}PNint_{\rho_1}(B^c)$, for the pairwise PN dense set B^c in (X, ρ_1, ρ_2) .

Then, we have $(PNcl_{\rho_1}PNint_{\rho_2}(B^c))^c = 0_X$ and

$(PNcl_{\rho_2}PNint_{\rho_1}(B^c))^c = 0_X$ and hence $PNint_{\rho_1}PNcl_{\rho_2}(B) = 0_X$ and

$PNint_{\rho_2}PNcl_{\rho_1}(B) = 0_X$.

Therefore B is a pairwise PN nowhere dense set in (X, ρ_1, ρ_2) . □

Proposition 4.5. If (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space if $PNcl_{\rho_1}PNint_{\rho_2}(A) \neq 1_X$ and $PNcl_{\rho_2}PNint_{\rho_1}(A) \neq 1_X$ for a PN A in (X, ρ_1, ρ_2) , then $PNcl_{\rho_1}PNcl_{\rho_2} \neq 1_X$ and $PNcl_{\rho_2}PNcl_{\rho_1}(A) \neq 1_X$ in (X, ρ_1, ρ_2) .

Proof. Let $PNcl_{\rho_1}PNint_{\rho_2}(A) \neq 1_X$ and $PNcl_{\rho_2}PNcl_{\rho_1}(A) \neq 1_X$, for a PN-set A in the PN bitopological space (X, ρ_1, ρ_2) .

Suppose that $PNcl_{\rho_1}PNcl_{\rho_2}(A) = 1_X$ and

$PNcl_{\rho_2}PNcl_{\rho_1}(A) = 1_X$ in (X, ρ_1, ρ_2) .

Since (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable

space, $PNcl_{\rho_1}PNcl_{\rho_2}(A) = 1_X$ and $PNcl_{\rho_2}PNcl_{\rho_1}(A) = 1_X$ in (X, ρ_1, ρ_2) , will imply that

$PNcl_{\rho_1}PNint_{\rho_2}(A) = 1_X$ and $PNcl_{\rho_2}PNint_{\rho_1}(A) = 1_X$

in (X, ρ_1, ρ_2) , a contradiction to the hypothesis.

Hence we must have $PNcl_{\rho_1}PNcl_{\rho_2}(A) \neq 1_X$ and

$PNcl_{\rho_2}PNcl_{\rho_1}(A) \neq 1_X$ in (X, ρ_1, ρ_2) . □

Proposition 4.6. If $PNint_{\rho_1}PNcl_{\rho_2}(A) \neq 0_X$ and $PNint_{\rho_2}PNcl_{\rho_1} \neq 0_X$, for a PN A in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) , then $PNint_{\rho_1} \neq 0_X$ and $PNint_{\rho_2}(A) \neq 0_X$ in (X, ρ_1, ρ_2) .

Proof. Suppose that $PNint_{\rho_1}(A) = 0_X$ and $PNint_{\rho_2}(A) = 0_X$ for a PN-set A in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) .

Then, $(PNint_{\rho_1}PNint_{\rho_2}(A))^c = (PNint_{\rho_1}(0_X))^c = 1_X$ and

$(PNint_{\rho_2}PNint_{\rho_1}(A))^c = (PNint_{\rho_2}(0_X))^c = 1_X$.

Then $PNcl_{\rho_1}PNcl_{\rho_2}(A^c) = 1_X$ and $PNcl_{\rho_2}PNcl_{\rho_1}(A^c) = 1_X$.

Hence (A^c) is a pairwise PN dense set in (X, ρ_1, ρ_2) . Since (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space, $PNcl_{\rho_1}PNint_{\rho_2}(A^c) = 1_X$ and $PNcl_{\rho_2}PNint_{\rho_1}(A^c) = 1_X$.

Then, we will have $(PNint_{\rho_1}PNcl_{\rho_2}(A))^c = 1_X$ and

$(PNint_{\rho_2}PNcl_{\rho_1}(A))^c = 1_X$ and

hence $PNint_{\rho_1}PNcl_{\rho_2}(A) = 0_X$ and

$PNint_{\rho_2}PNcl_{\rho_1}(A) = 0_X$, a contradiction.

Hence we must have $PNint_{\rho_1} \neq 0_X$ and $PNint_{\rho_2}(A) \neq 0_X$ in (X, ρ_1, ρ_2) . □

Theorem 4.7. If A is a pairwise PN σ -nowhere dense set in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) , then A is a pairwise PN nowhere dense set and pairwise PN F_σ -set in (X, ρ_1, ρ_2) .

Proof. Let A be a pairwise PN σ -nowhere dense set in (X, ρ_1, ρ_2) .

Then A is a pairwise PN F_σ -set in (X, ρ_1, ρ_2) .

Since $PNint_{\rho_1}PNint_{\rho_2}(A) = 0_X = PNint_{\rho_2}PNint_{\rho_1}(A)$.

Then $(PNint_{\rho_1}PNint_{\rho_2}(A))^c = 1_X = (PNint_{\rho_2}PNint_{\rho_1}(A))^c$ implies that $PNcl_{\rho_1}PNcl_{\rho_2}(A^c) = 1_X = PNcl_{\rho_2}PNcl_{\rho_1}(A^c)$.

Hence, (A^c) is a pairwise fuzzy dense set in (X, ρ_1, ρ_2) . Since (X, ρ_1, ρ_2) is a pairwise fuzzy strongly irresolvable space, for the pairwise fuzzy dense set (A^c) in (X, ρ_1, ρ_2) , we have

$PNcl_{\rho_1}PNint_{\rho_2}(A^c) = 1_X = PNcl_{\rho_2}PNint_{\rho_1}(A^c)$.

Then $(PNint_{\rho_1}PNcl_{\rho_2}(A))^c = 0_X = (PNint_{\rho_2}PNcl_{\rho_1}(A))^c$ implies that $PNint_{\rho_1}PNcl_{\rho_2}(A)$ and $PNint_{\rho_2}PNcl_{\rho_1}(A) = 0_X$.

Therefore A is a pairwise PN nowhere dense set and pairwise PN F_σ -set in (X, ρ_1, ρ_2) . □

Proposition 4.8. If A is a pairwise PN σ -nowhere dense set in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) , then A is a pairwise PN semi closed set in (X, ρ_1, ρ_2) .

Proof. Let A be a pairwise PN σ -nowhere dense set in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) . Then, by theorem 4.7, A is a pairwise PN nowhere dense set in (X, ρ_1, ρ_2) and hence $PNint_{\rho_1}PNcl_{\rho_2}(A)$

$= 0_X$ and $PNint_{\rho_2}PNcl_{\rho_1}(A) = 0_X$.

Then, we have $PNint_{\rho_1}PNcl_{\rho_2}(A) \subseteq A$ and

$PNint_{\rho_2}PNcl_{\rho_1}(A) \subseteq A$.

Therefore A is a pairwise PN semi-closed set in (X, ρ_1, ρ_2) . □

Proposition 4.9. If A is a pairwise PN dense and pairwise PN G_δ -set in a pairwise strongly irresolvable space (X, ρ_1, ρ_2) , then A is a pairwise PN semi-open set in (X, ρ_1, ρ_2) .

Proof. Let A be a pairwise PN dense and pairwise PN G_δ -set in (X, ρ_1, ρ_2) .

Then, by theorem 4.3, (A^c) is a pairwise PN σ -nowhere dense set in (X, ρ_1, ρ_2) .

Since (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space, by proposition 4.8, (A^c) is a pairwise PN semi-closed set in (X, ρ_1, ρ_2) . Hence A is a pairwise PN semi-open set in (X, ρ_1, ρ_2) . □

Proposition 4.10. If $A_i \subseteq (A_j)^c (i \neq j)$, where A_i is a pairwise PN dense set in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) , then A_j is a pairwise PN nowhere dense set in (X, ρ_1, ρ_2) .

Proof. Let A_i be a pairwise PN dense set in (X, ρ_1, ρ_2) .

Since (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space, for the pairwise PN dense set A_i , we have

$PNcl_{\rho_1}PNint_{\rho_2}(A_i) = 1_X$ and $PNcl_{\rho_2}PNint_{\rho_1}(A_i) = 1_X$.

Now $A_i \subseteq (A_j)^c (i \neq j)$ implies that

$PNcl_{\rho_1}PNint_{\rho_2}(A_i) \subseteq PNcl_{\rho_1}PNint_{\rho_2}((A_j)^c)$ and

$PNcl_{\rho_2}PNint_{\rho_1}(A_i) \subseteq PNcl_{\rho_2}PNint_{\rho_1}(A_j)^c$.

Then, we have $1_X \subseteq PNcl_{\rho_1}PNint_{\rho_2}(A_j)^c$ and

$1_X \subseteq PNcl_{\rho_2}PNint_{\rho_1}(A_j)^c$.

That is, $PNcl_{\rho_1}PNint_{\rho_2}((A_j)^c) = 1_X$ and

$PNcl_{\rho_2}PNint_{\rho_1}((A_j)^c) = 1_X$.

Hence, $PNint_{\rho_1}PNcl_{\rho_2}(A_j) = 0_X$ and $PNint_{\rho_2}PNcl_{\rho_1}(A_j) = 0_X$.

Therefore A_j is a pairwise PN nowhere dense set in (X, ρ_1, ρ_2) . □

Definition 4.11. A PN bitopological space (X, ρ_1, ρ_2) is called a pairwise PN σ -Baire space

if $PNint_{\rho_i}(\bigcup_{k=1}^\infty (A_k)) = 0_X, (i = 1, 2)$

where A_k 's are pairwise PN σ -nowhere dense sets in (X, ρ_1, ρ_2) .

Proposition 4.12. If (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space and pairwise PN σ -Baire space, then $PNcl_{\rho_i}(\bigcap_{k=1}^\infty (B_k)) = 1_X (i = 1, 2)$ where B_k 's are pairwise PN semi-open sets in (X, ρ_1, ρ_2) .

Proof. Let (X, ρ_1, ρ_2) be a pairwise PN strongly irresolvable space and pairwise PN σ -Baire space. Let A_k 's

be pairwise PN σ -nowhere dense sets in (X, ρ_1, ρ_2) . Since (X, ρ_1, ρ_2) is a pairwise PN σ -Baire space,

$PNint_{\rho_i}(\bigcup_{k=1}^\infty (A_k)) = 0_X, (i = 1, 2)$ where A_k 's are pairwise PN σ -nowhere dense sets in (X, ρ_1, ρ_2) .

Then, $(PNint_{\rho_i}(\bigcup_{k=1}^\infty (A_k)))^c$

$= 1_X$. This implies that $PNcl_{\rho_i}(\bigcap_{k=1}^\infty ((A_k)^c)) = 1_X$. By proposition 4.8, the pairwise PN σ -nowhere dense sets A_k 's in the pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) , are pairwise PN semi-closed sets in

(X, ρ_1, ρ_2) and hence $((A_k)^c)$'s are pairwise PN semi-open sets in (X, ρ_1, ρ_2) . Let $B_k = (A_k)^c$. Hence we have $PNcl_{\rho_i}(\bigcap_{k=1}^{\infty} (B_k)) = 1_X, (i = 1, 2)$, where B_k 's are pairwise PN semi-open sets in (X, ρ_1, ρ_2) . \square

Definition 4.13. A PN bitopological space (X, ρ_1, ρ_2) is called a pairwise PN Baire space if

$$PNint_{\rho_i}(\bigcup_{k=1}^{\infty} (A_k)) = 0_X, (i = 1, 2)$$

where A_k 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) .

Proposition 4.14. If $A_i \subseteq (A_j)^c (i \neq j)$, where A_i is a pairwise PN dense set in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) and if

$$PNcl_{PN\rho_k}(\bigcap_{i=1}^{\infty} (A_i)) = 1_X (k = 1, 2),$$

then (X, ρ_1, ρ_2) is a pairwise PN Baire space.

Proof. Let $A_i \subseteq (A_j)^c (i \neq j)$, where A_i is a pairwise PN dense set in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) . Then, by proposition 4.10, A_i 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) . Now $A_i \subseteq (A_j)^c$, implies that $PNcl_{PN\rho_k}(\bigcap_{i=1}^{\infty} (A_i)) \subseteq PNcl_{PN\rho_k}(\bigcap_{j=1}^{\infty} ((A_j)^c))$. Then we have $1_X \subseteq PNcl_{PN\rho_k}(\bigcap_{j=1}^{\infty} ((A_j)^c))$ and hence $1_X \subseteq (PNint_{PN\rho_k}(\bigcup_{j=1}^{\infty} (A_j)))^c$. This implies that $PNint_{PN\rho_k}(\bigcup_{j=1}^{\infty} (A_j)) \subseteq 0_X$. That is, $PNint_{PN\rho_k}(\bigcap_{j=1}^{\infty} (A_j)) = 0_X$. Hence $PNint_{PN\rho_k}(\bigcup_{j=1}^{\infty} (A_j)) = 0_X$, where A_j 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) , implies that (X, ρ_1, ρ_2) is a pairwise PN Baire space. \square

Proposition 4.15. If $PNcl_{\rho_i}(\bigcap_{k=1}^{\infty} (A_k)) = 1_X (i = 1, 2)$ where A_k 's are pairwise PN dense and pairwise PN G_σ -sets in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) , then (X, ρ_1, ρ_2) is a pairwise PN Baire space.

Proof. Now $PNcl_{\rho_i}(\bigcap_{k=1}^{\infty} (A_k)) = 1_X (i = 1, 2)$, implies that $PNint_{\rho_i}(\bigcup_{k=1}^{\infty} ((A_k)^c)) = 0_X$. Since A_k 's are pairwise PN dense and pairwise PN G_σ -sets in a pairwise PN strongly irresolvable space (X, ρ_1, ρ_2) , by theorem 4.3, $((A_k)^c)$'s are pairwise PN σ -nowhere dense sets in (X, ρ_1, ρ_2) . Also, by theorem 4.7, $((A_k)^c)$'s are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) . Hence $PNint_{\rho_i}(\bigcup_{k=1}^{\infty} ((A_k)^c)) = 0_X$, where $((A_k)^c)$'s are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) , implies that (X, ρ_1, ρ_2) is a pairwise PN Baire space. \square

Definition 4.16. A PN bitopological space (X, ρ_1, ρ_2) is a pairwise PN almost resolvable space if $\bigcup_{k=1}^{\infty} (A_k) = 1_X$, where A_k 's in (X, ρ_1, ρ_2) are such that

$$PNint_{\rho_1}PNint_{\rho_2}(A_k) = PNint_{\rho_2}PNint_{\rho_1}(A_k) = 0_X.$$

Otherwise (X, ρ_1, ρ_2) is called a pairwise PN almost irresolvable space.

Theorem 4.17. If a PN bitopological space (X, ρ_1, ρ_2) is a pairwise PN second category space, then (X, ρ_1, ρ_2) is a pairwise PN almost irresolvable space.

Proof. Let (X, ρ_1, ρ_2) be a second category space. Then, $\bigcup_{i=1}^{\infty} (A_i) \neq 1_X$, where A_i 's are PN nowhere dense sets in (X, ρ_1, ρ_2) . That is, $\bigcup_{i=1}^{\infty} (A_i) \neq 1_X$, where $PNint_{\rho_1}PNcl_{\rho_2}(A_i) = 0_X = PNint_{\rho_2}PNcl_{\rho_1}(A_i)$, Now, $PNint_{\rho_1}PNint_{\rho_2}(A_i) \subseteq PNint_{\rho_1}PNcl_{\rho_2}(A_i)$ and $PNint_{\rho_2}PNint_{\rho_1}(A_i) \subseteq PNint_{\rho_2}PNcl_{\rho_1}(A_i)$, implies that $PNint_{\rho_1}PNint_{\rho_2}(A_i) = 0_X$ and $PNint_{\rho_2}PNint_{\rho_1}(A_i) = 0_X$. Hence $\bigcup_{i=1}^{\infty} (A_i) \neq 1_X$, where $PNint_{\rho_1}PNint_{\rho_2}(A_i) = 0_X$ and $PNint_{\rho_2}PNint_{\rho_1}(A_i) = 0_X$ and therefore (X, ρ_1, ρ_2) is a pairwise PN almost irresolvable spaces. \square

Proposition 4.18. Let the PN bitopological space (X, ρ_1, ρ_2) be a pairwise PN strongly irresolvable space. Then, we have the following:

- (i) (X, ρ_1, ρ_2) is a pairwise PN almost irresolvable space, then (X, ρ_1, ρ_2) is a pairwise PN second category space.
- (ii) (X, ρ_1, ρ_2) is a pairwise PN almost resolvable space, then (X, ρ_1, ρ_2) is a pairwise PN first category space.

Proof. (i) Let (X, ρ_1, ρ_2) be a pairwise PN almost irresolvable space.

Then, $\bigcup_{k=1}^{\infty} (A_k) \neq 1_X$, where A_k 's in (X, ρ_1, ρ_2) are such that

$$PNint_{\rho_1}PNint_{\rho_2}(A_k) = PNint_{\rho_2}PNint_{\rho_1}(A_k) = 0_X.$$

Now, $(PNint_{\rho_1}PNint_{\rho_2}(A_k))^c = 1_X$ and

$$(PNint_{\rho_2}PNint_{\rho_1}(A_k))^c = 1_X.$$

Then, $PNcl_{\rho_1}PNcl_{\rho_2}((A_k)^c) = 1_X$ and $PNcl_{\rho_2}PNcl_{\rho_1}((A_k)^c) = 1_X$ and

hence $((A_k)^c)$'s are pairwise PN dense sets in (X, ρ_1, ρ_2) .

Since (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space, for the pairwise PN dense set A_k , we have

$$PNcl_{\rho_1}PNint_{\rho_2}((A_k)^c) = 1_X \text{ and } PNcl_{\rho_2}PNint_{\rho_1}((A_k)^c) = 1_X.$$

This implies that $PNint_{\rho_1}PNcl_{\rho_2}(A_k) = PNint_{\rho_2}PNcl_{\rho_1}(A_k) = 0_X$ and hence A_k 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) . Thus, $\bigcup_{k=1}^{\infty}(A_k) \neq 1_X$, where A_k 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) , implies that (X, ρ_1, ρ_2) is a pairwise PN second category space.

(ii) Let (X, ρ_1, ρ_2) be a pairwise PN almost resolvable space. Then, $\bigcup_{k=1}^{\infty}(A_k) = 1_X$, where A_k 's in (X, ρ_1, ρ_2) are such that $PNint_{\rho_1}PNint_{\rho_2}(A_k) = PNint_{\rho_2}PNint_{\rho_1}(A_k) = 0_X$. Now, $(PNint_{\rho_1}PNint_{\rho_2}(A_k))^c = 1_X$ and $(PNint_{\rho_2}PNint_{\rho_1}(A_k))^c = 1_X$. Then, $PNcl_{\rho_1}PNcl_{\rho_2}((A_k)^c) = 1_X$ and $PNcl_{\rho_2}PNcl_{\rho_1}((A_k)^c) = 1_X$ and hence $((A_k)^c)$'s are pairwise PN dense sets in (X, ρ_1, ρ_2) . Since (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space, for the pairwise PN dense set A_k , we have $PNcl_{\rho_1}PNint_{\rho_2}((A_k)^c) = 1_X$ and $PNcl_{\rho_2}PNint_{\rho_1}(A_k)^c = 1_X$. This implies that $PNint_{\rho_1}PNcl_{\rho_2}(A_k) = PNint_{\rho_2}PNcl_{\rho_1}(A_k) = 0_X$ and hence A_k 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) . Thus, $\bigcup_{k=1}^{\infty}(A_k) = 1_X$, where A_k 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) , implies that (X, ρ_1, ρ_2) is a pairwise PN first category space. \square

Theorem 4.19. If $\bigcup_{k=1}^{\infty}(A_k) = 1_X$, where A_k 's are pairwise PN σ -nowhere dense sets in a PN bitopological space (X, ρ_1, ρ_2) , then (X, ρ_1, ρ_2) is a pairwise PN almost resolvable space.

Definition 4.20. A PN bitopological space (X, ρ_1, ρ_2) is called pairwise PN σ -first category space if the PN 1_X is a pairwise PN σ -first category set in (X, ρ_1, ρ_2) . That is, $1_X = \bigcup_{i=1}^{\infty}(A_i)$, where A_i 's are pairwise PN σ -nowhere dense sets in (X, ρ_1, ρ_2) . Otherwise, (X, ρ_1, ρ_2) will be called a pairwise PN σ -second category space.

Proposition 4.21. If (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable and pairwise PN σ -first category space, then (X, ρ_1, ρ_2) is a pairwise PN first category space.

Proof. Let (X, ρ_1, ρ_2) be a pairwise PN σ -first category space. Then, $\bigcup_{k=1}^{\infty}(A_k) = 1_X$, where A_k 's are pairwise PN σ -nowhere dense sets in (X, ρ_1, ρ_2) . Since (X, ρ_1, ρ_2) is a pairwise PN strongly irresolvable space, by theorem 4.7, then A_k 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) . Hence $\bigcup_{k=1}^{\infty}(A_k) = 1_X$, where the A_k 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) , implies that (X, ρ_1, ρ_2) is a pairwise PN first category space. \square

Proposition 4.22. If (X, ρ_1, ρ_2) is a pairwise PN Baire and pairwise PN strongly irresolvable space, then $PNcl_{\rho_i}(\bigcap_{k=1}^{\infty}(A_k)) = 1_X$ ($i = 1, 2$) where A_k 's are pairwise PN dense in (X, ρ_1, ρ_2) .

Proof. Let (X, ρ_1, ρ_2) be a pairwise PN Baire space. Then, $PNint_{\rho_i}(\bigcup_{k=1}^{\infty}(A_k)) = 0_X$, ($i = 1, 2$), where A_k 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) . Since A_k 's are pairwise PN nowhere dense sets in (X, ρ_1, ρ_2) , by theorem 4.2, $(A_k)^c$'s are pairwise PN dense sets in (X, ρ_1, ρ_2) . Now $PNint_{\rho_i}(\bigcup_{k=1}^{\infty}(A_k)) = 0_X$, implies that $(PNint_{\rho_i}(\bigcup_{k=1}^{\infty}(A_k)))^c = 1_X$. Then, we have $PNcl_{\rho_i}(\bigcap_{k=1}^{\infty}(A_k)^c) = 1_X$. Let $(A_k)^c = B_k$. Then $PNcl_{\rho_i}(\bigcap_{k=1}^{\infty}(B_k)) = 1_X$, where B_k 's are pairwise PN dense sets in (X, ρ_1, ρ_2) . \square

References

- [1] E.O.Adeleke, A.A.A. Agboola, F.Smarandache, "Refined Neutrosophic Rings I", International journal of neutrosophic science, Vol 2, PP 77-81, 2020.
- [2] E.O.Adeleke, A.A.A. Agboola, F.Smarandache, "Refined Neutrosophic Rings II", International journal of neutrosophic science, Vol 2, PP 89-94, 2020.
- [3] Ahmed Hatip, "The special Neutrosophic function", International journal of neutrosophic science, Vol 4, PP 104-116, 2020.
- [4] K.T. Atanassov, "Intuitionistic fuzzy sets", Fuzzy Sets Syst, Vol 20, PP 87-96, 1986.

- [5] P.Bosc,O.Pivert, "On a fuzzy bipolar relational algebra", Information Sciences,Vol 219,PP 1-16,2013.
- [6] C.L.Chang," Fuzzy Topological Spaces",J.Math.Anal.Appl.,Vol 24,PP 182-190,1968.
- [7] D.Coker,"An introduction to fuzzy Topological Spaces", Fuzzy Sets and Systems, Vol 88,PP 81-89,1997.
- [8] R.Jansi,K.Mohana ,Florentin Smarandache,"Correlation Measure for Pythagorean Neutrosophic Fuzzy Sets with T and F as Dependent Neutrosophic Components" ,Neutrosophic Sets and Systems,Vol 30,PP 202-212,2019.
- [9] A.Kandil," Biproximities and fuzzy bitopological spaces", Simon Stevin, Vol 63, PP 45-66,1989.
- [10] R.Levy,"Almost P-spaces",Canad.J.Math., Vol XXIX(2),PP 284-288,1977.
- [11] A.K.Misra," A topological view of P-spaces",Gen.Tology Appl.,Vol 2,PP 349-362 ,1972.
- [12] M.Parimala,M.Karthika,Florentin Smarandache,Said Broumi," On $\alpha\omega$ -closed sets and its connectedness in terms of neutrosophic topological spaces",International journal of neutrosophic science,Vol 2,PP 82-88,2020.
- [13] F.Smarandache,"A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set",Neutrosophic Probability; American Research Press: Rehoboth, DE, USA, 1999.
- [14] F. Smarandache," Degree of dependence and independence of the (sub)components of fuzzy set and Neutrosophic set", Neutrosophic Sets Syst.,Vol 11,PP 95-97,2016.
- [15] G.Thangaraj, G.Balasubramanian,"on fuzzy resolvable and fuzzy irresolvable spaces",Fuzzy sets,Rough sets and Multivalued Operations and Application,Vol 1(2),PP 173-180, 2009.
- [16] R.R.Yager,"Pythagorean fuzzy subsets",In:Proc Joint IFSA World Congress and NAFIPS Annual Meeting,Edmonton,Canada,PP57-61,2013.
- [17] R.R.Yager,A.M.Abbasov, "Pythagorean membership grades,complex numbers and decision making",Int J Intell Syst,Vol 28,PP 436-452,2013.
- [18] L.A.Zadeh,"Fuzzy sets",Inf Control ,Vol 8,338-353,1965.