δ-open sets in Neutrosophic Hypersoft Topological Spaces

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Abstract

In this paper, neutrosphic hypersoft δ-open sets are introduced by defining the neutrosophic hypersoft regular open sets, pre-open sets, δ-interior and δ-closure. Under the guidance of these definitions, neutrosophic hypersoft δ semi-open sets and δ pre-open sets are also introduced. Further, we have deduced inuitionistic hypersoft topology and fuzzy hypersoft topology from the neutrosophic hypersoft topology. Moreover, we discuss about the relations between neutrosphic hypersoft δ-open sets, δ semi-open sets, δ pre-open sets, semi-open sets and pre-open sets and their properties with examples.

Keywords: neutrosophic hypersoft pre-open sets, neutrosophic hypersoft δ open sets, neutrosophic hypersoft δ-semi open sets, neutrosophic hypersoft δ-pre open sets.

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1 Introduction

The real world decision making problems in medical diagnosis, engineering, economics, management, computer science, artificial intelligence, social sciences, environmental science and sociology contains more uncertain and inadequate data. The traditional mathematical methods cannot deal with these kind of problems due to the imprecise data. To deal the problems with uncertainty, Zadeh [20] introduced the fuzzy set in 1965 which contains the membership value in [0,1]. The topological structure on fuzzy set was developed by Chang [6] as fuzzy topological space. Then Atanassov [4] extended this idea as Intuitionistic fuzzy set in 1983 which includes both membership and non-membership values. Coker [7] introduced intuitionistic fuzzy set in a topology as intuitionistic fuzzy topological space. Nevertheless, it can deal only with the incomplete data but not with the inconsistent or indeterminate data. To overcome this issue, Smarandache [16, 17] introduced the neutrosophic set which contains membership, indeterminacy and non-membership values which are independent to each other. It can handle all kind of real life situations containing inconsistent, incomplete and indeterminate data. Salama and Alblowi [13] in 2012, developed neutrosophic topological space. A new mathematical tool, soft set theory was introduced by Molodstov [10] in 1999 to deal uncertainties in which a soft set is a collection of imprecise interpretations of an object. Soft set is a parameterized family of subsets where parameters are the properties, attributes or characteristics of the objects. The soft set theory have several applications in different fields such as decision making, optimization, forecasting, data analysis etc. Shabir and Naz [15] developed soft topological spaces.

Maji [9] combined the neutrosophic structure and the soft set concept to develop neutrosophic soft sets and the same was modified by Deli and Broumi [8]. Later neutrosophic soft topological spaces were presented by Bera [5]. Smarandache [18] extended the notion of a soft set to a hypersoft set and then to plithogenic set by replacing function with a multi-argument function described in the cartesian product with a different set of attributes. This new concept of hypersoft set is more flexible than the soft set and more suitable in the decision-making issues involving different kind of attributes. Saqlain et.al. [14] defined the aggregate operators of neutrosophic hypersoft set. Ozturk and Yolcu [11] redefined the same and developed the neutrosophic hypersoft topological spaces. Ajay and Charisma [2] introduced fuzzy hypersoft topology, intuitionistic hypersoft topology and neutrosophic hypersoft topology. Ajay et.al. [3] defined neutrosophic hypersoft semi-open sets and developed an application in multiattribute group decision making.

Saha [12] defined δ-open sets in fuzzy topological spaces. Vadivel et al. [19] introduced δ-open sets in neutrosophic topological spaces. In 2019, Acikgoz and Esenbel [1] defined neutrosophic soft δ-topology.

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In this article, we develop the concept of neutrosophic pre-open sets, δ open sets, δ semi-open sets and δ pre-open sets thereby defining the neutrosophic hypersoft regular open set, δ-interior and δ-closure and also the relations between them are discussed with the theorems and examples. Added to that, we have proved with an example that neutrosophic hypersoft topology is the generalized framework which generalizes intuitionistic hypersoft topology and fuzzy hypersoft topology.

2 Preliminaries

Definition 2.1 [13] Let Y be an initial universe. A neutrosophic set (briefly Ns) L is an object having the form $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$ where $\mu_L \to [0,1]$ denote the degree of membership function, $\sigma_L \to [0,1]$ denote the degree of indeterminacy function and $\nu_L \to [0,1]$ denote the degree of non-membership function respectively of each element $y \in Y$ to the set L and $0 \leq \mu_L(y) + \sigma_L(y) + \nu_L(y) \leq 3$ for each $y \in Y$.

Definition 2.2 [10] Let Y be an initial universe, Q be a set of parameters and $\mathcal{P}(Y)$ be the power set of Y. A pair $(\tilde{H}, Q)$ is called the a soft set over Y where $\tilde{H}$ is a mapping $\tilde{H} : Q \to \mathcal{P}(Y)$. In other words, the soft set is a parametrized family of subsets of the set Y.

Definition 2.3 [8] Let Y be an initial universe, Q be a set of parameters. Let $\mathcal{P}(Y)$ denotes the set of all neutrosophic sets of Y. Then a neutrosophic soft set $(\tilde{H}, Q)$ over Y (briefly N_sSs) is defined by $(\tilde{H}, Q) = \{(q, \langle y, \mu_{\tilde{H}(q)}(y), \sigma_{\tilde{H}(q)}(y), \nu_{\tilde{H}(q)}(y)\rangle : y \in Y) : q \in Q\}$, where $\mu_{\tilde{H}(q)}(y), \sigma_{\tilde{H}(q)}(y), \nu_{\tilde{H}(q)}(y) \in [0,1]$ respectively called the degree of membership function, the degree of indeterminacy function and the degree of non-membership function of $\tilde{H}(q)$. Since the supremum of each μ, σ, ν is 1, the inequality $0 \leq \mu_{\tilde{H}(q)}(y) + \sigma_{\tilde{H}(q)}(y) + \nu_{\tilde{H}(q)}(y) \leq 3$ is obvious.

Definition 2.4 [18] Let Y be an initial universe and $\mathcal{P}(Y)$ be the power set of Y. Consider $q_1, q_2, q_3, ..., q_n$ for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $Q_1,\ Q_2,\ ...,\ Q_n$ with $Q_i \cap Q_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$. Then the pair $(\tilde{H},\ Q_1 \times Q_2 \times ... \times Q_n)$ where $\tilde{H} : Q_1 \times Q_2 \times ... \times Q_n \to \mathcal{P}(Y)$ is called a Hypersoft set over Y.

Definition 2.5 [14] Let Y be an initial universal set and $Q_1, Q_2, ..., Q_n$ be pairwise disjoint sets of parameters. Let $\mathcal{P}(Y)$ be the set of all neutrosophic sets of Y. Let E_i be the nonempty subset of the pair Q_i for each $i = 1, 2, ..., n$. A neutrosophic hypersoft set (briefly, N_sHSs) over Y is defined as the pair $(\tilde{H}, E_1 \times E_2 \times ... \times E_n)$ where $\tilde{H} : E_1 \times E_2 \times ... \times E_n \to \mathcal{P}(Y)$ and $\tilde{H}(E_1 \times E_2 \times ... \times E_n) = \{(q, \langle y, \mu_{\tilde{H}(q)}(y), \sigma_{\tilde{H}(q)}(y), \nu_{\tilde{H}(q)}(y)\rangle : y \in Y) : q \in E_1 \times E_2 \times ... \times E_n \subseteq Q_1 \times Q_2 \times ... \times Q_n\}$ where $\mu_{\tilde{H}(q)}(y)$ is the membership value of truthiness, $\sigma_{\tilde{H}(q)}(y)$ is the membership value of indeterminacy and $\nu_{\tilde{H}(q)}(y)$ is the membership value of falsity such that $\mu_{\tilde{H}(q)}(y), \sigma_{\tilde{H}(q)}(y), \nu_{\tilde{H}(q)}(y) \in [0,1]$. Also, $0 \leq \mu_{\tilde{H}(q)}(y) + \sigma_{\tilde{H}(q)}(y) + \nu_{\tilde{H}(q)}(y) \leq 3$.

Definition 2.6 [14] Let Y be an universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be two N_sHSs's over Y. Then $(\tilde{H}, \wedge_1)$ is the neutrosophic hypersoft subset of $(\tilde{G}, \wedge_2)$ if $\mu_{\tilde{H}(q)}(y) \leq \mu_{\tilde{G}(q)}(y), \sigma_{\tilde{H}(q)}(y) \leq \sigma_{\tilde{G}(q)}(y), \nu_{\tilde{H}(q)}(y) \leq \nu_{\tilde{G}(q)}(y)$.

It is denoted by $(\tilde{H}, \wedge_1) \subseteq (\tilde{G}, \wedge_2)$.

Definition 2.7 [14] Let Y be an universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be N_sHSs's over Y. $(\tilde{H}, \wedge_1)$ is equal to $(\tilde{G}, \wedge_1)$ if $\mu_{\tilde{H}(q)}(y) = \mu_{\tilde{G}(q)}(y), \sigma_{\tilde{H}(q)}(y) = \sigma_{\tilde{G}(q)}(y), \nu_{\tilde{H}(q)}(y) = \nu_{\tilde{G}(q)}(y)$.

Definition 2.8 [11] Let Y be an universal set and $(\tilde{H}, \wedge)$ be N_sHSs over Y. $(\tilde{H}, \wedge)^c$ is the complement of N_sHSs of $(\tilde{H}, \wedge)$ if $\mu^c_{\tilde{H}(q)}(y) = \nu_{\tilde{H}(q)}(y)$, $\sigma^c_{\tilde{H}(q)}(y) = 1 - \sigma_{\tilde{H}(q)}(y)$, $\nu^c_{\tilde{H}(q)}(y) = \mu_{\tilde{H}(q)}(y)$ where $\forall q \in \wedge$ and $\forall y \in Y$. It is clear that $((\tilde{H}, \wedge)^c)^c = (\tilde{H}, \wedge)$.

Definition 2.9 [11] A N_sHSs $(\tilde{H}, \wedge)$ over the universe set Y is said to be null neutrosophic hypersoft set if $\mu_{\tilde{H}(q)}(y) = 0,\ \sigma_{\tilde{H}(q)}(y) = 0,\ \nu_{\tilde{H}(q)}(y) = 1\ \forall q \in \wedge$ and $y \in Y$. It is denoted by $\tilde{0}_{(Y,Q)}$.

A N_sHSs $(\tilde{G}, \wedge)$ over the universal set Y is said to be absolute neutrosophic hypersoft set if $\mu_{\tilde{H}(q)}(y) = 1, \sigma_{\tilde{H}(q)}(y) = 1, \nu_{\tilde{H}(q)}(y) = 0\ \forall q \in \wedge$ and $y \in Y$. It is denoted by $\tilde{1}_{(Y,Q)}$.

Clearly, $\tilde{0}^c_{(Y,Q)} = \tilde{1}_{(Y,Q)}$ and $\tilde{1}^c_{(Y,Q)} = \tilde{0}_{(Y,Q)}$.

Definition 2.10 [11] Let Y be the universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be N_sHSs's over Y. Extended union $(\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)$ is defined as

$$\mu((\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)) = \begin{cases} \mu_{\tilde{H}(q)}(y) & \text{if } q \in \wedge_1 - \wedge_2 \\ \mu_{\tilde{G}(q)}(y) & \text{if } q \in \wedge_2 - \wedge_1 \\ max\{\mu_{\tilde{H}(q)}(y), \mu_{\tilde{G}(q)}(y)\} & \text{if } q \in \wedge_1 \cap \wedge_2 \end{cases}$$

$$\sigma((\tilde{H},\wedge_1)\cup(\tilde{G},\wedge_2)) = \begin{cases} \sigma_{\tilde{H}(q)}(y) & \text{if } q\in\wedge_1-\wedge_2 \\ \sigma_{\tilde{G}(q)}(y) & \text{if } q\in\wedge_2-\wedge_1 \\ max\{\sigma_{\tilde{H}(q)}(y),\sigma_{\tilde{G}(q)}(y)\} & \text{if } q\in\wedge_1\cap\wedge_2 \end{cases}$$

$$\nu((\tilde{H},\wedge_1)\cup(\tilde{G},\wedge_2)) = \begin{cases} \nu_{\tilde{H}(q)}(y) & \text{if } q\in\wedge_1-\wedge_2 \\ \nu_{\tilde{G}(q)}(y) & \text{if } q\in\wedge_2-\wedge_1 \\ min\{\nu_{\tilde{H}(q)}(y),\nu_{\tilde{G}(q)}(y)\} & \text{if } q\in\wedge_1\cap\wedge_2 \end{cases}$$

Definition 2.11 [11] Let Y be the universal set and $(\tilde{H},\wedge_1)$ and $(\tilde{G},\wedge_2)$ be N_sHS's over Y. Extended intersection $(\tilde{H},\wedge_1)\cap(\tilde{G},\wedge_2)$ is defined as

$$\mu((\tilde{H},\wedge_1)\cap(\tilde{G},\wedge_2)) = \begin{cases} \mu_{\tilde{H}(q)}(y) & \text{if } q\in\wedge_1-\wedge_2 \\ \mu_{\tilde{G}(q)}(y) & \text{if } q\in\wedge_2-\wedge_1 \\ min\{\mu_{\tilde{H}(q)}(y),\mu_{\tilde{G}(q)}(y)\} & \text{if } q\in\wedge_1\cap\wedge_2 \end{cases}$$

$$\sigma((\tilde{H},\wedge_1)\cap(\tilde{G},\wedge_2)) = \begin{cases} \sigma_{\tilde{H}(q)}(y) & \text{if } q\in\wedge_1-\wedge_2 \\ \sigma_{\tilde{G}(q)}(y) & \text{if } q\in\wedge_2-\wedge_1 \\ min\{\sigma_{\tilde{H}(q)}(y),\sigma_{\tilde{G}(q)}(y)\} & \text{if } q\in\wedge_1\cap\wedge_2 \end{cases}$$

$$\nu((\tilde{H},\wedge_1)\cap(\tilde{G},\wedge_2)) = \begin{cases} \nu_{\tilde{H}(q)}(y) & \text{if } q\in\wedge_1-\wedge_2 \\ \nu_{\tilde{G}(q)}(y) & \text{if } q\in\wedge_2-\wedge_1 \\ max\{\nu_{\tilde{H}(q)}(y),\nu_{\tilde{G}(q)}(y)\} & \text{if } q\in\wedge_1\cap\wedge_2 \end{cases}$$

Definition 2.12 [11] Let $\{(\tilde{H}_i,\wedge_i)|i\in I\}$ be a family of N_sHSs's over the universe set Y. Then

$\bigcup\limits_{i\in I}(\tilde{H}_i,\wedge_i) = \{\langle y, sup[\mu_{\tilde{H}_i(q)}(y)]_{i\in I}, sup[\sigma_{\tilde{H}_i(q)}(y)]_{i\in I}, inf[\nu_{\tilde{H}_i(q)}(y)]_{i\in I}\rangle : y\in Y\}$

$\bigcap\limits_{i\in I}(\tilde{H}_i,\wedge_i) = \{\langle y, inf[\mu_{\tilde{H}_i(q)}(y)]_{i\in I}, inf[\sigma_{\tilde{H}_i(q)}(y)]_{i\in I}, sup[\nu_{\tilde{H}_i(q)}(y)]_{i\in I}\rangle : y\in Y\}$.

Definition 2.13 [11] Let (Y,Q) be the family of all N_sHSs's over the universe set Y and $\tau\subseteq N_sHSs(Y,Q)$. Then τ is said to be a neutrosophic hypersoft topology (briefly, N_sHSt) on Y if

(i) $\tilde{0}_{(Y,Q)}$ and $\tilde{1}_{(Y,Q)}$ belongs to τ

(ii) the union of any number of N_sHSs's in τ belongs to τ

(iii) the intersection of finite number of N_sHSs's in τ belongs to τ.

Then (Y,Q,τ) is called a neutrosophic hypersoft toplogical space (briefly, N_sHSts) over Y. Each member of τ is said to be neutrosophic hypersoft open set (briefly, N_sHSos). A $N_sHSs(\tilde{H},\wedge)$ is called a neutrosophic hypersoft closed set (briefly, N_sHScs) if its complement $(\tilde{H},\wedge)^C$ is N_sHSos.

The intuitionisic hypersoft topological space and fuzzy topological space are defined in [2].

Definition 2.14 [11] Let (Y,Q,τ) be a N_sHSts over Y and $(\tilde{H},\wedge)\in N_sHSs(Y,Q)$ be a N_sHSs. Then, the neutrosophic hypersoft interior (briefly, N_sHSint) of $(\tilde{H},\wedge)$ is defined as $N_sHSint(\tilde{H},\wedge) = \cup\{(\tilde{G},\wedge) : (\tilde{G},\wedge)\subseteq(\tilde{H},\wedge)$ where $(\tilde{G},\wedge) is N_sHSos\}$.

Definition 2.15 [11] Let (Y,Q,τ) be a N_sHSts over Y and $(\tilde{H},\wedge)\in N_sHSs(Y,Q)$ be a N_sHSs. Then, the neutrosophic hypersoft closure (briefly, N_sHScl) of $(\tilde{H},\wedge)$ is defined as $N_sHScl(\tilde{H},\wedge) = \cap\{(\tilde{G},\wedge) : (\tilde{G},\wedge)\supseteq(\tilde{H},\wedge)$ where $(\tilde{G},\wedge) is N_sHScs\}$.

Definition 2.16 [3] Let (Y,Q,τ) be a N_sHSts over Y and $(\tilde{H},\wedge)\in N_sHSs(Y,Q)$ be a N_sHSs. Then, $(\tilde{H},\wedge)$ is called the neutrosophic hypersoft semiopen set (briefly, N_sHSSos) if $(\tilde{H},\wedge)\subseteq N_sHScl(int(\tilde{H},\wedge))$.

A $N_sHSs(\tilde{H},\wedge)$ is called a neutrosophic hypersoft semiclosed set (briefly, N_sHSScs) if its complement $(\tilde{H},\wedge)^c$ is a N_sHSSos.

3 δ-open sets in Neutrosophic Hypersoft Topological Spaces

Definition 3.1 Let (Y,Q,τ) be a N_sHSts over Y. A N_sHSs $(\tilde{H},\wedge)$ is said to be a neutrosophic hypersoft regular open set (briefly, N_sHSros) if $(\tilde{H},\wedge) = N_sHSint(N_sHScl(\tilde{H},\wedge))$. The complement of N_sHSros is called a neutrosophic hypersoft regular closed set (briefly, N_sHSrcs) in Y.

Definition 3.2 Let (Y,Q,τ) be a N_sHSts over Y and $(\tilde{H},\wedge)$ be a N_sHSs on Y. Then the neutrosophic hypersoft

(i) δ-interior (briefly, N_sHSint) of $(\tilde{H}, \wedge)$ is defined by
$N_sHS\delta int(\tilde{H}, \wedge) = \bigcup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a N_sHSros in $Y\}$

(ii) δ-closure (briefly, N_sHScl) of $(\tilde{H}, \wedge)$ is defined by
$N_sHS\delta cl(\tilde{H}, \wedge) = \bigcap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a N_sHSrcs in $Y\}$

Definition 3.3 Let (Y, Q, τ) be a N_sHSts over Y. A N_sHSs $(\tilde{H}, \wedge)$ is said to be a neutrosophic hypersoft

(i) semi-regular if $(\tilde{H}, \wedge)$ is both $N_sHS\mathcal{S}os$ and $N_sHS\mathcal{S}cs$.

(ii) pre open set (briefly, $N_sHS\mathcal{P}os$) if $(\tilde{H}, \wedge) \subseteq N_sHSint(N_sHScl(\tilde{H}, \wedge)$

(iii) δ-open set (briefly, $N_sHS\delta os$) if $(\tilde{H}, \wedge) = N_sHS\delta int(\tilde{H}, \wedge)$

(iv) δ-pre open set (briefly, $N_sHS\delta\mathcal{P}os$) if $(\tilde{H}, \wedge) \subseteq N_sHSint(N_sHS\delta cl(\tilde{H}, \wedge))$

(v) δ-semi open set (briefly, $N_sHS\delta\mathcal{S}os$) if $(\tilde{H}, \wedge) \subseteq N_sHScl(N_sHS\delta int(\tilde{H}, \wedge))$

The complement of $N_sHS\delta os$ (resp. $N_sHS\mathcal{P}os$, $N_sHS\delta\mathcal{P}os$ & $N_sHS\delta\mathcal{S}os$) is called a $N_sHS\delta$ (resp. N_sHS pre, $N_sHS\delta$ pre & $N_sHS\delta$ semi) closed set (briefly, $N_sHS\delta cs$ (resp. $N_sHS\mathcal{P}cs$, $N_sHS\delta\mathcal{P}cs$ & $N_sHS\delta\mathcal{S}cs$)) in Y.

The family of all $N_sHS\delta os$ (resp. $N_sHS\delta cs$, N_sHSros, N_sHSrcs, $N_sHS\mathcal{P}os$, $N_sHS\mathcal{P}cs$ $N_sHS\delta\mathcal{P}os$, $N_sHS\delta\mathcal{P}cs$, $N_sHS\delta\mathcal{S}os$ & $N_sHS\delta\mathcal{S}cs$) of Y is denoted by $N_sHS\delta OS(Y)$ (resp. $N_sHS\delta CS$(Y), $N_sHSrOS(Y)$, $N_sHSrOS(Y)$, $N_sHS\mathcal{P}OS(Y)$, $N_sHS\mathcal{P}CS(Y)$, $N_sHS\delta\mathcal{P}OS(Y)$, $N_sHS\delta\mathcal{P}CS(Y)$, $N_sHS\delta\mathcal{S}OS(Y)$ & $N_sHS\delta\mathcal{S}CS(Y)$).

Definition 3.4 Let (Y, Q, τ) be a N_sHSts over Y and $(\tilde{H}, \wedge)$ be a N_sHSs on Y. Then the neutrosophic hypersoft

(i) δ-pre (resp. δ-semi) interior (briefly, $N_sHS\delta\mathcal{P}int$ (resp. $N_sHS\delta\mathcal{S}int$)) of $(\tilde{H}, \wedge)$ is defined by
$N_sHS\delta\mathcal{P}int(\tilde{H}, \wedge) = \bigcup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $N_sHS\delta\mathcal{P}os$ (resp. $N_sHS\delta\mathcal{S}os$) in $Y\}$

(ii) δ-pre (resp. δ-semi) closure (briefly, $N_sHS\delta\mathcal{P}cl$ (resp. $N_sHS\delta\mathcal{S}cl$)) of $(\tilde{H}, \wedge)$ is defined by
$N_sHS\delta\mathcal{P}cl(\tilde{H}, \wedge) = \bigcap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $N_sHS\delta\mathcal{P}cs$ (resp. $N_sHS\delta\mathcal{S}cs$) in $Y\}$

Definition 3.5 Let (Y, Q, τ_I) be an intuitionistic hypersoft topological space (briefly, $IHSts$) over Y. An intuitionistic hypersoft set (briefly, $IHSs$) $(\tilde{H}, \wedge)$ is said to be an intuitionistic hypersoft regular open set (briefly, $IHSros$) if $(\tilde{H}, \wedge) = IHSint(IHScl(\tilde{H}, \wedge))$. The complement of $IHSros$ is called an intuitionistic hypersoft regular closed set (briefly, $IHSrcs$) in Y.

Definition 3.6 Let (Y, Q, τ_I) be an $IHSts$ over Y and $(\tilde{H}, \wedge)$ be an $IHSs$ on Y. Then the intuitionistic hypersoft (briefly, IHS)

(i) δ-interior (briefly, $IHSint$) of $(\tilde{H}, \wedge)$ is defined by
$IHS\delta int(\tilde{H}, \wedge) = \bigcup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $IHSros$ in $Y\}$

(ii) δ-closure (briefly, $IHScl$) of $(\tilde{H}, \wedge)$ is defined by
$IHS\delta cl(\tilde{H}, \wedge) = \bigcap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $IHSrcs$ in $Y\}$

Definition 3.7 Let (Y, Q, τ_I) be an $IHSts$ over Y. An $IHSs$ $(\tilde{H}, \wedge)$ is said to be an intuitionistic hypersoft

(i) semi-regular if $(\tilde{H}, \wedge)$ is both $IHS\mathcal{S}os$ and $IHS\mathcal{S}cs$.

(ii) pre open set (briefly, $IHS\mathcal{P}os$) if $(\tilde{H}, \wedge) \subseteq IHSint(IHScl(\tilde{H}, \wedge)$

(iii) δ-open set (briefly, $IHS\delta os$) if $(\tilde{H}, \wedge) = IHS\delta int(\tilde{H}, \wedge)$

(iv) δ-pre open set (briefly, $IHS\delta\mathcal{P}os$) if $(\tilde{H}, \wedge) \subseteq IHSint(IHS\delta cl(\tilde{H}, \wedge))$

(v) δ-semi open set (briefly, $IHS\delta\mathcal{S}os$) if $(\tilde{H}, \wedge) \subseteq IHScl(IHS\delta int(\tilde{H}, \wedge))$

The complement of $IHS\delta os$ (resp. $IHS\mathcal{P}os$, $IHS\delta\mathcal{P}os$ & $IHS\delta\mathcal{S}os$) is called a $IHS\delta$ (resp. IHS pre, $IHS\delta$ pre & $IHS\delta$ semi) closed set (briefly, $IHS\delta cs$ (resp. $IHS\mathcal{P}cs$, $IHS\delta\mathcal{P}cs$ & $IHS\delta\mathcal{S}cs$)) in Y.

The family of all $IHS\delta os$ (resp. $IHS\delta cs$, $IHSros$, $IHSrcs$, $IHS\mathcal{P}os$, $IHS\mathcal{P}cs$ $IHS\delta\mathcal{P}os$, $IHS\delta\mathcal{P}cs$, $IHS\delta\mathcal{S}os$ & $IHS\delta\mathcal{S}cs$) of Y is denoted by $IHS\delta OS(Y)$ (resp. $IHS\delta CS$(Y), $IHSrOS(Y)$, $IHSrOS(Y)$, $IHS\mathcal{P}OS(Y)$, $IHS\mathcal{P}CS(Y)$, $IHS\delta\mathcal{P}OS(Y)$, $IHS\delta\mathcal{P}CS(Y)$, $IHS\delta\mathcal{S}OS(Y)$ & $IHS\delta\mathcal{S}CS(Y)$).

Definition 3.8 Let (Y, Q, τ_I) be a $IHSts$ over Y and $(\tilde{H}, \wedge)$ be a $IHSs$ on Y. Then the intuitionistic hypersoft

(i) δ-pre (resp. δ-semi) interior (briefly, $IHS\delta\mathcal{P}int$ (resp. $IHS\delta\mathcal{S}int$)) of $(\tilde{H}, \wedge)$ is defined by $IHS\delta\mathcal{P}int(\tilde{H}, \wedge) = \bigcup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $IHS\delta\mathcal{P}os$ (resp. $IHS\delta\mathcal{S}os$) in $Y\}$

(ii) δ-pre (resp. δ-semi) closure (briefly, $IHS\delta\mathcal{P}cl$ (resp. $IHS\delta\mathcal{S}cl$)) of $(\tilde{H}, \wedge)$ is defined by $IHS\delta\mathcal{P}cl(\tilde{H}, \wedge) = \bigcap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $IHS\delta\mathcal{P}cs$ (resp. $IHS\delta\mathcal{S}cs$) in $Y\}$

Definition 3.9 Let (Y, Q, τ_F) be a fuzzy hypersoft topological space (briefly, $FHSts$) over Y. An fuzzy hypersoft set (briefly, $FHSs$) $(\tilde{H}, \wedge)$ is said to be a fuzzy hypersoft regular open set (briefly, $FHSros$) if $(\tilde{H}, \wedge) = FHSint(FHScl(\tilde{H}, \wedge))$. The complement of $FHSros$ is called a fuzzy hypersoft regular closed set (briefly, $FHSrcs$) in Y.

Definition 3.10 Let (Y, Q, τ_F) be a $FHSts$ over Y and $(\tilde{H}, \wedge)$ be a $FHSs$ on Y. Then the fuzzy hypersoft (briefly, FHS)

(i) δ-interior (briefly, $FHSint$) of $(\tilde{H}, \wedge)$ is defined by $FHS\delta int(\tilde{H}, \wedge) = \bigcup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $FHSros$ in $Y\}$

(ii) δ-closure (briefly, $FHScl$) of $(\tilde{H}, \wedge)$ is defined by $FHS\delta cl(\tilde{H}, \wedge) = \bigcap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $FHSrcs$ in $Y\}$

Definition 3.11 Let (Y, Q, τ_F) be a $FHSts$ over Y. An $FHSs$ $(\tilde{H}, \wedge)$ is said to be a fuzzy hypersoft

(i) semi-regular if $(\tilde{H}, \wedge)$ is both $FHS\mathcal{S}os$ and $FHS\mathcal{S}cs$.

(ii) pre open set (briefly, $FHS\mathcal{P}os$) if $(\tilde{H}, \wedge) \subseteq FHSint(FHScl(\tilde{H}, \wedge)$

(iii) δ-open set (briefly, $FHS\delta os$) if $(\tilde{H}, \wedge) = FHS\delta int(\tilde{H}, \wedge)$

(iv) δ-pre open set (briefly, $FHS\delta\mathcal{P}os$) if $(\tilde{H}, \wedge) \subseteq FHSint(FHS\delta cl(\tilde{H}, \wedge))$

(v) δ-semi open set (briefly, $FHS\delta\mathcal{S}os$) if $(\tilde{H}, \wedge) \subseteq FHScl(FHS\delta int(\tilde{H}, \wedge))$

The complement of $FHS\delta os$ (resp. $FHS\mathcal{P}os$, $FHS\delta\mathcal{P}os$ & $FHS\delta\mathcal{S}os$) is called a $FHS\delta$ (resp. FHS pre, $FHS\delta$ pre & $FHS\delta$ semi) closed set (briefly, $FHS\delta cs$ (resp. $FHS\mathcal{P}cs$, $FHS\delta\mathcal{P}cs$ & $FHS\delta\mathcal{S}cs$)) in Y.

The family of all $FHS\delta os$ (resp. $FHS\delta cs$, $FHSros$, $FHSrcs$, $FHS\mathcal{P}os$, $FHS\mathcal{P}cs$ $FHS\delta\mathcal{P}os$, $FHS\delta\mathcal{P}cs$, $FHS\delta\mathcal{S}os$ & $FHS\delta\mathcal{S}cs$) of Y is denoted by $FHS\delta OS(Y)$ (resp. $FHS\delta CS(\mathrm{Y})$, $FHSrOS(Y)$, $FHSrOS(Y)$, $FHS\mathcal{P}OS(Y)$, $FHS\mathcal{P}CS(Y)$, $FHS\delta\mathcal{P}OS(Y)$, $FHS\delta\mathcal{P}CS(Y)$, $FHS\delta\mathcal{S}OS(Y)$ & $FHS\delta\mathcal{S}CS(Y)$).

Definition 3.12 Let (Y, Q, τ_F) be a $FHSts$ over Y and $(\tilde{H}, \wedge)$ be a $FHSs$ on Y. Then the fuzzy hypersoft

(i) δ-pre (resp. δ-semi) interior (briefly, $FHS\delta\mathcal{P}int$ (resp. $FHS\delta\mathcal{S}int$)) of $(\tilde{H}, \wedge)$ is defined by $FHS\delta\mathcal{P}int(\tilde{H}, \wedge) = \bigcup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $FHS\delta\mathcal{P}os$ (resp. $FHS\delta\mathcal{S}os$) in $Y\}$

(ii) δ-pre (resp. δ-semi) closure (briefly, $FHS\delta\mathcal{P}cl$ (resp. $FHS\delta\mathcal{S}cl$)) of $(\tilde{H}, \wedge)$ is defined by $FHS\delta\mathcal{P}cl(\tilde{H}, \wedge) = \bigcap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq (\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $FHS\delta\mathcal{P}cs$ (resp. $FHS\delta\mathcal{S}cs$) in $Y\}$

Example 3.1 Let $Y = \{y_1, y_2\}$ be a $NsHS$ initial universe and the attributes be Q_1, Q_2. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\},\ Q_2 = \{b_1, b_2\}.$$

Suppose that

$$E_1 = \{a_1, a_2\}, E_2 = \{b_1\}$$
$$D_1 = \{a_1\}, D_2 = \{b_1, b_2\}$$

are subsets of Q_i for each $i = 1, 2$. Then the N_sHSs $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_3)$ over the universe Y are as follows.

$$(\tilde{H}_1, \wedge_1) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.8,0.8,0.2}, \frac{y_2}{0.6,0.8,0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{y_1}{0.7,0.8,0.3}, \frac{y_2}{0.5,0.5,0.2}\}\rangle \end{matrix}\right\}$$

$$(\tilde{H}_2, \wedge_2) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.2,0.4,0.6}, \frac{y_2}{0.3,0.5,0.6}\}\rangle, \\ \langle (a_1, b_2), \{\frac{y_1}{0.5,0.5,0.4}, \frac{y_2}{0.4,0.5,0.5}\}\rangle \end{matrix}\right\}$$

$$(\tilde{H}_3, \wedge_3) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.2,0.4,0.6}, \frac{y_2}{0.3,0.5,0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{y_1}{0.7,0.8,0.3}, \frac{y_2}{0.5,0.5,0.2}\}\rangle, \\ \langle (a_1, b_2), \{\frac{y_1}{0.5,0.5,0.4}, \frac{y_2}{0.4,0.5,0.5}\}\rangle \end{matrix}\right\}$$

$$(\tilde{H}_4, \wedge_3) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.8,0.8,0.2}, \frac{y_2}{0.6,0.8,0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{y_1}{0.7,0.8,0.3}, \frac{y_2}{0.5,0.5,0.2}\}\rangle, \\ \langle (a_1, b_2), \{\frac{y_1}{0.5,0.5,0.4}, \frac{y_2}{0.4,0.5,0.5}\}\rangle \end{matrix}\right\}$$

Then $\tau = \{0_{(Y,Q)}, 1_{(Y,Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_3), (\tilde{H}_4, \wedge_3)\}$ is a N_sHSts.

Remark 3.1 From N_sHSt we can deduce $IHSt$ and $FHSt$. $IHSt$ is obtained by considering the membership values and non membership values whereas $FHSt$ is obtained by considering only membership values. For example,

Example 3.2 Let $Y = \{y_1, y_2\}$ be an IHS initial universe and the attributes be Q_1, Q_2. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\},\ Q_2 = \{b_1, b_2\}$$

Suppose that

$$\begin{matrix} E_1 = \{a_1, a_2\}, E_2 = \{b_1\} \\ D_1 = \{a_1\}, D_2 = \{b_1, b_2\} \end{matrix}$$

are subsets of Q_i for each $i = 1, 2$. Then the $IHSs$ $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_4)$ over the universe Y are as follows.

$$(\tilde{H}_1, \wedge_1) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.8,0.2}, \frac{y_2}{0.6,0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{y_1}{0.7,0.3}, \frac{y_2}{0.5,0.2}\} \rangle \end{matrix}\right\}$$
$$(\tilde{H}_2, \wedge_2) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.2,0.6}, \frac{y_2}{0.3,0.6}\} \rangle, \\ \langle (a_1, b_2), \{\frac{y_1}{0.5,0.4}, \frac{y_2}{0.4,0.5}\} \rangle \end{matrix}\right\}$$
$$(\tilde{H}_3, \wedge_3) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.2,0.6}, \frac{y_2}{0.3,0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{y_1}{0.7,0.3}, \frac{y_2}{0.5,0.2}\} \rangle, \\ \langle (a_1, b_2), \{\frac{y_1}{0.5,0.4}, \frac{y_2}{0.4,0.5}\} \rangle \end{matrix}\right\}$$
$$(\tilde{H}_4, \wedge_3) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.8,0.2}, \frac{y_2}{0.6,0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{y_1}{0.7,0.3}, \frac{y_2}{0.5,0.2}\} \rangle, \\ \langle (a_1, b_2), \{\frac{y_1}{0.5,0.4}, \frac{y_2}{0.4,0.5}\} \rangle \end{matrix}\right\}$$

Then $\tau = \{0_{(Y,Q)}, 1_{(Y,Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_3), (\tilde{H}_4, \wedge_3)\}$ is a $IHSts$.

Example 3.3 Let $Y = \{y_1, y_2\}$ be an FHS initial universe and the attributes be Q_1, Q_2. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\},\ Q_2 = \{b_1, b_2\}$$

Suppose that

$$\begin{matrix} E_1 = \{a_1, a_2\}, E_2 = \{b_1\} \\ D_1 = \{a_1\}, D_2 = \{b_1, b_2\} \end{matrix}$$

are subsets of Q_i for each $i = 1, 2$. Then the $FHSs$ $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_4)$ over the universe Y are as follows.

$$(\tilde{H}_1, \wedge_1) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.8}, \frac{y_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{y_1}{0.7}, \frac{y_2}{0.5}\} \rangle \end{matrix}\right\}$$
$$(\tilde{H}_2, \wedge_2) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.2}, \frac{y_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{y_1}{0.5}, \frac{y_2}{0.4}\} \rangle \end{matrix}\right\}$$
$$(\tilde{H}_3, \wedge_3) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.2}, \frac{y_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{y_1}{0.7}, \frac{y_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{y_1}{0.5}, \frac{y_2}{0.4}\} \rangle \end{matrix}\right\}$$
$$(\tilde{H}_4, \wedge_3) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{y_1}{0.8}, \frac{y_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{y_1}{0.7}, \frac{y_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{y_1}{0.5}, \frac{y_2}{0.4}\} \rangle \end{matrix}\right\}$$

Then $\tau = \{0_{(Y,Q)}, 1_{(Y,Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_3), (\tilde{H}_4, \wedge_3)\}$ is a $FHSts$.

Theorem 3.1 The $N_sHS\delta$-interior operator satisfies,

(i) $N_sHS\delta int(\tilde{H}, \wedge) \subseteq (\tilde{H}, \wedge)$

(ii) $(\tilde{H}, \wedge) \subseteq (\tilde{G}, \wedge) \Rightarrow N_sHS\delta int(\tilde{H}, \wedge) \subseteq N_sHS\delta int(\tilde{G}, \wedge)$

(iii) $N_sHS\delta int((\tilde{H}, \wedge) \cap (\tilde{G}, \wedge)) = N_sHS\delta int(\tilde{H}, \wedge) \cap N_sHS\delta int(\tilde{G}, \wedge)$

(iv) $N_sHS\delta int(\tilde{H}, \wedge)$ is the greatest $N_sHS\delta os$ containing $(\tilde{H}, \wedge)$

(v) $N_sHS\delta int(\tilde{H},\wedge)=(\tilde{H},\wedge)$ iff $(\tilde{H},\wedge)$ is $N_sHS\delta os$

(vi) $N_sHS\delta int(N_sHS\delta int(\tilde{H},\wedge))=N_sHS\delta int(\tilde{H},\wedge)$

(vii) $(Y-N_sHS\delta int(\tilde{H},\wedge))=N_sHS\delta cl(Y-(\tilde{H},\wedge))$.

Proof.

(i) $N_sHS\delta int(\tilde{H},\wedge)=\cup\{(\tilde{G},\wedge):(\tilde{G},\wedge)\subseteq(\tilde{H},\wedge)$ & $(\tilde{G},\wedge)$ is a $N_sHSros\}$. Thus, $N_sHS\delta int(\tilde{H},\wedge)\subseteq(\tilde{H},\wedge)$.

(ii) $N_sHS\delta int(\tilde{G},\wedge)=\cup\{(\tilde{L},\wedge):(\tilde{L},\wedge)\subseteq(\tilde{H},\wedge)$ & $(\tilde{L},\wedge)$ is a $N_sHSros\}\supseteq N_sHS\delta int(\tilde{H},\wedge)$. Thus, $N_sHS\delta int(\tilde{H},\wedge)\subseteq N_sHS\delta int(\tilde{G},\wedge)$.

(iii) $N_sHS\delta int((\tilde{H},\wedge)\cap(\tilde{G},\wedge))=\cup\{(\tilde{L},\wedge):(\tilde{L},\wedge)\subseteq((\tilde{H},\wedge)\cap(\tilde{G},\wedge))$ & $(\tilde{L},\wedge)$ is a $N_sHSros\}=(\{\cup(\tilde{L},\wedge):(\tilde{L},\wedge)\subseteq(\tilde{H},\wedge)\&(\tilde{L},\wedge)$ is a $N_sHSros\})\cap(\cup\{(\tilde{L},\wedge):(\tilde{L},\wedge)\subseteq(\tilde{G},\wedge)$ & $(\tilde{L},\wedge)$ is a $N_sHSros\}=N_sHS\delta int(\tilde{H},\wedge)\cap N_sHS\delta int(\tilde{G},\wedge)$. Thus, $N_sHS\delta int((\tilde{H},\wedge)\cap(\tilde{G},\wedge))=N_sHS\delta int(\tilde{H},\wedge)\cap N_sHS\delta int(\tilde{G},\wedge)$.

(iv) If $(\tilde{G},\wedge)$ is a any $N_sHS\delta os$ contained in $(\tilde{H},\wedge)$, then $(\tilde{H},\wedge)\subseteq N_sHS\delta int(\tilde{H},\wedge)$. Hence, $N_sHS\delta int(\tilde{H},\wedge)$ is the greatest $N_sHS\delta os$ containing $(\tilde{H},\wedge)$.

(v) Suppose $(\tilde{H},\wedge)$ is any $N_sHS\delta os$ of Y. Then, the greatest $N_sHS\delta os$ contained in $(\tilde{H},\wedge)$ is itself. Therefore, $N_sHS\delta int(\tilde{H},\wedge)=(\tilde{H},\wedge)$.

(vi) By (iv), the greatest $N_sHS\delta os$ containing $N_sHS\delta int(\tilde{H},\wedge)$ is itself. Hence, $N_sHS\delta int(N_sHS\delta int(\tilde{H},\wedge)=N_sHS\delta int(\tilde{H},\wedge)$.

(vii) $N_sHS\delta int(\tilde{H},\wedge)$ is the greatest $N_sHS\delta os$ containing $(\tilde{H},\wedge)$. The complement is the smallest $N_sHS\delta cs$ contained in $Y-(\tilde{H},\wedge)$. Therefore, $Y-N_sHS\delta int(\tilde{H},\wedge))=N_sHS\delta cl(Y-(\tilde{H},\wedge))$. ■

Theorem 3.2 The $N_sHS\delta$-closure operator satisfies,

(i) $(\tilde{H},\wedge)\subseteq N_sHS\delta cl(\tilde{H},\wedge)$.

(ii) $(\tilde{H},\wedge)\subseteq(\tilde{G},\wedge)\Rightarrow N_sHS\delta cl(\tilde{H},\wedge)\subseteq N_sHS\delta cl(\tilde{G},\wedge)$.

(iii) $N_sHS\delta cl((\tilde{H},\wedge)\cup(\tilde{G},\wedge))=N_sHS\delta cl(\tilde{H},\wedge)\cup N_sHS\delta cl(\tilde{G},\wedge)$.

(iv) $N_sHS\delta cl(\tilde{H},\wedge)$ is the smallest $N_sHS\delta cs\subseteq(\tilde{H},\wedge)$.

(v) $N_sHS\delta cl(\tilde{H},\wedge)=(\tilde{H},\wedge)$ iff $(\tilde{H},\wedge)$ is a $N_sHS\delta cs$.

(vi) $N_sHS\delta cl(N_sHS\delta cl(\tilde{H},\wedge))=N_sHS\delta cl(\tilde{H},\wedge)$.

(vii) $(Y-N_sHS\delta cl(\tilde{H},\wedge))=N_sHS\delta int(Y-(\tilde{H},\wedge))$.

(viii) $y\in N_sHS\delta cl(\tilde{H},\wedge)$ iff $(\tilde{H},\wedge)\cap(\tilde{L},\wedge)=\tilde{0}_{(Y,Q)}$ for every $N_sHS\delta os$ $(\tilde{L},\wedge)$ containing y.

Proof.

(viii) Suppose, $y\in N_sHS\delta cl(\tilde{H},\wedge)$. Let $(\tilde{L},\wedge)$ be a $N_sHS\delta os$ containing y. If $(\tilde{H},\wedge)\cap(\tilde{L},\wedge)=\tilde{0}_{(Y,Q)}$, then $Y-(\tilde{L},\wedge)$ is a $N_sHS\delta cs$ containing $(\tilde{H},\wedge)$ and so $y\notin(\tilde{H},\wedge)$, a contradiction. Therefore, $(\tilde{H},\wedge)\cap(\tilde{L},\wedge)\neq\tilde{0}_{(Y,Q)}$. If $y\notin N_sHS\delta cl(\tilde{H},\wedge)$, then there exists a $N_sHS\delta cs(\tilde{L},\wedge)^c$ containing $(\tilde{H},\wedge)$ which contains $y\notin(\tilde{L},\wedge)^c$. Then $(\tilde{L},\wedge)=Y-(\tilde{L},\wedge)^c$ is a $N_sHS\delta os$ containing y such that $(\tilde{H},\wedge)\cap(\tilde{L},\wedge)=\tilde{0}_{(Y,Q)}$, a contradiction. Therefore, $y\in N_sHS\delta cl(\tilde{H},\wedge)$.

The other cases follow from the Theorem 3.1. ■

Remark 3.2 The Theorems 3.1 and 3.2 are also true for $N_sHS\delta\mathcal{P}os$ & $N_sHS\delta\mathcal{S}os$ of their respective interior and closure operators.

Theorem 3.3 The statements are hold for N_sHSts, but not the converse.

(i) Every N_sHSros (resp. N_sHSrcs) is semi-regular.

(ii) Every N_sHSros (resp. N_sHSrcs) is $N_sHS\delta os$ (resp. $N_sHS\delta cs$).

(iii) Every $N_sHS\delta os$ (resp. $N_sHS\delta cs$) is $N_sHS\delta\mathcal{S}os$ (resp. $N_sHS\delta\mathcal{S}cs$).

(iv) Every $N_sHS\delta os$ (resp. $N_sHS\delta cs$) is N_sHSos (resp. N_sHScs).

(v) Every N_sHSos (resp. N_sHScs) is $N_sHS\delta os$ (resp. $N_sHS\delta cs$).

(vi) Every N_sHSos (resp. N_sHScs) is $N_sHS\mathcal{P}os$ (resp. $N_sHS\mathcal{P}cs$).

(vii) Every $N_sHS\delta\mathcal{S}os$ (resp. $N_sHS\delta\mathcal{S}cs$) is $N_sHS\delta os$ (resp. $N_sHS\delta cs$).

(viii) Every $N_sHS\mathcal{P}os$ (resp. $N_sHS\mathcal{P}cs$) is $N_sHS\delta\mathcal{P}os$ (resp. $N_sHS\mathcal{S}cs$).

(ix) Every N_sHS semi-regular set is $N_sHS\delta\mathcal{S}os$.

Proof.

(i) If $(\tilde{H},\wedge)$ is N_sHSros, then $N_sHSint(N_sHScl(\tilde{H},\wedge)) = (\tilde{H},\wedge)$. Then, $N_sHSint(N_sHScl(\tilde{H},\wedge)) \subseteq (\tilde{H},\wedge)$. Hence, $(\tilde{H},\wedge)$ is $N_sHS\mathcal{S}cs$. Now $N_sHSint(N_sHScl(\tilde{H},\wedge)) \subseteq (\tilde{H},\wedge) \subseteq N_sHScl(N_sHSint(\tilde{H},\wedge))$. Hence $(\tilde{H},\wedge)$ is $N_sHS\mathcal{S}os$. So, $(\tilde{H},\wedge)$ is N_sHS semi-regular.

(iii) If $(\tilde{H},\wedge)$ is $N_sHS\delta os$, then $(\tilde{H},\wedge) = N_sHS\delta int(\tilde{H},\wedge) \subseteq N_sHScl(N_sHS\delta int(\tilde{H},\wedge))$. Hence, $(\tilde{H},\wedge) \subseteq N_sHScl(N_sHS\delta int(\tilde{H},\wedge))$. So, $(\tilde{H},\wedge)$ is $N_sHS\delta\mathcal{S}os$.

(iv) If $(\tilde{H},\wedge)$ is $N_sHS\delta os$, then $(\tilde{H},\wedge) = N_sHS\delta int(\tilde{H},\wedge) \subseteq N_sHSint(\tilde{H},\wedge)$. So, $(\tilde{H},\wedge)$ is N_sHSos.

(v) If $(\tilde{H},\wedge)$ is N_sHSos, then $(\tilde{H},\wedge) = N_sHSint(\tilde{H},\wedge) \subseteq N_sHScl(N_sHSint(\tilde{H},\wedge))$. So $(\tilde{H},\wedge)$ is $N_sHS\mathcal{S}os$.

(vi) If $(\tilde{H},\wedge)$ is N_sHSos, then $(\tilde{H},\wedge) = N_sHSint(\tilde{H},\wedge) \subseteq N_sHSint(N_sHScl(\tilde{H},\wedge))$. So,$(\tilde{H},\wedge)$ is $N_sHS\mathcal{P}os$.

(vii) If $(\tilde{H},\wedge)$ is $N_sHS\delta\mathcal{S}os$, then $(\tilde{H},\wedge) \subseteq N_sHSclN_sHS(\delta int(\tilde{H},\wedge)) \subseteq N_sHScl(N_sHSint(\tilde{H},\wedge))$. So, $(\tilde{H},\wedge)$ is $N_sHS\mathcal{S}os$.

(viii) If $(\tilde{H},\wedge)$ is $N_sHS\mathcal{P}os$, then $(\tilde{H},\wedge) \subseteq N_sHSint(N_sHScl(\tilde{H},\wedge)) \subseteq N_sHSint(N_sHS\delta cl(\tilde{H},\wedge))$. So $(\tilde{H},\wedge)$ is $N_sHS\delta\mathcal{P}os$.

The proofs of (ii) and (ix) are obvious. ■

Example 3.4 Let $Y = \{y_1, y_2, y_3\}$ be a $NsHS$ initial universe and the attributes be Q_1, Q_2, Q_3. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\},\ Q_2 = \{b_1, b_2\},\ Q_3 = \{c_1, c_2, c_3\}$$

Suppose that

$$\begin{gathered} E_1 = \{a_1, a_2\}, E_2 = \{b_1, b_2\}, E_3 = \{c_1, c2\} \\ C_1 = \{a_1, a_2, a_3\}, C_2 = \{b_1, b_2\}, C_3 = \{c_1, c_2\} \\ D_1 = \{a_2, a_3\}, D_2 = \{b_1, b_2\}, D_3 = \{c_1\} \end{gathered}$$

are subsets of Q_i for each $i = 1, 2, 3$. Then the N_sHSs $(\tilde{H}_1,\wedge_1)$, $(\tilde{H}_2,\wedge_2)$, $(\tilde{H}_3,\wedge_2)$, $(\tilde{H}_4,\wedge_2)$, $(\tilde{H}_5,\wedge_2)$ over the universe Y are as follows.

$$(\tilde{H}_1,\wedge_1) = \left\{ \begin{array}{l} \langle (a_1,b_1,c_1), \{\frac{y_1}{0.8,0.1,0.9}, \frac{y_2}{0.3,0.2,0.3}, \frac{y_3}{0.2,0.2,0.6}\}\rangle, \\ \langle (a_2,b_2,c_2), \{\frac{y_1}{0.7,0.4,0.8}, \frac{y_2}{0.7,0.3,0.8}, \frac{y_3}{0.5,0.5,0.8}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_2,\wedge_2) = \left\{ \begin{array}{l} \langle (a_1,b_1,c_1), \{\frac{y_1}{0.7,0.5,0.8}, \frac{y_2}{0.3,0.4,0.3}, \frac{y_3}{0.3,0.5,0.3}\}\rangle, \\ \langle (a_2,b_2,c_2), \{\frac{y_1}{0.5,0.5,0.7}, \frac{y_2}{0.8,0.3,0.8}, \frac{y_3}{0.6,0.4,0.7}\}\rangle, \\ \langle (a_3,b_1,c_1), \{\frac{y_1}{0.4,0.5,0.7}, \frac{y_2}{0.4,0.4,0.6}, \frac{y_3}{0.1,0.3,0.6}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_3,\wedge_2) = \left\{ \begin{array}{l} \langle (a_1,b_1,c_1), \{\frac{y_1}{0.8,0.5,0.8}, \frac{y_2}{0.3,0.4,0.3}, \frac{y_3}{0.3,0.5,0.3}\}\rangle, \\ \langle (a_2,b_2,c_2), \{\frac{y_1}{0.7,0.5,0.7}, \frac{y_2}{0.8,0.3,0.8}, \frac{y_3}{0.6,0.5,0.7}\}\rangle, \\ \langle (a_3,b_1,c_1), \{\frac{y_1}{0.4,0.5,0.7}, \frac{y_2}{0.4,0.4,0.6}, \frac{y_3}{0.1,0.3,0.6}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_4,\wedge_2) = \left\{ \begin{array}{l} \langle (a_1,b_1,c_1), \{\frac{y_1}{0.7,0.1,0.9}, \frac{y_2}{0.3,0.2,0.3}, \frac{y_3}{0.2,0.2,0.6}\}\rangle, \\ \langle (a_2,b_2,c_2), \{\frac{y_1}{0.5,0.4,0.8}, \frac{y_2}{0.7,0.3,0.8}, \frac{y_3}{0.5,0.4,0.8}\}\rangle, \\ \langle (a_3,b_1,c_1), \{\frac{y_1}{0.4,0.5,0.7}, \frac{y_2}{0.4,0.4,0.6}, \frac{y_3}{0.1,0.3,0.6}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_5,\wedge_2) = \left\{ \begin{array}{l} \langle (a_1,b_1,c_1), \{\frac{y_1}{0.8,0.1,0.9}, \frac{y_2}{0.3,0.2,0.3}, \frac{y_3}{0.2,0.2,0.6}\}\rangle, \\ \langle (a_2,b_2,c_2), \{\frac{y_1}{0.7,0.4,0.8}, \frac{y_2}{0.7,0.3,0.8}, \frac{y_3}{0.5,0.5,0.8}\}\rangle, \\ \langle (a_3,b_1,c_1), \{\frac{y_1}{0.4,0.5,0.7}, \frac{y_2}{0.4,0.4,0.6}, \frac{y_3}{0.1,0.3,0.6}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_6,\wedge_3) = \left\{ \begin{array}{l} \langle (a_2,b_1,c_1), \{\frac{y_1}{0.9,0.3,0.2}, \frac{y_2}{0.8,0.5,0.2}, \frac{y_3}{0.7,0.5,0.2}\}\rangle, \\ \langle (a_3,b_2,c_1), \{\frac{y_1}{0.8,0.5,0.3}, \frac{y_2}{0.7,0.6,0.1}, \frac{y_3}{0.9,0.6,0.2}\}\rangle \end{array} \right\}$$

Then $\tau = \{0_{(Y,Q)}, 1_{(Y,Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_2), (\tilde{H}_5, \wedge_2)\}$ is a N_sHSts. Then,

(i) $(\tilde{H}_3, \wedge_2)^c$ is $N_sHS\delta\mathcal{S}os$ but not N_sHSos

(ii) $(\tilde{H}_2, \wedge_2)^c$ is $N_sHS\delta\mathcal{P}os$ but not N_sHSos

(iii) $(\tilde{H}_4, \wedge_2)$ is N_sHSos but not $N_sHS\delta os$

(iv) $(\tilde{H}_3, \wedge_2)^c$ is $N_sHS\delta\mathcal{S}os$ but not $N_sHS\delta os$

(v) $(\tilde{H}_2, \wedge_2)^c$ is $N_sHS\delta\mathcal{P}os$ but not $N_sHS\mathcal{P}os$

(vi) $(\tilde{H}_3, \wedge_2)^c$ is $N_sHS\mathcal{S}os$ but not N_sHSos

(vii) $(\tilde{H}_2, \wedge_2)$ is $N_sHS\mathcal{S}os$ but not $N_sHS\delta\mathcal{S}os$

(viii) $(\tilde{H}_6, \wedge_3)$ is $N_sHS\mathcal{P}os$ but not N_sHSos

(ix) $(\tilde{H}_3, \wedge_2)^c$ is N_sHS semi-regular but not N_sHSros

Remark 3.3 From the results discussed above, we obtain the following diagram in N_sHSts.

$$\begin{tikzcd}
N_sHSros \arrow[r] \arrow[d] & N_sHS\delta os \arrow[r] \arrow[d] & N_sHSos \arrow[r] \arrow[d] & N_sHSPos \arrow[d] \\
N_sHSsemi\text{-}regular \arrow[r] & N_sHS\delta Sos \arrow[r] & N_sHSSos \arrow[r] & N_sHS\delta Pos
\end{tikzcd}$$

Proposition 3.1 The union (resp. intersection) of any family of $N_sHS\delta\mathcal{S}OS(Y)$ (resp. $N_sHS\delta\mathcal{S}CS(Y)$) is a $N_sHS\delta\mathcal{S}OS(Y)$ (resp. $N_sHS\delta\mathcal{S}CS(Y)$).

Proof. Let $((\tilde{H}, \wedge))_a : a \in \tau))$ be a family of $N_sHS\delta\mathcal{S}os$'s. For each $a \in \tau$, $(\tilde{H}, \wedge))_a \subseteq N_sHScl(N_sHS\delta int(\tilde{H}, \wedge))_a)$.
$\bigcup_{a\in\tau} (\tilde{H}, \wedge)_a \subseteq \bigcup_{a\in\tau} N_sHScl(N_sHS\delta int(\tilde{H}, \wedge)_a) \subseteq N_sHScl(N_sHS\delta int(\tilde{H}, \wedge)_a)$.
The other case is similar. ■

Remark 3.4 Proposition 3.1 is also true for $N_sHS\delta\mathcal{P}OS(Y)$, $N_sHS\delta\mathcal{P}CS(Y)$, $N_sHS\delta OS(Y)$, $N_sHS\delta CS(Y)$, $N_sHS\mathcal{P}OS(Y)$ & $\mathcal{P}CS(Y)$.

Proposition 3.2 The intersection of any family of $N_sHS\mathcal{P}OS(Y)$ (resp. $N_sHS\mathcal{S}OS(Y)$, $N_sHS\delta\mathcal{P}OS(Y)$ & $N_sHS\delta\mathcal{S}OS(Y)$) need not be a $N_sHS\mathcal{P}OS(Y)$ (resp. $N_sHS\mathcal{S}OS(Y)$, $N_sHS\delta\mathcal{P}OS(Y)$ & $N_sHS\delta\mathcal{S}OS(Y)$).

Example 3.5 Consider the N_sHSts in the Example 3.4. Suppose there are two N_sHSs's $(\tilde{H}_7, \wedge_1)$ and $(\tilde{H}_8, \wedge_1)$ as follows:

$$(\tilde{H}_7, \wedge_1) = \begin{Bmatrix} \langle (a_1, b_1, c_1), \{\frac{y_1}{0.8,0.5,0.8}, \frac{y_2}{0.3,0.4,0.4}, \frac{y_3}{0.4,0.3,0.5}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{y_1}{0.8,0.6,0.3}, \frac{y_2}{0.7,0.4,0.8}, \frac{y_3}{0.5,0.6,0.7}\}\rangle \end{Bmatrix}$$
$$(\tilde{H}_8, \wedge_1) = \begin{Bmatrix} \langle (a_1, b_1, c_1), \{\frac{y_1}{0.7,0.5,0.2}, \frac{y_2}{0.4,0.4,0.3}, \frac{y_3}{0.3,0.6,0.2}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{y_1}{0.6,0.5,0.5}, \frac{y_2}{0.8,0.3,0.1}, \frac{y_3}{0.7,0.5,0.3}\}\rangle \end{Bmatrix}$$

Then, $(\tilde{H}_7, \wedge_1)$ and $(\tilde{H}_8, \wedge_1)$ are $N_sHS\mathcal{P}os$'s but $(\tilde{H}_7, \wedge_1) \cap (\tilde{H}_8, \wedge_1)$ is not $N_sHS\mathcal{P}os$.

Example 3.6 Let $Y = \{y_1, y_2\}$ be a $NsHS$ initial universe and the attributes be Q_1, Q_2. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\},\ Q_2 = \{b_1, b_2\}.$$

Suppose that

$$E_1 = \{a_1, a_2\}, E_2 = \{b_1\}$$
$$D_1 = \{a_1\}, D_2 = \{b_1, b_2\}$$

are subsets of Q_i for each $i = 1, 2$. Then the N_sHSs $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_3)$, $(\tilde{H}_5, \wedge_1)$, $(\tilde{H}_6, \wedge_2)$ over the universe Y are as follows.

$$(\tilde{H}_1,\wedge_1)=\left\{\begin{matrix}\langle(a_1,b_1),\{\frac{y_1}{0.8,0.8,0.2},\frac{y_2}{0.6,0.8,0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{y_1}{0.7,0.8,0.5},\frac{y_2}{0.5,0.5,0.2}\}\rangle\end{matrix}\right\}$$

$$(\tilde{H}_2,\wedge_2)=\left\{\begin{matrix}\langle(a_1,b_1),\{\frac{y_1}{0.2,0.4,0.6},\frac{y_2}{0.3,0.5,0.6}\}\rangle,\\ \langle(a_1,b_2),\{\frac{y_1}{0.5,0.5,0.5},\frac{y_2}{0.5,0.5,0.5}\}\rangle\end{matrix}\right\}$$

$$(\tilde{H}_3,\wedge_3)=\left\{\begin{matrix}\langle(a_1,b_1),\{\frac{y_1}{0.2,0.4,0.6},\frac{y_2}{0.3,0.5,0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{y_1}{0.7,0.8,0.5},\frac{y_2}{0.5,0.5,0.2}\}\rangle,\\ \langle(a_1,b_2),\{\frac{y_1}{0.5,0.5,0.5},\frac{y_2}{0.5,0.5,0.5}\}\rangle\end{matrix}\right\}$$

$$(\tilde{H}_4,\wedge_3)=\left\{\begin{matrix}\langle(a_1,b_1),\{\frac{y_1}{0.8,0.8,0.2},\frac{y_2}{0.6,0.8,0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{y_1}{0.7,0.8,0.5},\frac{y_2}{0.5,0.5,0.2}\}\rangle,\\ \langle(a_1,b_2),\{\frac{y_1}{0.5,0.5,0.5},\frac{y_2}{0.5,0.5,0.5}\}\rangle\end{matrix}\right\}$$

$$(\tilde{H}_5,\wedge_1)=\left\{\begin{matrix}\langle(a_1,b_1),\{\frac{y_1}{0.2,0.5,0.7},\frac{y_2}{0.5,0.2,0.4}\}\rangle,\\ \langle(a_2,b_1),\{\frac{y_1}{0.5,0.4,0.6},\frac{y_2}{0.5,0.5,0.4}\}\rangle\end{matrix}\right\}$$

$$(\tilde{H}_6,\wedge_2)=\left\{\begin{matrix}\langle(a_1,b_1),\{\frac{y_1}{0.3,0.2,0.7},\frac{y_2}{0.4,0.2,0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{y_1}{0.5,0.3,0.4},\frac{y_2}{0.6,0.5,0.8}\}\rangle\end{matrix}\right\}$$

Then $\tau=\{0_{(Y,Q)},1_{(Y,Q)},(\tilde{H}_1,\wedge_1),(\tilde{H}_2,\wedge_2),(\tilde{H}_3,\wedge_3),(\tilde{H}_4,\wedge_3)\}$ is a N_sHSts. Here $(\tilde{H}_5,\wedge_1)$ and $(\tilde{H}_6,\wedge_2)$ are $N_sHS\mathcal{S}os$'s but $(\tilde{H}_5,\wedge_1)\cap(\tilde{H}_6,\wedge_2)$ is not $N_sHS\mathcal{S}os$.

Proposition 3.3 If $(\tilde{H},\wedge)$ is a $N_sHS\delta os$ and $(\tilde{G},\wedge)$ is $N_sHS\delta\mathcal{P}os$, then $(\tilde{H},\wedge)\cap(\tilde{G},\wedge)$ is a $N_sHS\delta\mathcal{P}os$.

Proof. $(\tilde{H},\wedge)\cap(\tilde{G},\wedge)\subseteq N_sHS\delta int(\tilde{H},\wedge)\cap N_sHSint(N_sHS\delta cl(\tilde{G},\wedge))\subseteq N_sHSint((\tilde{H},\wedge)\cap N_sHS\delta cl(\tilde{G},\wedge))\subseteq N_sHSint(N_sHS\delta cl(\tilde{H},\wedge)\cap(\tilde{G},\wedge))$. Therefore,$(\tilde{H},\wedge)\cap(\tilde{G},\wedge)$ is a $N_sHS\delta P$os. ■

Remark 3.5 The proposition is also true if $(\tilde{G},\wedge)$ is a $N_sHS\delta\mathcal{S}os$, $N_sHS\mathcal{S}os$ & $N_sHS\mathcal{P}os$.

4 Conclusion

In this study, neutrosphic hypersoft pre-open sets, δ open sets, δ semi-open sets and δ pre-open sets are introduced and their properties are discussed with the examples. Neutrosophic hypersoft topology is a general framework which generalizes the concept of intuitionistic hypersoft topology and fuzzy hypersoft topology which we have proved with an example. Since the real world is full of indeterminacy, uncertain and inadequate data, neutrosophic hypersoft toplogy have a major role in the emerging research field. This work can be applied in real world decision making problems involving more attributes for making a better decision.

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