Fuzzy Hypersoft Set theory For Decision Making and its properties

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Abstract Hypersoft set is an extension of soft set where there is more than one set of attributes occur and it is very much helpful in multi criteria group decision making problem. In Hypersoft set, the function F is a multi-argument function.In this paper we have used the notion of Fuzzy Hypersoft Set(FHSS) which is a combination of Fuzzy set and Hypersoft set and in earlier research work the concept of Fuzzy Hypersoft set was missing. Fuzzy Hypersoft Set gives more flexibility as compared to Fuzzy Soft Set(FSS) to tackel the parameterized problems of uncertainty.To overcome the issue where FSS cannot be used, there is a dire need of another environment which is known as FHSS. It works well when there is more complexity involved in the parametric data.This work includes some basic set theoretic operations and for the reliability and the authenticity of these operations, we have successfully applied them in some suitable examples.All these operations are very useful in various decision making problems.

Keywords: Fuzzy set, Soft set, Hypersoft set , Decision making

1 Introduction

Uncertainty is a part and parcel of our life and it exist in various forms . The classical set theoretical approach can't find out any way to deal with incomplete information i.e the information which is blurred.For scientific computation of such types of data we need a powerful tool which gives us a precise idea of an object. Finally, Fuzzy set theory was introduced by L.A.Zadeh[1]in 1965 for dealing with uncertain, incomlete information in a systematic way.In fuzzy set theory, every object of the set has some membership value and it is called degree of membership and it always lies in the unit closed interval[0,1]. In classical set theory, there is only two possibilities of an object i.e we choose 1 for belongingness and 0 for non belongingness of an object. So, Fuzzy set theory is the extension of classical set theory in which we are enabled to extend the range of a domain under fuzzy environment where each object is a fuzzy word or fuzzy sentence, fuzzy axiom etc.An element belongs to a fuzzy set means it takes any value which ranges from 0 to 1.It has been successfully applied in different branches of Mathematics such as game theory, pattern recognition, image processing, optimization, probability,logic etc. It gives a general framework for modeling the vague concept.But, we know that every theory has its own limitations.Using fuzzy set theory ,we only determine the degree of membership of an object,but there is no scope of non-membership degree. But we have experienced with the co-existance of agreement-disagreement, truth-falsity, success-failure, action-reaction etc in real life scenerio to make a balance between two opposite concept.To realise the importance of non-membership value along with membership value another set theoretical notion whics is known as Intuitionistic fuzzy set(IFS) was introduced by K.Attanosov[2]in 1986.In IFS, every object has two values such as membership and non-membership values which depend on each other and their sum always less or equal to 1.After the introduction of fuzzy set theory, many researchers and mathematicians work on it in several directions and applied it successfully in decision making problem and developed new theories, propositions , axioms etc. Some of their contributions are given in [3, 4, 5]

Due to the more complexity in the nature of uncertainty in a data the above mentioned types of sets are not sufficient for mathematical modeling. Which leads to the introduction of Soft set(SS). Soft set was introduced by Molodtsov[6] in 1999.It is the more general framework of modeling the vagueness in parametric manner. It has been successfully applied in different fields with great success.Some of the novel works on soft set theory given in [7, 8, 9, 10, 11, 12]. By combining the fuzzy set and soft set a new concept called Fuzzy soft set(FSS) was introduced by Maji et al.[13] in 2001.In [14, 15],FSS theory has been successfully applied.

Recently, Smarandache [16] generalize the concept of Soft set to Hypersoft set(HSS), where the function F is transformed into multi-attribute function. The main motivation of using HSS is that when the attributes are more than one and further bisected, the soft set environment cannot be applied to handle such types of cases.So, there is a worth need to define a new approach to solve such type of problems.

In this work we have introduced the concept of Fuzzy Hypersoft Set(FHSS) which is a combination of Fuzzy set and Hypersoft set. Then we have defined different set theoretic operations on them and then there is an attempt to use these operations successfully in multi criteria decision making problem with the help of a suitable example.

2 Preliminaries

This section includes some basic operations with examples which are useful for subsequent discussions.

2.1 Definition [1]

Let X be a non empty set. Then a fuzzy set A is a set having the form $A = \{(x, \mu_A(x)) : x \in X\}$, where the function $\mu_A : X \to [0, 1]$ is called the membership function and $\mu_A(x)$ is called the degree of membership of each element $x \in X$.

2.2 Definition [6]

Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes power set of U and $A \subseteq E$. Then the pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

2.3 Definition [13]

Let U be an initial universe and E be a set of para meters. Let I^U be the set of all fuzzy subsets of U and $A \subseteq E$. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to I^U$.

2.4 Definition [16]

Let ξ be the set of universe and $P(\xi)$ be a power set of ξ. Consider l^1,l^2,......,l^n, for $n \geq 1$, be n well defined attributes, whose corresponding values are respectively the set L^1,L^2,......,L^n with $L^i \cap L^j = \phi$, for i$\neq$j and i,j $\in \{1, 2, ..., n\}$ then the pair$(F, L^1 \times L^2 \times L^3 \times$ .... $\times L^n)$ is said to be Hypersoft set over ξ, where F: $L^1 \times L^2 \times ..... \times L^n \to P(\xi)$.

2.4.1 Example

Let $U = \{c_1, c_2, c_3, c_4\}$ be the set of universe and $N = \{c_1, c_3\} \subset U$. We consider the attributes be $x_1 = Size$, $x_2 = Color$, $x_3 = Costprice(in\ dollar)$, $x_4 = Mileage$ and $x_5 = Model$ and their respective values are respectively given by
$Size = X_1 = \{small, medium, big\}$
$Color = X_2 = \{white, black, red, blue\}$
$Costprice(in\ dollar) = X_3 = \{1000, 1050, 1080, 2000\}$
$Mileage(in\ kmpl) = X_4 = \{24, 21, 27, 19\}$
$Model = X_5 = \{Maruti\ \ Suzuki\ Dzire, Honda\ Amaze, Tata\ Tigor, Ford\ Figo\}$
Let the function be :
$F\colon X_1 \times X_2 \times X_3 \times X_4 \times X_5 \to P(U)$
In respect of N, one has assume that
$F = (\{big, red, 1080, 27, Honda\ Amaze\}) = \{c_1, c_3\}$

3 Fuzzy Hypersoft set

In this section we have defined Fuzzy Hypersoft set and some set theoretic operations along with suitable examples.

3.1 Definition

Let ς be the universe of discourse and P(ς) be the power set of ς, suppose l^1,l^2,......,l^n, for $n \geq 1$, be n well defined attributes, whose corresponding values are respectively the set L^1,L^2,......,L^n with $L^i \cap L^j = \phi$, for i$\neq$j and i,j $\in \{1, 2, ..., n\}$ and $L^1 \times L^2 \times ..... \times L^n = S$, then the pair (F, S) is said to be Fuzzy Hypersoft set($FHSS$) over ς, where $F\colon L^1 \times L^2 \times ..... \times L^n \to P(\varsigma)$ and Γ_S=F($L^1 \times L^2 \times ..... \times L^n$)=$\{< x, \mu(F(S))) >\colon x \in \varsigma\}$, where μ is the membership function which determines the value of the degree of belongingness and $\mu : \varsigma \to [0, 1]$.

3.1.1 Example

Let us consider an example where we have proposed a data which is suitable for selecting a plot of land by the decision makers. Suppose ς be the set of decision makers to decide the best plot given as ς=$\{d^1, d^2, d^3, d^4, d^5\}$ and A=$\{d^1, d^3, d^5\} \subset \varsigma$.
Consider the set of attributes as P^1=Plot size(in sq.ft), P^2= Plot location,

P^3=Cost of plot(in sq.ft) and P^4=Surrounding of a plot and their corresponding values are given as
P^1=$\{2000, 1745, 1100, 900, 1245\}$, P^2=$\{Ramnagar, Badharghat, AkhauraRoad, KrishnaNagar,$
P^3=$\{4135, 3812, 3907, 2547\}$,$P^4$=$\{Shoppingmall, Railwaystation, Airport, Multispecialisthospi$
Therefore, $F\colon P^1 \times P^2 \times P^3 \times P^4 \to P(\varsigma)$
We have the following tables for their fuzzy values

Table 1: Decision making fuzzy values for the size of the plot

P^1(Plot size in sq.ft)	d^1	d^2	d^3	d^4	d^5
2000	0.6	0.5	0.7	0.4	0.8
1745	0.6	0.9	0.1	0.5	0.5
1100	0.6	o.7	0.4	0.8	0.5
900	0.3	0.2	0.5	0.6	0.2
1245	0.5	0.7	0.8	0.6	0.8

Table 2: Decision making fuzzy values for the location of the plot

P^2(Plot location)	d^1	d^2	d^3	d^4	d^5
Ramnagar	0.8	0.9	0.7	0.8	0.9
Badharghat	0.7	0.8	0.6	0.7	0.6
Akhaura Road	0.6	o.8	0.5	0.6	0.8
Krishna Nagar	0.4	0.3	0.2	0.3	0.2
Narsingarh	0.8	0.6	0.5	0.7	0.9

Table 3: Decision making fuzzy values for the cost of the plot

P^3(Plot cost in per sq.ft)	d^1	d^2	d^3	d^4	d^5
4135	0.5	0.4	0.8	0.5	0.7
3812	0.8	0.6	0.8	0.6	0.8
3907	0.8	o.4	0.8	0.6	0.9
2547	0.6	0.6	0.5	0.7	0.6

Table 4: Decision making fuzzy values for the surrounding of the plot

P^4(Plot surrounding)	d^1	d^2	d^3	d^4	d^5
Shopping Mall	0.7	0.6	0.8	0.5	0.8
Railway Station	0.6	0.4	0.5	0.8	0.7
Airport	0.5	o.7	0.6	0.8	0.7
Multispecialist Hospital	0.3	0.7	0.5	0.4	0.7
Highway	0.4	0.7	0.6	0.6	0.8

Then for the set A=$\{d^1, d^3, d^5\}$ we define the Fuzzy Hypersoft set in the following form:

F=$\{1100, Ramnagar, 3812, Shoppingmall\}$=$\{< d^1, (1100, 0.6), (Ramnagar, 0.8), (3812, 0.8), (sho$

$, < d^3, (1100, 0.4), (Ramnagar, 0.7), (3812, 0.8), (shoppingmall, 0.8) >, < d^5, (1100, 0.5), (Ramnag$

$\}$

In a similar manner we can construct $5 \times 5 \times 4 \times 5 = 500$ such fuzzy Hypersoft set for set A as far as the attribute values are concered. Though it is large number but with the help of Data Science both the computation process and its stroage are a very easy practice. So, their is a lot of choices made by the decision makers among which we can choose the one which is suitable for us in all the perspective. Such type of facility is not available if we use fuzzy soft set. So, the concept of Fuzzy Hypersoft set gives us a scope to extend our critical thinking in a systematic manner and it also the need of the hour and by using it we also redefine or extend the earlier concept by introducing various properties on Fuzzy Hypersoft sets and all these properties are more valid and more reliable as compared to the existing one.

It is to be noted that the set of all Fuzzy Hypersoft Set over ς is denoted by FHSS(ς.)

3.2 Definition

Let $\Gamma_S \in FHSS(\varsigma)$, where $L^1 \times L^2 \times ..... \times L^n = S$. If $\forall x \in \varsigma$, S=Φ then Γ_S is called an FHS-null set and it is denoted by Γ_Φ.

3.3 Definition

Let $\Gamma_S \in FHSS(\varsigma)$, where $L^1 \times L^2 \times ..... \times L^n = S$. If S is a crisp set and $\forall x \in \varsigma$ $\Gamma_S = \varsigma$ then Γ_S is called the FHS-universal set.

3.4 Definition

Let $\Gamma_S, \Gamma_T \in FHSS(\varsigma)$. Then Γ_S is said to be a FHS-sub set of Γ_T i.e $\Gamma_S \subseteq \Gamma_T$ if S$\subseteq$T and $\mu(F(S)) \leq \mu(F(T))$.

3.5 Proposition

For $\Gamma_S, \Gamma_T, \Gamma_R \in FHSS(\varsigma)$, we have the following results:

1. For $\Gamma_S \subseteq \Gamma_T$ and $\Gamma_T \subseteq \Gamma_S \implies \Gamma_S = \Gamma_T$
2. For $\Gamma_S \subseteq \Gamma_T$ and $\Gamma_T \subseteq \Gamma_R \implies \Gamma_S \subseteq \Gamma_R$

3. $\Gamma_\Phi \subseteq \Gamma_S$

3.6 Definition

Let $\Gamma_S \in FHSS(\varsigma)$.Then the complement of Γ_S is denoted by$(\Gamma_S)^c$ and it is defined as $(\Gamma_S)^c$=$\{< x, 1-\mu(F(S)) >: x \in \varsigma\}$.

3.7 Proposition

We have the following propositions which are based on FHS-complement set

1. $(\Gamma_S^c)^c = \Gamma_S$
2. $(\Gamma_\Phi)^c = \varsigma$ (in case of crisp set)
3. $(\Gamma_S \cup \Gamma_T)^c = \Gamma_S^c \cap \Gamma_T^c$ (De Morgans law)
4. $(\Gamma_S \cap \Gamma_T)^c = \Gamma_S^c \cup \Gamma_T^c$ (De Morgans law)

3.8 Definition

Let $\Gamma_S, \Gamma_T \in FHSS(\varsigma)$. Then their union is denoted by $\Gamma_S \cup \Gamma_T$ and it is defined as
$\Gamma_S \cup \Gamma_T = \{< x, max(\mu(F(S)), \mu(F(T))) >: \forall x \in S \cap T\}$

3.9 Definition

Let $\Gamma_S, \Gamma_T \in FHSS(\varsigma)$. Then their intersection is denoted by $\Gamma_S \cap \Gamma_T$ and it is defined as
$\Gamma_S \cap \Gamma_T = \{< x, min(\mu(F(S)), \mu(F(T))) >: \forall x \in S \cap T\}$

3.10 Proposition

For any $\Gamma_S, \Gamma_T, \Gamma_R \in FHSS(\varsigma)$ we have the following:

1. $\Gamma_S \cup \Gamma_T = \Gamma_T \cup \Gamma_S$ and $\Gamma_S \cap \Gamma_T = \Gamma_T \cap \Gamma_S$
2. $\Gamma_S \cup (\Gamma_T \cup \Gamma_R) = (\Gamma_S \cup \Gamma_T) \cup \Gamma_R$ and $\Gamma_S \cap (\Gamma_T \cap \Gamma_R) = (\Gamma_S \cap \Gamma_T) \cup \Gamma_R$
3. $\Gamma_S \cup (\Gamma_S \cap \Gamma_T) = \Gamma_S$ and $\Gamma_S \cap (\Gamma_S \cup \Gamma_T) = \Gamma_S$
4. $\Gamma_S \cup (\Gamma_T \cap \Gamma_R) = (\Gamma_S \cup \Gamma_T) \cap (\Gamma_S \cup \Gamma_R)$ and
$\Gamma_S \cap (\Gamma_T \cup \Gamma_R) = (\Gamma_S \cap \Gamma_T) \cup (\Gamma_S \cap \Gamma_R)$

4 Weightage of Fuzzy Hypersoft Set in Decision Making

In example 3.1.1, we have determined only one Fuzzy Hypersoft set though there are 500 different choices are available.As we know that Computer science is an integral part of Mathematics then by using suitable software we can easily enumerate all these 500 cases within few minutes as it is quite difficult to do manually, thats why we avoid such lengthy calculation. But one thing is there among these 500 cases, which is the best choice for the customer? Without any proper decision making of all the items we donot say, this or that is the best choice or there may be a multiple choices for the customer. For that purpose we need to find the weitage of each Fuzzy Hypersoft set by using the following formula:

$$\mathcal{W}=\frac{1}{|FHSS(\varsigma)|}\sum_{i=1}^{n} max(\mu(d^i)) \mid \Xi_{FV} \mid$$

where $\mid FHSS(\varsigma) \mid$= number of Fuzzy Hypersoft sets over ς
$max(\mu(d^i))$= maximum fuzzy membership value with respect to the decision maker
$\mid \Xi_{FV} \mid$= number of fuzzy attribute values corresponding to the set of attribute

We have to determine $\mathcal{W}$ for each and every Fuzzy Hypersoft set of the given problem. After that we choose the best choice for the customer which has the maximum weightage. In case of a tie there may be a multiple choice for a customer. In such case it depends on him or her to pick up the suitable choice as he or she wish to pick.

5 Application

There is a huge scope of the use of Fuzzy Hypersoft set in various areas such as forecasting,bussiness management,traffic control,pattern recognition,similarity measures,cryptography,neural networking,computer science ,data science,sociology etc.

6 Conclusion

Here we have used the nevel concept known as Hypersoft set which was introduced by F.Smarandache in 2018(168-170).By combining the fuzzy set and hypersoft set a new concept Fuzzy Hypersoft set is introduced.Fuzzy Hypersoft set is the more generalized form of Fuzzy soft set.So in this paper an attempt has been made to study the new concept of Fuzzy Hypersoft set and its types and discuss some set theoretic operations and proposition on them.With the help of an reliable and valid example it also be shown that how this concept is more effective in multicriteria decision making problem as compared to the earlier existing concepts.It also be discussed in which area we can apply this concept successfully.In future we also extend this concept by introducing intuitionistic fuzzy set,interval valued fuzzy set,complex fuzzy set,interval valued intuitionistic fuzzy set,rough set etc.

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