A new multi-attribute decision making method with single-valued neutrosophic graphs

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Abstract

In most realistic situations, the theory and method of multi-attribute decision-making have been widely used in different fields, such as engineering, economy, management, military, and others. Although many studies in some extended fuzzy contexts have been explored with multi-attribute decision-making, it is widely recognized that single-valued neutrosophic sets can describe incomplete, indeterminate and inconsistent information more easier. In this paper, aiming at addressing multi-attribute decision making problems with single-valued neutrosophic information, related models and multi-attribute decision making approaches based on the fuzzy graph theory are studied. In specific, we first introduce the notion of single-valued neutrosophic sets and graphs together with several common operational laws. Then a multi-attribute decision making method based on single-valued neutrosophic graphs is established. Finally, an illustrative example and a comparative analysis are conducted to verify the feasibility and efficiency of the proposed method.

Keywords: single-valued neutrosophic sets; multi-attribute decision making; fuzzy graph theory; single-valued neutrosophic graphs

1.Introduction

As an important part of modern decision-making sciences, multi-attribute decision-making aims to make decision-making analysis of a limited number of options from the perspective of multiple attributes, and then choose an optimal choice of alternatives by means of information integration rules. Its theories and methods have been widely used in many areas, and it has effectively promoted the development of social economy [1]. Due to the limitation of decision-makers' cognitions and the increasing complexity of decision-making problems, it is difficult for decision-makers to process decision-making information accurately. Since Zadeh put forward the fuzzy set theory [2], fuzzy multi-attribute decision-making has become an important research direction. With the progress of fuzzy sets, a variety of generalized fuzzy sets have been proposed. Among them, Atanassoy's intuitionistic fuzzy sets [3] are more flexible and practical than traditional fuzzy sets in addressing uncertainties. Further, Smarandache [4] proposed the concept of neutrosophic sets. A neutrosophic set contains three types of membership functions (the truth, indeterminacy and falsity ones). Afterwards, single-valued neutrosophic sets were introduced by Smarandache [4-5] and Wang et al. [6], which can be seen as a generalized form of intuitionistic fuzzy sets and have numerous applications in real-life applications. At present, single-valued neutrosophic multi-attribute problems have been widely studied by scholars and practitioners [7-9].

Rosenfeld [10] proposed the notion of fuzzy graphs. Later on, Mordeson and Peng [11] provided several common operations of fuzzy graphs. Ye [12] gave the idea of single-valued neutrosophic minimum spanning trees and introduced a corresponding clustering method. Yang et al. [13] elaborated single-valued neutrosophic relations. Dhavaseelan et al. [14] defined strong neutrosophic graphs. Akram and Shahzadi [15] introduced the notion of neutrosophic soft graphs in 2016. Recently, Akram and Shahzadi [16] studied properties of single-valued neutrosophic graphs by level graphs. They also developed some operations according to presented novel notions in Broumi et al. [17] and Shah-Hussain [18]. Akram and Sitara [19] studied single-valued neutrosophic graph structures for decision-making issues.

In this paper, we first review the concept, operators and score functions of single-valued neutrosophic sets and graphs, we refer to single-valued neutrosophic graph structures. Then we propose a multi-attribute decision rule by means of single-valued neutrosophic graphs. Finally an illustrative example and a comparative analysis are conducted to verify the feasibility and efficiency of the proposed method.

2. Basic knowledge

2.1 Single-valued neutrosophic sets

Definition 1 [4-6] Let X be a universe of discourse. A single-valued neutrosophic set A on X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then a single-valued neutrosophic set A on X is provided by:

$$A=\left\{x\left(T_A(x), I_A(x), F_A(x)\right) \middle| x \in X\right\},$$

where $T_A(x), I_A(x), F_A(x) \rightarrow \operatorname{int}[0,1]$ for all $x \in X$ to the set A, then we have the following condition $0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3$.

Definition 2 [4-6] Let X be the universe of discourse. $\forall A, B \in SVN(X)$, then the following operations are defined as follows:

1. $A \oplus B=\left\{\left\langle x, T_A(x)+T_B(x)-T_A(x) T_B(x), I_A(x) I_B(x), F_A(x) F_B(x)\right\rangle\right\}$;

2. $A \otimes B=\left\{\left\langle x, T_A(x) T_B(x), I_A(x)+I_B(x)-I_A(x) I_B(x),\right.\right.$

$\left.\left.F_A(x)+F_B(x)-F_A(x) F_B(x)\right\rangle\right\}$;

3. $\lambda \cdot A=\left\{\left\langle x, 1-\left(1-T_A(x)^\lambda\right), I_A(x)^\lambda, F_A(x)^\lambda\right\rangle\right\}$;

4. $A^\lambda=\left\{\left\langle x, T_A(x)^\lambda, 1-\left(1-I_A(x)^\lambda\right), 1-\left(1-F_A(x)^\lambda\right)\right\rangle\right\}$;

5. $A \boxminus B = \left\{ \left\langle x, \frac{T_A(x) - T_B(x)}{1 - T_B(x)}, \frac{I_A(x)}{I_B(x)}, \frac{F_A(x)}{F_B(x)} \right\rangle \right\}$;

6. $A \boxdot B = \left\{ \left\langle x, \frac{T_A(x)}{T_B(x)}, \frac{I_A(x) - I_B(x)}{1 - I_B(x)}, \frac{F_A(x) - F_B(x)}{1 - F_B(x)} \right\rangle \right\}$;

7. $A^c = \left\{ \left\langle x, F_A(x), 1 - I_A(x), T_A(x) \right\rangle \right\}$;

8. $A \cap B = \left\{ \left\langle x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x) \right\rangle \right\}$;

9. $A \cup B = \left\{ \left\langle x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x) \right\rangle \right\}$.

Definition 3 [4-6] Suppose $A = \left\{ x\left(T_A(x), I_A(x), F_A(x)\right) \middle| x \in X \right\}$ is a single-valued neutrosophic number. A score function with regard to x is provided as the following mathematical expression:

$$s(x) = \left\{ \left\langle x, T_A(x) + 1 - I_A(x) + 1 - F_A(x) \right\rangle \middle| x \in X \right\}.$$

2.2 Single-valued neutrosophic graphs

Definition 4 [19] Suppose V is a finite universe of discourse. A single-valued neutrosophic graph G is defined as the following form:

$$G = (C, D),$$

where C is a single-valued neutrosophic set on V with $T_C, I_C, F_C : V \to \text{int}[0,1]$, D is a single-valued neutrosophic set on $V \times V$ with $T_C, I_C, F_C : V \times V \to \text{int}[0,1]$. For any a_x and a_y, we have

$$T_D(a_x a_y) \le \min\left(T_C(a_x), T_C(a_y)\right),$$

$$I_D(a_x a_y) \le \max\left(I_C(a_x), I_C(a_y)\right),$$

$$F_D(a_x a_y) \le \max\left(F_C(a_x), F_C(a_y)\right).$$

Then $G = (C, D)$ is a single-valued neutrosophic graph of $G^* = (V, E)$. In specific, C is single-valued neutrosophic vertices on G, and D is single-valued neutrosophic edges on G.

3. Multi-attribute decision making based on single-valued neutrosophic graphs

In this section, for single-valued neutrosophic multi-attribute decision making problems with correlations and priorities among attributes, a multi-attribute decision making approach based on single-valued neutrosophic graphs is established. Firstly, we describe the basic model of the problem. After that, a single-valued neutrosophic multi-attribute decision making algorithm with correlations and priorities is given.

3.1. The model building

In the problem of single-valued neutrosophic multi-attribute decision making with correlations and priorities among attributes at the same time, let a set of alternatives be $Q=\{q_1,q_2,\cdots q_m\}$, a set of attributes be $V=\{a_1,a_2,\cdots,a_n\}$, and a set of attribute weights be $W=(w_1,w_2,\cdots w_n)^T$ that satisfies $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$. In addition, there is a linear priority relationship $a_1\succ a_2\succ\cdots\succ a_n$ among all attributes, where $a_1\succ a_2$ refers to a_1 is more important than a_2. Then, a decision maker can evaluate an alternative $q_k\,(k=1,2,\cdots,m)$ by using the attribute $a_j\,(j=1,2,\cdots,n)$, and the evaluation result is given in the form of single-valued neutrosophic numbers, thus we form a single-valued neutrosophic decision making matrix $D=(d_{kj})_{m\times n}$. In order to solve the above-stated problem by means of advantages of single-valued neutrosophic graphs, let C be a single-valued neutrosophic set on V with $T_C,I_C,F_C:V\to\mathrm{int}[0,1]$, and D be a single-valued neutrosophic set on $V\times V$ with $T_D,I_D,F_D:V\to\mathrm{int}[0,1]$. For any a_i and a_j, we have

$$T_D(a_ia_j)\le\min\left(T_C(a_i),T_C(a_j)\right),$$

$$I_D(a_ia_j)\le\max\left(I_C(a_i),I_C(a_j)\right),$$

$$F_D(a_ia_j)\le\max\left(F_C(a_i),F_C(a_j)\right),$$

thus a single-valued neutrosophic graph $G=(C,D)$ of graph $G^*=(V,E)$ is built. At last, after integrating several single-valued neutrosophic information under multiple attributes, overall attribute values that correspond to alternatives $q_1,q_2,\cdots q_m$ are analyzed and an optimal alternative is selected. On the whole, the above multi-attribute decision making process includes representations, constructions, analysis and other stages of decision making information.

3.2 The model algorithm

In what follows, a single-valued neutrosophic multi-attribute decision making algorithm with correlations and priority relationships is given. The core of this algorithm lies in handling the relevance and priority relationships among attributes effectively.

Stage 1: In terms of dealing with correlations among attributes, we develop an energy coefficient in the background of single-valued neutrosophic sets for solving the degree of interactions among attributes. For two related attributes a_i and $a_j\,(i,j=1,2,\ldots,n)$, the energy coefficient of single-valued neutrosophic information is expressed as follows:

$$\varphi_{ij}=\left\langle \sum_{\alpha\in T_D(a_ia_j)}\alpha^2, \sum_{\eta\in I_D(a_ia_j)}\eta^2, \sum_{\beta\in F_D(a_ia_j)}\beta^2 \right\rangle,$$

it is easy to see if $i=j$, then $\varphi_{ij}=\varphi_{ji}$ is true. At this time, the energy coefficient of the single-valued neutrosophic information reaches the maximum value $\langle 1,0,0\rangle$; if all attributes are independent of each other, the energy coefficient of single-valued neutrosophic information reaches the minimum value $\langle 0,0,1\rangle$ at this time. In most cases, the single-valued neutrosophic information energy coefficient is presented between $\langle 0,0,1\rangle$ and $\langle 1,0,0\rangle$.

Stage 2: In terms of solving the priority relationship, to obtain attribute weights objectively, we develop a concept of eccentricities in the background of single-valued neutrosophic graphs, and integrate the idea of linear priority relations among attributes. At first, we calculate the eccentricity of every single-valued neutrosophic vertices correspondingly. Suppose $G=(C,D)$ is a single-valued neutrosophic graph of $G^*=(V,E)$, and there are $n+1$ single-valued neutrosophic vertices $u=a_0,a_1,\cdots,a_{n-1},a_n=v$. Then, the single-valued neutrosophic eccentricity for every attribute of V is $e(a_j)=\left\langle \max_{x\in V}\{F_D(a_ia_j)\}, \min_{x\in V}\{1-I_D(a_ia_j)\}, \min_{x\in V}\{T_D(a_ia_j)\}\right\rangle$ $(j=1,2.\cdots,n)$. If there is a liner priority relationship among attributes $a_1\succ a_2\succ\cdots\succ a_n$, then the attribute weight w_j can be derived from the above single-valued neutrosophic eccentricity, thus we can obtain $w_j=\prod_{k=1}^{j}S_{k-1}$; $S_j=e(a_j)$ when $j\neq 0$; $S_j=\langle 1,0,0\rangle$ when $j=0$. Therefore, when all attribute weights are completely unknown, attribute weights can be obtained by using the priority relationship between attributes.

At last, based on the above idea of coping with correlations and priority relationships among attributes, we eventually establish a single-valued neutrosophic multi-attribute decision making algorithm that is shown in Algorithm 1 below.

Input A single-valued neutrosophic decision making matrix $D=(d_{kj})_{m\times n}$, the linear priority relationship among attributes $a_1\succ a_2\succ\cdots\succ a_n$, and a single-valued neutrosophic graph $G=(C,D)$ that describes the correlation among attributes.

Output The best alternative.

Step 1 Calculate the energy coefficient φ_{ij} of single-valued neutrosophic information between two related attributes a_i and a_j.

Step 2 Calculate the attribute weight w_j by using the priority relationship among attributes.

Step 3 Calculate the overall attribute value $\widetilde{q_k} = \sum_{j=1}^{n} w_j \left(\sum_{x=1}^{n} d_{kx}\varphi_{xj} \right)$ of an alternative $q_k \left(k = 1, 2, \cdots, m\right)$.

Step 4 Calculate the score function $s\left(\widetilde{q_k}\right)$ that corresponds to the overall attribute value of an alternative q_k.

Step 5 Determine the best alternative $q^* = \max_{k=1}^{m} \left\{ s\left(\widetilde{q_k}\right) \right\}$.

4. An illustrative example

This section intends to present detailed processes of addressing a single-valued neutrosophic multi-attribute decision making problem by utilizing the case study from Literature [20], then a comparative analysis is arranged to demonstrate the effectiveness of the proposed single-valued neutrosophic multi-attribute decision making algorithm.

4.1 Case descriptions

Indian government had been issued a global tender to select the contractor for these projects in the newspaper and considered five attributes required, i.e., Technology Expertise$\left(a_1\right)$, Service quality$\left(a_2\right)$, Bandwidth$\left(a_3\right)$, Internet speed $\left(a_4\right)$ and Customer Services $\left(a_5\right)$, and the importance of attributes is set by $W = \left\{w_1, w_2, w_3, w_4, w_5,\right\}^T$. In addition, suppose there is a priority relationship $a_1 \succ a_2 \succ a_3 \succ a_4 \succ a_5$ among these five attributes, that is, the person responsible for business gives the highest priority to technology expertise, then other four attributes are concerned by the same individual in succession. The five contractors (i.e., alternatives) namely, "Jaihind Road Builders private (Pvt.) limited (Ltd.) "$\left(q_1\right)$, "J.K. Construction" $\left(q_2\right)$, "Build quick Infrastructure Pvt. Ltd."$\left(q_3\right)$,"Relcon Infra projects Ltd." $\left(q_4\right)$, and" Tata Infrastructure Ltd." $\left(q_5\right)$ bid for these projects. In order to reasonably describe the incompleteness in the above-stated MADM problem, a single-valued neutrosophic decision making matrix $D = \left(d_{kj}\right)_{5\times5}$ is presented in Table 1. Afterwards, a single-valued neutrosophic graph $G = \left(C, D\right)$ of $G^* = \left(V, E\right)$ is established to describe correlations between attributes, where $E = \left\{a_1a_2, a_1a_3, a_1a_4, a_1a_5, a_2a_3, a_2a_4, a_2a_5, a_3a_4, a_3a_5, a_4a_5\right\}$, and we present the established single-valued neutrosophic graph in the following Figure 2. In light of the above expressions, the proposed single-valued neutrosophic multi-attribute decision making algorithm by virtue of single-valued neutrosophic graphs will be used to obtain the best contractor for Indian government.

4.2 Processes of single-valued neutrosophic multi-attribute decision making

According to the proposed single-valued neutrosophic multi-attribute decision making algorithm, we first calculate the energy coefficient of the single-valued neutrosophic information between attributes as follows.

$$\varphi_{12} = \left\langle \sum_{\alpha \in T_D\left(a_i a_j\right)} \alpha^2, \sum_{\eta \in I_D\left(a_i a_j\right)} \eta^2, \sum_{\beta \in F_D\left(a_i a_j\right)} \beta^2 \right\rangle = \left\langle 0.16, 0.09, 0.16 \right\rangle.$$

Similarity, it is not difficult to get

$\varphi_{13} = \langle 0.04, 0.25, 0.36 \rangle$, $\varphi_{14} = \langle 0.09, 0.16, 0.36 \rangle$, $\varphi_{15} = \langle 0.09, 0.25, 0.16 \rangle$,
$\varphi_{23} = \langle 0.04, 0.25, 0.25 \rangle$, $\varphi_{24} = \langle 0.09, 0.16, 0.36 \rangle$, $\varphi_{25} = \langle 0.09, 0.25, 0.16 \rangle$,
$\varphi_{34} = \langle 0.04, 0.25, 0.16 \rangle$, $\varphi_{35} = \langle 0.04, 0.25, 0.36 \rangle$, $\varphi_{45} = \langle 0.09, 0.25, 0.36 \rangle$.

Table 1. The single-valued neutrosophic decision making matrix

	a_1	a_2	a_3	a_4	a_5
q_1	$\langle 0.5, 0.3, 0.4 \rangle$	$\langle 0.5, 0.2, 0.3 \rangle$	$\langle 0.2, 0.2, 0.6 \rangle$	$\langle 0.3, 0.2, 0.4 \rangle$	$\langle 0.3, 0.3, 0.4 \rangle$
q_2	$\langle 0.7, 0.1, 0.3 \rangle$	$\langle 0.6, 0.2, 0.3 \rangle$	$\langle 0.6, 0.3, 0.2 \rangle$	$\langle 0.6, 0.4, 0.2 \rangle$	$\langle 0.7, 0.1, 0.2 \rangle$
q_1	$\langle 0.5, 0.3, 0.4 \rangle$	$\langle 0.6, 0.2, 0.4 \rangle$	$\langle 0.6, 0.1, 0.2 \rangle$	$\langle 0.5, 0.1, 0.3 \rangle$	$\langle 0.6, 0.4, 0.3 \rangle$
q_1	$\langle 0.7, 0.3, 0.2 \rangle$	$\langle 0.7, 0.2, 0.2 \rangle$	$\langle 0.4, 0.5, 0.2 \rangle$	$\langle 0.5, 0.2, 0.2 \rangle$	$\langle 0.4, 0.5, 0.4 \rangle$
q_1	$\langle 0.4, 0.1, 0.3 \rangle$	$\langle 0.5, 0.1, 0.2 \rangle$	$\langle 0.4, 0.1, 0.5 \rangle$	$\langle 0.4, 0.3, 0.6 \rangle$	$\langle 0.3, 0.2, 0.4 \rangle$

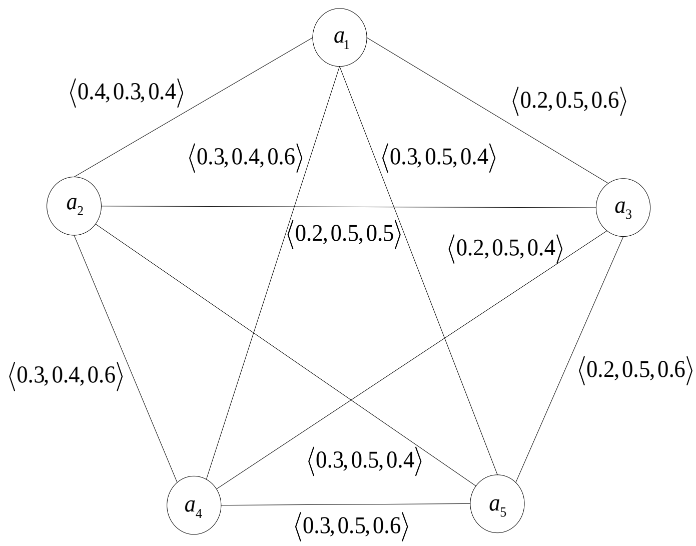

Figure 1: The single-valued neutrosophic graph for describing correlations between attributes

Then, we calculate attribute weights by virtue of the linear priority relationship $a_1 \succ a_2 \succ a_3 \succ a_4 \succ a_5$, and the following single-valued neutrosophic eccentricities for every attributes are calculated at first.

$e(a_1) = \langle 0.6, 0.5, 0.2 \rangle$, $e(a_2) = \langle 0.6, 0.5, 0.2 \rangle$, $e(a_3) = \langle 0.6, 0.5, 0.2 \rangle$, $e(a_4) = \langle 0.6, 0.5, 0.2 \rangle$,
$e(a_5) = \langle 0.6, 0.5, 0.2 \rangle$.

In light of the above results, we further obtain $S_0 = \langle 1, 0, 0 \rangle$. In a similar manner, $S_1 = \langle 0.6, 0.5, 0.2 \rangle$, $S_2 = \langle 0.6, 0.5, 0.2 \rangle$, $S_3 = \langle 0.6, 0.5, 0.2 \rangle$, $S_4 = \langle 0.6, 0.5, 0.2 \rangle$, and $S_5 = \langle 0.6, 0.5, 0.2 \rangle$. Hence, the attribute weight can be obtained via the above results: $w_1 = \langle 1,0,0 \rangle$, $w_2 = \langle 0.6, 0.5, 0.2 \rangle$, $w_3 = \langle 0.36, 0.75, 0.36 \rangle$, $w_4 = \langle 0.216, 0.875, 0.488 \rangle$, $w_5 = \langle 0.1296, 0.9375, 0.5904 \rangle$.

Afterwards, we calculate the following overall attribute values.

$$\begin{aligned}\widetilde{q_1} = & w_1 \otimes \left(d_{11} \otimes \varphi_{11} \oplus d_{12} \otimes \varphi_{21} \oplus d_{13} \otimes \varphi_{31} \oplus d_{14} \otimes \varphi_{41} \right) \oplus \\ & w_2 \otimes \left(d_{11} \otimes \varphi_{12} \oplus d_{12} \otimes \varphi_{22} \oplus d_{13} \otimes \varphi_{32} \oplus d_{14} \otimes \varphi_{42} \right) \oplus \\ & w_3 \otimes \left(d_{11} \otimes \varphi_{13} \oplus d_{12} \otimes \varphi_{23} \oplus d_{13} \otimes \varphi_{33} \oplus d_{14} \otimes \varphi_{43} \right) \oplus \\ & w_4 \otimes \left(d_{11} \otimes \varphi_{14} \oplus d_{12} \otimes \varphi_{24} \oplus d_{13} \otimes \varphi_{34} \oplus d_{14} \otimes \varphi_{44} \right) = \langle 0.7741, 0.0017, 0.001 \rangle.\end{aligned}$$

Similarly, it is not difficult to get

$\widetilde{q_2} = \langle 0.9266, 0.0007, 0.0002 \rangle$, $\widetilde{q_3} = \langle 0.8539, 0.0011, 0.0005 \rangle$,
$\widetilde{q_4} = \langle 0.9102, 0.0033, 0.0002 \rangle$, $\widetilde{q_5} = \langle 0.7587, 0.0003, 0.0007 \rangle$.

At last, the score function $s\left(\widetilde{q_z}\right)$ can be calculated as $s\left(\widetilde{q_1}\right) = 2.7714$, $s\left(\widetilde{q_2}\right) = 2.9257$, $s\left(\widetilde{q_3}\right) = 2.8523$, $s\left(\widetilde{q_4}\right) = 2.9067$, $s\left(\widetilde{q_5}\right) = 2.7577$. According to the order of $s\left(\widetilde{q_z}\right)$ from large to small, we get the ranking of emerging technology enterprise $q_2 \succ q_4 \succ q_3 \succ q_1 \succ q_5$, so the best contractor is q_2.

4.3 Comparative analysis

For the sake of presenting validity and effectiveness of the constructed single-valued neutrosophic multi-attribute decision making algorithm, a comparative analysis is studied by using single-valued neutrosophic multi-attribute decision making approaches proposed in Literature [20], both specific calculation processes and discussions are presented below.

According to the method of logarithmic single-valued neutrosophic weighted average (L-SVNWA) operators and logarithmic single-valued neutrosophic weighted geometric (L-SVNWG) operators, we will make a comparison analysis by using them in the above presented case study. Suppose a single-valued neutrosophic making decision matrix is $D = \left(d_{kj} \right)_{m \times n} \left(k = 1, 2, \cdots, m, j = 1, 2, \cdots, n \right)$, then we present the following L-SVNWA operators and L-SVNWG operators.

(1) The L-SVNWA operator is provided as follows:

$$\begin{aligned}\widetilde{q_k} &= L - SVNWA\left(\widetilde{q_{k1}}, \widetilde{q_{k2}}, \ldots, \widetilde{q_{kn}} \right) \\ &= \left\langle 1 - \prod_{j=1}^{n} \left(\log_{\lambda_{kj}} \alpha_{kj} \right)^{w_j}, \prod_{j=1}^{n} \left(\log_{\lambda_{kj}} \left(1 - \eta_{kj} \right) \right)^{w_j}, \prod_{j=1}^{n} \left(\log_{\lambda_{kj}} \left(1 - \beta_{kj} \right) \right)^{w_j} \right\rangle.\end{aligned}$$

(2) The L-SVNWG operator is provided as follows:

$$\widetilde{q_k} = L - SVNWA\left(\widetilde{q_{k1}}, \widetilde{q_{k2}}, \ldots, \widetilde{q_{kn}}\right)$$
$$= \left\langle \prod_{j=1}^{n}\left(\log_{\lambda_{kj}} \alpha_{kj}\right)^{w_j}, 1-\prod_{j=1}^{n}\left(\log_{\lambda_{kj}}\left(1-\eta_{kj}\right)\right)^{w_j}, 1-\prod_{j=1}^{n}\left(\log_{\lambda_{kj}}\left(1-\beta_{kj}\right)\right)^{w_j} \right\rangle.$$

Compared with previous comparative analysis, the single-valued neutrosophic numbers of each contractor under corresponding attributes are integrated and the score of single-valued neutrosophic numbers after integrations is obtained. The contractor with larger score value is the best candidate for J.K. construction. By using L-SVNWA, the ranking of contractors is $q_2 \succ q_4 \succ q_3 \succ q_5 \succ q_1$, which is not completely consistent with the results obtained by using the method proposed in this paper. It is easy to see that there are differences in the ranking between q_5 and q_1. However, the above ranking results do not affect the selection of the best contractor, the best contractor is still the contractor q_2. By using L-SVNWG, the ranking of contractors is $q_2 \succ q_4 \succ q_3 \succ q_1 \succ q_5$, the results by using the method proposed in this paper are consistent.

Compared with the decision results obtained from the method proposed in Literature [20], the advantages of the constructed single-valued neutrosophic multi-attribute decision making algorithm are mainly reflected as follows:

(1) The constructed single-valued neutrosophic multi-attribute decision making algorithm utilizes the framework of fuzzy graphs to address multi-attribute decision making, which makes single-valued neutrosophic graphs excel in expressing correlations between attributes via edges between vertices in single-valued neutrosophic information systems, thus the addressing of correlational single-valued neutrosophic multi-attribute decision making becomes more efficient.

(2) In light of the proposed theoretical aspects of single-valued neutrosophic graphs, the constructed single-valued neutrosophic single-valued neutrosophic algorithm provides a two-stage problem solving approach by integrating the strategies with correlations and prioritization relationships at the same time, which is beneficial for completing a complicated multi-attribute decision making with high qualities.

5. Conclusions

In real world, single-valued neutrosophic sets have many advantages in dealing with uncertainties compared to fuzzy sets and intuitionistic fuzzy sets, it is easy to see that single-valued neutrosophic sets play important part in information depictions in single-valued neutrosophic, thus it is necessary to establish efficient information analysis tools for single-valued neutrosophic in single-valued neutrosophic information systems. In this paper, we first introduce the concept and operation rules of single-value neutrosophic sets and graphs. Then we propose a single-valued neutrosophic algorithm based on single-valued neutrosophic graphs. Finally, we use a practical case study and a corresponding comparative analysis to show the applicability and effectiveness of the presented single-valued neutrosophic single-valued neutrosophic algorithm.

In terms of future works, it is noted that there still exist some interesting topics that are worth exploring. First, discussing more theoretical issues for single-valued neutrosophic graphs is necessary, such as hypergraph structures and vague hypergraph structures. Second, it is meaningful to further extend single-valued neutrosophic graphs to more realistic decision making contexts, such as incomplete information systems, hybrid information systems, dynamic information systems, etc.

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