



Nonagonal Neutrosophic Linear Non-Linear Numbers, Alpha Cuts and Their Applications using TOPSIS

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Abstract

The concept of neutrosophic become really handy now a days and based on non-standard analysis to mention mathematical outcomes, uncertainty, non-completed situations, inconsistency, distinctness. The main concept of Neutrosophic set based on membership values of truth, indeterminacy and falsity, which are independent and which play vital role in situations like uncertainty, incomplete and inconsistency. From triangular to octagonal neutrosophic number. They play vital role in modeling problems, science, biology and many more. Hence it is clear that these are necessary and have real life applications, but some real-life problems have more edges and their triangular to octagonal fail to overcome this situation (mention in table 1). Hence, nonagonal neutrosophic numbers give a wide scope of utilizations while managing more variances in the decision-making condition with nine edges for membership values of truth, indeterminacy and falsity. In this current article we present comparison between triangular to nonagonal neutrosophic number and their requirement, explore differential equations in neutrosophic environment as Linear, symmetric and asymmetric types further, their α – cut and then we present a real-life problem and solved it with TOPSIS technique of MCDM.

Keywords: Accuracy function, Neutrosophic number, Nonagonal Neutrosophic numbers (NNN), MCDM, TOPSIS.

1- Introduction

Analysts and mathematicians from throughout the world created significant expository aptitudes and critical thinking systems to survey an expansive scope of issues in human asset, medication, determination issues and so on but the most testing issues was related with MCDM.

In this way, the need to deal with unsure circumstances and dubiousness, mathematicians generate a new way, called fuzzy which can fulfil their decision-making requirements. The very first attempt in this race was soft set theory which was introduced by Molodtsov [1] in 1999 to deal with uncertainty. The further effort was fuzzy set theory [2-4] then intuitionistic fuzzy numbers [5] and finally neutrosophic sets [6] as figure 2. Neutrosophic fuzzy numbers [7] is really

quintessential regards to desission making and triangular neutrosophic numbers [8] was commencing then Trapezoidal neutrosophic numbers then Pentagonal neutrosophic numbers [9] then Hexagonal neutrosophic numbers then heptagonal neutrosophic numbers then octagonal neutrosophic numbers [10], his representations and $\tilde{\alpha}$ – cut was given as [11] as well as his aggregate, arithmetic and geometric operators of octagonal neutrosophic numbers and applications also proposed [12] and finally we proposed nonagonal neutrosophic numbers, as well as researchers keep working and introduce new possibilities [13-15]. Wang [16] extended the idea and present single-value neutrosophic sets as extension of NSs. Simplified neutrosophic sets given by Ye [17] and novel operations and aggregation operation was given by Peng et al [16,17]. Multi-valued [18], bipolar [19] and interval neutrosophic sets [20] are different extensions of neutrosophic sets.

Smarandache and different researchers [21-28] from all over the world work on various extension of neutrosophic sets in MCDM and TOPSIS technique. A lot work available regards to selection problem [29-49] and MCDM. Properties as well as applications of triangular and pentagonal neutrosophic number given by [50-54]. With these unique concepts of Nonagonal Neutrosophic numbers (NNN), we can work on many fluctuations at the same time, that proceedable due to more edges found in nonagonal as compare to triangular, pentagonal and octagonal. Now with this current epoch, we can convert neutrosophic numbers into fuzzy numbers and also the power to deal with more fluctuations move on a new level. With these great efforts of mathematicians, we are able to perform these productive and value able decision-making efforts.

Neutrosophic numbers also really useful in other mathematical fields such as transportation problem in neutrosophic environment of operational research [55]. Then, Pratihar further introduce modified vogel’s approximation method [56] in the field of operational research, a method to increase income. The concept of single inputs and outputs of pipelines further improve by Mohapatra [57]. In the line of progress of operational research Kumar introduce optimal path selection by fuzzy reliable shortest path [58] and neutrosophic shortest path [59]. Gayen introduce anti-fuzzy subgroup [60] and plithogenic subgroup [61-62], which further promote the plithogenic algebraic structure as well as introduce notation of plithogenic subgroup. Gayen with smarandache introduce interval-valued triple T-norm [63], which have both pure and applied math applications.

In the case of nonagonal neutrosophic number, we have nine edges regards to truthness, Indeterminacy and Falsity membership value. So, provide us more range to deal with more fluctuations. As decision making is really diverse and come with endless possibilities, hence we required a system which can be useable in these tuff situations. Detail example available as table below:

Neutrosophic numbers	Black color	High mileage	Price	Design	Speed	Safety Features	4×4	Resale value	Break system
Triangular Neutrosophic numbers	Determinable (D)	(D)	(D)	*	*	*	*	*	*
Trapezoidal Neutrosophic numbers	(D)	(D)	(D)	(D)	*	*	*	*	*
Pentagonal Neutrosophic numbers	(D)	(D)	(D)	(D)	(D)	*	*	*	*
Hexagonal Neutrosophic numbers	(D)	(D)	(D)	(D)	(D)	(D)	*	*	*
Heptagonal NNs	(D)	(D)	(D)	(D)	(D)	(D)	(D)	*	*
Octagonal Neutrosophic numbers	(D)	(D)	(D)	(D)	(D)	(D)	(D)	(D)	*

Nonagonal Neutrosophic numbers	(D)	(D)	(D)	(D)	(D)	(D)	(D)	(D)	(D)
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Table 1: This current table shows, if we have to take a decision on nine different criteria then only nonagonal neutrosophic numbers can do that job for us.

1.1 Motivation

A lot of research articles regards to neutrosophic arena available, which they apply and exaggerated the concept of MCDM. Here we have a ground braking concept, nonagonal neutrosophic numbers (NNN), which is totally new. Our main approach is to define the concept of nonagonal for that cenerio, where triangular to octangonal neutrosophic fail to overcome. With the nine edges and wide range of membership values of truth, indeterminacy and falsity, it can proceed better and overcome more real life problems, based on uncertainty.

1.2 The Paper presentation

We extended the concept of Nonagonal Neutrosophic Numbers (NNN).

- Introduced Linear, Non-Linear, Linear symmetric, Non-Linear symmetric Nonagonal neutrosophic Numbers.
- $\tilde{\alpha}$ – cut is defined for all type.
- A case study which is defined from real-life selection problem, solved by TOPSIS.

1.3 Structure of Paper

The structure of this current epoch is defined in following figure.

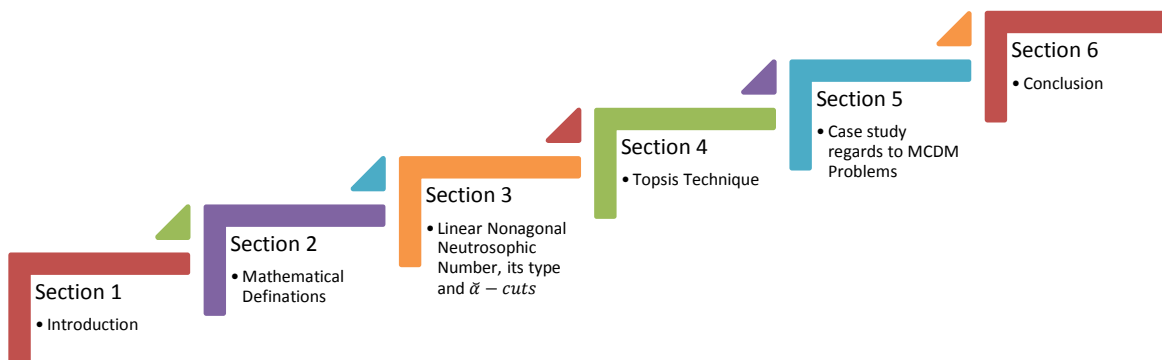


Figure 1: Structure of article with a pictorial view

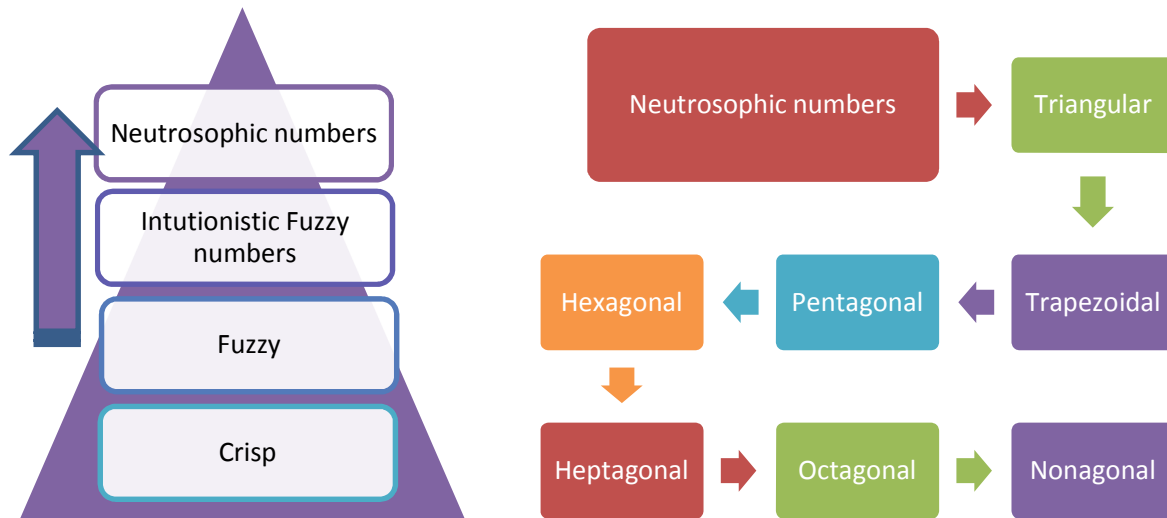


Figure 2. Extensions of intuitionistic fuzzy numbers as flow chart.

2 Mathematical Definitions

In that section, we will mention necessary required definitions, which we will use throughout the paper.

Definition 2.1: Neutrosophic Sets: Set \widetilde{nA} is neutrosophic if $\widetilde{nA} = \{x; \{[T_{\widetilde{nA}}(x), I_{\widetilde{nA}}(x), F_{\widetilde{nA}}(x)]\}; x \in X\}$

Where $T_{\widetilde{nA}}(x) \mapsto [0, 1]$ as truth membership function (TMF), indeterminacy IMF $I_{\widetilde{nA}}(x)$, and falsity (FMF) $F_{\widetilde{nA}}(x)$ as well as $T_{\widetilde{nA}}(x), I_{\widetilde{nA}}(x), F_{\widetilde{nA}}(x)$ mentions following relation:

$$0^- \leq T_{\widetilde{nA}}(x) + I_{\widetilde{nA}}(x) + F_{\widetilde{nA}}(x) \leq 3^+$$

Definition 2.2: Triangular Neutrosophic numbers: Triangular single value neutrosophic number is given as $A_{\widetilde{Neu}} = (\check{p}_1, \check{p}_2, \check{p}_3; \check{r}_1, \check{r}_2, \check{r}_3)$ as well as truth, indeterminacy and falsity is given as:

$$\check{T}_{A_{\widetilde{Neu}}} = \begin{cases} \frac{x - \check{p}_1}{\check{p}_1 - \check{p}_2} & \text{for } \check{p}_1 \leq x < \check{p}_2 \\ 1 & \text{when } x = \check{p}_2 \\ \frac{\check{p}_3 - x}{\check{p}_3 - \check{p}_2} & \text{for } \check{p}_2 < x \leq \check{p}_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\check{I}_{A_{\widetilde{Neu}}} = \begin{cases} \frac{\check{q}_2 - x}{\check{q}_2 - \check{q}_1} & \text{for } \check{q}_1 \leq x < \check{q}_2 \\ 0 & \text{when } x = \check{q}_2 \\ \frac{x - \check{q}_2}{\check{q}_3 - \check{q}_2} & \text{for } \check{q}_2 < x \leq \check{q}_3 \\ 1 & \text{otherwise} \end{cases}$$

$$\check{F}_{A_{\widetilde{Neu}}} = \begin{cases} \frac{x - \check{p}_1}{\check{p}_1 - \check{p}_2} & \text{for } \check{p}_1 \leq x < \check{p}_2 \\ 1 & \text{when } x = \check{p}_2 \\ \frac{\check{p}_3 - x}{\check{p}_3 - \check{p}_2} & \text{for } \check{p}_2 < x \leq \check{p}_3 \\ 0 & \text{otherwise} \end{cases}$$

Where $0 \leq T_{A_{\overline{Neu}}}(x) + I_{A_{\overline{Neu}}}(x) + F_{A_{\overline{Neu}}}(x) \leq 3, x \in A_{\overline{Neu}}$

Parametric foam for this type is $(A_{\overline{Neu}})_{\alpha,\beta,\gamma} = [\check{T}_{Neu1}(\check{\alpha}), \check{T}_{Neu2}(\check{\alpha}); \check{I}_{Neu1}(\check{\alpha}), \check{I}_{Neu2}(\check{\alpha}); \check{F}_{Neu1}(\check{\alpha}), \check{F}_{Neu2}(\check{\alpha})]$, where, $\check{T}_{Neu1}(\check{\alpha}) = P_1 + \check{\alpha}(\check{P}_2 - P_1)$, $\check{T}_{Neu2}(\check{\alpha}) = P_3 - \check{\alpha}(\check{P}_3 - P_2)$, $\check{I}_{Neu1}(\check{\beta}) = q_2 - \check{\beta}(q_2 - q_1)$, $\check{I}_{Neu2}(\check{\beta}) = q_2 + \check{\beta}(q_3 - q_2)$, $\check{F}_{Neu1}(\check{\gamma}) = r_2 - \check{\gamma}(r_2 - r_1)$, $\check{F}_{Neu2}(\check{\gamma}) = r_2 + \check{\gamma}(r_3 - r_2)$, here $0 < \check{\alpha} \leq 1, 0 < \check{\beta} \leq 1, 0 < \check{\gamma} \leq 1$ and $0 < \check{\alpha} + \check{\beta} + \check{\gamma} < 3$.

Definition 2.3: Trapezoidal Neutrosophic Number: Let \check{X} as universe if discourse, A trapezoidal neutrosophic set \check{A} in \check{X} is defined as: $\check{N} = \{(\check{X}, \check{T}_{\check{N}}(x), \check{I}_{\check{N}}(x), \check{F}_{\check{N}}(x)) | x \in \check{X}\}$ where $\check{T}_{\check{N}}(x) \subset [0,1], \check{I}_{\check{N}}(x) \subset [0,1], \check{F}_{\check{N}}(x) \subset [0,1]$ as three trapezoidal number, $\check{T}_{\check{N}}(x) = (\check{t}_N^1(x), \check{t}_N^2(x), \check{t}_N^3(x), \check{t}_N^4(x)) : x \mapsto [0,1], \check{I}_{\check{N}}(x) = (\check{i}_N^1(x), \check{i}_N^2(x), \check{i}_N^3(x), \check{i}_N^4(x)) : x \mapsto [0,1], \check{F}_{\check{N}}(x) = (\check{f}_N^1(x), \check{f}_N^2(x), \check{f}_N^3(x), \check{f}_N^4(x)) : x \mapsto [0,1]$ with condition $0 \leq \check{t}_N^4(x) + \check{i}_N^4(x) + \check{f}_N^4(x) \leq 3, x \in \check{X}$.

Definition 2.4: Pentagonal Neutrosophic Number: For single valued, pentagonal neutrosophic number (\check{S}) given as $\check{S} = \langle [m^1, n^1, o^1, p^1, q^1; \check{\pi}], [m^2, n^2, o^2, p^2, q^2; \check{\xi}], [m^3, n^3, o^3, p^3, q^3; \check{\delta}] \rangle$ where $\check{\pi}, \check{\xi}, \check{\delta} \in [0,1]$. Truth membership function (\check{T}_S): $\mathbb{R} \mapsto [0, \check{\pi}]$, the indeterminacy (\check{I}_S): $\mathbb{R} \mapsto [0, \check{\xi}]$ and falsity (\check{F}_S): $\mathbb{R} \mapsto [0, \check{\delta}]$ as well as given as:

$$T_{A_{\overline{Neu}}} = \begin{cases} \check{T}_{St1}(x) & m^1 \leq x < n^1 \\ \check{T}_{St2}(x) & n^1 \leq x < o^1 \\ \check{\pi} & x = o^1 \\ \check{T}_{St2}(x) & o^1 \leq x < p^1 \\ \check{T}_{St1}(x) & p^1 \leq x < q^1 \\ 0 & \text{otherwise} \end{cases} \quad I_{A_{\overline{Neu}}} = \begin{cases} \check{I}_{St1}(x) & m^2 \leq x < n^2 \\ \check{I}_{St2}(x) & n^2 \leq x < o^2 \\ \check{\xi} & x = o^2 \\ \check{I}_{St2}(x) & o^2 \leq x < p^2 \\ \check{I}_{St1}(x) & p^2 \leq x < q^2 \\ 1 & \text{otherwise} \end{cases} \quad F_{A_{\overline{Neu}}} = \begin{cases} \check{F}_{St1}(x) & m^3 \leq x < n^3 \\ \check{F}_{St2}(x) & n^3 \leq x < o^3 \\ \check{\delta} & x = o^3 \\ \check{F}_{St2}(x) & o^3 \leq x < p^3 \\ \check{F}_{St1}(x) & p^3 \leq x < q^3 \\ 1 & \text{otherwise} \end{cases}$$

Where $\langle [m^1 < n^1 < o^1 < p^1 < q^1; \check{\pi}], [m^2 < n^2 < o^2 < p^2 < q^2; \check{\xi}], [m^3 < n^3 < o^3 < p^3 < q^3; \check{\delta}] \rangle$

Definition 2.5: Octagonal Neutrosophic Number [ONN]: Neutrosophic as \check{S} will defined as,

$\check{S} = \langle [(a, b, c, d, e, f, g, h): \check{\theta}][(\check{a}^1, \check{b}^1, \check{c}^1, \check{d}^1, \check{e}^1, \check{f}^1, \check{g}^1, \check{h}^1): \psi][(\check{a}^2, \check{b}^2, \check{c}^2, \check{d}^2, \check{e}^2, \check{f}^2, \check{g}^2, \check{h}^2): \varphi] \rangle$ where $\check{\theta}, \check{\psi}, \check{\varphi} \in [0,1]$.

The truth membership function ($\check{\theta}_S$): $\mathbb{R} \mapsto [0,1]$,

The indeterminacy membership function ($\check{\psi}_S$): $\mathbb{R} \mapsto [0,1]$,

The falsity membership function ($\check{\varphi}_S$): $\mathbb{R} \mapsto [0,1]$ as well as given follows:

$$\check{\theta}_S(x) = \begin{cases} \check{\theta}_{S0}(x) & a \leq x < b \\ \check{\theta}_{S1}(x) & b \leq x < c \\ \check{\theta}_{S2}(x) & c \leq x < d \\ \check{\theta}_{S3}(x) & d \leq x < e \\ \check{\theta} & x = e \\ \check{\theta}_{S3}(x) & e \leq x < f \\ \check{\theta}_{S2}(x) & f \leq x < g \\ \check{\theta}_{S1}(x) & g \leq x < h \\ 0 & \text{otherwise} \end{cases} \quad \check{\psi}_S(x) = \begin{cases} \check{\psi}_{S0}(x) & \check{a}^1 \leq x < \check{b}^1 \\ \check{\psi}_{S1}(x) & \check{b}^1 \leq x < \check{c}^1 \\ \check{\psi}_{S2}(x) & \check{c}^1 \leq x < \check{d}^1 \\ \check{\psi}_{S3}(x) & \check{d}^1 \leq x < \check{e}^1 \\ \check{\psi} & x = \check{e}^1 \\ \check{\psi}_{S3}(x) & \check{e}^1 \leq x < \check{f}^1 \\ \check{\psi}_{S2}(x) & \check{f}^1 \leq x < \check{g}^1 \\ \check{\psi}_{S1}(x) & \check{g}^1 \leq x < \check{h}^1 \\ 1 & \text{otherwise} \end{cases} \quad \check{\varphi}_S(x) = \begin{cases} \check{\varphi}_{S0}(x) & \check{a}^2 \leq x < \check{b}^2 \\ \check{\varphi}_{S1}(x) & \check{b}^2 \leq x < \check{c}^2 \\ \check{\varphi}_{S2}(x) & \check{c}^2 \leq x < \check{d}^2 \\ \check{\varphi}_{S3}(x) & \check{d}^2 \leq x < \check{e}^2 \\ \check{\varphi} & x = \check{e}^2 \\ \check{\varphi}_{S3}(x) & \check{e}^2 \leq x < \check{f}^2 \\ \check{\varphi}_{S2}(x) & \check{f}^2 \leq x < \check{g}^2 \\ \check{\varphi}_{S1}(x) & \check{g}^2 \leq x < \check{h}^2 \\ 1 & \text{otherwise} \end{cases}$$

Where $\check{S} = \langle [(\check{a} < \check{b} < \check{c} < \check{d} < \check{e} < \check{f} < \check{g} < \check{h}): \check{\theta}] [(\check{a}^1 < \check{b}^1 < \check{c}^1 < \check{d}^1 < \check{e}^1 < \check{f}^1 < \check{g}^1 < \check{h}^1): \psi] [(\check{a}^2 < \check{b}^2 < \check{c}^2 < \check{d}^2 < \check{e}^2 < \check{f}^2 < \check{g}^2 < \check{h}^2): \varphi] \rangle$.

Definition 2.6: Nonagonal Neutrosophic Number [NNN]: Neutrosophic as \check{S} will defined as,

$$\check{S} = \langle [(\check{a}, \check{b}, \check{c}, \check{d}, \check{e}, \check{f}, \check{g}, \check{h}, \check{i}): \check{\theta}] [(\check{a}^1, \check{b}^1, \check{c}^1, \check{d}^1, \check{e}^1, \check{f}^1, \check{g}^1, \check{h}^1, \check{i}^1): \psi] [(\check{a}^2, \check{b}^2, \check{c}^2, \check{d}^2, \check{e}^2, \check{f}^2, \check{g}^2, \check{h}^2, \check{i}^2): \varphi] \rangle \quad \text{where } \check{\theta}, \check{\psi}, \check{\varphi} \in [0,1].$$

The truth membership function $(\check{\theta}_S): \mathbb{R} \mapsto [0,1]$,

The indeterminacy membership function $(\check{\psi}_S): \mathbb{R} \mapsto [0,1]$,

The falsity membership function $(\check{\varphi}_S): \mathbb{R} \mapsto [0,1]$ as well as given follows:

$$\check{\theta}_S(x) = \begin{cases} \check{\theta}_{S0}(x) & \check{a} \leq x < \check{b} \\ \check{\theta}_{S1}(x) & \check{b} \leq x < \check{c} \\ \check{\theta}_{S2}(x) & \check{c} \leq x < \check{d} \\ \check{\theta}_{S3}(x) & \check{d} \leq x < \check{e} \\ \vartheta & x = \check{e} \\ \check{\theta}_{S3}(x) & \check{e} \leq x < \check{f} \\ \check{\theta}_{S2}(x) & \check{f} \leq x < \check{g} \\ \check{\theta}_{S1}(x) & \check{g} \leq x < \check{h} \\ \check{\theta}_{S0}(x) & \check{h} \leq x < \check{i} \\ 0 & \text{otherwise} \end{cases} \quad \check{\psi}_S(x) = \begin{cases} \check{\psi}_{S0}(x) & \check{a}^1 \leq x < \check{b}^1 \\ \check{\psi}_{S1}(x) & \check{b}^1 \leq x < \check{c}^1 \\ \check{\psi}_{S2}(x) & \check{c}^1 \leq x < \check{d}^1 \\ \check{\psi}_{S3}(x) & \check{d}^1 \leq x < \check{e}^1 \\ \vartheta & x = \check{e}^1 \\ \check{\psi}_{S3}(x) & \check{e}^1 \leq x < \check{f}^1 \\ \check{\psi}_{S2}(x) & \check{f}^1 \leq x < \check{g}^1 \\ \check{\psi}_{S1}(x) & \check{g}^1 \leq x < \check{h}^1 \\ \check{\psi}_{S0}(x) & \check{h}^1 \leq x < \check{i}^1 \\ 1 & \text{otherwise} \end{cases} \quad \check{\varphi}_S(x) = \begin{cases} \check{\varphi}_{S0}(x) & \check{a}^2 \leq x < \check{b}^2 \\ \check{\varphi}_{S1}(x) & \check{b}^2 \leq x < \check{c}^2 \\ \check{\varphi}_{S2}(x) & \check{c}^2 \leq x < \check{d}^2 \\ \check{\varphi}_{S3}(x) & \check{d}^2 \leq x < \check{e}^2 \\ \varrho & x = \check{e}^2 \\ \check{\varphi}_{S3}(x) & \check{e}^2 \leq x < \check{f}^2 \\ \check{\varphi}_{S2}(x) & \check{f}^2 \leq x < \check{g}^2 \\ \check{\varphi}_{S1}(x) & \check{g}^2 \leq x < \check{h}^2 \\ \check{\varphi}_{S0}(x) & \check{h}^2 \leq x < \check{i}^2 \\ 1 & \text{otherwise} \end{cases}$$

Where $\check{S} = \langle [(\check{a} < \check{b} < \check{c} < \check{d} < \check{e} < \check{f} < \check{g} < \check{h} < \check{i}): \check{\theta}] [(\check{a}^1 < \check{b}^1 < \check{c}^1 < \check{d}^1 < \check{e}^1 < \check{f}^1 < \check{g}^1 < \check{h}^1 < \check{i}^1): \psi] [(\check{a}^2 < \check{b}^2 < \check{c}^2 < \check{d}^2 < \check{e}^2 < \check{f}^2 < \check{g}^2 < \check{h}^2 < \check{i}^2): \varphi] \rangle$.

3. In that section we investigate its representation and discuss its properties.

3.1 Linear NNN with symmetry

Let $\check{A}_{LS} = (\check{a}, \check{b}, \check{c}, \check{d}, \check{e}, \check{f}, \check{g}, \check{h}, \check{i})$ as linear NNN with these membership function:

$$\text{Truth} = \check{T}_L(X) = \begin{cases} 0 & x < \check{a} \\ k \left(\frac{x-\check{a}}{\check{b}-\check{a}} \right) & \check{a} < x < \check{b} \\ k \left(\frac{x-\check{b}}{\check{c}-\check{b}} \right) & \check{b} < x < \check{c} \\ k & \check{c} < x < \check{d} \\ k + (1-k) \left(\frac{x-\check{d}}{\check{e}-\check{d}} \right) & \check{d} < x < \check{e} \\ 1 & \check{e} < x < \check{f} \\ k + (1-k) \left(\frac{\check{g}-x}{\check{g}-\check{f}} \right) & \check{f} < x < \check{g} \\ k & \check{g} < x < \check{h} \\ k \left(\frac{\check{i}-x}{\check{i}-\check{h}} \right) & \check{h} < x < \check{i} \\ 0 & x > \check{i} \end{cases}$$

$$\text{Falsity} = \hat{F}_L(X) = \begin{cases} 0 & x < \hat{a}^1 \\ k \left(\frac{x - \hat{a}^1}{\hat{b}^1 - \hat{a}^1} \right) & \hat{a}^1 < x < \hat{b}^1 \\ k \left(\frac{x - \hat{b}^1}{\hat{c}^1 - \hat{b}^1} \right) & \hat{b}^1 < x < \hat{c}^1 \\ k & \hat{c}^1 < x < \hat{d}^1 \\ k + (1 - k) \left(\frac{x - \hat{d}^1}{\hat{e}^1 - \hat{d}^1} \right) & \hat{d}^1 < x < \hat{e}^1 \\ 1 & \hat{e}^1 < x < \hat{f}^1 \\ k + (1 - k) \left(\frac{\hat{g}^1 - x}{\hat{g}^1 - \hat{f}^1} \right) & \hat{f}^1 < x < \hat{g}^1 \\ k & \hat{g}^1 < x < \hat{h}^1 \\ k \left(\frac{\hat{i}^1 - x}{\hat{i}^1 - \hat{h}^1} \right) & \hat{h}^1 < x < \hat{i}^1 \\ 1 & x > \hat{i}^1 \end{cases}$$

$$\text{Indeterminacy} = \hat{I}_L(X) = \begin{cases} 0 & x < \hat{a}^2 \\ k \left(\frac{x - \hat{a}^2}{\hat{b}^2 - \hat{a}^2} \right) & \hat{a}^2 < x < \hat{b}^2 \\ k \left(\frac{x - \hat{b}^2}{\hat{c}^2 - \hat{b}^2} \right) & \hat{b}^2 < x < \hat{c}^2 \\ k & \hat{c}^2 < x < \hat{d}^2 \\ k + (1 - k) \left(\frac{x - \hat{d}^2}{\hat{e}^2 - \hat{d}^2} \right) & \hat{d}^2 < x < \hat{e}^2 \\ 1 & \hat{e}^2 < x < \hat{f}^2 \\ k + (1 - k) \left(\frac{\hat{g}^2 - x}{\hat{g}^2 - \hat{f}^2} \right) & \hat{f}^2 < x < \hat{g}^2 \\ k & \hat{g}^2 < x < \hat{h}^2 \\ k \left(\frac{\hat{i}^2 - x}{\hat{i}^2 - \hat{h}^2} \right) & \hat{h}^2 < x < \hat{i}^2 \\ 1 & x > \hat{i}^2 \end{cases}$$

As, $0 < k < 1$

$$\mathcal{A}_{\hat{\alpha}} = \{x \in \mathcal{X} \mid \hat{T}_L(X), \hat{F}_L(X), \hat{I}_L(X) \geq \hat{\alpha}\}$$

3.2 α - cut of Linear ONN with symmetry: α - cut can be express as:

$$\text{Truth} = \hat{T}_L(X) = \begin{cases} \hat{\mathcal{A}}_{1L}(\hat{\alpha}) = \hat{a} + \frac{\hat{\alpha}}{\hat{b}_1} (\hat{b} - \hat{a}) \text{ for } \hat{\alpha} \in [0, \hat{b}_1] \\ \hat{\mathcal{A}}_{2L}(\hat{\alpha}) = \hat{b} + \frac{1 - \hat{\alpha}}{1 - \hat{b}_2} (\hat{c} - \hat{b}) \text{ for } \hat{\alpha} \in [\hat{b}_2, 1] \\ \hat{\mathcal{A}}_{3L}(\hat{\alpha}) = \hat{c} + \frac{1 - \hat{\alpha}}{1 - \hat{b}_3} (\hat{d} - \hat{c}) \text{ for } \hat{\alpha} \in [\hat{b}_3, 1] \\ \hat{\mathcal{A}}_{4L}(\hat{\alpha}) = \hat{d} + \frac{1 - \hat{\alpha}}{1 - \hat{b}_4} (\hat{e} - \hat{d}) \text{ for } \hat{\alpha} \in [\hat{b}_4, 1] \\ \hat{\mathcal{A}}_{4R}(\hat{\alpha}) = \hat{e} - \frac{\hat{\alpha}}{\hat{b}_4} (\hat{f} - \hat{e}) \text{ for } \hat{\alpha} \in [0, \hat{b}_4] \\ \hat{\mathcal{A}}_{3R}(\hat{\alpha}) = \hat{f} - \frac{\hat{\alpha}}{\hat{b}_3} (\hat{g} - \hat{f}) \text{ for } \hat{\alpha} \in [0, \hat{b}_3] \\ \hat{\mathcal{A}}_{2R}(\hat{\alpha}) = \hat{g} - \frac{\hat{\alpha}}{\hat{b}_2} (\hat{h} - \hat{g}) \text{ for } \hat{\alpha} \in [0, \hat{b}_2] \\ \hat{\mathcal{A}}_{1R}(\hat{\alpha}) = \hat{h} - \frac{\hat{\alpha}}{\hat{b}_1} (\hat{i} - \hat{h}) \text{ for } \hat{\alpha} \in [0, \hat{b}_1] \end{cases}$$

$$\begin{aligned}
 \text{Falsity} = \hat{F}_L(X) &= \begin{cases} \hat{\mathcal{A}}_{1L}(\alpha) = \hat{a}^1 + \frac{\alpha}{\hat{b}_1}(\hat{b}^1 - \hat{a}^1) \text{ for } \alpha \in [0, \hat{b}_1] \\ \hat{\mathcal{A}}_{2L}(\alpha) = \hat{b}^1 + \frac{1-\alpha}{1-\hat{b}_2}(\hat{c}^1 - \hat{b}^1) \text{ for } \alpha \in [\hat{b}_2, 1] \\ \hat{\mathcal{A}}_{3L}(\alpha) = \hat{c}^1 + \frac{1-\alpha}{1-\hat{b}_3}(\hat{d}^1 - \hat{c}^1) \text{ for } \alpha \in [\hat{b}_3, 1] \\ \hat{\mathcal{A}}_{4L}(\alpha) = \hat{d}^1 + \frac{1-\alpha}{1-\hat{b}_4}(\hat{e}^1 - \hat{d}^1) \text{ for } \alpha \in [\hat{b}_4, 1] \\ \hat{\mathcal{A}}_{3R}(\alpha) = \hat{e}^1 - \frac{\alpha}{\hat{b}_4}(\hat{f}^1 - \hat{e}^1) \text{ for } \alpha \in [0, \hat{b}_4] \\ \hat{\mathcal{A}}_{3R}(\alpha) = \hat{f}^1 - \frac{\alpha}{\hat{b}_3}(\hat{g}^1 - \hat{f}^1) \text{ for } \alpha \in [0, \hat{b}_3] \\ \hat{\mathcal{A}}_{2R}(\alpha) = \hat{g}^1 - \frac{\alpha}{\hat{b}_2}(\hat{h}^1 - \hat{g}^1) \text{ for } \alpha \in [0, \hat{b}_2] \\ \hat{\mathcal{A}}_{1R}(\alpha) = \hat{h}^1 - \frac{\alpha}{\hat{b}_1}(\hat{i}^1 - \hat{h}^1) \text{ for } \alpha \in [0, \hat{b}_1] \end{cases} \\
 \text{Indeterminacy} = \hat{I}_L(X) &= \begin{cases} \hat{\mathcal{A}}_{1L}(\alpha) = \hat{a}^2 + \frac{\alpha}{\hat{b}_1}(\hat{b}^2 - \hat{a}^2) \text{ for } \alpha \in [0, \hat{b}_1] \\ \hat{\mathcal{A}}_{2L}(\alpha) = \hat{b}^2 + \frac{1-\alpha}{1-\hat{b}_2}(\hat{c}^2 - \hat{b}^2) \text{ for } \alpha \in [\hat{b}_2, 1] \\ \hat{\mathcal{A}}_{3L}(\alpha) = \hat{c}^2 + \frac{1-\alpha}{1-\hat{b}_3}(\hat{d}^2 - \hat{c}^2) \text{ for } \alpha \in [\hat{b}_3, 1] \\ \hat{\mathcal{A}}_{4L}(\alpha) = \hat{d}^2 + \frac{1-\alpha}{1-\hat{b}_4}(\hat{e}^2 - \hat{d}^2) \text{ for } \alpha \in [\hat{b}_4, 1] \\ \hat{\mathcal{A}}_{4R}(\alpha) = \hat{e}^2 - \frac{\alpha}{\hat{b}_4}(\hat{f}^2 - \hat{e}^2) \text{ for } \alpha \in [0, \hat{b}_4] \\ \hat{\mathcal{A}}_{3R}(\alpha) = \hat{f}^2 - \frac{\alpha}{\hat{b}_3}(\hat{g}^2 - \hat{f}^2) \text{ for } \alpha \in [0, \hat{b}_3] \\ \hat{\mathcal{A}}_{2R}(\alpha) = \hat{g}^2 - \frac{\alpha}{\hat{b}_2}(\hat{h}^2 - \hat{g}^2) \text{ for } \alpha \in [0, \hat{b}_2] \\ \hat{\mathcal{A}}_{1R}(\alpha) = \hat{h}^2 - \frac{\alpha}{\hat{b}_1}(\hat{i}^2 - \hat{h}^2) \text{ for } \alpha \in [0, \hat{b}_1] \end{cases}
 \end{aligned}$$

There we have $\hat{\mathcal{A}}_{1L}(\alpha), \hat{\mathcal{A}}_{2L}(\alpha), \hat{\mathcal{A}}_{3L}(\alpha), \hat{\mathcal{A}}_{4L}(\alpha)$ are increasing and $\hat{\mathcal{A}}_{4R}(\alpha), \hat{\mathcal{A}}_{3R}(\alpha), \hat{\mathcal{A}}_{2R}(\alpha), \hat{\mathcal{A}}_{1R}(\alpha)$ are decreasing.

3.3 Non-Linear Nonagonal neutrosophic numbers with symmetry: is give as $\hat{\mathcal{A}}_{LS} = (\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}, \hat{f}, \hat{g}, \hat{h}, \hat{i})_{(\hat{n}_1, \hat{n}_2, \hat{m}_1, \hat{m}_2)}$ as well as membership function as following:

$$\text{Truth} = \hat{T}_L(X) = \begin{cases} 0 & x < \hat{a} \\ k \left(\frac{x-\hat{a}}{\hat{b}-\hat{a}} \right)^{n_1} & \hat{a} < x < \hat{b} \\ k \left(\frac{x-\hat{b}}{\hat{c}-\hat{b}} \right)^{n_2} & \hat{b} < x < \hat{c} \\ k & \hat{c} < x < \hat{d} \\ k + (1-k) \left(\frac{x-\hat{d}}{\hat{e}-\hat{d}} \right)^{n_3} & \hat{d} < x < \hat{e} \\ 1 & \hat{e} < x < \hat{f} \\ k + (1-k) \left(\frac{\hat{g}-x}{\hat{g}-\hat{f}} \right)^{m_1} & \hat{f} < x < \hat{g} \\ k & \hat{g} < x < \hat{h} \\ k \left(\frac{\hat{i}-x}{\hat{i}-\hat{h}} \right)^{m_2} & \hat{h} < x < \hat{i} \\ 0 & x > \hat{i} \end{cases}$$

$$\text{Indeterminacy} = \hat{I}_L(X) = \begin{cases} 0 & x < a^{\cdot 2} \\ k \left(\frac{x - a^{\cdot 2}}{b^{\cdot 2} - a^{\cdot 2}} \right)^{m_1} & a^{\cdot 2} < x < b^{\cdot 2} \\ k \left(\frac{x - b^{\cdot 2}}{c^{\cdot 2} - b^{\cdot 2}} \right)^{m_2} & b^{\cdot 2} < x < c^{\cdot 2} \\ k & c^{\cdot 2} < x < d^{\cdot 2} \\ k + (1 - k) \left(\frac{x - d^{\cdot 2}}{e^{\cdot 2} - d^{\cdot 2}} \right)^{m_3} & d^{\cdot 2} < x < e^{\cdot 2} \\ 1 & e^{\cdot 2} < x < f^{\cdot 2} \\ k + (1 - k) \left(\frac{g^{\cdot 2} - x}{g^{\cdot 2} - f^{\cdot 2}} \right)^{n_1} & f^{\cdot 2} < x < g^{\cdot 2} \\ k & g^{\cdot 2} < x < h^{\cdot 2} \\ k \left(\frac{i^{\cdot 2} - x}{i^{\cdot 2} - h^{\cdot 2}} \right)^{n_2} & h^{\cdot 2} < x < i^{\cdot 2} \\ 1 & x > i^{\cdot 2} \end{cases} \quad \text{As, } 0 < k < 1$$

$$\text{Falsity} = \hat{F}_L(X) = \begin{cases} 0 & x < a^{\cdot 1} \\ k \left(\frac{x - a^{\cdot 1}}{b^{\cdot 1} - a^{\cdot 1}} \right)^{m_1} & a^{\cdot 1} < x < b^{\cdot 1} \\ k \left(\frac{x - b^{\cdot 1}}{c^{\cdot 1} - b^{\cdot 1}} \right)^{m_2} & b^{\cdot 1} < x < c^{\cdot 1} \\ k & c^{\cdot 1} < x < d^{\cdot 1} \\ k + (1 - k) \left(\frac{x - d^{\cdot 1}}{e^{\cdot 1} - d^{\cdot 1}} \right)^{m_3} & d^{\cdot 1} < x < e^{\cdot 1} \\ 1 & e^{\cdot 1} < x < f^{\cdot 1} \\ k + (1 - k) \left(\frac{g^{\cdot 1} - x}{g^{\cdot 1} - f^{\cdot 1}} \right)^{n_1} & f^{\cdot 1} < x < g^{\cdot 1} \\ k & g^{\cdot 1} < x < h^{\cdot 1} \\ k \left(\frac{i^{\cdot 1} - x}{i^{\cdot 1} - h^{\cdot 1}} \right)^{n_2} & h^{\cdot 1} < x < i^{\cdot 1} \\ 1 & x > i^{\cdot 1} \end{cases} \quad , \mathcal{A}_{\hat{\alpha}} = \{x \in \mathcal{X} \mid \hat{T}_L(X), \hat{F}_L(X), \hat{I}_L(X) \geq \hat{\alpha}\}$$

3.4 α - cut of Non-Linear NFN with symmetry:

α - cut of non-LONNS can be express by $\mathcal{A}_{\hat{\alpha}} = \{x \in \mathcal{X} \mid \hat{T}_L(X), \hat{F}_L(X), \hat{I}_L(X) \geq \hat{\alpha}\}$

$$\text{Truth} = \hat{T}_L(X) = \begin{cases} \hat{\mathcal{A}}_{1L}(\hat{\alpha}) = \hat{a} + \left(\frac{\hat{\alpha}}{\hat{b}_1} \right)^{\hat{n}_1} (\hat{b} - \hat{a}) \text{ for } \hat{\alpha} \in [0, \hat{b}_1] \\ \hat{\mathcal{A}}_{2L}(\hat{\alpha}) = \hat{b} + \left(\frac{1 - \hat{\alpha}}{1 - \hat{b}_2} \right)^{\hat{n}_2} (\hat{c} - \hat{b}) \text{ for } \hat{\alpha} \in [\hat{b}_2, 1] \\ \hat{\mathcal{A}}_{3L}(\hat{\alpha}) = \hat{c} + \left(\frac{1 - \hat{\alpha}}{1 - \hat{b}_3} \right)^{\hat{n}_3} (\hat{d} - \hat{c}) \text{ for } \hat{\alpha} \in [\hat{b}_3, 1] \\ \hat{\mathcal{A}}_{4L}(\hat{\alpha}) = \hat{d} + \left(\frac{1 - \hat{\alpha}}{1 - \hat{b}_4} \right)^{\hat{n}_4} (\hat{e} - \hat{d}) \text{ for } \hat{\alpha} \in [\hat{b}_4, 1] \\ \hat{\mathcal{A}}_{4R}(\hat{\alpha}) = \hat{e} - \left(\frac{\hat{\alpha}}{\hat{b}_4} \right)^{\hat{m}_1} (\hat{f} - \hat{e}) \text{ for } \hat{\alpha} \in [0, \hat{b}_4] \\ \hat{\mathcal{A}}_{3R}(\hat{\alpha}) = \hat{f} - \left(\frac{\hat{\alpha}}{\hat{b}_3} \right)^{\hat{m}_2} (\hat{g} - \hat{f}) \text{ for } \hat{\alpha} \in [0, \hat{b}_3] \\ \hat{\mathcal{A}}_{2R}(\hat{\alpha}) = \hat{g} - \left(\frac{\hat{\alpha}}{\hat{b}_2} \right)^{\hat{m}_3} (\hat{h} - \hat{g}) \text{ for } \hat{\alpha} \in [0, \hat{b}_2] \\ \hat{\mathcal{A}}_{1R}(\hat{\alpha}) = \hat{h} - \left(\frac{\hat{\alpha}}{\hat{b}_1} \right)^{\hat{m}_4} (\hat{i} - \hat{h}) \text{ for } \hat{\alpha} \in [0, \hat{b}_1] \end{cases}$$

$$\text{Indeterminacy} = \hat{I}_L(X) = \begin{cases} \hat{\mathcal{A}}_{1L}(\alpha) = a^{\cdot 2} + \left(\frac{\alpha}{b_1}\right)^{m_1} (b^{\cdot 2} - a^{\cdot 2}) \text{ for } \alpha \in [0, b_1] \\ \hat{\mathcal{A}}_{2L}(\alpha) = b^{\cdot 2} + \left(\frac{1-\alpha}{1-b_2}\right)^{m_2} (c^{\cdot 2} - b^{\cdot 2}) \text{ for } \alpha \in [b_2, 1] \\ \hat{\mathcal{A}}_{3L}(\alpha) = c^{\cdot 2} + \left(\frac{1-\alpha}{1-b_3}\right)^{m_3} (d^{\cdot 2} - c^{\cdot 2}) \text{ for } \alpha \in [b_3, 1] \\ \hat{\mathcal{A}}_{4L}(\alpha) = d^{\cdot 2} + \left(\frac{1-\alpha}{1-b_4}\right)^{m_4} (e^{\cdot 2} - d^{\cdot 2}) \text{ for } \alpha \in [b_4, 1] \\ \hat{\mathcal{A}}_{3R}(\alpha) = e^{\cdot 2} - \left(\frac{\alpha}{b_4}\right)^{n_1} (f^{\cdot 2} - e^{\cdot 2}) \text{ for } \alpha \in [0, b_4] \\ \hat{\mathcal{A}}_{3R}(\alpha) = f^{\cdot 2} - \left(\frac{\alpha}{b_3}\right)^{n_2} (g^{\cdot 2} - f^{\cdot 2}) \text{ for } \alpha \in [0, b_3] \\ \hat{\mathcal{A}}_{2R}(\alpha) = g^{\cdot 2} - \left(\frac{\alpha}{b_2}\right)^{n_3} (h^{\cdot 2} - g^{\cdot 2}) \text{ for } \alpha \in [0, b_2] \\ \hat{\mathcal{A}}_{1R}(\alpha) = h^{\cdot 2} - \left(\frac{\alpha}{b_1}\right)^{n_4} (i^{\cdot 2} - h^{\cdot 2}) \text{ for } \alpha \in [0, b_1] \end{cases}$$

$$\text{Falsity} = \hat{F}_L(X) = \begin{cases} \hat{\mathcal{A}}_{1L}(\alpha) = a^{\cdot 1} + \left(\frac{\alpha}{b_1}\right)^{m_1} (b^{\cdot 1} - a^{\cdot 1}) \text{ for } \alpha \in [0, b_1] \\ \hat{\mathcal{A}}_{2L}(\alpha) = b^{\cdot 1} + \left(\frac{1-\alpha}{1-b_2}\right)^{m_2} (c^{\cdot 1} - b^{\cdot 1}) \text{ for } \alpha \in [b_2, 1] \\ \hat{\mathcal{A}}_{3L}(\alpha) = c^{\cdot 1} + \left(\frac{1-\alpha}{1-b_3}\right)^{m_3} (d^{\cdot 1} - c^{\cdot 1}) \text{ for } \alpha \in [b_3, 1] \\ \hat{\mathcal{A}}_{4L}(\alpha) = d^{\cdot 1} + \left(\frac{1-\alpha}{1-b_4}\right)^{m_4} (e^{\cdot 1} - d^{\cdot 1}) \text{ for } \alpha \in [b_4, 1] \\ \hat{\mathcal{A}}_{4R}(\alpha) = e^{\cdot 1} - \left(\frac{\alpha}{b_4}\right)^{n_1} (f^{\cdot 1} - e^{\cdot 1}) \text{ for } \alpha \in [0, b_4] \\ \hat{\mathcal{A}}_{3R}(\alpha) = f^{\cdot 1} - \left(\frac{\alpha}{b_3}\right)^{n_2} (g^{\cdot 1} - f^{\cdot 1}) \text{ for } \alpha \in [0, b_3] \\ \hat{\mathcal{A}}_{2R}(\alpha) = g^{\cdot 1} - \left(\frac{\alpha}{b_2}\right)^{n_3} (h^{\cdot 1} - g^{\cdot 1}) \text{ for } \alpha \in [0, b_2] \\ \hat{\mathcal{A}}_{1R}(\alpha) = h^{\cdot 1} - \left(\frac{\alpha}{b_1}\right)^{n_4} (i^{\cdot 1} - h^{\cdot 1}) \text{ for } \alpha \in [0, b_1] \end{cases}$$

The function which are increasing are $\hat{\mathcal{A}}_{1L}(\alpha), \hat{\mathcal{A}}_{2L}(\alpha), \hat{\mathcal{A}}_{3L}(\alpha), \hat{\mathcal{A}}_{4L}(\alpha)$ with respect to α and $\hat{\mathcal{A}}_{4R}(\alpha), \hat{\mathcal{A}}_{3R}(\alpha), \hat{\mathcal{A}}_{2R}(\alpha), \hat{\mathcal{A}}_{1R}(\alpha)$ are decreasing with respect to α .

$$\text{Truth} = \hat{T}_L(X) = \begin{cases} 0 & x < \hat{a} \\ p \left(\frac{x-\hat{a}}{\hat{b}-\hat{a}}\right)^{n_1} & \hat{a} < x < \hat{b} \\ p \left(\frac{x-\hat{b}}{\hat{c}-\hat{b}}\right)^{n_2} & \hat{b} < x < \hat{c} \\ k & \hat{c} < x < \hat{d} \\ k - (k - P) \left(\frac{x-\hat{d}}{\hat{e}-\hat{d}}\right)^{n_3} & \hat{d} < x < \hat{e} \\ 1 & \hat{e} < x < \hat{f} \\ k - (k - r) \left(\frac{\hat{g}-x}{\hat{g}-\hat{f}}\right)^{m_1} & \hat{f} < x < \hat{g} \\ k & \hat{g} < x < \hat{h} \\ r \left(\frac{\hat{i}-x}{\hat{i}-\hat{h}}\right)^{m_2} & \hat{h} < x < \hat{i} \\ 0 & x > \hat{i} \end{cases}$$

$$Indeterminacy = \hat{I}_L(X) = \begin{cases} 0 & x < a^{\cdot 2} \\ y \left(\frac{x - a^{\cdot 2}}{b^{\cdot 2} - a^{\cdot 2}} \right)^{m_1} & a^{\cdot 2} < x < b^{\cdot 2} \\ y \left(\frac{x - b^{\cdot 2}}{c^{\cdot 2} - b^{\cdot 2}} \right)^{m_2} & b^{\cdot 2} < x < c^{\cdot 2} \\ X & c^{\cdot 2} < x < d^{\cdot 2} \\ X - (X - Y) \left(\frac{x - d^{\cdot 2}}{e^{\cdot 2} - d^{\cdot 2}} \right)^{m_3} & d^{\cdot 2} < x < e^{\cdot 2} \\ 1 & e^{\cdot 2} < x < f^{\cdot 2} \\ x - (X - z) \left(\frac{g^{\cdot 2} - x}{g^{\cdot 2} - f^{\cdot 2}} \right)^{n_1} & f^{\cdot 2} < x < g^{\cdot 2} \\ z & g^{\cdot 2} < x < h^{\cdot 2} \\ z \left(\frac{i^{\cdot 2} - x}{i^{\cdot 2} - h^{\cdot 2}} \right)^{n_2} & h^{\cdot 2} < x < i^{\cdot 2} \\ 1 & x > i^{\cdot 2} \end{cases}$$

$$Falsity = \hat{F}_L(X) = \begin{cases} 0 & x < a^{\cdot 1} \\ q \left(\frac{x - a^{\cdot 1}}{b^{\cdot 1} - a^{\cdot 1}} \right)^{m_1} & a^{\cdot 1} < x < b^{\cdot 1} \\ q \left(\frac{x - b^{\cdot 1}}{c^{\cdot 1} - b^{\cdot 1}} \right)^{m_2} & b^{\cdot 1} < x < c^{\cdot 1} \\ k & c^{\cdot 1} < x < d^{\cdot 1} \\ w - (w - q) \left(\frac{x - d^{\cdot 1}}{e^{\cdot 1} - d^{\cdot 1}} \right)^{m_3} & d^{\cdot 1} < x < e^{\cdot 1} \\ 1 & e^{\cdot 1} < x < f^{\cdot 1} \\ w - (w - s) \left(\frac{g^{\cdot 1} - x}{g^{\cdot 1} - f^{\cdot 1}} \right)^{n_1} & f^{\cdot 1} < x < g^{\cdot 1} \\ w & g^{\cdot 1} < x < h^{\cdot 1} \\ s \left(\frac{i^{\cdot 1} - x}{i^{\cdot 1} - h^{\cdot 1}} \right)^{n_2} & h^{\cdot 1} < x < i^{\cdot 1} \\ 1 & x > i^{\cdot 1} \end{cases} \text{ As, } 0 < k < 1$$

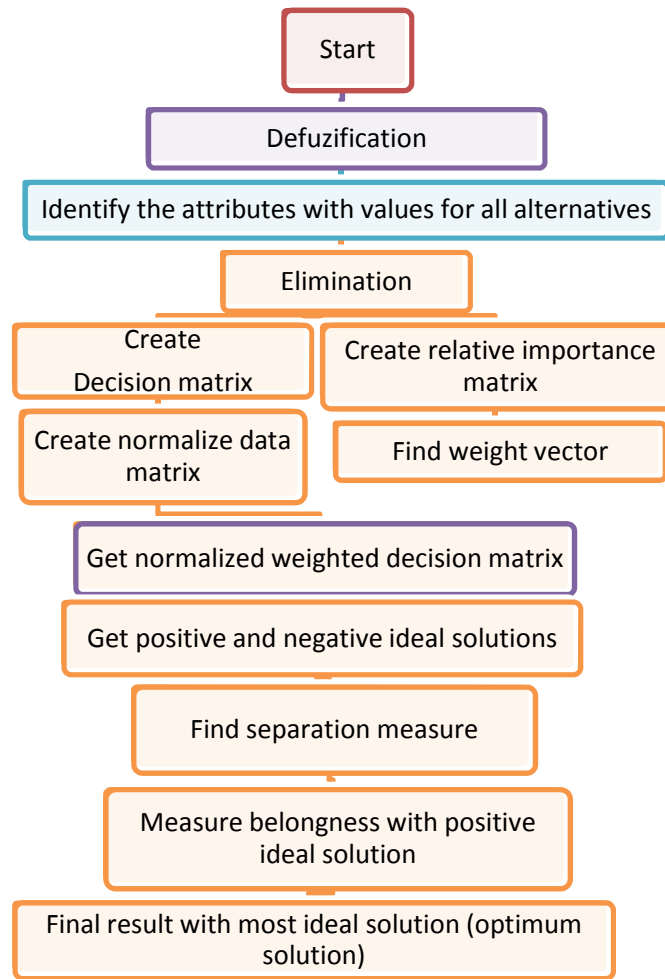
4. TOPSIS Technique

TOPSIS is a technique used for order performance by the help of similarity to the ideal solution. This idea is given by Hwang and Yoon. This is one of the most common technique for daily life decision making problem. This method is really close to the concept of PIS and NIS. Other methods like VIKOR also suitable for that kind of situation. We defuzzied the octagonal Fuzzy number by

$$\hat{D}^{TNO_N} = \left(\frac{\hat{a} + \hat{b} + \hat{c} + \hat{d} + \hat{e} + \hat{f} + \hat{g} + \hat{h} + \hat{i}}{9} \right), \quad \hat{D}^{INO_N} = \left(\frac{\hat{a}^1 + \hat{b}^1 + \hat{c}^1 + \hat{d}^1 + \hat{e}^1 + \hat{f}^1 + \hat{g}^1 + \hat{h}^1 + \hat{i}^1}{9} \right) \text{ and}$$

$$\hat{D}^{FNO_N} = \left(\frac{\hat{a}^2 + \hat{b}^2 + \hat{c}^2 + \hat{d}^2 + \hat{e}^2 + \hat{f}^2 + \hat{g}^2 + \hat{h}^2 + \hat{i}^2}{9} \right)$$

$$d(\bar{x}, \bar{y}) = \sqrt{\frac{1}{3} (\mathbf{a}_1 - \hat{\mathbf{a}}_2)^2 + (\mathbf{b}_1 - \hat{\mathbf{b}}_2)^2 + (\mathbf{c}_1 - \hat{\mathbf{c}}_2)^2}$$



Step 1: We will assigned the rating to the criteria and by given alternatives. Here we have a supposition that we have a decision group with K members. The alternative is shown as \mathcal{A}_i with respect to criteria \mathcal{C}_j is denoted $\bar{x}_{ij}^k = (\bar{a}_{ij}^k, b_{ij}^k, \bar{c}_{ij}^k)$ and the weight criteria is given as $\bar{w}_j^k = (\bar{w}_{j1}^k, w_{j2}^k, \bar{w}_{j3}^k)$.

Step 2: we will make a relation between fuzzy rating with alternatives and fuzzy weighting with criteria.

Linguistic value	Nonagonal FN \bar{a}_{ij}
Absolutely Recommended	(7,8,9,8,6,8,6,7,9)
Strongly Recommended	(6,7,8,7,8,6,5,7,8)
Recommended	(5,6,7,7,7,6,8,7,7)
May be Recommended	(4,5,7,4,5,7,4,7,5)
Weakly Recommended	(2,3,5,5,6,3,4,3,4)
Rarely Recommended	(1,2,4,3,2,4,4,3,5)
Not Sure to be Recommended	(1,1,3,2,2,1,3,2,3)

Fuzzy Rating as $\bar{x}_{ij}^k = (a_{ij}, b_{ij}, \bar{c}_{ij})$ of i^{th} alternative with respect to j^{th} criteria.

$$\bar{a}_{ij} = \min_k \{a_{ij}^k\}, \quad \bar{b}_{ij} = \frac{1}{K} \sum_{k=1}^K b_{ij}^k, \quad c_{ij} = \max_k \{c_{ij}^k\}.$$

The weight as $\bar{w}_j = (\bar{w}_{j1}, w_{j2}, \bar{w}_{j3})$ for criteria \bar{C}_j are given by these formulas:

$$\bar{w}_{j1} = \min_k \{w_{j1}^k\}, \quad \bar{w}_{j2} = \frac{1}{K} \sum_{k=1}^K w_{j2}^k, \quad w_{j3} = \max_k \{w_{j3}^k\}.$$

Step 3: The normalized fuzzy decision matrix as $\bar{R} = [\bar{r}_{ij}]$,

$$\bar{r}_{ij} = \left(\frac{\bar{a}_{ij}}{c_j^*}, \frac{\bar{b}_{ij}}{c_j^*}, \frac{\bar{c}_{ij}}{c_j^*} \right) \text{ and } c_j^* = \max_i \{c_{ij}\} \text{ beneficial criteria}$$

$$\bar{r}_{ij} = \left(\frac{\bar{a}_j^-}{c_{ij}}, \frac{\bar{a}_j^-}{b_{ij}}, \frac{\bar{a}_j^-}{a_{ij}} \right) \text{ and } c_j^- = \min_i \{a_{ij}\} \text{ non beneficial criteria.}$$

Step 4: Now weighted normalized fuzzy decision matrix will compute.

Step 5: Now compute the fuzzy negative ideal solution FNIS and fuzzy positive ideal solution FPIS.

Step 6: get a solution of each distance of alternative to FPIS and FNIS.

$$\bar{d}_i^* = \sum_{j=1}^n \check{d}(v_{ij}, v_j^*), \quad d_i^- = \sum_{j=1}^n \check{d}(v_{ij}, v_j^-) \text{ be the distance.}$$

Step 7: Closeness Coefficient $\bar{C}C_i = \frac{d_i^-}{d_i^- + d_i^*}$

Step 8: Alternatives will be ranked.

5 Case Study: We will check the productivity and flexibility of this proposed TOPSIS method, here we have most useful real-life issue. Suppose that, A person is going to buy a car, here a lot of possibilities available and decision based on three different criteria. We have to select best one.

Numerical problem: Here U is the universe. Person is going to buy a car, So, give a look on possible options, mention in Table 1. Three different brands ($\hat{A}, \hat{B}, \hat{C}$) applied for this opportunity, with different opportunity. Choice parameter are $\{\hat{C}_1, \hat{C}_2, \hat{C}_3\}$ and detailed mention in table 1.

\hat{A}	\hat{B}	\hat{C}
$\{\hat{C}_1(0.71,0.35,0.71,0.77,0.31,0.73,0.67,0.61,0.3)$ $(0.93,0.43,0.93,0.88,0.84,0.99,0.96,0.90,0.4)$ $(0.86,0.95,0.89,0.97,0.94,0.93,0.75,0.81,0.99)\}$	$\{\hat{C}_1(0.63,0.73,0.36,0.94,0.85,0.86,0.84,0.85,0.89)$ $(0.33,0.46,0.79,0.91,0.79,0.75,0.79,0.74,0.96)$ $(0.98,0.93,0.95,0.85,0.97,0.98,0.74,0.86,0.90)\}$	$\{\hat{C}_1(0.63,0.83,0.83,0.66,0.85,0.44,0.85,0.63,0.98)$ $(0.76,0.75,0.69,0.84,0.94,0.97,0.63,0.75,0.63)\}$ $(0.88,0.93,0.95,0.95,0.77,0.98,0.84,0.86,0.90)$
$\{\hat{C}_2(0.85,0.75,0.96,0.54,0.83,0.75,0.63,0.56,0.97)$ $(0.85,0.45,0.65,0.38,0.78,0.79,0.57,0.13,0.9)$ $(0.96,0.89,0.98,0.99,0.97,0.96,0.97,0.95,0.11)\}$	$\{\hat{C}_2(0.66,0.49,0.68,0.99,0.67,0.11,0.56,0.87,0.95)$ $(0.93,0.73,0.83,0.68,0.84,0.79,0.45,0.76,0.70)$ $(0.85,0.95,0.96,0.64,0.93,0.75,0.73,0.55,0.96)\}$	$\{\hat{C}_2(0.98,0.69,0.98,0.88,0.79,0.97,0.96,0.97,0.85)$ $(0.75,0.45,0.55,0.45,0.28,0.78,0.59,0.67,0.23)$ $(0.89,0.95,0.86,0.94,0.93,0.95,0.91,0.99,0.99)\}$
$\{\hat{C}_3(0.74,0.83,0.94,0.75,0.96,0.34,0.75,0.69,0.95)$ $(0.35,0.46,0.98,0.59,0.65,0.71,0.74,0.76,0.25)$ $(0.84,0.73,0.75,0.85,0.98,0.74,0.86,0.84,0.91)\}$	$\{\hat{C}_3(0.94,0.93,0.84,0.95,0.96,0.84,0.95,0.91,0.99)$ $(0.28,0.36,0.58,0.25,0.65,0.61,0.54,0.26,0.88)$ $(0.98,0.93,0.95,0.95,0.78,0.94,0.96,0.94,0.91)\}$	$\{\hat{C}_3(0.84,0.96,0.33,0.35,0.52,0.97,0.93,0.21,0)$ $(0.45,0.95,0.54,0.78,0.39,0.75,0.61,0.44,0.76)$ $(0.21,0.74,0.33,0.64,0.85,0.76,0.34,0.65,0.99)\}$

[Now, this given matrix depend on $(\hat{C}_1, \hat{C}_2, \hat{C}_3)$ as row and $(\hat{A}, \hat{B}, \hat{C})$ as column]

STEP 1. We defuzzified the Octagonal Fuzzy number by

$$\hat{D}^{TNO_N} = \left(\frac{\hat{a} + \hat{b} + \hat{c} + \hat{d} + \hat{e} + \hat{f} + \hat{g} + \hat{h} + \hat{i}}{9} \right), \quad \hat{D}^{INO_N} = \left(\frac{\hat{a}^{-1} + \hat{b}^{-1} + \hat{c}^{-1} + \hat{d}^{-1} + \hat{e}^{-1} + \hat{f}^{-1} + \hat{g}^{-1} + \hat{h}^{-1} + \hat{i}^{-1}}{9} \right) \text{ and}$$

$$\hat{D}^{FNO_N} = \left(\frac{\hat{a}^2 + \hat{b}^2 + \hat{c}^2 + \hat{d}^2 + \hat{e}^2 + \hat{f}^2 + \hat{g}^2 + \hat{h}^2 + \hat{i}^2}{9} \right)$$

By applying these formulas, Neutrosophic soft matrix as following:

Criteria	A	B	C
\hat{C}_1	(0.6, 0.8, 0.9)	(0.8, 0.7, 0.9)	(0.7, 0.8, 0.9)
\hat{C}_2	(0.7, 0.6, 0.9)	(0.7, 0.8, 0.8)	(0.8, 0.5, 0.9)
\hat{C}_3	(0.8, 0.6, 0.8)	(0.9, 0.5, 0.9)	(0.6, 0.6, 0.6)

STEP 2: For normalized aggregate fuzzy decision matrix.

$$\check{r}_{ij} = \left(\frac{\check{a}_{ij}}{\check{c}_{ij}}, \frac{\check{b}_{ij}}{\check{c}_{ij}}, \frac{\check{c}_{ij}}{\check{c}_{ij}} \right)$$

Criteria	A	B	C
\hat{C}_1	(0.7, 0.9, 1.0)	(0.9, 0.8, 1.0)	(0.8, 0.9, 1.0)
\hat{C}_2	(0.8, 0.7, 1.0)	(0.9, 1.0, 1.0)	(0.9, 0.5, 1.0)
\hat{C}_3	(1.0, 0.7, 1.0)	(1.0, 0.5, 1.0)	(1.0, 1.0, 1.0)

Then for criteria weighting there is a aggregate decision matrix

$$\bar{W}_1 = (0.6, 0.6, 0.7) , \bar{W}_2 = (0.1, 0.2, 0.4) , \bar{W}_3 = (0.3, 0.5, 0.5)$$

STEP 3: $\check{p}_{ij} = \check{r}_{ij}$ will multiply by \bar{w}_j as well as weighted normalized fuzzy decision matrix.

Criteria	A	B	C
\hat{C}_1	(0.4, 0.5, 0.7)	(0.09, 0.1, 0.4)	(0.2, 0.4, 0.5)
\hat{C}_2	(0.4, 0.6, 0.7)	(0.09, 0.2, 0.4)	(0.2, 0.2, 0.5)
\hat{C}_3	(0.6, 0.4, 0.7)	(0.1, 0.1, 0.4)	(0.3, 0.5, 0.5)

STEP 4: Find $\hat{F}NIS$ and $\hat{F}PIS$

$$\hat{A}^+ = (\hat{\mathcal{P}}_1^+, \hat{\mathcal{P}}_2^+, \hat{\mathcal{P}}_3^+ \dots \dots \hat{\mathcal{P}}_n^+)$$

$$\hat{\mathcal{P}}_j^+ = \max (\hat{\mathcal{P}}_{ij3}) \quad i=1,2,\dots,m \quad , \quad j=1,2,3,\dots,n$$

$$\hat{A}^- = (\hat{\mathcal{P}}_1^-, \hat{\mathcal{P}}_2^-, \hat{\mathcal{P}}_3^- \dots \dots \hat{\mathcal{P}}_n^-)$$

$$\hat{\mathcal{P}}_j^- = \min (\hat{\mathcal{P}}_{ij3}) \quad i=1,2,\dots,m \quad , \quad j=1,2,3,\dots,n$$

$$\hat{A}^+ = \hat{\mathcal{P}}_1^+(0.7, 0.4, 0.5), \hat{\mathcal{P}}_2^+(0.7, 0.4, 0.5), \hat{\mathcal{P}}_3^+(0.7, 0.4, 0.5)$$

$$\hat{A}^- = \hat{\mathcal{P}}_1^-(0.4, 0.09, 0.2), \hat{\mathcal{P}}_2^-(0.4, 0.09, 0.2), \hat{\mathcal{P}}_3^-(0.4, 0.1, 0.3)$$

$$\text{Now by } \hat{d}(\hat{x}, \hat{y}) = \sqrt{\frac{1}{3} (\hat{\alpha}_1 - \hat{\alpha}_2)^2 + (\hat{b}_1 - \hat{b}_2)^2 + (\hat{c}_1 - \hat{c}_2)^2}$$

STEP 5 to STEP 8 mentioned below

Table (Positive ideal solution)

Criteria	\hat{A}	\hat{B}	\hat{C}
\hat{C}_1	0.4472	0.2733	0.2886
\hat{C}_2	0.3316	0.2171	0.3511
\hat{C}_3	0.2081	0.3690	0.2516

FNIS Table (Negative ideal solution)

Criteria	\hat{A}	\hat{B}	\hat{C}
\hat{C}_1	0.2466	0.2685	0.2465
\hat{C}_2	0.3142	0.2900	0.3397
\hat{C}_3	0.3131	0.2000	0.2509

Now estimate the distance between each weighted alternative.

$$d_i^* = \sum_{j=1}^n d(v_{ij}, v_j^*), d_i^- = \sum_{j=1}^n d(v_{ij}, v_j^-)$$

$$d_1^* = 0.9869 \quad d_1^- = 0.8739$$

$$d_2^* = 0.8594 \quad d_2^- = 0.7585$$

$$d_3^* = 0.8913 \quad d_3^- = 0.8371$$

Closeness coefficient

$$\hat{C}\hat{C}_i = \frac{d_i^-}{d_i^- + d_i^*}$$

$$\hat{C}\hat{C}_1 = \frac{0.8739}{0.8739 + 0.9869} = 0.4696,$$

$$\hat{C}\hat{C}_2 = \frac{0.7585}{0.7585 + 0.8594} = 0.4688$$

$$\hat{C}\hat{C}_3 = \frac{0.8371}{0.8371 + 0.8913} = 0.4578$$

Strategy	Results	Rank
$\hat{C}\hat{C}_1$	0.4696	1
$\hat{C}\hat{C}_2$	0.4688	2
$\hat{C}\hat{C}_3$	0.4578	3

Clearly

$\hat{A} > \hat{B} > \hat{C}$. The best car for this person is \hat{A} .

6 Conclusion:

In this current article, we present type of nonagonal neutrosophic number (Linear, Non-Linear, Symmetric, Asymmetric) as well as their $\hat{\alpha}$ – cuts also proposed. Nonagonal neutrosophic number will be really useful in the field of multi-criteria decision making MCDM problems of daily life as well as it can deal more fluctuations. To estimate reliability and productivity, we also present a daily life problem and solved it with TOPSIS technique of MCDM. At the very first we convert nonagonal to fuzzy using accuracy function and then we use it in existing method. Further, we will present aggregate operators of nonagonal neutrosophic number as well as matrix notation with operations.

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