A Note on Neutrosophic Polynomials and Some of Its Properties

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Abstract

The purpose of this article is to study neutrosophic polynomials i.e. polynomials which are Neutrosophic in nature and study its properties with the help of neutrosophic numbers. Apart from this we discuss different types of neutrosophic polynomials with concrete examples and establish some theorems and results which will be useful for the further study. We also give a solution method to find the approximate roots of a neutrosophic polynomial equation.

Keywords: Neutrosophic numbers; Neutrosophic polynomials; Synthetic division; Multiple roots

1.Introduction

Zadeh [1] is the initiator of fuzzy set. It is a such kind of mathematical tool by which we can generalize the classical concepts by introducing membership function. Fuzziness is determined by a membership function. So it plays a vital role for the fuzzy representation. Fuzzy set mainly concerned with membership function and there are different methods for the determination of membership function . So sometimes to choose the best method to get the best result is a tedious job. Embedding the idea of uncertainty, Gau and Buehrer introduced vague set [2] , Goguen initiated L-fuzzy set [3] and Pawlak introduced Rough set [4]. Some other extensions of fuzzy sets are mentioned in [5-7]. These generalized concepts enhance the scope to solve the uncertain problems appropriately.

Sometimes it is difficult to represent uncertainty by assigning a single real value , that's why interval -valued fuzzy sets was proposed . In expert system, in belief system etc there is a need to consider both truth-membership and the falsity membership for actual elucidation of an element in undetermined, ambivalent domain. For such situation both fuzzy set and interval-valued fuzzy set are irrelevant to use. In suchlike circumstances only intuitionistic fuzzy set is recommended. It can operate the deficient data with the help of falsity-membership. But there exists indeterminate and inconsistent data in uncertain environment which can't be handled by such type of sets.

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From logical view if somebody ask me to read the mind of a person then how it can be done. We know that human mind mainly divided into three parts which are responsive mind, senseless mind and dormant mind. Dormant means it is partially responsive and partially senseless. So it can't be measured by neither membership value nor non-membership value. It is a type of indeterminacy. In real life problems also we come across with such a situation. So we need another convenient tool which can be able to measure the level of indeterminacy along with membership and non-membership. Which leads to the introduction of Neutrosophic sets. Smarandache [8] is the pioneer of Neutrosophic set. By adding indeterminacy with intuitionistic fuzzy set , Smarandache defined neutrosophic set . Pertaining the idea of Neutrosophic sets several theories , results and applications are developed by the researchers and Mathematicians of all over the world. Some of their contributions are mentioned in [9-13] . For parametric nature in a data one more numerical tool developed by Molodtsov [14]. In [15], Maji et al. study and describe various properties of soft set. It has been successfully used in various aspects. It is considered even as more general framework of modeling and manipulating the vague concept. In 2012, Maji [16] introduced Neutrosophic soft set (NSS). Application of neutrosophic soft set shown in [17]. Several works related to NSSs have been done effectively over various discipline by the neutrosophication in data .

In this work we mainly concerned with Neutrosophic polynomials and its types. We also contribute some theorems and results based on Neutrosophic polynomials and try to develop the earlier concept of classical polynomials so that we have a bright scope to use it further and to establish new theories and results related to polynomials which are indeterministic in nature. For practical purpose we include [18-21].

The present article is arranged in the following manner :

Section-2 comprised with Neutrosophic numbers and basic operations on them. In section-3, firstly defined neutosophic polynomials and its types and then deduce some theorems and give examples of each type to justify the results.In the last section(in section-4), the main purpose of the paper is discussed briefly.

2. Classical Neutrosophic Numbers and basic operations

Here, we give the basic definition of Neutrosophic Numbers and its properties.

Definition 1. A classical Neutrosophic number is of the form $x+y\Im$, where x , y are real or complex coefficients and $\Im$=indeterminacy, such that $0.\Im=0$ and $\Im^2=\Im$. In general $\Im^n=\Im$, for all positive integers n. If x, y are real then $x+y\Im$ is called Neutrosophic real number otherwise it is known as Neutrosophic complex number. $3-5\Im$, $6+8\Im$ etc are examples of Neutrosophic real numbers and $(4+3i)+(4-2i)\Im, (3-2i)+i\Im$ etc are examples of Neutrosophic complex numbers where $i \equiv \sqrt{-1}$ and $\Im$ denotes indeterminacy.

Generally a Neutrosophic complex number can be written as $x+iy+z\Im+ti\Im$, where x, y, z and t are reals and $i \equiv \sqrt{-1}$. Indeterminacy $\Im$ (with non-zero coefficient) in a Neutrosophic number is known as a true Neutrosophic number . In our work we mainly concerned with Neutrosophic real numbers.

Definition 2. If $p_1+\Im q_1$ and $p_2+\Im q_2$ be two classical neutrosophic real numbers then their division is denoted by $\frac{p_1+\Im q_1}{p_2+\Im q_2} = a+\Im b$, where p_2 and q_2 are real and they never zero together and $\Im$ denotes indeterminacy.

On comparing both sides

$p_2 a = p_1$ and $q_2 a + (p_2+q_2)b = q_1$

For one and only one solution , we have

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$$\begin{vmatrix} p_2 & 0 \\ q_2 & p_2+q_2 \end{vmatrix} \neq 0$$

or, $p_2(p_2+q_2) \neq 0$

or, $p_2 \neq 0$ and $p_2 \neq -q_2$

Then, $a = \frac{p_1}{p_2}$ and $b = \frac{p_2q_1 - p_1q_2}{p_2\left(p_2+q_2\right)}$

Thus, $\frac{p_1 + \Im q_1}{p_2 + \Im q_2} = \frac{p_1}{p_2} + \Im \frac{p_2q_1 - p_1q_2}{p_2\left(p_2+q_2\right)}$

Results 1. For the division of neutrosophic real numbers we have the following results:

1. $\frac{p+q\Im}{p\lambda + q\lambda\Im} \frac{p+q\Im}{\lambda(p+q\Im)} = \frac{1}{\lambda}$, for $\lambda \neq 0$ and $p+q\Im \neq 0$
2. $\frac{\Im}{p+q\Im} = \frac{p}{p(p+q\Im)}.\Im = \frac{1}{p+q\Im}.\Im$, for $p \neq 0$ and $p+q\Im \neq 0$
3. $\frac{p+\Im q}{\lambda\Im}$ = undefined, for either λ=0 or $\Im$ doesnot take any non zero value.

In particular: $\frac{\Im}{\Im}$ = undefined

4. $\frac{p+\Im q}{\lambda} = \frac{p}{\lambda} + \frac{q}{\lambda}.\Im$, for $\lambda \neq 0$
5. $\frac{r}{p+q\Im} = \frac{r}{p} - \frac{qr}{p(p+q\Im)}.\Im$, for $p \neq 0$ and $(p+q\Im) \neq 0$

3. Neutrosophic Polynomials

Firstly we give the notion of neutrosophic polynomials with a concrete example then some results based on it are established.

Definition 3. Neutrosophic polynomial is a polynomial whose coefficients (atleast one of them contain $\Im$) are neutrosophic numbers . If its coefficients are neutrosophic real numbers then it is called neutrosophic real polynomial otherwise it is called neutrosophic complex polynomial.

$P(y) = 3y^2 + (3+\Im)y - (5+3\Im)$ and $Q(x) = 4x^3 + (1-6i\Im)\,x^2 + 7\Im x - 6i\Im$ are examples of neutrosophic real polynomial and neutrosophic complex polynomial respectively.

In this work we mainly concerned with neutrosophic real polynomials.

In general any expression of the form $P_N\left(x\right) = p_0x^n + p_1x^{n-1} + p_2x^{n-2} + ... + p_n$ where $p_0, p_1, p_2, ..., p_n$ are neutrosophic real numbers in which at least one of the coefficient contains indeterminacy($\Im$) and n is a

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positive integer is called neutrosophic real polynomial of degree n if $p_0 \neq 0$. The terms of a neutrosophic real polynomial with zero coefficient are normally omitted.

From logical view we give a real example of Neutrosophic polynomial in the following way:

We brought a cake from a bakery which has a volume 120 cu.units. Length of the cake is 2 units more than its breadth and its hight is slightly more than its breadth. We consider a,b and c for length, breadth and height respectively. Using the given condition we write $a = b + 2, c = b + \Im$, where $\Im$ is the indeterminacy involved in this problem. So such a situation can be shown as

$$120 = (b+2)b(b+\Im)$$
$$or, (b^2 + 2b)(b+\Im) = 120$$
$$or, (b^3 + b^2\Im + 2b^2 + 2\Im b) = 120$$
$$or, b^3 + b^2(\Im + 2) + 2\Im b = 120$$

Which is the required neutrosophic polynomial for the above problem.

Now the question arises how we solve this type of equation. To do so we consider some examples.

Example 1. Find the roots of the neutrosophic real polynomial $6x^2 + (10+\Im)x + 4\Im = 0$.

Solution. We have,

$$6x^2 + (10+\Im)x + 4\Im = 0$$
$$or, x = \frac{-(10+\Im) \pm \sqrt{(10+\Im)^2 - 4.6.4\Im}}{12} \text{ (using the quadratic formula)}$$
$$or, x = \frac{-10-\Im \pm 5\sqrt{4-3\Im}}{12}$$

Let, $\sqrt{4-3\Im} = a + \Im b$, where a and b are real and $\Im \leq \frac{4}{3}$.

$$or, 4 - 3\Im = a^2 + 2ab\Im + b^2\Im$$
$$or, 4 - 3\Im = a^2 + (2ab + b^2)\Im$$
$$a^2 = 4 \quad \Big| \quad 2ab + b^2 = -3$$
$$a = \pm 2$$
$$\text{when } a = 2,\ b^2 + 4b + 3 = 0 \Rightarrow b = -1, -3$$
$$\text{and when } a = -2, b^2 - 4b + 3 = 0 \Rightarrow b = 3, 1$$

Putting the values of a and b in the above equation we get the values of x which are $-\frac{\Im}{2}, \frac{-5+\Im}{3}, \frac{-4\Im}{3}$ and $\frac{-\Im-10}{6}$

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Therefore, $\left\{-\frac{\Im}{2}, \frac{-5+\Im}{3}, \frac{-4\Im}{3}, \frac{-\Im-10}{6}\right\}$ is the solution set, where $\Im \leq \frac{4}{3}$.

Now we check the other properties of the polynomial.

Clearly, $P_N\left(-\frac{\Im}{2}\right) = P_N\left(\frac{-5+\Im}{3}\right) = P_N\left(\frac{-4\Im}{3}\right) = P_N\left(\frac{-\Im-10}{6}\right) = 0$

Let, $P_N(x) = 6x^2 + (10+\Im)x + 4\Im$ then

$$P_N(x) = 6\left[x - \left(-\frac{\Im}{2}\right)\right]\left[x - \left(\frac{-5+\Im}{3}\right)\right] = 6\left[x - \left(\frac{-4\Im}{3}\right)\right]\left[x - \left(\frac{-\Im-10}{6}\right)\right]$$

Clearly, the above neutrosophic polynomial has more than one factorization i.e it is not unique and the number of roots are more than that of the degree of the neutrosophic polynomial.

Let us take one more example to get more concrete result

Example 2. Find the roots of the neutrosophic polynomial $x^2 - 6\Im = 0$

Solution. $x^2 - 6\Im = 0 \Rightarrow x^2 = 6\Im \Rightarrow x = \pm\sqrt{6\Im}$

Let, $\sqrt{6\Im} = a + \Im b$, where a and b are real and $\Im \geq 0$.

$$or, 6\Im = a^2 + (2ab + b^2)\Im$$
$$or, a = 0 \quad \Big| b^2 = 6 \Rightarrow b = \pm\sqrt{6}$$

OR, $\sqrt{6\Im} = \sqrt{6}\,.\sqrt{\Im} = \sqrt{6}\ \Im$

Therefore, the solutions are $\pm\sqrt{6}\ \Im$

In this case it obeys the property of classical polynomial as the degree of the polynomial is equal to the no of roots. So there is no such proper relation between the degree and the number of roots of a neutrosophic polynomial . But one thing is clear from these two examples that number of roots will be either the degree of the polynomial or double of its degree. It is for the reader to verify the roots of the higher degree polynomial to get more concrete results.

Remark 1. Number of roots of a neutrosophic polynomial of degree $n \geq 1$ will be n or 2n or 3n……..and so on.

Definition4. A neutrosophic polynomial without any zero coefficient is called a complete Neutrosophic polynomial otherwise it is incomplete.

Definition5. A neutrosophic polynomial whose coefficients are zero and it is represented by $0=0.\Im$ is called vanishing Neutrosophic polynomial.

Definition6. Two neutrosophic polynomials are said to be equal iff their corresponding coefficients are alike and their corresponding indeterminacies converges to a fixed value.

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3.1 Division algorithm

If $P_N(x)$ and $\phi_N(x)$ be two neutrosophic polynomials where $\deg(\phi_N(x)) \le \deg(P_N(x))$, then to divide $P_N(x)$ by $\phi_N(x)$ is to establish a relation of the form

$$P_N(x) = \phi_N(x).Q_N + R_N,$$

Where Q_N and R_N are respectively the quotient and the remainder and $\deg(R_N) < \deg(\phi_N(x))$.Here Q_N and R_N are unique. If R_N =0, then $P_N(x)$ is completely divisible by $\phi_N(x)$.

3.2 Synthetic division method

It is a process in which we divide a polynomial of degree n by a binomial under neutrosophic environment.

Let,

$P_N(x) = (a_0 + \Im)x^n + (a_1 + \Im)x^{n-1} + (a_2 + \Im)x^{n-2} + ... + (a_n + \Im)$, where $(a_0 + \Im) \ne 0$

and the binomial is $(x - (\Im + h))$

By division algorithm, we have

$$(a_0 + \Im)x^n + (a_1 + \Im)x^{n-1} + (a_2 + \Im)x^{n-2} + ... + (a_n + \Im) = (x - (\Im + h))\left[(b_0 + \Im)x^{n-1} + (b_1 + \Im)x^{n-2} + (b_2 + \Im)x^{n-3} + ... + (b_{n-1} + \Im) + (R + \Im)\right]$$

Equating the coefficients of like powers, we have

$a_0 + \Im = b_0 + \Im$

$a_1 + \Im = (b_1 + \Im - b_0\Im - \Im - hb_0 - \Im h) \Rightarrow b_1 = a_1 + hb_0 + \Im(1 + h + b_0)$

Similarly,

$b_2 = a_2 + hb_1 + \Im(1 + h + b_1)$

$b_3 = a_3 + hb_2 + \Im(1 + h + b_2)$

…...............................

$R = a_n + hb_{n-1} + \Im(1 + h + b_{n-1})$

Thus, we establish an easy approach by which we calculate the coefficients of the quotient and the remainder as follows:

$a_0 + \Im$	$a_1 + \Im$	$a_2 + \Im$	....	$a_n + \Im$
	$b_0(\Im + h) + \Im(\Im + h)$	$b_1(\Im + h) + \Im(\Im + h)$	....	$b_{n-1}(\Im + h) + \Im(\Im + h)$
$b_0 + \Im$	$b_1 + \Im$	$b_2 + \Im$	……………..	$R + \Im$

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Note1. If we divide a neutrosophic polynomial $P_N\left(x\right)$ by $\left(ax-(b+\Im\right)$, we first to find the quotient(□) and remainder($\Re$) in the division of $P_N\left(x\right)$ by $\left(x-\frac{(b+\Im)}{a}\right)$, then the quotient and remainder in the division of $P_N\left(x\right)$ by $\left(ax-(b+\Im\right)$ will be $\frac{□}{a}$ and $\Re$ respectively.

Example3. Using synthetic division method divide $x^4+(5+\Im)x^3+(4+\Im)x^2+(8+\Im)x-(20+\Im)$ by $(\mathrm{x}-(1+\Im))$

Solution. By synthetic division method, we have

1	$5+\Im$	$4+\Im$	$8+\Im$	$-(20+\Im)$
	$(1+\Im)$	$6+8\Im$	$10+18\Im$	$18+56\Im$
1	$6+\Im$	$10+9\Im$	$18+19\Im$	$-2+55\Im$

$Q=x^3+(6+\Im)x^2+(10+9\Im)x+(18+19\Im)$ and $\mathrm{R}=-2+55\Im$

3.3 Remainder theorem

Statement. If a Neutrosophic real polynomial be divided by a neutrosophic binomial $\left(x-(\Im+h)\right)$ then the remainder is $P_N\left(\Im+h\right)$

Proof. By division algorithm,

$$P_n\left(x\right)=\left(x-(\Im+h)\right)Q+(R+\Im)$$

Puttting $x=\Im+h$, we have $(R+\Im)=P_N\left(\Im+h\right)$

Example4. Find the remainder when $\mathrm{x}^3-(3+\Im)x^2+4x-3$ is divided by $(x-(2+\Im))$

Solution. By remainder theorem,

$P_n\left(2+\Im\right)=1-\Im$, which is the required remainder.

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Example5. Find a relation between r and s when $2x^3-(7+\Im)x^2+(r+\Im)x+(s+\Im)$ is exactly divisible by $(x-(3+I))$

Solution. By remainder theorem,

$P_n(3+\Im)=0 \Rightarrow r(3+\Im)+s=9-14\Im$, which is the required relation

Results2. We have the following results

1. Let $P_N(x)$ be a Neutrosophic Polynomial which vanishes when x takes the values $p_1+\Im$, $p_2+\Im$, $p_3+\Im$,……, $p_n+\Im$ where no two of which are equal then the product $(x-(p_1+\Im))(x-(p_2+\Im)),........(x-(p_n+\Im))=0$.

2. A Neutrosophic polynomial $P_N(x)$ of n-th degree $(n\geq 1)$ can be vanish for more than n values of x.

3.4 Method to express a given neutrosopic real polynomial $P_N(x)$ as a function of $(x-(\Im+h))$

Procedure.

$$\text{Let } P_N(x)=(a_0+\Im)x^n+(a_1+\Im)x^{n-1}+(a_2+\Im)x^{n-2}+........+(a_n+\Im) \quad (1)$$

$$=b_0(x-(\Im+h))^n+b_1(x-(\Im+h))^{n-1}+..........+b_{n-1}(x-(\Im+h))+(b_n+\Im) \quad (2)$$

$$=(x-(\Im+h))\left[b_0(x-(\Im+h))^{n-1}+b_1(x-(\Im+h))^{n-2}+..........+b_{n-1}\right]+(b_n+\Im)$$

$$=(x-(\Im+h))\,Q\left[(x-(\Im+h))\right]+(b_n+\Im)$$

Where $Q\left[(x-(\Im+h))\right]$ is the quotient and $(b_n+\Im)$ is the remainder.Therefore, the remainder is the last coefficient of (2). If $Q\left[(x-(\Im+h))\right]$ divided by $(x-(\Im+h))$ the remainder will be $(b_{n-1}+\Im)$,which is the coefficient of $(x-(\Im+h))$ in (2).

If we continue like this , we obtain, after each successive division $(b_n+\Im)$, $(b_{n-1}+\Im)$,….., $(b_1+\Im)$ and $(b_0+\Im)=(a_0+\Im)$.

Thus, the synthetic division method is applied for successive division as at each stage we get the quotient and the remainder.

Example 6. Express $P_N(x)=x^5+(5+\Im)x^3+(3+\Im)x$ as a polynomial of $(x-(1+\Im))$
Solution. By using synthetic division method successively, we have

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$1+\Im$	1	0	$5+\Im$	0	$3+\Im$	0	
		$1+\Im$	$1+3\Im$	$6+14\Im$	$6+34\Im$	$9+79\Im$	
	1	$1+\Im$	$6+4\Im$	$6+14\Im$	$9+35\Im$	$9+79\Im$	
		$1+\Im$	$2+6\Im$	$8+28\Im$	$14+98\Im$		
	1	$2+2\Im$	$8+10\Im$	$14+42\Im$	$23+133\Im$		
		$1+\Im$	$3+9\Im$	$11+49\Im$			
	1	$3+3\Im$	$11+19\Im$	$25+91\Im$			
		$1+\Im$	$4+12\Im$				
	1	$4+4\Im$	$15+31\Im$				
		$1+\Im$					
	1	$5+5\Im$					
	1						

$$P_N(x) = x^5 + (5+\Im)x^3 + (3+\Im)x$$
$$= \left(x-(1+\Im)\right)^5 + (5+5\Im)\left(x-(1+\Im)\right)^4 + (15+31\Im)\left(x-(1+\Im)\right)^3 + (25+91\Im)\left(x-(1+\Im)\right)^2$$
$$+ (23+133\Im)\left(x-(1+\Im)\right) + (9+79\Im)$$

Theorem1. In the neutrosophic polynomial $(a_0+\Im)x^n + (a_1+\Im)x^{n-1} + (a_2+\Im)x^{n-2} + ... + (a_n+\Im)$, *where* $(a_0+\Im) \neq 0$ the leading term $(a_0+\Im)x^n$ exceeds the sum of the remaining terms for $x \geq \frac{a_k+\Im}{a_0+\Im} + 1$, where $a_k+\Im$ is the greatest coefficient irrespective of sign.

Proof.

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$(a_0+\Im)x^n > (a_1+\Im)x^{n-1}+(a_2+\Im)x^{n-2}+........+(a_n+\Im)$

if $(a_0+\Im)x^n > (a_k+\Im)(x^{n-1}+x^{n-2}+........+(x+1)),(a_k+\Im)$ being the greatest coefficient

i.e., *if* $(a_0+\Im)x^n > (a_k+\Im)\dfrac{x^n-1}{x-1}$

if $x^n > \dfrac{(a_k+\Im)}{(a_0+\Im)}(x^n-1)$

This holds if $\dfrac{a_k+\Im}{(a_0+\Im)(x-1)} \le 1$

If $x \ge \dfrac{(a_k+\Im)}{(a_0+\Im)}+1$

Hence the theorem.

3.5 Multiple roots

Let $(\alpha_1+\Im),\ (\alpha_2+\Im),......,(\alpha_n+\Im)$ be the roots of the equation $P_N(x)=0$ in which first r(r<n) quantities of them be equal to $(\alpha_1+\Im)$, then we write $P_N(x)=(x_1-(\alpha_1+\Im))^r.\phi(x),\ \phi(\alpha_1+\Im)\ne 0$ and $(\alpha_1+\Im)$ is a root of the equation $P_N(x)=0$ of multiplicity r.

Theorem2. If $(\alpha_1+\Im)$ be a root of $P_N(x)=0$ of multiplicity r, then $(\alpha_1+\Im)$ is a root of $P_N^{/}(x)$ of multiplicity (r-1), where $P_N^{/}(x)$ is the first order derivative of $P_N(x)$.

Proof. It is straight forward.

3.6 Newtons method of approximation

Let $P_N(x)=0$ be a given Neutrosophic polynomial equation in x and it has a root nearly equal to $\alpha_1+\Im$ suppose it is $\alpha_1+\Im+h$, where h is very very small.

$$P_N(\alpha_1+\Im+h)=P_N(\alpha_1+\Im)+\text{h}\,P_N^{/}(\alpha_1+\Im)+\frac{h^2}{2!}P_N^{//}(\alpha_1+\Im)+..........$$

Since $\alpha_1+\Im+h$ is a root then $P_N(\alpha_1+\Im+h)=0$

$$P_N(\alpha_1+\Im)+\text{h}\,P_N^{/}(\alpha_1+\Im)+\frac{h^2}{2!}P_N^{//}(\alpha_1+\Im)+..........=0$$

Neglecting the square and higher powers of h , it becomes

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$$P_N(\alpha_1+\Im)+\text{h}\,P_N^{/}(\alpha_1+\Im)=0$$

or, $h=\dfrac{-P_N(\alpha_1+\Im)}{P_N^{/}(\alpha_1+\Im)}$

Hence the first approximation of the root will be $\beta=\alpha_1+\Im-\dfrac{P_N(\alpha_1+\Im)}{P_N^{/}(\alpha_1+\Im)}$

Then its closer approximation will be $\beta-\dfrac{P_N(\beta+\Im)}{P_N^{/}(\beta+\Im)}$

Proceeding in this manner we may obtain an approximation which is very close to the root upto any desired degree of accuracy. In this way approximation is usually rapid.

Example7. Find the real root of $(1-\Im)x^3+(3+\Im)x+(4+\Im)=0$ correct upto 3 significant places.

Solution. It is left for the reader.

3.7 Symmetric Neutrosophic Functions of Roots

By symmetric neutrosophic functions we mean those functions which remained unchanged in value when any two of its roots are interchanged. We can find the values of symmetric neutrosophic functions of roots interms of the coefficients.

Example 8. If $(\alpha+\Im),(\beta+\Im),(\gamma+\Im)$ be the roots of $x^3-(p+\Im)x^2+(r+\Im)=0$ then, find the equation whose roots are $\dfrac{(\beta+\Im)+(\gamma+\Im)}{(\alpha+\Im)}$, $\dfrac{(\gamma+\Im)+(\alpha+\Im)}{(\beta+\Im)}$, $\dfrac{(\alpha+\Im)+(\beta+\Im)}{(\gamma+\Im)}$

Solution.

Let $y=\dfrac{(\beta+\Im)+(\gamma+\Im)}{(\alpha+\Im)}=\dfrac{(\beta+\Im)+(\gamma+\Im)+(\alpha+\Im)-(\alpha+\Im)}{(\alpha+\Im)}=\dfrac{(p+\Im)}{(\alpha+\Im)}-1$

$$\Rightarrow y+1=\frac{(p+\Im)}{(\alpha+\Im)}\Rightarrow \alpha+\Im=\frac{p+\Im}{y+1}$$

Replacing x by $\dfrac{p+\Im}{y+1}$ in the equation then it becomes

$$(r+\Im)y^3+3(r+1)y^2+\left\{\left(3r-p^3\right)+\Im\left(4-3p^2+3p\right)\right\}y+(r+\Im)=0$$

Here we note that due to the symmetric nature of the roots we have obtained the required equation so easily.

3.8 Relation between roots and coefficients of a of a Neutrosophic polynomial equation

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Let $P_N\left(x\right)=\left(a_0+\Im\right)x^n+\left(a_1+\Im\right)x^{n-1}+.........+\left(a_{n-1}+\Im\right)x+\left(a_n+\Im\right)$ be a Neutrosophic polynomial of degree n. Let $P_N\left(x\right)=0$ have n roots $\left(\alpha_1+\Im\right)$, $\left(\alpha_2+\Im\right)$,........,$\left(\alpha_n+\Im\right)$.

$$\left(a_0+\Im\right)x^n+\left(a_1+\Im\right)x^{n-1}+.........+\left(a_{n-1}+\Im\right)x+\left(a_n+\Im\right)=\left(a_0+\Im\right)\begin{bmatrix}(x-(\alpha_1+\Im))(x-(\alpha_2+\Im)).......\\(x-(\alpha_n+\Im))\end{bmatrix}$$

$$\Rightarrow x^n+\frac{\left(a_1+\Im\right)}{\left(a_0+\Im\right)}x^{n-1}+.........+\frac{\left(a_{n-1}+\Im\right)}{\left(a_0+\Im\right)}x+\frac{\left(a_n+\Im\right)}{\left(a_0+\Im\right)}=\left[(x-(\alpha_1+\Im))(x-(\alpha_2+\Im)).......(x-(\alpha_n+\Im))\right]$$

$$\Rightarrow x^n+\frac{\left(a_1+\Im\right)}{\left(a_0+\Im\right)}x^{n-1}+.........+\frac{\left(a_{n-1}+\Im\right)}{\left(a_0+\Im\right)}x+\frac{\left(a_n+\Im\right)}{\left(a_0+\Im\right)}=x^n-\Sigma(\alpha_1+\Im)x^{n-1}+\Sigma(\alpha_1+\Im)(\alpha_2+\Im)x^{n-2}-\Sigma(\alpha_1+\Im)(\alpha_2+\Im)(\alpha_3+\Im)x^{n-3}+......$$
$$+\left(-1\right)^n(\alpha_1+\Im)(\alpha_2+\Im).....(\alpha_n+\Im)$$

Equating both sides we write the following

$$\Sigma(\alpha_1+\Im)=-\frac{\left(a_1+\Im\right)}{\left(a_0+\Im\right)}$$

$$\Sigma(\alpha_1+\Im)(\alpha_2+\Im)=\frac{\left(a_2+\Im\right)}{\left(a_0+\Im\right)}$$

....

$$(\alpha_1+\Im)(\alpha_2+\Im).....(\alpha_n+\Im)=\left(-1\right)^n\frac{\left(a_n+\Im\right)}{\left(a_0+\Im\right)}$$

4. Conclusions

In this article we have studied neutrosophic real polynomial whose coefficients are neutrosophic numbers . Then we study some of its properties and introduce synthetic method of division to find the quotient and remainder easily. We also give a solution method to find the approximate roots of a neutrosophic polynomial equation. Some concrete examples are given. There is some possibility to use this concept to find the real roots by using different approximation method. Under neutrosophic environment we will use this concept to solve real life problems based on mensuration, trigonometry, geometry etc. We also include some papers in reference section for practical purpose.

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