On Refined Neutrosophic Hypergroup

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Abstract

This paper presents the refinement of neutrosophic hypergroup and studies some of its properties. Several interesting results and examples are presented. The existence of a good homomorphism between a refined neutrosophic hypergroup $H(I_1, I_2)$ and a neutrosophic hypergroup $H(I)$ is established.
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1 Introduction and Preliminaries

Neutrosophy is a new branch of philosophy that studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic set and neutrosophic logic were introduced in 1995 by Smarandache as generalizations of fuzzy set [21] and respectively intuitionistic fuzzy logic [10]. In neutrosophic logic, each proposition has a degree of truth (T), a degree of indeterminancy (I) and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^{-}0, 1^{+}[$, for more detailed information, the reader should see [15,17]. In 2013, Florentin Smarandache in [16] introduced refined neutrosophic components of the form $< T_1, T_2, \cdots, T_p; I_1, I_2, \cdot, I_r; F_1, F_2, \cdots, F_s >$. The birth of refinement of the neutrosophic components $< T, I, F >$ has led to the extension of neutrosophic numbers $a + bI$ into refined neutrosophic numbers of the form $(a+b_1I_1+b_2I_2+\cdots+b_nI_n)$ where $a, b_1, b_2, \cdots, b_n$ are real or complex numbers. Using these refined neutrosophic numbers, the concept of refined neutrosophic set was introduced and this paved way for the development of refined neutrosophic algebraic structures. Agboola in [4] introduced the concept of refined neutrosophic structure and he studied refined neutrosophic groups in particular and presented their fundamental properties. Since then, several researchers in this field have studied this concept and a great deal of results have been published. In [1], Adeleke et al. presented results on refined neutrosophic rings, refined neutrosophic subrings and in [2], they presented results on refined neutrosophic ideals and refined neutrosophic homomorphisms. A comprehensive review of neutrosophy, neutrosophic triplet set and neutrosophic algebraic structures can be found in [12,18–20].

In [13], Marty, introduced the concept of hypergroups by considering the quotient of a group by its subgroup. And this was the birth of an interesting new branch of Mathematics known as “Algebraic hyperstructures” which is considered as a generalization of classical algebraic structures. In the classical algebraic structure, the composition of two elements is an element whereas in algebraic hyperstructure, the composition of two elements is a non-empty set. Since then, many different kinds of hyperstructures (hyperrings, hypermodules, hypervector spaces,$\cdots$) have been introduced and studied. Also, many theories of algebraic hyperstructures have been propounded as well as their applications to various areas of sciences and technology. For comprehensive details on hyperstructures, the reader should see [11,14]. The concept of neutrosophic hypergroup and their properties was introduced by Agboola and Davvaz in [7]. More connections between algebraic hyperstructures and neutrosophic set can be found in many recent publications, see [3,5,6,8,9].
The present paper is concerned with the development of connections between algebraic hyperstructures and neutrosphic algebraic structures and again concerned with studying the refinement of neutrosophic hypergroups in particular and present some of their basic properties.

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For the purposes of this paper, it will be assumed that I splits into two indeterminacies I_1 [contradiction (true (T) and false (F))] and I_2 [ignorance (true (T) or false (F))]. It then follows logically that:

$$\begin{aligned} I_1 I_1 &= I_1^2 &= I_1, \\ I_2 I_2 &= I_2^2 &= I_2, \text{ and} \\ I_1 I_2 &= I_2 I_1 &= I_1. \end{aligned}$$

Definition 1.1. [4] If $* : X(I_1, I_2) \times X(I_1, I_2) \mapsto X(I_1, I_2)$ is a binary operation defined on $X(I_1, I_2)$, then the couple $(X(I_1, I_2), *)$ is called a refined neutrosophic algebraic structure and it is named according to the laws (axioms) satisfied by $*$.

Definition 1.2. [4] Let $(X(I_1, I_2), +, \cdot)$ be any refined neutrosophic algebraic structure where $+$ and . are ordinary addition and multiplication respectively.
For any two elements $(a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$, we define

$$(a, bI_1, cI_2) + (d, eI_1, fI_2) = (a + d, (b + e)I_1, (c + f)I_2),$$
$$(a, bI_1, cI_2) \cdot (d, eI_1, fI_2) = (ad, (ae + bd + be + bf + ce)I_1, (af + cd + cf)I_2).$$

Definition 1.3. [4] If $''+''$ and $''.''$ are ordinary addition and multiplication, I_k with $k = 1, 2$ have the following properties:

1. $I_k + I_k + \cdots + I_k = nI_k$.
2. $I_k + (-I_k) = 0$.
3. $I_k \cdot I_k \cdot \cdots \cdot I_k = I_k^n = I_k$ for all positive integers $n > 1$.
4. $0 \cdot I_k = 0$.
5. I_k^{-1} is undefined and therefore does not exist.

Definition 1.4. [4] Let $(G, *)$ be any group. The couple $(G(I_1, I_2), *)$ is called a refined neutrosophic group generated by G, I_1 and I_2. $(G(I_1, I_2), *)$ is said to be commutative if for all $x, y \in G(I_1, I_2)$, we have $x * y = y * x$. Otherwise, we call $(G(I_1, I_2), *)$ a non -commutative refined neutrosophic group.

Definition 1.5. [4] If $(X(I_1, I_2), *)$ and $(Y(I_1, I_2), *')$ are two refined neutrosophic algebraic structures, the mapping

$$\phi : (X(I_1, I_2), *) \longrightarrow (Y(I_1, I_2), *')$$

is called a neutrosophic homomorphism if the following conditions hold:

1. $\phi((a, bI_1, cI_2) * (d, eI_1, fI_2)) = \phi((a, bI_1, cI_2)) *' \phi((d, eI_1, fI_2))$.
2. $\phi(I_k) = I_k$ for all $(a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$ and $k = 1, 2$.

Example 1.6. [4] Let
$\mathbb{Z}_2(I_1, I_2) = \{(0, 0, 0), (1, 0, 0), (0, I_1, 0), (0, 0, I_2), (0, I_1, I_2), (1, I_1, 0), (1, 0, I_2), (1, I_1, I_2)\}$.
Then $(\mathbb{Z}_2(I_1, I_2), +)$ is a commutative refined neutrosophic group of integers modulo 2.
Generally for a positive integer $n \geq 2$, $(\mathbb{Z}_n(I_1, I_2), +)$ is a finite commutative refined neutrosophic group of integers modulo n.

Example 1.7. [4] Let $(G(I_1, I_2), *)$ and and $(H(I_1, I_2), *')$ be two refined neutrosophic groups.
Let $\phi : G(I_1, I_2) \times H(I_1, I_2) \to G(I_1, I_2)$ be a mapping defined by $\phi(x, y) = x$ and let
$\psi : G(I_1, I_2) \times H(I_1, I_2) \to H(I_1, I_2)$ be a mapping defined by $\psi(x, y) = y$. Then ϕ and ψ are refined neutrosophic group homomorphisms.

Definition 1.8. [11] Let H be a non-empty set and $\circ : H \times H \longrightarrow P^*(H)$ be a hyperoperation. The couple $(H, \circ)$ is called a hypergroupoid. For any two non-empty subsets A and B of H and $x \in H$, we define

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \quad A \circ x = A \circ \{x\} \quad \text{and} \quad x \circ B = \{x\} \circ B.$$

Definition 1.9. [11] A hypergroupoid $(H, \circ)$ is called a semihypergroup if for all a, b, c of H we have $(a \circ b) \circ c = a \circ (b \circ c)$, which means that

$$\bigcup_{u \in a \circ b} u \circ c = \bigcup_{v \in b \circ c} a \circ v.$$

A hypergroupoid $(H, \circ)$ is called a quasihypergroup if for all a of H we have $a \circ H = H \circ a = H$. This condition is also called the reproduction axiom.

Definition 1.10. [11] A hypergroupoid $(H, \circ)$ which is both a semihypergroup and a quasi- hypergroup is called a hypergroup.

Definition 1.11. [11] Let $(H, \circ)$ and $(H', \circ')$ be two hypergroupoids. A map $\phi : H \longrightarrow H'$, is called

1. an inclusion homomorphism if for all x, y of H, we have $\phi(x \circ y) \subseteq \phi(x) \circ' \phi(y)$;
2. a good homomorphism if for all x, y of H, we have $\phi(x \circ y) = \phi(x) \circ' \phi(y)$.

Definition 1.12. [11] Let H be a non-empty set and let + be a hyperoperation on H. The couple $(H, +)$ is called a canonical hypergroup if the following conditions hold:

1. $x + y = y + x$, for all $x, y \in H$,
2. $x + (y + z) = (x + y) + z$, for all $x, y, z \in H$,
3. there exists a neutral element $0 \in H$ such that $x + 0 = \{x\} = 0 + x$, for all $x \in H$,
4. for every $x \in H$, there exists a unique element $-x \in H$ such that $0 \in x + (-x) \cap (-x) + x$,
5. $z \in x + y$ implies $y \in -x + z$ and $x \in z - y$, for all $x, y, z \in H$.

Definition 1.13. [7] Let $(H, \star)$ be any hypergroup and let $< H \cup I >= \{x = (a, bI) : a, b \in H\}$. The couple $N(H) = (< H \cup I >, \star)$ is called a neutrosophic hypergroup generated by H and I under the hyperoperation $\star$. The part a is called the non-neutrosophic part of x and the part b is called the neutrosophic part of x. If $x = (a, bI)$ and $y = (c, dI)$ are any two elements of $N(H)$, where $a, b, c, d \in H$, we define

$$x \star y = (a, bI) \star (c, dI) = \{(u, vI) | u \in a \star c, v \in a \star d \cup b \star c \cup b \star d\} = (a \star c, (a \star d \cup b \star c \cup b \star d)I).$$

Note that $a \star c \subseteq H$ and $(a \star d \cup b \star c \cup b \star d) \subseteq H$.

2 Formulation of Refined Neutrosophic Hypergroup

Definition 2.1. Let $(H, \star)$ be any hypergroup and let $< H \cup (I_1, I_2) >= \{x = (a, bI_1, cI_2) : a, b, c \in H\}$. The couple $(H(I_1, I_2), \star)$, is called a refined neutrosophic hypergroup generated by H, I_1 and I_2 under the hyperoperation $\star$. The part a is called the non-neutrosophic part of x and the part b and c are called the neutrosophic parts of x.

If $x = (a, bI_1, cI_2)$ and $y = (d, eI_1, fI_2)$ are any two elements of $H(I_1, I_2)$, where $a, b, c, d \in H$, we define

$$\begin{aligned} x \star y &= (a, bI_1, cI_2) \star (d, eI_1, fI_2) \\ &= \{(u, vI_1, wI_2) : u \in a \star d, v \in a \star e \cup b \star d \cup b \star e \cup b \star f \cup c \star e, w \in a \star f \cup c \star d \cup c \star f\} \\ &= (a \star d, (a \star e \cup b \star d \cup b \star e \cup b \star f \cup c \star e)I_1, (a \star f \cup c \star d \cup c \star f)I_2). \end{aligned}$$

Note that $a \star d \subseteq H$, $(a \star e \cup b \star d \cup b \star e \cup b \star f \cup c \star e) \subseteq H$ and $(a \star f \cup c \star d \cup c \star f) \subseteq H$.

Note 1. *If the operation on $H(I_1, I_2)$ is hyperaddition $(+')$ then for all $x = (a, bI_1, cI_2)$ and $y = (d, eI_1, fI_2)$ elements of $H(I_1, I_2)$, with $a, b, c, d \in H$, we define*

$$x +' y = (a, bI_1, cI_2) +' (d, eI_1, fI_2) = (a + d, (b + e)I_1, (c + f)I_2).$$

Here the addition on the right is the hyperaddition in H.

Proposition 2.2. *Let $(H, +)$ be a hypergroupoid, then, the refined neutrosophic hypergroup $(H(I_1, I_2), +')$ is a hypergroup with identity element $\theta = (0, 0I_1, 0I_2)$ iff $(H, +)$ is a hypergroup with identity element* 0.

Proof. Suppose $(H,+)$ is a hypergroup and $x=(a,bI_1,cI_2), y=(d,eI_1,fI_2), z=(g,hI_1,kI_2)\in H(I_1,I_2)$.
Then we show that $(H(I_1,I_2),+')$ is a hypergroup.
First, we shall show that $(H(I_1,I_2),+')$ is a semihypergroup.

$$\begin{aligned} x+'(y+'z) &= (a,bI_1,cI_2)+'((d,eI_1,fI_2)+'(g,hI_1,kI_2) \\ &= (a,bI_1,cI_2)+'((d+g,(e+h)I_1,(f+k)I_2)) \\ &= (a+(d+g),(b+(e+h))I_1,(c+(f+k))I_2) \\ &= ((a+d)+g,((b+e)+h)I_1,((c+f)+k)I_2) \\ &= (a+d,(b+e)I_1,(c+f)I_2)+'(g,hI_1,kI_2) \\ &= ((a,bI_1,cI_2)+'(d.eI_1,fI_2))+'(g,hI_1,kI_2) \\ &= (x+'y)+'z. \end{aligned}$$

Secondly, we shall show that $(H(I_1,I_2),+')$ is a quasihypergroup.
That is , we want to show that $x+'H(I_1,I_2)=H(I_1,I_2)+'x=H(I_1,I_2)$.

$$\begin{aligned} x+'H(I_1,I_2) &= (a,bI_1,cI_2)+'\{(d,eI_1,fI_2):(d,eI_1,fI_2)\in H(I_1,I_2)\} \\ &= (a+d,(b+e)I_1,(c+f)I_2) \\ &\subseteq H(I_1,I_2). \\ &\Longrightarrow x+'H(I_1,I_2)\subseteq H(I_1,I_2). \end{aligned}$$

Now we show that $H(I_1,I_2)\subseteq x+'H(I_1,I_2)$, let $z=(g,hI_1,kI_2)\in H(I_1,I_2)$ with $g,h,k\in H$.
There exist $a_1,a_2,a_3\in H$ such that $g\in a_1+H, h\in a_2+H$ and $k\in a_3+H$, since H is a hypergroup.
Hence $(g,hI_1,kI_2)\in(a_1,a_2I_1,a_3I_2)+H$, which implies that $H(I_1,I_2)\subseteq x+'H(I_1,I_2)$.
Accordingly, $H(I_1,I_2)=x+'H(I_1,I_2)$. Similarly, we can show that $H(I_1,I_2)=H(I_1,I_2)+'x$.
$\therefore$ We can conclude that $(H(I_1,I_2),+')$ is a hypergroup.
Conversely, suppose $(H(I_1,I_2),+')$ is a hypergroup and $x=(a,bI_1,cI_2), y=(d,eI_1,fI_2)$,
$z=(g,hI_1,kI_2)\in H(I_1,I_2)$, with $a,d,g,b=c=e=f=h=k=0\in H$.
Then we show that $(H,+)$ is a hypergroup.
Since $H(I_1,I_2)$ is a hypergroup, $x+'(y+'z)=(x+'y)+'z$.
But

$$\begin{aligned} x+'(y+'z) &= (a,0I_1,0I_2)+'((d,0I_1,0I_2)+'(g,0I_1,0I_2)) \\ &= (a,0I_1,0I_2)+'((d+g),(0+0)I_1,(0+0)I_2) \\ &= (a+(d+g),0I_1,0I_2) \\ &= a+(d+g)) \text{ and} \\ (x+'y)+'z &= ((a,0I_1,0I_2)+'(d,0I_1,0I_2))+'(g,0I_1,0I_2) \\ &= ((a+d,(0+0)I_1,(0+0)I_2)+'((d,0I_1,0I_2) \\ &= ((a+d)+g,0I_1,0I_2)=(a+d)+g. \end{aligned}$$

$$\therefore x+'(y+'z)=(x+'y)+'z\Longrightarrow a+(d+g)=(a+d)+g.$$

Thus $(H,+)$ is a semihypergroup.

Since $H(I_1,I_2)$ is a quasihypergroup, for $x=(a,bI_1,cI_2)\in H(I_1,I_2)$ with $a,b=c=0\in H$ we have that $x+'H(I_1,I_2)=H(I_1,I_2)+'x=H(I_1,I_2)$.
But

$$\begin{aligned} x+'H(I_1,I_2) &= (a,0I_1,0I_2)+'H(I_1,I_2) \\ &= (a,0I_1,0I_2)+'\{(h,0I_1,0I_2):h\in H\} \\ &= \{(a+h,0I_1,0I_2):h\in H\} \\ &= \{a+h:h\in H\}=a+H \text{ and} \\ H(I_1,I_2)+'x &= H(I_1,I_2)+'(a,0I_1,0I_2) \\ &= \{(h,0I_1,0I_2):h\in H\}+'(a,0I_1,0I_2) \\ &= \{(h+a,0I_1,0I_2):h\in H\} \\ &= \{h+a:h\in H\}=H+a. \end{aligned}$$

$\therefore x+'H(I_1,I_2)=H(I_1,I_2)+'x\Longrightarrow a+H=H+a$.
Since $a\in H$, $a+H=H$ and $H+a=H$ which implies that $a+H=H+a=H$.
Hence, we can conclude that $(H,+)$ is a hypergroup.

□

Proposition 2.3. *Every refined neutrosophic hypergroup is a semihypergroup.*

Proof. Let $(H(I_1,I_2),\star)$ be any refined neutrosophic hypergroup and let $x=(a,bI_1,cI_2)$,
$y=(d,eI_1,fI_2), z=(g,hI_1,kI_2)$ be arbitrary elements of $H(I_1,I_2)$, where $a,b,c,d,e,f,g,h,k\in H$.

Then,

$$\begin{aligned} x \star y &= (a, bI_1, cI_2) \star (d, eI_1, fI_2) \\ &= \{(u, vI_1, wI_2) : u \in a \star d, v \in a \star e \cup b \star d \cup b \star e \cup b \star f \cup c \star e, w \in a \star f \cup c \star d \cup c \star f\} \\ &= (a \star d, (a \star e \cup b \star d \cup b \star e \cup b \star f \cup c \star e)I_1, (a \star f \cup c \star d \cup c \star f)I_2). \\ &\subseteq H(I_1, I_2). \end{aligned}$$

Hence, $(H(I_1, I_2), \star)$ is a hypergroupoid.
Next

$$\begin{aligned} x \star (y \star z) &= (a, bI_1, cI_2) \star ((d, eI_1, fI_2) \star (g, hI_1, kI_2)) \\ &= (a, bI_1, cI_2) \star ((d \star g, (d \star h \cup e \star g \cup e \star h \cup e \star k \cup f \star h)I_1, \\ &\quad (d \star k \cup f \star g \cup f \star k)I_2) \\ &= a \star (d \star g), (a \star (d \star h) \cup a \star (e \star g) \cup a \star (e \star h) \cup a \star (e \star k) \cup a \star (f \star h) \\ &\quad \cup b \star (d \star g) \cup b \star (d \star h) \cup b \star (e \star g) \cup b \star (e \star h) \cup b \star (e \star k) \\ &\quad \cup b \star (f \star h) \cup b \star (d \star k) \cup b \star (f \star g) \cup b \star (f \star k) \cup c \star (d \star h) \cup c \star (e \star g) \\ &\quad \cup c \star (e \star h) \cup c \star (e \star k) \cup c \star (f \star h))I_1, \\ &\quad (a \star (d \star k) \cup a \star (f \star g) \cup a \star (f \star k) \cup c \star (d \star g) \cup c \star (d \star k) \cup c \star (f \star g) \\ &\quad \cup c \star (f \star k))I_2) \\ &= (a \star d) \star g, ((a \star d) \star h \cup (a \star e) \star g \cup (a \star e) \star h \cup (a \star e) \star k \cup (a \star f) \star h \\ &\quad \cup (b \star d) \star g \cup (b \star d) \star h \cup (b \star e) \star g \cup (b \star e) \star h \cup (b \star e) \star k \cup (b \star f) \star h \\ &\quad \cup (b \star d) \star k \cup (b \star f) \star g \cup (b \star f) \star k \cup (c \star d) \star h \cup (c \star e) \star g \cup (c \star e) \star h \\ &\quad \cup (c \star e) \star k \cup (c \star f) \star h)I_1, \\ &\quad ((a \star d) \star k \cup (a \star f) \star g \cup (a \star f) \star k \cup (c \star d) \star g \cup (c \star d) \star k \cup (c \star f) \star g \\ &\quad \cup (c \star f) \star k)I_2) \\ &= ((a, bI_1, cI_2) \star (d, eI_1, fI_2)) \star (g, hI_1, kI_2) \\ &= (x \star y) \star z. \end{aligned}$$

Accordingly, $(H(I_1, I_2), \star)$ is a semihypergroup. □

Proposition 2.4. *A refined neutrosophic hypergroup is not always a quasihypergroup.*

Proof. To see this, consider a refined neutrosophic hypergroup, say $(H(I_1, I_2), \star)$, where $(0, 0I_1, 0I_2) \notin H(I_1, I_2)$. Then for $x = (a, bI_1, cI_2) \in H(I_1, I_2)$ we have that

$$\begin{aligned} x \star H(I_1, I_2) &= (a, bI_1, cI_2) \star H(I_1, I_2) \\ &= (a, bI_1, cI_2) \star \{(h_1, h_2I_1, h_3I_2) : h_1, h_2, h_3 \in H\} \\ &= \{a \star h_1, (a \star h_2 \cup b \star h_1 \cup b \star h_2 \cup b \star h_3 \cup c \star h_2)I_1, (a \star h_3 \cup c \star h_1 \cup c \star h_3)I_2\} \\ &= \{(u, vI_1, wI_2) : u \in a \star h_1, v \in (a \star h_2 \cup b \star h_1 \cup b \star h_2 \cup b \star h_3 \cup c \star h_2), \\ &\quad w \in (a \star h_3 \cup c \star h_1 \cup c \star h_3)\} \\ &\subset H(I_1, I_2). \end{aligned}$$

$$\begin{aligned} H(I_1, I_2) \star x &= H(I_1, I_2) \star (a, bI_1, cI_2) \\ &= \{(h_1, h_2I_1, h_3I_2) : h_1, h_2, h_3 \in H\} \star (a, bI_1, cI_2) \\ &= \{h_1 \star a, (h_1 \star b \cup h_2 \star a \cup h_2 \star b \cup h_2 \star c \cup h_3 \star b)I_1, (h_1 \star c \cup h_3 \star a \cup h_3 \star c)I_2\} \\ &= \{(u, vI_1, wI_2) : u \in h_1 \star a, , v \in (h_1 \star b \cup h_2 \star a \cup h_2 \star b \cup h_2 \star c \cup h_3 \star b), \\ &\quad w \in (h_1 \star c \cup h_3 \star a \cup h_3 \star c)\} \\ &\subset H(I_1, I_2). \end{aligned}$$

We can see that $x \star H(I_1, I_2) = H(I_1, I_2) \star x \neq H(I_1, I_2)$. This implies that reproduction axioms fails to hold in this case. □

We note that the reproduction axioms fails to hold in some refined neutrosophic hypergroup, hence there exist some neutrosophic hypergroups that are not hypergroup. This observation is recorded in the next proposition.

Proposition 2.5. *Let $(H(I_1, I_2), \star)$ be a refined neutrosophic hypergroup, then*

1. *$(H(I_1, I_2), \star)$ in general is not a hypergroup ;*
2. *$(H(I_1, I_2), \star)$ always contain a hypergroup.*

Proof. 1. From Proposition 2.4 above, we can see that the reproduction axiom is not always satisfied. Then the proof follows.

2. It follows from the definition of a neutrosophic hypergroup.

□

Example 2.6. Let $H = \{a, b\}$ be a set with the hyperoperation defined as follows

$$a \star a = a, \quad a \star b = b \star a = b \text{ and } b \star b = \{a, b\}.$$

Let
$H(I_1, I_2) = \{a, b, \alpha_1 = (a, aI_1, aI_2), \alpha_2 = (a, aI_1, bI_2), \alpha_3 = (a, bI_1, aI_2), \alpha_4 = (a, bI_1, bI_2), \beta_1 = (b, bI_1, bI_2), \beta_2 = (b, bI_1, aI_2), \beta_3 = (b, aI_1, bI_2), \beta_4 = (b, aI_1, aI_2)\}$ be a refined neutrosophic set and let $\star'$ be a hyperoperation on $H(I_1, I_2)$ defined in the table below.

Take $\alpha = \{\alpha_1 = (a, aI_1, aI_2), \alpha_2 = (a, aI_1, bI_2), \alpha_3 = (a, bI_1, aI_2), \alpha_4 = (a, bI_1, bI_2)\}$ and
$\beta = \{\beta_1 = (b, bI_1, bI_2), \beta_2 = (b, bI_1, aI_2), \beta_3 = (b, aI_1, bI_2), \beta_4 = (b, aI_1, aI_2)\}$.

Table 1: Cayley table for the binary operation " $\star'$ "

$\star'$	a	b	α_1	α_2	α_3	α_4	β_1	β_2	β_3	β_4
a	a	b	α_1	α_2	α_3	α_4	β_1	β_2	β_3	β_4
b	b	$\{a, b\}$	β_1	$\{\beta_1, \beta_2\}$	$\{\beta_1, \beta_3\}$	β	$\{\alpha, \beta\}$	$\{\alpha_2, \alpha_4, \beta_1, \beta_3\}$	$\{\alpha_3, \alpha_4, \beta_1, \beta_2\}$	$\{\alpha_4, \beta_1\}$
α_1	α_1	β_1	α_1	α	$\{\alpha_1, \alpha_3\}$	α	β_1	β	$\{\beta_1, \beta_3\}$	β
α_2	α_2	$\{\beta_1, \beta_2\}$	α	α	α	α	β	β	β	β
α_3	α_3	$\{\beta_1, \beta_3\}$	$\{\alpha_1, \alpha_3\}$	α	$\{\alpha_1, \alpha_3\}$	α	$\{\beta_1, \beta_3\}$	β	$\{\beta_1, \beta_3\}$	β
α_4	α_4	β	α	α	α	α	β	β	β	β
β_1	β_1	$\{\alpha, \beta\}$	β_1	β	$\{\beta_1, \beta_3\}$	β	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$
β_2	β_2	$\{\alpha_2, \alpha_4, \beta_1, \beta_3\}$	β	β	β	β	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$
β_3	β_3	$\{\alpha_3, \alpha_4, \beta_1, \beta_2\}$	$\{\beta_1, \beta_3\}$	β	$\{\beta_1, \beta_3\}$	β	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$
β_4	β_4	$\{\alpha_4, \beta_1\}$	β	β	β	β	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$	$\{\alpha, \beta\}$

It is clear from the table that $(H(I_1, I_2), \star)$ is a refined neutrosophic hypergroup since it contains a proper subset $\{a, b\}$ which is a hypergroup under $\star$.

Example 2.7. Let $H = \{a, b, c\}$ and define " $\star$ " on H as follows

Table 2: Cayley table for the binary operation " $\star$ "

$\star$	a	b	c
a	a	b	c
b	b	$\{a, b\}$	$\{b, c\}$
c	c	$\{b, c\}$	$\{a, b, c\}$

Let $H(I_1, I_2) = \{a, b, c, \alpha_1 = (a, aI_1, aI_2), \alpha_2 = (a, aI_1, bI_2), \alpha_3 = (a, aI_1, cI_2), \alpha_4 = (a, bI_1, aI_2), \alpha_5 = (a, cI_1, aI_2), \alpha_6 = (a, bI_1, cI_2), \alpha_7 = (a, cI_1, bI_2), \alpha_8 = (a, bI_1, bI_2), \alpha_9 = (a, cI_1, cI_2), \beta_1 = (b, bI_1, bI_2), \beta_2 = (b, bI_1, aI_2), \beta_3 = (b, bI_1, cI_2), \beta_4 = (b, aI_1, bI_2), \beta_5 = (b, cI_1, bI_2), \beta_6 = (b, aI_1, cI_2), \beta_7 = (b, cI_1, aI_2), \beta_8 = (b, aI_1, aI_2), \beta_9 = (b, cI_1, cI_2), \tau_1 = (c, cI_1, cI_2), \tau_2 = (c, cI_1, aI_2), \tau_3 = (c, cI_1, bI_2), \tau_4 = (c, aI_1, cI_2), \tau_5 =$

$(c, bI_1, cI_2), \tau_6 = (c, aI_1, bI_2), \tau_7 = (c, bI_1, aI_2), \tau_8 = (c, aI_1, aI_2), \tau_9 = (c, bI_1, bI_2)\}$ be a refined neutrosophic set and let $\star'$ be a hyperoperation on $H(I_1, I_2)$, then using the definition of $\star$ in Table 2 , $(H(I_1, I_2), \star')$ is a refined neutrosophic hypergroup.

Example 2.8. Let $H(I_1, I_2) = \{e, a, b, c, (I_1, I_2), (aI_1, aI_2), (bI_1, bI_2), (cI_1, cI_2)\}$ be a refined neutrosophic semi group where $a^2 = b^2 = c^2 = e, ab = ba = c$ and $ac = ca = b$ and let $P(I_1, I_2) = \{e, a, (aI_1, aI_2)\}$ be a refined neutrosophic subset of $H(I_1, I_2)$.Then for all $x, y \in H(I_1, I_2)$ define

$$xoy = xP(I_1, I_2)y.$$

Then $(H(I_1, I_2)), \circ)$ is a refined neutrosophic hypergroup.

Example 2.9. Let $V(I_1, I_2)$ be a weak refined neutrosophic vector space over a field K. Then for all $x = (a, bI_1, cI_2), y = (d, eI_1, fI_2) \in V(I_1, I_2)$ define

$$\begin{aligned} x \circ y &= \{k \bullet (x + y) : k \in K\} \\ &= \{k \bullet (a + d), k \bullet (b + e)I_1, k \in \bullet (c + f)I_2 : k \in K\} \\ &= \{k \bullet a + k \bullet d, (k \bullet b + k \bullet e)I_1, (k \bullet c + k \bullet f)I_2 : k \in K\}. \end{aligned}$$

Then $(V(I_1, I_2), \circ)$ is a hypergroup.

To see this we proceed as follows :
Firstly we show that $(V(I_1, I_2), \circ)$ is a hypergroupoid.
So, for $x = (a, bI_1, cI_2), y = (d, eI_1, fI_2) \in (V(I_1, I_2), \circ)$ and $k \in K$we have that

$$x \circ y = \{k \bullet (x + y) : k \in K\} = \{k \bullet (a + d), k \bullet (b + e)I_1, k \bullet (c + f)I_2 : k \in K\}$$
$$= \{(u, vI_1, wI_2) : u \in k \bullet (a + d), v \in k \bullet (b + e), w \in k \bullet (c + f)\} \in V(I_1, I_2).$$

Next we show $(V(I_1, I_2), \circ)$ is a semi-hypergroup, i.e., $\circ$ is associative.
Let $x = (a, bI_1, cI_2), y = (d, eI_1, fI_2)$ and $z = (g, hI_1, jI_2) \in V(I_1, I_2)$ then we want to show that $x \circ (y \circ z) = (x \circ y) \circ z$.
Consider $x \circ (y \circ z) = (a, bI_1, cI_2) \circ ((d, eI_1, fI_2)o(g, hI_1, jI_2))$
$= (a, bI_1, cI_2) \circ \{(u, vI_1, wI_2) : u \in k \bullet (d + g), v \in k \bullet (e + h), w \in k \bullet (f + j)\}$
$= \{(p, qI_1, rI_2) : p \in k \bullet (a + u), q \in k \bullet (b + v), r \in k \bullet (c + w)\}$
$= \{(p, qI_1, rI_2) : p \in k \bullet (a + k \bullet (d + g)), q \in k \bullet (b + k \bullet (e + h)), r \in k \bullet (c + k \bullet (f + j))\}$
$= \{(p, qI_1, rI_2) : p \in k \bullet (a + (d + g)), q \in k \bullet (b + (e + h)), r \in k \bullet (c + (f + j))\}$
$= \{(p, qI_1, rI_2) : p \in k \bullet ((a + d) + g), q \in k \bullet ((b + e) + h), r \in k \bullet ((c + f) + j)\}$
$= \{(p, qI_1, rI_2) : p \in k \bullet (k \bullet (a + d) + g), q \in k \bullet (k \bullet (b + e) + h), r \in k \bullet (k \bullet (c + f) + j)\}$
$= \{(p', q'I_1, r'I_2) : p' \in k \bullet (a + d), q' \in k \bullet (b + e), r' \in k \bullet (c + f)\} \circ (g, hI_1, jI_2)$
$= ((a, bI_1, cI_2) \circ (d, eI_1, fI_2)) \circ (g, hI_1, jI_2)$
$= (x \circ y) \circ z$.
Next, we show that $\circ$ satisfies the reproduction axiom.
Let $x = (a, bI_1, cI_2) \in V(I_1, I_2)$ with $a, b, c \in V$ then
$(a, bI_1, cI_2) \circ V(I_1, I_2) = \{(a, bI_1, cI_2) \circ (v_1, v_2I_1, v_3I_2) : v_1, v_2, v_3 \in V\}$
$= \{(p, qI_1, rI_2) : p \in k \bullet (a + v_1), q \in k \bullet (b + v_2)I_1, k \bullet (c + v_3)I_3\}$
$= \{(p, qI_1, rI_2) : p \in k \bullet (v_1 + a), q \in k \bullet (v_2 + b)I_1, k \bullet (v_3 + c)I_3\}$
$= \{(v_1, v_2I_1, v_3I_2) \circ (a, bI_1, cI_2) : v_1, v_2, v_3 \in V\} = V(I_1, I_2) \circ (a, bI_1, cI_2) = V(I_1, I_2)$.
Therefore $V(I_1, I_2) \circ (a, bI_1, cI_2) = (a, bI_1, cI_2) \circ V(I_1, I_2) = V(I_1, I_2)$.

Example 2.10. Let $V(I_1, I_2)$ be a strong refined neutrosophic vector space over a refined neutrosophic field $K(I_1, I_2)$. For all $x = (a, bI_1, cI_2), y = (d, eI_1, fI_2) \in V(I_1, I_2)$ and $k = (p, qI_1, rI_2) \in K(I_1, I_2)$ define

$$\begin{aligned} x \circ y &= \{(p, qI_1, rI_2) \bullet (a + d, (b + e)I_1, (c + f)I_2)\} \\ &= \{(u, vI_1, wI_2) : \\ &\quad u \in p \bullet (a + d), \\ &\quad v \in p \bullet (b + e) \cup q \bullet (a + d) \cup q \bullet (b + e) \cup q \bullet (c + f) \cup r \bullet (b + e), \\ &\quad w \in p \bullet (c + f) \cup r \bullet (a + d) \cup r \bullet (c + f)\}. \end{aligned}$$

Then $V(I_1, I_2, o)$ is refined neutrosophic hypergroup.

Proposition 2.11. *Let $(H(I_1, I_2), \star_1)$ and $(K(I_1, I_2), \star_2)$ be any two refined neutrosophic hypergroups. Then, $(H(I_1, I_2) \times K(I_1, I_2), \star)$ is a refined neutrosophic hypergroup, where*

$$(x_1, x_2) \star (y_1, y_2) = \{(x, y) : x \in x_1 \star_1 y_1, y \in x_2 \star_2 y_2, \forall (x_1, x_2), (y_1, y_2) \in H(I_1, I_2) \times K(I_1, I_2)\}.$$

Proof. Let $(x_1,x_2),(y_1,y_2)\in H(I_1,I_2)\times K(I_1,I_2)$, where $x=(a,bI_1,cI_2)$ and $y=(d,eI_1,fI_2)$ then

$$\begin{array}{rcl}(x_1,x_2)\star(y_1,y_2) & = & ((a_1,b_1I_1,c_1I_2),(a_2,b_2I_1,c_2I_2))\star((d_1,e_1I_1,f_1I_2),(d_2,e_2I_1,f_1I_2))\\ & = & \{((k_1,m_1I_1,t_1I_2),(k_2,m_2I_1,t_2I_2)):\\ & & (k_1,m_1I_1,t_1I_2)\in(a_1,b_1I_1,c_1I_2)\star(d_1,e_1I_1,f_1I_2),\\ & & (k_2,m_2I_1,t_2I_2)\in(a_2,b_2I_1,c_2I_2)\star(d_2,e_2I_1,f_2I_2)\}\\ & = & \{k_1\in a_1\star d_1,m_1\in a_1\star e_1\cup b_1\star d_1\cup b_1\star e_1\cup b_1\star f_1\cup c_1\star e_1,\\ & & t_1\in a_1\star f_1\cup c_1\star d_1\cup c_1\star f_1,\\ & & k_2\in a_2\star d_2,m_2\in a_2\star e_2\cup b_2\star d_2\cup b_2\star e_2\cup b_2\star f_2\cup c_2\star e_2,\\ & & t_2\in a_2\star f_2\cup c_2\star d_2\cup c_2\star f_2\}\\ & = & \{((a_1\star d_1,(a_1\star e_1\cup b_1\star d_1\cup b_1\star e_1\cup b_1\star f_1\cup c_1\star e_1)I_1\\ & & (a_1\star f_1\cup c_1\star d_1\cup c_1\star f_1)I_2),\\ & & (a_2\star d_2,(a_2\star e_2\cup b_2\star d_2\cup b_2\star e_2\cup b_2\star f_2\cup c_2\star e_2)I_2,\\ & & (a_2\star f_2\cup c_2\star d_2\cup c_2\star f_2)I_2)\}\subseteq H(I_1,I_2)\times K(I_1,I_2).\end{array}$$

Then $(H(I_1,I_2))\times K(I_1,I_2),\star)$ is a refine neutrosophic hypergroupoid.
Let, $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in H(I_1,I_2)\times K(I_1,I_2)$, where $x=(a,bI_1,cI_2)$, $y=(d,eI_1,fI_2)$ and $z=(g,hI_1jI_2)$ then
$((x_1,x_2)\star(y_1,y_2))\star(z_1,z_2)=(((a_1,b_1I_1,c_1I_2),(a_2,b_2I_1,c_2I_2))\star((d_1,e_1I_1,f_1I_2),(d_2,e_2I_1,f_2I_2)))$
$\star((g_1,h_1I_1,j_1I_1),(g_2,h_2I_1,j_2I_2))$
$=\{((k_1,m_1I_1,t_1I_2),(k_2,m_2I_1,t_2I_2)):k_1\in a_1\star d_1,m_1\in a_1\star e_1\cup b_1\star d_1\cup b_1\star e_1\cup b_1\star f_1\cup c_1\star e_1,$
$t_1\in a_1\star f_1\cup c_1\star d_1\cup c_1\star f_1,\quad k_2\in a_2\star d_2,m_2\in a_2\star e_2\cup b_2\star d_2\cup b_2\star e_2\cup b_2\star f_2\cup c_2\star e_2,$
$t_2\in a_2\star f_2\cup c_2\star d_2\cup c_2\star f_2\}\star((g_1,h_1I_1,j_1I_1),(g_2,h_2I_1,j_2I_2))$
$=((k_1,m_1I_1,t_1I_2),(k_2,m_2I_1,t_2I_2))\star((g_1,h_1I_1,j_1I_1),(g_2,h_2I_1,j_2I_2))$
$=\{((p_1,q_1I_1,r_1I_2),(p_2,q_2I_1,r_2I_2)):p_1\in k_1\star g_1,q_1\in k_1\star h_1\cup m_1\star g_1\cup m_1\star h_1\cup m_1\star j_1\cup t_1\star h_1,$
$r_1\in k_1\star j_1\cup t_1\star g_1\cup t_1\star j_1,p_2\in k_2\star g_2,q_2\in k_2\star h_2\cup m_2\star g_2\cup m_2\star h_2\cup m_2\star j_2\cup t_2\star h_2,$
$r_2\in k_2\star j_2\cup t_2\star g_2\cup t_2\star j_2\}$
$=\{((p_1,q_1I_1,r_1I_2),(p_2,q_2I_1,r_2I_2)):p_1\in(a_1\star d_1)\star g_1,$
$q_1\in(a_1\star d_1)\star h_1\cup(a_1\star e_1)\star g_1\cup(b_1\star d_1)\star g_1\cup(b_1\star e_1)\star g_1\cup(b_1\star f_1)\star g_1\cup(c_1\star e_1)\star g_1\cup(a_1\star$
$e_1)\star h_1\cup(b_1\star d_1)\star h_1\cup(b_1\star e_1)\star h_1\cup(b_1\star f_1)\star h_1\cup(c_1\star e_1)\star h_1\cup(a_1\star e_1)\star j_1$
$\cup(b_1\star d_1)\star j_1\cup(b_1\star e_1)\star j_1\cup(b_1\star f_1)\star j_1\cup(c_1\star e_1)\star j_1\cup(a_1\star f_1)\star h_1\cup(c_1\star d_1)\star h_1\cup(c_1\star f_1)\star h_1,$
$r_1\in(a_1\star d_1)\star j_1\cup(a_1\star f_1)\star g_1\cup(c_1\star d_1)\star g_1\cup(c_1\star f_1)\star g_1\cup(a_1\star f_1)\star j_1\cup(c_1\star d_1)\star j_1\cup(c_1\star f_1)\star j_1$
$p_2\in(a_2\star d_2)\star g_2,$
$q_2\in(a_2\star d_2)\star h_2\cup(a_2\star e_2)\star g_2\cup(b_2\star d_2)\star g_2\cup(b_2\star e_2)\star g_2\cup(b_2\star f_2)\star g_2\cup(c_2\star e_2)\star g_2\cup(a_2\star$
$e_2)\star h_2\cup(b_2\star d_2)\star h_2\cup(b_2\star e_2)\star h_2\cup(b_2\star f_2)\star h_2\cup(c_2\star e_2)\star h_2\cup(a_2\star e_2)\star j_2\cup(b_2\star d_2)\star j_2\cup$
$(b_2\star e_2)\star j_2\cup(b_2\star f_2)\star j_2\cup(c_2\star e_2)\star j_2\cup,(a_2\star f_2)\star h_2\cup(c_2\star d_2)\star h_2\cup(c_2\star f_2)\star h_2,$
$r_2\in(a_2\star d_2)\star j_2\cup(a_2\star f_2)\star g_2\cup(c_2\star d_2)\star g_2\cup(c_2\star f_2)\star g_2\cup(a_2\star f_2)\star j_2\cup(c_2\star d_2)\star j_2\cup(c_2\star f_2)\star j_2\}$
$=\{((p_1,q_1I_1,r_1I_2),(p_2,q_2I_1,r_2I_2)):p_1\in a_1\star(d_1\star g_1),$
$q_1\in a_1\star(d_1\star h_1)\cup a_1\star(e_1\star g_1)\cup b_1\star d_1(\star g_1)\cup b_1\star(e_1\star g_1)\cup b_1\star(f_1\star g_1)\cup c_1\star(e_1\star g_1)\cup a_1\star(e_1\star$
$h_1)\cup b_1\star(d_1\star h_1)\cup b_1\star(e_1\star h_1)\cup b_1\star(f_1\star h_1)\cup c_1\star(e_1\star h_1)\cup a_1\star(e_1\star j_1)$
$\cup b_1\star(d_1\star j_1)\cup b_1\star(e_1\star j_1)\cup b_1\star(f_1\star j_1)\cup c_1\star(e_1\star j_1)\cup a_1\star(f_1\star h_1)\cup c_1\star(d_1\star h_1)\cup c_1\star(f_1\star h_1),$
$r_1\in a_1\star(d_1\star j_1)\cup a_1\star(f_1\star g_1)\cup c_1\star(d_1\star g_1)\cup c_1\star(f_1\star g_1)\cup a_1\star(f_1\star j_1)\cup c_1\star(d_1\star j_1)\cup c_1\star(f_1\star j_1)$
$p_2\in a_2\star(d_2\star g_2),$
$q_2\in a_2\star(d_2\star h_2)\cup a_2\star(e_2\star g_2)\cup b_2\star(d_2\star g_2)\cup b_2\star(e_2\star g_2)\cup b_2\star(f_2\star g_2)\cup c_2\star(e_2\star g_2)\cup a_2\star$
$(e_2\star h_2)\cup b_2\star(d_2\star h_2)\cup b_2\star(e_2\star h_2)\cup b_2\star(f_2\star h_2)\cup c_2\star(e_2\star h_2)\cup a_2\star(e_2\star j_2)\cup b_2\star(d_2\star j_2)\cup$
$b_2\star(e_2\star j_2)\cup b_2\star(f_2\star j_2)\cup c_2\star(e_2\star j_2)\cup,a_2\star(f_2\star h_2)\cup c_2\star(d_2\star h_2)\cup c_2\star(f_2\star h_2),$
$r_2\in a_2\star(d_2\star j_2)\cup a_2\star(f_2\star g_2)\cup c_2\star(d_2\star g_2)\cup c_2\star(f_2\star g_2)\cup a_2\star(f_2\star j_2)\cup c_2\star(d_2\star j_2)\cup c_2\star(f_2\star j_2)\}$
$=((a_1,b_1I_1,c_1I_2),(a_2,b_2I_1,c_2I_2))\star\{((u_1,v_1I_1,w_1I_2),(u_2,v_2I_1,w_2I_2)):$
$u_1\in d_1\star g_1,v_1\in d_1\star h_1\cup e_1\star g_1\cup e_1\star h_1\cup e_1\star j_1\cup f_1\star h_1,w_1\in d_1\star j_1\cup f_1\star g_1\cup f_1\star j_1,$
$u_2\in d_2\star g_2,v_2\in d_2\star h_2\cup e_2\star g_2\cup e_2\star h_2\cup e_2\star j_2\cup f_2\star h_2,w_2\in d_2\star j_2\cup f_2\star g_2\cup f_2\star j_2\}$
$=((a_1,b_1I_1,c_1I_2),(a_2,b_2I_1,c_2I_2))\star(((d_1,e_1I_1,f_1I_2),(d_2,e_2I_1f_2I_2))\star((g_1,h_1I_1,j_1I_2),(g_2,h_2I_1j_2I_2)))$
$=(x_1,x_2)\star((y_1,y_2)\star(z_1,z_2)).$
Hence, $(H(I_1,I_2)\times K(I_1,I_2),\star)$ is a refined neutrosophic semi-hypergroup.
Lastly, let $(x_1,x_2)\in H(I_1,I_2)\times K(I_1,I_2)$ then
$((a_1,b_1I_1,c_1I_2),(a_2,b_2I_1,c_2I_2))\star(H(I_1,I_2)\times K(I_1,I_2))$
$=\{((a_1,b_1I_1,c_1I_2),(a_2,b_2I_1,c_2I_2))\star((d_1,e_1I_1,e_2I_2),(d_2,e_2I_1,f_2I_2)):(d_1,e_1I_1,e_2I_2)\in H(I_1,I_2),$
$(d_2,e_2I_1,f_2I_2)\in K(I_1,I_2)\}$
$=\{(a_1\star d_1,(a_1\star e_1\cup b_1\star d_1\cup b_1\star e_1\cup b_1\star f_1\cup c_1\star e_1)I_1,(a_1\star f_1\cup c_1\star d_1\cup c_1\star f_1)I_2,$
$(a_2\star d_2,(a_2\star e_2\cup b_2\star d_2\cup b_2\star e_2\cup b_2\star f_2\cup c_2\star e_2)I_1(a_2\star f_2\cup c_2\star d_2\cup c_2\star f_2)I_2\}$

$= (a_1, b_1I_1, c_1I_2) \star H(I_1, I_2), (a_2, b_2I_1, c_2I_2) \star K(I_1, I_2)$
$= H(I_1, I_2)\star(a_1, b_1I_1, c_1I_2), K(I_1, I_2)\star(a_2, b_2I_1, c_2I_2)$ since $H(I_1, I_2)$ and $K(I_1, I_2)$ are hypergroups.
$= H(I_1, I_2) \times K(I_1, I_2)$.
Hence $(H(I_1, I_2)) \times K(I_1, I_2), \star)$ is a refined neutrosophic quasi hypergroup.
Then we can conclude that $(H(I_1, I_2)) \times K(I_1, I_2), \star)$ is a refined neutrososphic hypergroup. □

Proposition 2.12. *Let $(H(I_1, I_2), \star)$ be a refined neutrosophic hypergroup and let (K, o) be a hypergroup. Then, $(H(I_1, I_2) \times K, \star')$ is a refined neutrosophic hypergroup, where*

$$(h_1, k_1) \star' (h_2, k_2) = \{(h, k) : h \in h_1 \star' h_2, k \in k_1 \circ k_2, \forall (h_1, k_1), (h_2, k_2) \in N(H) \times K\}.$$

Proof : It follows from similar approach to the proof of Proposition 2.11 .

Proposition 2.13. *Let $(H(I_1, I_2), \star)$ be a refined neutrosophic hypergroup, then for all elements of $H(I_1, I_2)$ no two elements combine to give empty set.*

Proof. Let $(a, bI_1, cI_2), (x, yI_1, zI_2) \in H(I_1, I_2)$. Suppose $(a, bI_1, I_2) \star (x, yI_1, zI_2) = \emptyset$.
Then since $H(I_1, I_2)$ is a neutrosophic hypergroup, by reproduction axiom we have

$$\begin{aligned} H(I_1, I_2) &= (a, bI_1, cI_2) \star H(I_1, I_2) \\ &= (a, bI_1, cI_2) \star ((x, yI_1, I_2) \star H(I_1, I_2)) \\ &= ((a, bI_1, cI_2) \star (x, yI_1, I_2)) \star H(I_1, I_2) \\ &= \emptyset \star H(I_1, I_2) \\ &= \emptyset. \end{aligned}$$

This is absurd, hence there exist no two elements of $H(I_1, I_2)$ that combine to give empty set. □

Definition 2.14. Let $H(I_1, I_2)$ be a refined neutrosophic hypergroup and let $K[I_1, I_2]$ be a proper subset of $H(I_1, I_2)$. Then $K[I_1, I_2]$ is said to be a refined neutrosophic semi-subhypergroup of $H(I_1, I_2)$ if
$x \star y \subseteq K[I_1, I_2]$ for all $x, y \in K[I_1, I_2]$.

Definition 2.15. Let $H(I_1, I_2)$ be a refined neutrosophic hypergroup and let $K[I_1, I_2]$ be a proper subset of $H(I_1, I_2)$. Then

1. $K[I_1, I_2]$ is said to be a refined neutrosophic subhypergroup of $H(I_1, I_2)$ if $K[I_1, I_2]$ is a refined neutrosophic hypergroup, that is, $K[I_1, I_2]$ must contain a proper subset which is a hypergroup.

2. $K[I_1, I_2]$ is said to be a refined pseudo neutrosophic subhypergroup of $H(I_1, I_2)$ if $K[I_1, I_2]$ is a refined neutrosophic hypergroup which contains no proper subset which is a hypergroup.

Note 2. *A refined neutrosophic hypergroup is a much more complicated structure than the structure of a refined neutrosophic group. In a refined neutrosophic group, the intersection of any two refined neutrosophic subgroups is a refined neutrosophic subgroup, this is not usually so in the case of a refined neutrosophic hypergroups, since the reproductive axioms fails to hold in this case. This has led to the consideration of different kinds of refined neutrosophic subhypergroups, which are ; Closed, Ultraclosed and Conjugable.*

Proposition 2.16. *Let $M(I_1, I_2)$ and $N(I_1, I_2)$ be any refined neutrosophic subhypergroups of a refined neutrososphic hypergroup $H(I_1, I_2)$, then $M(I_1, I_2) \cap N(I_1, I_2)$ is a refined neutrosophic semi-subhypergroup.*

Proof. $M(I_1, I_2) \cap N(I_1, I_2) \neq \emptyset$, since $M(I_1, I_2)$ and $N(I_1, I_2)$ are non-empty subhypergroups of $H(I_1, I_2)$.
Now, let $x = (a_1, b_1I_1, c_1I_2), y = (a_2, b_2I_1, c_2I_2) \in M(I_1, I_2) \cap N(I_1, I_2)$.
Then $(a_1, b_1I_1, c_1I_2), (a_2, b_2I_1, c_2I_2) \in M(I_1, I_2)$ and $(a_1, b_1I_1, c_1I_2), (a_2, b_2I_1, c_2I_2) \in N(I_1, I_2)$. Since, $M(I_1, I_2)$ and $N(I_1, I_2)$ are refined neutrosophic subhypergroup, we have that
$(a_1, b_1I_1, c_1I_2) \star (a_2, b_2I_1, c_2I_2) \subseteq M(I_1, I_2)$ and
$(a_1, b_1I_1, c_1I_2)\star(a_2, b_2I_1, c_2I_2) \subseteq N(I_1, I_2) \Longrightarrow (a_1, b_1I_1, c_1I_2)\star(a_2, b_2I_1, c_2I_2) \subseteq M(I_1, I_2)\cap N(I_1, I_2)$.
Hence, $M(I_1, I_2) \cap N(I_1, I_2)$ is a refined neutrosophic semi-subhypergroup. □

Proposition 2.17. *Let $M(I_1, I_2)$ and $N(I_1, I_2)$ be any refined neutrosophic semi-subhypergroups of a refined neutrososphic commutative hypergroup $H(I_1, I_2)$, then the set*

$$M(I_1, I_2)N(I_1, I_2) = \{xy : x \in M(I_1, I_2), y \in N(I_1, I_2)\}$$

is a refined neutrosophic semi-subhypergroup of $H(I_1, I_2)$.

Definition 2.18. Let $K(I_1, I_2)$ be a refined neutrosophic subhypergroup of a refined neutrosophic hypergroup $(H(I_1, I_2), \star)$. Then,

1. $K(I_1, I_2)$ is said to be closed on the left (right) if for all $k_1, k_2 \in K(I_1, I_2), x \in H(I_1, I_2)$ we have $k_2 \in x \star k_1 (k_2 \in k_1 \star x)$ implies that $x \in K(I_1, I_2)$;
2. $K(I_1, I_2)$ is said to be ultraclosed on the left (right) if for all $x \in H(I_1, I_2)$ we have $x \star K(I_1, I_2) \cap x \star (H(I_1, I_2) \backslash K(I_1, I_2)) = \emptyset \;\; (K(I_1, I_2) \star x \cap (H(I_1, I_2) \backslash K(I_1, I_2)) \star x = \emptyset)$;
3. $K(I_1, I_2)$ is said to be left (right) conjugable if $K(I_1, I_2)$ is left (right) closed and if for all $x \in H(I_1, I_2)$, there exists $h \in H(I_1, I_2)$ such that $x \star h \subseteq K(I_1, I_2) \;\; (h \star x \subseteq K(I_1, I_2))$;
4. $K(I_1, I_2)$ is said to be (closed, ultraclosed, conjugable) if it is left and right (closed, ultraclosed, conjugable).

Proposition 2.19. *Let $K[I_1, I_2]$ be a refined neutrosophic subhypergroup of $H(I_1, I_2)$, $A[I_1, I_2] \subseteq K[I_1, I_2]$ and $B[I_1, I_2] \subseteq H(I_1, I_2)$, then*

1. $A[I_1, I_2](B[I_1, I_2] \cap K[I_1, I_2]) \subseteq A[I_1, I_2]B[I_1, I_2] \cap K[I_1, I_2]$ *and*
2. $(B[I_1, I_2] \cap K[I_1, I_2])A[I_1, I_2] \subseteq B[I_1, I_2]A[I_1, I_2] \cap K[I_1, I_2]$.

Proof. The proof is similar to the proof in classical case. □

Proposition 2.20.
1. *If $K[I_1, I_2]$ is a left closed refined neutrososphic subhypergroup in $H[I_1, I_2]$, $A[I_1, I_2] \subseteq K[I_1, I_2]$ and $B[I_1, I_2] \subseteq H[I_1, I_2]$, then $(B[I_1, I_2] \cap K[I_1, I_2])/A[I_1, I_2] = (B[I_1, I_2]/A[I_1, I_2]) \cap K[I_1, I_2]$.*
2. *If $K[I_1, I_2]$ is a right closed subhypergroup in $H[I_1, I_2]$, $A[I_1, I_2] \subseteq K[I_1, I_2]$ and $B[I_1, I_2] \subseteq H[I_1, I_2]$, then $(B[I_1, I_2] \cap K[I_1, I_2]) \backslash A[I_1, I_2] = B[I_1, I_2] \backslash A[I_1, I_2] \cap K[I_1, I_2]$.*

Proof. The proof is similar to the proof in classical case. □

Proposition 2.21. *Let $K[I_1, I_2], M[I_1, I_2]$ be two refined neutrosophic subhypergroups of a refined neutrosophic hypergroup $H[I_1, I_2]$ and suppose that $K[I_1, I_2]$ is left (or right) closed in $H[I_1, I_2]$. Then $K[I_1, I_2] \cap M[I_1, I_2]$ is left (or right) closed in $M[I_1, I_2]$.*

Proof. The proof is similar to the proof in classical case. □

Proposition 2.22. *Let $(H(I_1, I_2)), \star)$ be a refined neutrosophic hypergroup and let ρ be an equivalence relation on $H(I_1, I_2)$.*

1. *If ρ is regular, then $H(I_1, I_2)/\rho$ is a refined neutrosophic hypergroup.*
2. *If ρ is strongly regular, then $H(I_1, I_2)/\rho$ is a refined neutrosophic group.*

The proposition will be proved with the example provided below.

Example 2.23. If $(G(I_1, I_2), +)$ is a refined neutrosophic abelian hypergroup, ρ is an equivalence relation in $G(I_1, I_2)$, which has classes $\bar{x} = \{x, -x\}$, then for all $\bar{x}, \bar{y}$ of $G(I_1, I_2)/\rho$, we define

$$\bar{x} o \bar{y} = \{\overline{x + y}, \overline{x - y}\}.$$

Then $(G(I_1, I_2)/\rho, o)$ is a refined neutrosophic hypergroup.

Proof. Let $\bar{x}, \bar{y} \in G(I_1, I_2)/\rho$, where $\bar{x} = \overline{(a, bI_1, cI_2)}$, and $\bar{y} = \overline{(d, eI_1, fI_2)}$ then
$\bar{x} o \bar{y} = \overline{(a, bI_1, cI_2)} o \overline{(d, eI_1, fI_2)} = \{\overline{(a + d, (b + e)I_1, (c + f)I_2)}, \overline{(a - d, (b - e)I_1, (c - f)I_2)}\}$
$= \{\overline{x + y}, \overline{x - y}\} \in G(I_1, I_2)/\rho$.
Then $(G(I_1, I_2)/\rho, o)$ is a refined neutrosophic hypergroupoid.
Next we show that o satisfies the associative law. Let $\bar{x}, \bar{y}, \bar{z} \in G(I_1, I_2)/\rho$, where $\bar{x} = \overline{(a, bI_1, cI_2)}$, $\bar{y} = \overline{(d, eI_1, fI_2)}$ and $\bar{z} = \overline{(g, hI_1, jI_2)}$ then

$$\bar{x} o (\bar{y} o \bar{z}) = \overline{(a, bI_1, cI_2)} o \left(\overline{(d, eI_1, fI_2)} o \overline{(g, hI_1, jI_2)}\right)$$
$$= \overline{(a, bI_1, cI_2)} o \left\{\overline{(d + g, (e + h)I_1, (f + j)I_2)}, \quad \overline{(d - g, (e - h)I_1, (f - j)I_2)}\right\}$$
$$= \left\{\overline{(a, bI_1, cI_2)} o \overline{(d + g, (e + h)I_1, (f + j)I_2)}, \quad \overline{(a, bI_1, cI_2)} o \overline{(d - g, (e - h)I_1, (f - j)I_2)}\right\}$$

$= \Big\{\{\overline{(a+(d+g),(b+(e+h))I_1,(c+(f+j))I_2)}, \overline{(a-(d+g),(b-(e+h))I_1,(c-(f+j))I_2)}\},$
$\{\overline{(a+(d-g),(b+(e-h))I_1,(c+(f-j))I_2)}, \overline{(a-(d-g),(b-(e-h))I_1,(c-(f-j))I_2)}\}\Big\}$
$= \bigcup\Big\{\overline{(a+(d+g),(b+(e+h))I_1,(c+(f+j))I_2)}, \overline{(a-(d+g),(b-(e+h))I_1,(c-(f+j))I_2)},$
$\overline{(a+(d-g),(b+(e-h))I_1,(c+(f-j))I_2)}, \overline{(a-(d-g),(b-(e-h))I_1,(c-(f-j))I_2)}\Big\}$
$= \bigcup\Big\{\overline{(a+(d+g),(b+(e+h))I_1,(c+(f+j))I_2)}, \overline{(a+(d-g),(b+(e-h))I_1,(c+(f-j))I_2)},$
$\overline{(a-(d-g),(b-(e-h))I_1,(c-(f-j))I_2)}, \overline{(a-(d+g),(b-(e+h))I_1,(c-(f+j))I_2)}\Big\}$
$= \bigcup\Big\{\overline{((a+d)+g,((b+e)+h)I_1,((c+f)+j)I_2)}, \overline{((a+d)-g,((b+e)-h)I_1,((c+f)-j)I_2)}$
$\overline{((a-d)+g,((b-e)+h)I_1,((c-f)+j)I_2)}\ , \overline{((a-d)-g,((b-e)-h)I_1,((c-f)-j)I_2)}\Big\}$
$= \Big\{\overline{((a+d),(b+e)I_1,(c+f)I_2)}o\overline{(g,hI_1,jI_2)}, \overline{((a-d),(b-e)I_1,(c-f)I_2)}o\overline{(g,hI_1,jI_2)}\Big\}$
$= \Big\{\overline{((a+d),(b+e)I_1,(c+f)I_2)}, \overline{((a-d),(b-e)I_1,(c-f)I_2)}\Big\}\ o\overline{(g,hI_1,jI_2)}$
$= \Big(\overline{(a,bI_1,cI_2)}o\overline{(d,eI_1,fI_2)}\Big)\ o\overline{(g,hI_1,jI_2)}$
$= (\bar{x}o\bar{y})o\bar{z}.$
Now we show that o satisfies the reproduction axiom. Let $\bar{x} \in G(I_1,I_2)/\rho$ then
$\overline{(a,bI_1,cI_2)}oG(I_1,I_2)/\rho = \{\overline{(a,bI_1,cI_2)}o\{\overline{(d,eI_1,fI_2)} : (d,eI_1,fI_2) \in G(I_1,I_2)\}$
$= \{\overline{(a,bI_1,cI_2)}o\overline{(d,eI_1,fI_2)} : (a,bI_1,cI_2),(d,eI_1,fI_2) \in G(I_1,I_2)\}$
$= \{\overline{((a+d),(b+e)I_1,(c+f)I_2)}, \overline{((a-d),(b-e)I_1,(c-f)I_2)}\}$
$= \{((a+d),(b+e)I_1,(c+f)I_2), -((a+d),(b+e)I_1,(c+f)I_2),$
$((a-d),(b-e)I_1,(c-f)I_2), -((a-d),(b-e)I_1,(c-f)I_2)\}$
$= \{((d+a),(e+b)I_1,(f+e)I_2), -((d+a),(e+b)I_1,(f+e)I_2),$
$-((d-a),(e-b)I_1,(f-c)I_2), ((d-a),(e-b)I_1,(f-c)I_2)\}$
$= \{((d+a),(e+b)I_1,(f+e)I_2), -((d+a),(e+b)I_1,(f+e)I_2),$
$((d-a),(e-b)I_1,(f-c)I_2), -((d-a),(e-b)I_1,(f-c)I_2)\}$
$= \{\overline{((d+a),(e+b)I_1,(f+e)I_2)}, \overline{((d-a),(e-b)I_1,(f-c)I_2)}\}$
$= \{\overline{(d,eI_1,fI_2)}o\overline{(a,bI_1,cI_2)}\}$
$= \{\overline{(d,eI_1,fI_2)}o\overline{(a,bI_1,cI_2)} : (d,eI_1,fI_2) \in G(I_1,I_2)\}$
$= G(I_1,I_2)/\rho\ \ o\ \ \overline{(a,bI_1,cI_2)}$
$= G(I_1,I_2)/\rho.$
Hence we say that $(G(I_1,I_2)/\rho, o)$ is a refined neutrosophic hypergroup. □

Definition 2.24. Let $(H_1(I_1,I_2),\star_1)$ and $(H_2(I_1,I_2)),\star_2)$ be any two refined neutrosophic hypergroups and let $f : H_1(I_1,I_2) \longrightarrow H_2)(I_1,I_2)$ be a map. Then

1. f is called a refined neutrosophic homomorphism if:
   (a) for all x,y of $H_1(I_1,I_2)$, $f(x\star_1 y) \subseteq f(x)\star_2 f(y)$,
   (b) $f(I_k) = I_k$ for $\quad k = 1,2.$
2. f is called a good refined neutrosophic homomorphism if:
   (a) for all x,y of $H_1(I_1,I_2)$, $f(x\star_1 y) = f(x)\star_2 f(y)$,
   (b) $f(I_k) = I_k$ for $\quad k = 1,2.$
3. f is called a refined neutrosophic isomorphism if f is a refined neutrosophic homomorphism and f^{-1} is also a refined neutrosophic homomorphism.
4. f is called a 2-refined neutrosophic homomorphism if for all x,y of $H_1(I_1,I_2)$,
   $f^{-1}(f(x)\star_2 f(y)) = f^{-1}(f(x\star_1 y))$.
5. f is called an almost strong refined neutrosophic homomorphism if for all x,y of $H_1(I_1,I_2)$,
   $f^{-1}(f(x)\star_2 f(y)) = f^{-1}(f(x))\star_1 f^{-1}(f(y))$.

Proposition 2.25. *Let $(H(I_1,I_2)),\star)$ be a refined neutrosophic hypergroup and let ρ be a regular equivalence relation on $H(I_1,I_2)$. Then, the map $\phi : H(I_1,I_2) \longrightarrow H(I_1,I_2)/\rho$ defined by*
$\phi(x) = \bar{x}$ is not a refined neutrosophic homomorphism (good refined neutrosophic homomorphism).

Proof. It is clear since $I \in H(I_1, I_2)$ but $\phi(I_k) \neq I_k$. □

Note 3. *Suppose we wish to establish any relationship between the refined neutrosophic hypergroups and the parent neutrosophic hypergroups, or any other neutrosophic hypergroup. Then, our task will be to find a mapping ϕ say, such that*

$$\phi : H(I_1, I_2) \longrightarrow H(I).$$

For all $(x, yI_1, zI_2) \in H(I_1, I_2)$ define ϕ by

$$\phi((x, yI_1, zI_2)) = (x, (y + z)I). \tag{1}$$

In what follows we present some of the basic properties of such mapping.

Proposition 2.26. *Let $(H(I_1, I_2), +')$ be a refined neutrosophic hypergroup and let $(H(I), +)$ be a neutrosophic hypergroup. The mapping ϕ defined in 1 above is a good homomorphism.*

Proof. ϕ is well defined. Suppose $(x, yI_1, zI_2) = (x'y'I_1, z'I_2)$ then we that $x = x', y = y'$ and $z' = z'$. So,

$$\phi((x, yI_1, zI_2)) = (x, (y + z)I) = x' + (y' + z')I = \phi(x', y'I_1, z'I_2).$$

Now, suppose $(x, yI_1, zI_2), (x', y'I_1, 1z'I_2) \in H(I_1, I_2)$ then

$$\begin{aligned}
\phi((x, yI_1, zI_2) +' (x', y'I_1, z'I_2)) &= \phi((x + x'), (y + y')I_1, (z + z')I_2) \\
&= (x + x'), (y + y' + z + z')I \\
&= (x + x'), ((y + z) + (y' + z'))I \\
&= (x + x'), ((y + z)I + (y' + z')I) \\
&= (x, (y + z)I) + (x', (y' + z')I) \\
&= \phi(x, yI_1, zI_2) + \phi(x, yI_1, zI_2).
\end{aligned}$$

Hence ϕ is a good homomorphism. □

Definition 2.27. Let $(H(I_1, I_2), +')$ be a refined neutrosophic hypergroup with identity element $(0, 0I_1, 0I_2)$ and $(H(I_1, I_2), +)$ be a neutrosophic hypergroup with identity element $(0, 0I)$. Let $\phi : H(I_1, I_2) \longrightarrow H(I)$ be a good homomorphism, then

$$\begin{aligned}
\ker\phi &= \{(x, yI_1, zI_2) : \phi((x, yI_1, zI_2)) = (0, 0I)\} \\
&= \{(x, yI_1, zI_2) : (x, (y + z)I) = (0, 0I)\} \\
&= \{(0, yI_1, (-y)I_2)\}.
\end{aligned}$$

Proposition 2.28. *Let $\phi : H(I_1, I_2) \longrightarrow H(I)$ be a good homomorphism.*

1. $\ker\phi$ *is a semi-subhypergroup of* $H(I_1, I_2)$.
2. $Im\phi$ *is a subhypergroup of* $H(I)$.

Proof. 1. Let $(a, bI_1, cI_2), (x, yI_1, zI_2) \in ker\ \phi$, then

$$\begin{aligned}
\phi((a, bI_1, cI_2) +' (x, yI_1, zI_2)) &= \phi((a, bI_1, cI_2)) + \phi((x, yI_1, zI_2)) \\
&= (0, 0I) + (0, 0I) \\
&= (0, 0I) \\
&\Longrightarrow (a, bI_1, cI_1) +' (x, yI_1, zI_2) \subseteq ker\phi.
\end{aligned}$$

Hence, $ker\phi$ is a semi-subhypergroup.

2. Let $(a, bI_1, cI_2) \in H(I_1, I_2)$, then

$$\begin{aligned}
\phi((a, bI_1, cI_2)) + \phi(H(I_1, I_2)) &= \bigcup_{(x,yI_1,zI_2)\in H(I_1,I_2)} \phi((a, bI_1, cI_2) +' (x, yI_1, zI_2)) \\
&= \phi((a, bI_1, cI_2) +' H(I_1, I_2)) \\
&= \phi(H(I_1, I_2)).
\end{aligned}$$

Following similar approach we can show that $\phi(H(I_1, I_2)) + \phi((a, bI_1, cI_2)) = \phi(H(I_1, I_2))$.
Thus, $Im\phi$ is a subhypergroup of $H(I)$.

□

3 Conclusion

In this paper, we have studied the refinement of neutrosophic hyperstructures. In particular, we have studied refined neutrosophic hypergroups and presented several results and examples. Also, we have established the existence of a good homomorphism between a refined neutrosophic hypergroup $H(I_1, I_2)$ and a neutrosophic hypergroup $H(I)$. We hope to present and study more advance properties of refined neutrosophic Hypergroups in our future papers.

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