



Aggregation Operators of Bipolar Neutrosophic Soft Sets and It's Applications In Auto Car Selection

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Abstract: In this paper, it is intended to study the concept of bipolar Neutrosophic soft set (BNSS). It is aimed to defined bipolar Neutrosophic soft set. Definitions and operations have been presented the BNSS. Then we present an aggregation BNSS operator and decision making algorithm depend on the BNSS. Number-based examples discussed to show (ability to be done) and efficiency of the advanced method.

Keywords: Bipolar Soft Sets, Neutrosophic Sets, Aggregation Operators, Soft Sets, Car Selection

1. Introduction

The uncommon theory of fuzzy set Zadeh [1] was presented as the addition of a crisp set by expanding the truthfulness value set to $[0,1]$. In fuzzy set theory, if the association membership of an element is $\mu(x)$ then the non-association membership $1 - \mu(x)$ and so it fixed Intuitionistic fuzzy set presented by Atanassov in 1986 [2] and form an addition of fuzzy set by expanding the truth value to the lattice $[0,1] \times [0,1]$. In daily life, problems, engineering, medical diagnosis, in the economy, and social science in many areas have to do with facts that have in its uncertainties. This problem may not be positively demonstrated by the existing methodologies in Greek and Latin math. There are approximately well-known methodological theories for discuss with vagueness such as FSS [3], IFSS [2], rough set [4], etc. To try to deal with a problem which is pointed out in [5]. To grip with these complications, Molodtsov coined the concept of a SS as a new methodical implement for studying with hesitations.

BSS and basic operations defined by Shabir and Naz [6] in 2013. Lee [7], the idea of bipolar FS as a simplification of a FS. A bipolar fuzzy set (BFS) is an addition of fuzzy sets whose association degree range is $[1,-1]$. In a BFS, the association degree of a component means that the components are immaterial to the correspondent property, the association degree $[0, 1]$ of a component shows that the elements somewhat fulfils the property and the association degree $[-1, 0]$ of an element shows that the elements somewhat fulfil the implied counter property.

Neutrosophic sets suggested by Smarandache [8] (1998,1999,2002,2005,2006,2010) which is a extension of fuzzy set and the IFS is a great instrument to deal with incomplete information which happen in the real world Neutrosophic sets are categorized. Neutrosophic set proposed by Smarandache in by truthfulness (T), Indeterminacy (I), and falseness (F). This concept is very significant in many applications areas since indeterminacy is computed explicitly and truthfulness, Indeterminacy, and falseness are independent.

Wang et al [9] coined the idea of single-valued neutrosophic sets. The SVNS can express the truthness, degree, indeterminate and stable information. All the reason described by the SVNS is very suitable for human approach due to the deficiencies of knowledge that human accept.

Then Deli et al [10] advanced multi-criteria bipolar \mathcal{NS} and study their applications in DM. Ali et al [11] suggested the concept of cubic set with applications in specimen identification. Broumi et al [12,13], proposed BSVNS, graph theory and its shortest path problem.

Molodtsov in [14] has some possible applications of soft set theory. Furthermore, Maji in [4] presented some results as an application of neutrosophic SS in DM. Also, some writer planning the ideas of neutrosophic SS. Soft set to deal undeserving in a parameterized way. The soft set is a mathematical arrangement which has the capability of an independent state of parameterizations lack, conditions of fuzzy set, rough set, expectations, etc. Also, some writers studied the theory of neutrosophic soft set in [10, 15, 16, 4, and 17]. Jafar et al [20-24] worked on decision making using soft sets, fuzzy soft sets, intuitionistic soft sets and NS. Saqlain et al [28-30] worked a lot in Neutrosophic environment which help us a lot. Jafar et al [31] discussed a new technology in agriculture using Neutrosophic soft sets.

This article is committed to suggested bipolar neutrosophic sets which are a mixture of a SS and BNSS. Initially, then we launch the BNSS and discuss some fundamentals things with instructive examples. Moreover, we defined some algebraic functions of the BNSS such as the complement, union, intersection, etc. Then, we defined aggregation bipolar neutrosophic sets operators are bipolar neutrosophic soft set residents decision-making algorithms based on bipolar \mathcal{NSS} . Furthermore, we offer the DM method for DM difficulties including the bipolar neutrosophic SS and present a sample associated with this technique.

Motivation and Objectives

Motivation behind this paper is to present decision making issues in some different notions. Generally every one is discussing decision Making issues only with one direction we extened the work of Deli et al [10] and discussed the concept of NSS in bipolar environment. We discussed Agrigation operator and then use it to the selection of best car in market.

2. Prelimanaries

2.1 Definition-1: [8]

If \mathcal{U} is a universal set. Neutrosophic sets (\mathcal{NS}) \mathcal{K} in \mathcal{U} is categorized by truthfulness $\mathcal{T}_{\mathcal{K}}$, an indetermination $\mathcal{J}_{\mathcal{K}}$ and a falseness $\mathcal{F}_{\mathcal{K}}$. Standard or non- standard elements are $\mathcal{T}_{\mathcal{K}}, \mathcal{J}_{\mathcal{K}}, \mathcal{F}_{\mathcal{K}}$ of $]0^-, 1^+[$ is defined as:

$$\mathcal{K} = \{ \langle \wp, (\mathcal{T}_{\mathcal{K}}(\wp), \mathcal{J}_{\mathcal{K}}(\wp), \mathcal{F}_{\mathcal{K}}(\wp)) \rangle : \wp \in \mathcal{U}, \mathcal{T}_{\mathcal{K}}(\wp), \mathcal{J}_{\mathcal{K}}(\wp), \mathcal{F}_{\mathcal{K}}(\wp) \in]0^-, 1^+[\}.$$

So that there is no confinement to the sum of $\mathcal{T}_{\mathcal{K}}(\wp), \mathcal{J}_{\mathcal{K}}(\wp)$ and $\mathcal{F}_{\mathcal{K}}(\wp)$, so $0^- \leq \mathcal{T}_{\mathcal{K}}(\wp) + \mathcal{J}_{\mathcal{K}}(\wp) + \mathcal{F}_{\mathcal{K}}(\wp) \leq 3^+$.

2.2 Definition-2: [9]

If \mathcal{C} is a universal set. A single-valued neutrosophic sets (\mathcal{SVNS}) \mathcal{B} , that is applicable in engineering and scientific field, in \mathcal{C} is categorized by a truthfulness $\mathcal{T}_{\mathcal{B}}$, an indetermination $\mathcal{J}_{\mathcal{B}}$ and a falseness $\mathcal{F}_{\mathcal{B}}$. Standard elements are $\mathcal{T}_{\mathcal{B}}(\wp), \mathcal{J}_{\mathcal{B}}(\wp), \mathcal{F}_{\mathcal{B}}(\wp)$ of $[0, 1]$ is defined as:

$$\mathcal{B} = \{ \langle \wp, (\mathcal{T}_{\mathcal{B}}(\wp), \mathcal{J}_{\mathcal{B}}(\wp), \mathcal{F}_{\mathcal{B}}(\wp)) \rangle : \wp \in \mathcal{C}, \mathcal{T}_{\mathcal{B}}(\wp), \mathcal{J}_{\mathcal{B}}(\wp), \mathcal{F}_{\mathcal{B}}(\wp) \in [0, 1] \}.$$

2.3 Definition-3: [14]

If \mathcal{U} be a universal set, \mathbb{E} be a constraint that expresses the elements of \mathcal{U} , $\tilde{\mathbb{A}} \subseteq \mathbb{E}$. A function $\mathcal{F}_{\tilde{\mathbb{A}}}$ is known as soft set $\mathcal{F}_{\tilde{\mathbb{A}}}$ w.r.t the universal set \mathcal{U} and represented by:

$$\mathcal{F}_{\tilde{\mathbb{A}}} : \mathbb{E} \rightarrow \mathcal{P}(\mathcal{U}) \text{ s.t } \mathcal{F}_{\tilde{\mathbb{A}}}(\wp) = \emptyset \text{ if } \wp \in \mathbb{E} - \tilde{\mathbb{A}}$$

Everywhere $\mathcal{F}_{\tilde{\mathbb{A}}}$ is known as an approximation function $\mathcal{F}_{\tilde{\mathbb{A}}}$. Also, soft set is categorized by the family of the subset of the set \mathcal{U} , and consequently, it can be inscribed a set of well-ordered pairs.

$$\mathcal{F}_{\tilde{\mathbb{A}}} = \{(\wp, \mathcal{F}_{\tilde{\mathbb{A}}}(\wp)) : \wp \in \mathbb{E}, \mathcal{F}_{\tilde{\mathbb{A}}}(\wp) = \emptyset \text{ if } \wp \in \mathbb{E} - \tilde{\mathbb{A}}\}$$

2.4 Definition-4: [15]

If \mathcal{U} is a universal set, $\mathcal{N}(\mathcal{U})$ be a group of all Neutrosophic sets on \mathcal{U} , \mathbb{E} be constraint that express the components of \mathcal{U} . Then a Neutrosophic soft set \mathcal{N} above \mathcal{U} is clear by set-valued function represented by:

$$\mathcal{F}_{\mathcal{N}}: \mathbb{E} \rightarrow \mathcal{N}(\mathcal{U})$$

The set $\mathcal{F}_{\mathcal{N}}(\wp)$ is the set of \wp elements of (\mathcal{NSS}) that may be arbitrary few of them have empty and non-empty intersection.

$$\mathcal{N} = \{(\wp, \{< \mathcal{U}, \mathcal{J}_{\mathcal{F}_{\mathcal{N}}(\wp)}(l), \mathcal{J}_{\mathcal{F}_{\mathcal{N}}(\wp)}(l), \mathcal{F}_{\mathcal{F}_{\mathcal{N}}(\wp)}(l) > : \wp \in \mathcal{U}\} : \wp \in \mathbb{E})\}$$

Everywhere

$$\mathcal{J}_{\mathcal{F}_{\mathcal{N}}(\wp)}(l), \mathcal{J}_{\mathcal{F}_{\mathcal{N}}(\wp)}(l), \mathcal{F}_{\mathcal{F}_{\mathcal{N}}(\wp)}(l) \in [0,1]$$

2.5 Definition-5: [15]

If $\tilde{\mathcal{N}}_1$ and $\tilde{\mathcal{N}}_2$ be two (\mathcal{NSS}) over universal set \mathcal{U} , correspondingly.

- i. $\tilde{\mathcal{N}}_1$ is known as neutrosophic soft subset of $\tilde{\mathcal{N}}_2$ is $\tilde{\mathbb{A}} \subseteq \tilde{\mathbb{B}}$ and $\mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_1(\wp)}}(l) \leq \mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_2(\wp)}}(l)$, $\mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_1(\wp)}}(l) \leq \mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_2(\wp)}}(l)$, $\mathcal{F}_{\mathcal{F}_{\tilde{\mathcal{N}}_1(\wp)}}(l) \geq \mathcal{F}_{\mathcal{F}_{\tilde{\mathcal{N}}_2(\wp)}}(l)$, $\forall \wp \in \tilde{\mathbb{A}}, l \in \mathcal{U}$.
- ii. $\tilde{\mathcal{N}}_1$ And $\tilde{\mathcal{N}}_2$ are equal if $\tilde{\mathcal{N}}_1 \subseteq \tilde{\mathcal{N}}_2$ and $\tilde{\mathcal{N}}_2 \subseteq \tilde{\mathcal{N}}_1$.

2.6 Definition-6: [15]

If $\tilde{\mathcal{N}}_1$ and $\tilde{\mathcal{N}}_2$ be two (\mathcal{NSS}) . Now,

- 1) \mathcal{N}_1^c is said to be a complement of \mathcal{NSS} is defined by:

$$\mathcal{N}_1^c = \{(\wp, \{< u, \mathcal{F}_{\mathcal{F}_{\tilde{\mathcal{N}}_1(\wp)}}(l), 1 - \mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_1(\wp)}}(l), \mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_1(\wp)}}(l) > : \wp \in \mathcal{U}\} : \wp \in \mathbb{E})\}$$

- 2) The union of $\tilde{\mathcal{N}}_1$ and $\tilde{\mathcal{N}}_2$ can be defined as $\tilde{\mathcal{N}}_3 = \tilde{\mathcal{N}}_1 \cup \tilde{\mathcal{N}}_2$ and written as:

$$\tilde{\mathcal{N}}_3 = \{(\wp, \{< u, \mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_3(\wp)}}(l), \mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_3(\wp)}}(l), \mathcal{F}_{\mathcal{F}_{\tilde{\mathcal{N}}_3(\wp)}}(l) > : \wp \in \mathcal{U}\} : \wp \in \mathbb{E})\}$$

Everywhere

$$\mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_3(\wp)}}(l) = \max(\mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_1(\wp)}}(l), \mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_2(\wp)}}(l))$$

$$\mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_3(\wp)}}(l) = \min(\mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_1(\wp)}}(l), \mathcal{J}_{\mathcal{F}_{\tilde{\mathcal{N}}_2(\wp)}}(l))$$

$$\mathcal{F}_{\mathcal{F}_{\tilde{\mathcal{N}}_3(\wp)}}(l) = \min(\mathcal{F}_{\mathcal{F}_{\tilde{\mathcal{N}}_1(\wp)}}(l), \mathcal{F}_{\mathcal{F}_{\tilde{\mathcal{N}}_2(\wp)}}(l))$$

3) The intersection of $\tilde{\mathcal{N}}_1$ and $\tilde{\mathcal{N}}_2$ can be defined as $\tilde{\mathcal{N}}_4 = \tilde{\mathcal{N}}_1 \dot{\cap} \tilde{\mathcal{N}}_2$ and written as:

$$\tilde{\mathcal{N}}_4 = \left\{ \left(\wp, \left\{ \langle l, \mathcal{T}_{\# \tilde{\mathcal{N}}_4(\wp)}(l), \mathcal{J}_{\# \tilde{\mathcal{N}}_4(\wp)}(l), \mathcal{F}_{\# \tilde{\mathcal{N}}_4(\wp)}(l) \rangle : \wp \in \mathbb{U} \right\} : \wp \in \mathbb{E} \right) \right\}$$

Everywhere

$$\mathcal{T}_{\# \tilde{\mathcal{N}}_4(\wp)}(l) = \min \left(\mathcal{T}_{\# \tilde{\mathcal{N}}_1(\wp)}(l), \mathcal{T}_{\# \tilde{\mathcal{N}}_2(\wp)}(l) \right)$$

$$\mathcal{J}_{\# \tilde{\mathcal{N}}_4(\wp)}(l) = \max \left(\mathcal{J}_{\# \tilde{\mathcal{N}}_1(\wp)}(l), \mathcal{J}_{\# \tilde{\mathcal{N}}_2(\wp)}(l) \right)$$

$$\mathcal{F}_{\# \tilde{\mathcal{N}}_4(\wp)}(l) = \max \left(\mathcal{F}_{\# \tilde{\mathcal{N}}_1(\wp)}(l), \mathcal{F}_{\# \tilde{\mathcal{N}}_2(\wp)}(l) \right)$$

2.7 Definition-7: [7]

If \mathcal{U} is a universal set. Ω be Bipolar FS in \mathcal{U} . It can be describe as:

$$\Omega = \left\{ \left(l, \mathcal{T}^+(l), \mathcal{T}^-(l) \right) : l \in \mathcal{U} \right\}$$

Everywhere $\mathcal{T}^+ \rightarrow [0,1]$ and $\mathcal{T}^- \rightarrow [-1,0]$.The positive truthfulness $\mathcal{T}^+(l)$ correspondent to bipolar fuzzy set and negative truthfulness $\mathcal{T}^-(l)$ of a component $l \in \mathcal{U}$ to about implied counter-property correspondent to Ω .

2.8 Definition-8: [13]

If \mathcal{U} be a universal set. \mathbb{E} be a constraint that express the element of \mathcal{U} . A bipolar fuzzy soft set \odot in \mathcal{U} . It can be written as:

$$\odot = \left\{ \left(\check{e}, \left\{ \left(l, \mathcal{T}^+(l), \mathcal{T}^-(l) \right) : l \in \mathcal{U} \right\} \right) : \check{e} \in \mathbb{E} \right\}$$

Everywhere $\mathcal{T}^+ \rightarrow [0,1]$ and $\mathcal{T}^- \rightarrow [-1,0]$.The positive truthfulness $\mathcal{T}^+(l)$, correspondent to bipolar fuzzy set \odot and negative truthfulness $\mathcal{T}^-(l)$ of a component $l \in \mathcal{U}$ to about implied counter-property correspondent to \odot .

2.9 Definition-9: [10]

If \mathcal{U} is a universal set. A Bipolar neutrosophic set in \mathcal{U} . It is denoted by $\hat{\mathcal{A}}$.It can be written as:

$$\hat{\mathcal{A}} = \left\{ \left(l, \mathcal{T}^+(l), \mathcal{J}^+(l), \mathcal{F}^+(l), \mathcal{T}^-(l), \mathcal{J}^-(l), \mathcal{F}^-(l) \right) : l \in \mathcal{U} \right\}$$

Everywhere $\mathcal{T}^+, \mathcal{J}^+, \mathcal{F}^+ \rightarrow [0,1]$ and $\mathcal{T}^-, \mathcal{J}^-, \mathcal{F}^- \rightarrow [-1,0]$. The positive degrees truthfulness, indeterminacy and falseness are denoted by $\mathcal{T}^+(l), \mathcal{J}^+(l), \mathcal{F}^+(l)$ correspondent to bipolar $\mathcal{NS} \hat{\mathcal{A}}$ and negative degrees truthfulness, indeterminacy and falseness are denoted by $\mathcal{T}^-(l), \mathcal{J}^-(l), \mathcal{F}^-(l)$ of a component $l \in \mathcal{U}$ to about implied counter- property correspondent to bipolar neutrosophic set $\hat{\mathcal{A}}$.

3. Bipolar Neutrosophic Soft Sets

In this segment, we propose the concept of \mathcal{NSS} and its operations.

3.1 Definition-10

If \mathcal{U} is a universal set. \mathbb{E} be a constraint that expresses the element of \mathcal{U} . A bipolar neutrosophic soft set \mathfrak{B} in \mathcal{U} .It can be describe as:

$$\mathfrak{B} = \left\{ \left(\check{e}, \left\{ \left(l, \mathcal{T}^+(l), \mathcal{J}^+(l), \mathcal{F}^+(l), \mathcal{T}^-(l), \mathcal{J}^-(l), \mathcal{F}^-(l) \right) : l \in \mathcal{U} \right\} \right) : \check{e} \in \mathbb{E} \right\}$$

Everywhere $\mathcal{T}^+, \mathcal{J}^+, \mathcal{F}^+ \rightarrow [0, 1]$ and $\mathcal{T}^-, \mathcal{J}^-, \mathcal{F}^- \rightarrow [-1, 0]$. The positive degrees truthfulness, indeterminacy and falseness are denoted by $\mathcal{T}^+(l), \mathcal{J}^+(l), \mathcal{F}^+(l)$ corresponding to bipolar NSS \mathfrak{B} and negative degrees truthfulness, indeterminacy, and falseness are denoted by $\mathcal{T}^-(l), \mathcal{J}^-(l), \mathcal{F}^-(l)$ of a component $l \in \mathcal{U}$ to about implied counter- property correspondent to bipolar NSS \mathfrak{B} .

Example-1: If $\mathcal{U} = \{l_1, l_2, l_3\}, \mathbb{E} = \{\check{e}_1, \check{e}_2\}$. After that bipolar NSS \mathfrak{B}_1 and \mathfrak{B}_2 above \mathcal{U} is given as, respectively;

$$\mathfrak{B}_1 = \{(\check{e}_1, \{(l_1, .6, .9, .2, -.6, -.8, -.4), (l_2, .7, .9, .8, -.6, -.8, -.3), (l_3, .7, .8, .2, -.6, -.9, -.9)\}), (\check{e}_2, \{(l_1, .9, .6, .7, -.6, -.8, -.4), (l_2, .5, .4, .8, -.6, -.8, -.3), (l_3, .8, .6, .2, -.5, -.8, -.7)\})\}$$

And

$$\mathfrak{B}_2 = \{(\check{e}_1, \{(l_1, .3, .8, .6, -.7, -.8, -.3), (l_2, .4, .6, .8, -.2, -.8, -.3), (l_3, .7, .3, .6, -.6, -.5, -.1)\}), (\check{e}_2, \{(l_1, .2, .6, .7, -.1, -.8, -.3), (l_2, .3, .9, .8, -.6, -.4, -.6), (l_3, .8, .5, .2, -.5, -.8, -.1)\})\}$$

3.2 Definition-11

An empty bipolar Neutrosophic soft set \mathfrak{B}^\emptyset with respect to universal set \mathcal{U} . it can be written as:

$$\mathfrak{B}^\emptyset = \{(\check{e}, \{(l, 0, 0, 1, -1, 0, 0)\}): l \in \mathcal{U}\}: \check{e} \in \mathbb{E}$$

3.3 Definition-12

An absolute bipolar Neutrosophic soft set \mathfrak{B}^u with respect to universal set \mathcal{U} . it can be written as:

$$\mathfrak{B}^u = \{(\check{e}, \{(l, 1, 1, 0, 0, -1, -1)\}): l \in \mathcal{U}\}: \check{e} \in \mathbb{E}$$

Example-2: If $\mathcal{U} = \{l_1, l_2, l_3\}, \mathbb{E} = \{\check{e}_1, \check{e}_2\}$

1. Empty bipolar NSS \mathfrak{B}^\emptyset in \mathcal{U} is given as:

$$\mathfrak{B}^\emptyset = \{(\check{e}_1, \{(l_1, 0, 0, 1, -1, 0, 0), (l_2, 0, 0, 1, -1, 0, 0), (l_3, 0, 0, 1, -1, 0, 0)\}), (\check{e}_2, \{(l_1, 0, 0, 1, -1, 0, 0), (l_2, 0, 0, 1, -1, 0, 0), (l_3, 0, 0, 1, -1, 0, 0)\})\}$$

2. Absolute bipolar NSS \mathfrak{B}^u in \mathcal{U} is given as:

$$\mathfrak{B}^u = \{(\check{e}_1, \{(l_1, 1, 1, 0, 0, -1, -1), (l_2, 1, 1, 0, 0, -1, -1), (l_3, 1, 1, 0, 0, -1, -1)\}), (\check{e}_2, \{(l_1, 1, 1, 0, 0, -1, -1), (l_2, 1, 1, 0, 0, -1, -1), (l_3, 1, 1, 0, 0, -1, -1)\})\}$$

3.4 Definition-13

If $\mathfrak{B}_j = \{(\check{e}, \{(l, \mathcal{T}_j^+(l), \mathcal{J}_j^+(l), \mathcal{F}_j^+(l), \mathcal{T}_j^-(l), \mathcal{J}_j^-(l), \mathcal{F}_j^-(l)\}): l \in \mathcal{U}\}): \check{e} \in \mathbb{E}\}$ for $j = 1, 2, \dots, n$ be two bipolar NSS with respect to universal set. Then, \mathfrak{B}_1 is a bipolar neutrosophic soft subset \mathfrak{B}_2 and It is represented by $\mathfrak{B}_1 \subseteq \mathfrak{B}_2$, if $\mathcal{T}_1^+(l) \leq \mathcal{T}_2^+(l), \mathcal{J}_1^+(l) \geq \mathcal{J}_2^+(l), \mathcal{F}_1^+(l) \geq \mathcal{F}_2^+(l), \mathcal{T}_1^-(l) \geq \mathcal{T}_2^-(l), \mathcal{J}_1^-(l) \leq \mathcal{J}_2^-(l), \mathcal{F}_1^-(l) \leq \mathcal{F}_2^-(l)$ for all $(\check{e}, l) \in \mathbb{E} \times \mathcal{U}$.

Example-3: If $\mathcal{U} = \{l_1, l_2\}, \mathbb{E} = \{\check{e}_1, \check{e}_2\}$. If

$$\mathfrak{B}_1 = \{(\check{e}_1, \{(l_1, .8, .7, .2, -.4, -.8, -.3), (l_2, .7, .6, .7, -.4, -.8, -.3)\}), (\check{e}_2, \{(l_1, .6, .7, .5, -.4, -.6, -.3), (l_2, .3, .8, .4, -.4, -.7, -.2)\})\}$$

$$\text{And } \mathfrak{B}_2 = \{(\check{e}_1, \{(l_1, .9, .3, .2, -.6, -.7, -.3), (l_2, .9, .2, .4, -.8, -.6, -.1)\}), \\ (\check{e}_2, \{(l_1, .8, .7, .5, -.4, -.6, -.3), (l_2, .5, .8, .4, -.9, -.7, -.1)\})\}$$

Then, we have $\mathfrak{B}_1 \subseteq \mathfrak{B}_2$.

3.5 Definition-14

If $\mathfrak{B}_j = \{(\check{e}, \{(l, \mathcal{T}_j^+(l), \mathcal{J}_j^+(l), \mathcal{F}_j^+(l), \mathcal{T}_j^-(l), \mathcal{J}_j^-(l), \mathcal{F}_j^-(l)) : l \in \mathcal{U}\}) : \check{e} \in \mathbb{E}\}$ for $j = 1, 2, \dots, n$ stand two bipolar NSS with respect to universal set. Then, \mathfrak{B}_1 is bipolar Neutrosophic soft equal to \mathfrak{B}_2 , is denoted by $\mathfrak{B}_1 = \mathfrak{B}_2$, if $\mathcal{T}_1^+(l) = \mathcal{T}_2^+(l), \mathcal{J}_1^+(l) = \mathcal{J}_2^+(l), \mathcal{F}_1^+(l) = \mathcal{F}_2^+(l), \mathcal{T}_1^-(l) = \mathcal{T}_2^-(l), \mathcal{J}_1^-(l) = \mathcal{J}_2^-(l), \mathcal{F}_1^-(l) = \mathcal{F}_2^-(l)$ for all $(\check{e}, l) \in \mathbb{E} \times \mathcal{U}$.

3.6 Definition-15

If \mathfrak{B} is a bipolar NSS with respect to universal set \mathcal{U} . then, \mathfrak{B}^c be a complement of a bipolar NSS. It can be written as;

$$\mathfrak{B}^c = \{(\check{e}, \{(l, \mathcal{F}^+(l), 1 - \mathcal{T}^+(l), \mathcal{T}^+(l), \mathcal{F}^-(l), -1 - \mathcal{T}^-(l), \mathcal{T}^-(l)) : l \in \mathcal{U}\}) : \check{e} \in \mathbb{E}\}$$

Example-4: consider the example 1

$$\mathfrak{B}^c = \{(\check{e}_1, \{(l_1, .2, .1, .6, -.4, -.2, -.6), (l_2, .8, .1, .7, -.3, -.2, -.6), (l_3, .2, .2, .7, -.9, -.1, -.6)\}), \\ (\check{e}_2, \{(l_1, .7, .4, .9, -.4, -.2, -.6), (l_2, .8, .6, .5, -.3, -.2, -.6), (l_3, .2, .4, .8, -.7, -.2, -.5)\})\}$$

3.7 Definition-16

If $\mathfrak{B}_j = \{(\check{e}, \{(l, \mathcal{T}_j^+(l), \mathcal{J}_j^+(l), \mathcal{F}_j^+(l), \mathcal{T}_j^-(l), \mathcal{J}_j^-(l), \mathcal{F}_j^-(l)) : l \in \mathcal{U}\}) : \check{e} \in \mathbb{E}\}$ for $j = 1, 2, \dots, n$ be two bipolar NSS above \mathcal{U} . After that \mathfrak{B}_1 and \mathfrak{B}_2 are union also denoted by $\mathfrak{B}_1 \cup \mathfrak{B}_2$. It can be written as;

$$\mathfrak{B}_1 \cup \mathfrak{B}_2 = \{(\check{e}, \{(l, \max_j\{\mathcal{T}_j^+(l)\}, \min_j\{\mathcal{J}_j^+(l)\}, \min_j\{\mathcal{F}_j^+(l)\}, \min_j\{\mathcal{T}_j^-(l)\}, \max_j\{\mathcal{J}_j^-(l)\}, \max_j\{\mathcal{F}_j^-(l)\}) : l \in \mathcal{U}\}) : \check{e} \in \mathbb{E}, \text{ and } j = 1, 2, \dots, n\}$$

Example-5: consider the example 1

$$\mathfrak{B}_1 \cup \mathfrak{B}_2 = \{(\check{e}_1, \{(l_1, .6, .8, .2, -.7, -.8, -.3), (l_2, .7, .6, .8, -.6, -.8, -.3), (l_3, .7, .3, .2, -.6, -.5, -.1)\}), \\ (\check{e}_2, \{(l_1, .9, .6, .7, -.6, -.8, -.3), (l_2, .5, .4, .8, -.6, -.4, -.3), (l_3, .8, .5, .2, -.5, -.8, -.1)\})\}$$

3.8 Definition-17

If $\mathfrak{B}_j = \{(\check{e}, \{(l, \mathcal{T}_j^+(l), \mathcal{J}_j^+(l), \mathcal{F}_j^+(l), \mathcal{T}_j^-(l), \mathcal{J}_j^-(l), \mathcal{F}_j^-(l)) : l \in \mathcal{U}\}) : \check{e} \in \mathbb{E}\}$ for $j = 1, 2, \dots, n$ stand n Bipolar NSS above \mathcal{U} . After that \mathfrak{B}_j are the union of n bipolar NSS is denoted by $\cup_{j=1}^n \mathfrak{B}_j$, can be written as;

$$\cup_{j=1}^n \mathfrak{B}_j = \{(\check{e}, \{(l, \max_j\{\mathcal{T}_j^+(l)\}, \min_j\{\mathcal{J}_j^+(l)\}, \min_j\{\mathcal{F}_j^+(l)\}, \min_j\{\mathcal{T}_j^-(l)\}, \max_j\{\mathcal{J}_j^-(l)\}, \max_j\{\mathcal{F}_j^-(l)\}) : l \in \mathcal{U}\}) : \check{e} \in \mathbb{E}, \text{ and } j = 1, 2, \dots, n\}$$

3.9 Definition-18

If $\mathfrak{B}_j = \{(\check{e}, \{(l, \mathcal{T}_j^+(l), \mathcal{J}_j^+(l), \mathcal{F}_j^+(l), \mathcal{T}_j^-(l), \mathcal{J}_j^-(l), \mathcal{F}_j^-(l)) : l \in \mathcal{U}\}) : \check{e} \in \mathbb{E}\}$ for $j = 1, 2, \dots, n$ be two bipolar NSS above \mathcal{U} . After that \mathfrak{B}_1 and \mathfrak{B}_2 are union also denoted by $\mathfrak{B}_1 \cap \mathfrak{B}_2$. It can be written as;

$$\mathfrak{B}_1 \hat{\cap} \mathfrak{B}_2 = \{(\tilde{e}, \{(\underline{l}, \min_j \{T_j^+(\underline{l})\}, \max_j \{T_j^+(\underline{l})\}, \max_j \{F_j^+(\underline{l})\}, \max_j \{T_j^-(\underline{l})\}, \min_j \{T_j^-(\underline{l})\}, \min_j \{F_j^-(\underline{l})\}) : \underline{l} \in \mathcal{U}\}) : \tilde{e} \in \mathbb{E}, \text{ and } j = 1, 2, \dots, n\}$$

Example-6: consider the example 1

$$\mathfrak{B}_1 \hat{\cap} \mathfrak{B}_2 = \{(\tilde{e}_1, \{(l_1, .3, .9, .6, -.6, -.8, -.4), (l_2, .4, .9, .8, -.2, -.8, -.3), (l_3, .7, .8, .6, -.6, -.9, -.9)\}), (\tilde{e}_2, \{(l_1, .2, .6, .7, -.1, -.8, -.4), (l_2, .3, .9, .8, -.6, -.4, -.6), (l_3, .8, .6, .2, -.5, -.8, -.7)\})\}$$

3.10 Definition-19

If $\mathfrak{B}_j = \{(\tilde{e}, \{(\underline{l}, T_j^+(\underline{l}), J_j^+(\underline{l}), F_j^+(\underline{l}), T_j^-(\underline{l}), J_j^-(\underline{l}), F_j^-(\underline{l})) : \underline{l} \in \mathcal{U}\}) : \tilde{e} \in \mathbb{E}\}$ for $j = 1, 2, \dots, n$ be n bipolar NSS above \mathcal{U} . After that \mathfrak{B}_j are the intersection of n bipolar NSS is denoted by $\bigcap_{j=1}^n \mathfrak{B}_j$, can be written as;

$$\bigcap_{j=1}^n \mathfrak{B}_j = \left\{ \tilde{e}, \left\{ (\underline{l}, \min_j \{T_j^+(\underline{l})\}, \max_j \{T_j^+(\underline{l})\}, \max_j \{F_j^+(\underline{l})\}, \max_j \{T_j^-(\underline{l})\}, \min_j \{T_j^-(\underline{l})\}, \min_j \{F_j^-(\underline{l})\}) : \underline{l} \in \mathcal{U} \right\} : \tilde{e} \in \mathbb{E}, \text{ and } j = 1, 2, \dots, n \right\}$$

4. Methodology

AGGREGATION BIPOLAR NEUTROSOPHIC SOFT OPERATOR

The following portion presents an aggregation bipolar soft operator for implementing in bipolar NSS further demonstration of an algorithm developed based on bipolar NSS is given by arithmetical example to apply the appropriate usage and applicability of proposition.

Definition

If $\mathfrak{B} = \{(\tilde{e}, \{(\underline{l}, T^+(\underline{l}), J^+(\underline{l}), F^+(\underline{l}), T^-(\underline{l}), J^-(\underline{l}), F^-(\underline{l})) : \underline{l} \in \mathcal{U}\}) : \tilde{e} \in \mathbb{E}\} = \{(\tilde{e}, \{(\underline{l}, T_{\tilde{e}}^+(\underline{l}), J_{\tilde{e}}^+(\underline{l}), F_{\tilde{e}}^+(\underline{l}), T_{\tilde{e}}^-(\underline{l}), J_{\tilde{e}}^-(\underline{l}), F_{\tilde{e}}^-(\underline{l})) : \underline{l} \in \mathcal{U}\}) : \tilde{e} \in \mathbb{E}\}$ be a bipolar NSS above \mathcal{U} . After that, the aggregation bipolar NSS operator is denoted by \mathfrak{B}_{agg} , can be written as;

$$\mathfrak{B}_{agg} = \{\eta_{\mathfrak{B}}(\underline{l}) / \underline{l} : \underline{l} \in \mathcal{U}\}$$

$$\eta_{\mathfrak{B}}(\underline{l}) = \frac{1}{2|\mathbb{E} \times \mathcal{U}|} \sum_{\tilde{e} \in \mathbb{E}} (|1 - J_{\tilde{e}}^+(\underline{l})(T_{\tilde{e}}^+(\underline{l}) - F_{\tilde{e}}^+(\underline{l})) + J_{\tilde{e}}^-(\underline{l})(T_{\tilde{e}}^-(\underline{l}) - F_{\tilde{e}}^-(\underline{l}))|)$$

Everywhere $|\mathbb{E} \times \mathcal{U}|$ is the cardinality of $\mathbb{E} \times \mathcal{U}$.

5. Algorithm

1. Make the bipolar NSS on \mathcal{U} .
2. Calculate the aggregation bipolar NS operator.
3. Find an alternative set on \mathcal{U} .

Example-7: Bipolar condition is a serious psychological disease especially if not treated early that can exceed to dangerous performance, challenging careers etc. A bipolar mood chard representing the condition of patient's every month. Bipolar teenagers and their families will greatly advantage from mood plotting and can suppose initial finding the signs and purpose of proper cures by their doctors. We make mood plan according to algorithm. Let $\mathcal{U} = \{l_1, l_2, l_3, l_4\}$ present the set of day in which data has been maintain and $\mathbb{E} = \{\tilde{e}_1 = \text{severe depression}, \tilde{e}_2 = \text{anxiety}, \tilde{e}_3 = \text{medication}\}$ be set of parameters. Now we apply a set of rules as follows

1- Decision making of bipolar NS \mathfrak{B} above another set \mathcal{U} as:

$$\mathfrak{B} = \{(\tilde{e}_1, \{(l_1, .3, .5, .6, -.8, -.9, -.8)(l_2, .4, .2, .9, -.7, -.3, -.1), (l_3, .7, .8, .3, -.9, -.6, -.2), (l_4, .2, .9, .1, -.6, -.9, -.8)\}), (\tilde{e}_2, \{(l_1, .9, .6, .8, -.2, -.7, -.9), (l_2, .6, .2, .9, -.7, -.1, -.4), (l_3, .1, .7, .4, -.3, -.6, -.9), (l_4, .6, .9, .3, -.1, -.3, -.7)\}), (\tilde{e}_3, \{(l_1, .7, .1, .3, -.9, -.2, -.8), (l_2, .8, .6, .9, -.1, -.5, -.8), (l_3, .1, .5, .9, -.6, -.9, -.7), (l_4, .8, .4, .7, -.3, -.5, -.1)\})\}$$

2- find the decision making of aggregation bipolar NS operator \mathfrak{B}_{agg} of \mathfrak{B} as:

$$\mathfrak{B}_{agg} = \{.1075/ l_1, .1283/ l_2, .1358/ l_3, .0891/ l_4\}$$

3- Choose the maximum degree of l_3 is .1358 from \mathfrak{B}_{agg} amongst the other.

Example-8: Let $\mathcal{U} = \{a_1, a_2, a_3, a_4, a_5\}$ present the set of AC inverters and $\mathbb{E} = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4\}$ be the set of constraints in which " $\tilde{e}_1 = company$ ", " $\tilde{e}_2 = functions$ ", " $\tilde{e}_3 = cheap$ ", " $\tilde{e}_4 = AC capacity$ ". Now we apply a set of rules as follows:

1. Decision making of bipolar NS \mathfrak{B} above another set \mathcal{U} as:

$$\mathfrak{B} = \{(\tilde{e}_1, \{(a_1, .2, .3, .7, -.5, -.8, -.6)(a_2, .6, .8, .9, -.3, -.9, -.7), (a_3, .5, .4, .8, -.6, -.8, -.3), (a_4, .3, .9, .6, -.6, -.8, -.4), (a_5, .2, .5, .6, -.8, -.5, -.7)\}), (\tilde{e}_2, \{(a_1, .3, .7, .5, -.9, -.1, -.9), (a_2, .5, .9, .8, -.6, -.8, -.1), (a_3, .8, .6, .2, -.5, -.8, -.7), (a_4, .9, .6, .7, -.6, -.7, -.9), (a_5, .8, .5, .2, -.5, -.8, -.8)\}), (\tilde{e}_3, \{(a_1, .3, .5, .8, -.9, -.7, -.1), (a_2, .6, .3, .1, -.8, -.9, -.9), (a_3, .9, .9, .5, -.3, -.2, -.1), (a_4, .3, .1, .7, -.2, -.7, -.3), (a_5, .4, .6, .8, -.1, -.9, -.7)\}), (\tilde{e}_4, \{(a_1, .9, .8, .3, -.1, -.2, -.7), (a_2, .3, .7, .8, -.3, -.5, -.4), (a_3, .5, .1, .3, -.7, -.4, -.3), (a_4, .6, .9, .7, -.9, -.8, -.4), (a_5, .9, .5, .2, -.1, -.1, -.6)\})\}$$

2. find the decision making of aggregation bipolar NS operator \mathfrak{B}_{agg} of \mathfrak{B} as:

$$\mathfrak{B}_{agg} = \{.1105/a_1, .11525/a_2, .0915/a_3, .114/a_4, .07525/a_5\}$$

3. Choose the maximum degree of a_2 is .11525 from \mathfrak{B}_{agg} amongst the other.

Example-9: Let $\mathcal{U} = \{c_1, c_2, c_3\}$ present the set of auto car and $\mathbb{E} = \{\tilde{e}_1 = cheap, \tilde{e}_2 = features, \tilde{e}_3 = metallic colour\}$ be a set of constraints. Now we set of rules as follows:

1. Decision making of bipolar NS \mathfrak{B} above another set \mathcal{U} as:

$$\mathfrak{B} = \{(\tilde{e}_1, \{(c_1, .1, .3, .5, -.4, -.2, -.5)(c_2, .3, .1, .6, -.3, -.5, -.6), (c_3, .2, .4, .5, -.3, -.1, -.2)\}), (\tilde{e}_2, \{(c_1, .6, .1, .3, -.2, -.1, -.4), (c_2, .7, .3, .1, -.2, -.3, -.5), (c_3, .1, .5, .4, -.3, -.2, -.1)\}), (\tilde{e}_3, \{(c_1, .5, .2, .1, -.7, -.1, -.3), (c_2, .3, .4, .1, -.5, -.2, -.1), (c_3, .4, .1, .6, -.3, -.2, -.5)\})\}$$

2. Find the decision making of aggregation bipolar NS operator \mathfrak{B}_{agg} of \mathfrak{B} as:

$$\mathfrak{B}_{agg} = \mathfrak{B}_{agg} = \{.1672/c_1, .145/c_2, .1833/c_3\}$$

3. Choose the maximum degree of c_3 is .1833 from \mathfrak{B}_{agg} amongst the other.

6. Conclusion

In this paper, we discussed about the concept of bipolar neutrosophic soft sets and redefined some features of that particular concepts. Then we have employed bipolar neutrosophic soft sets in auto car selection with the help of aggregation operators. So we reached at the decision that what type of car is selected on what characteristics.

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