Pythagorean Neutrosophic [PN] P-Spaces (with T and F are dependent neutrosophic components

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Abstract

In this paper, we study the Pythagorean Neutrosophic Sets with T and F are dependent neutrosophic components [PN] topological spaces. We also study the Pythagorean Neutrosophic [PN] P-spaces (with T and F are dependent neutrosophic components) , weak P-spaces and almost P-spaces.Also we study the inter-relations between the spaces are introduced.

Keywords: PN topology,PN P-spaces,PN weak P-spaces,PN almost P-spaces,PN weakly Lindelof ,PN Baire space

1 Introduction

Fuzzy sets were introduced by Zadeh [17] and he discussed only membership function. The fuzzy topology concept was first introduced by C.L.Chang [3] in 1968.After the extensions of fuzzy set theory Atanassov [1] generalized this concept and introduced a new concept called intuitionistic fuzzy set (IFS). Yager [13] familiarized the model of Pythagorean fuzzy set. However, in the some practical problems, the sum of membership degree and non-membership degree to which an alternative satisfying attribute provided by decision maker(DM) may be bigger than 1, but their square sum is less than or equal to 1.

IFS was failed to deal with indeterminate and inconsistent information which exist in beliefs system, therefore, Smarandache [9] in 1995 introduced new concept known as neutrosophic set(NS) which generalizes fuzzy sets and intuitionistic fuzzy sets and so on. A neutrosophic set includes truth membership, falsity membership and indeterminacy membership. In 2006, F.Smarandache introduced, for the first time, the degree of dependence (and consequently the degree of independence) between the components of the fuzzy set, and also between the components of the neutrosophic set. In 2016, the refined neutrosophic set was generalized to the degree of dependence or independence of subcomponents [10]. In neutrosophic set [10], if truth membership and falsity membership are dependent and indeterminacy is independent.Sometimes in real life, we face many problems which cannot be handled by using neutrosophic for example when $T_A(x) + I_A(x) + F_A(x) > 2$. Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PN] of condition is as their square sum does not exceeds 2.In 2003, A.K.Mishra [7] introduced the concept of P-spaces.The concept of P-spaces in fuzzy setting was defined by G.Balasubramanian[11].Almost P-spaces in classical topology was introduced by R.Levy[6] .

In this paper we study the Pythagorean Neutrosophic [PN] P-spaces (with T and F are dependent neutrosophic components) ,Pythagorean Neutrosophic [PN] weak P-spaces (with T and F are dependent neutrosophic components) and Pythagorean Neutrosophic [PN]almost P-spaces (with T and F are dependent neutrosophic components) .Also we studied the inter relations between the spaces.

2 preliminaries

Definition 2.1. **(Pythagorean Fuzzy Set)**[13] Let X be a non-empty set and I the unit interval $[0, 1]$. A PF set P is an object having the form $P = \{(x, \mu_P(x), v_P(x)) : x \in X\}$ where the function $\mu_P : X \to [0, 1]$ and $v_P : X \to [0, 1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P, and $0 \leq (\mu_P(x))^2 + (v_P(x))^2 \leq 1$ for each $x \in X$.

Definition 2.2. [10] Let X be a non-empty set (universe). A neutrosophic set A on X is an object of the form: $A = (x, T_A(x), I_A(x), F_A(x)) : x \in X$, Where $T_A(x), I_A(x), F_A(x) \in [0, 1]$,$0 \leq T_A(x)+I_A(x)+F_A(x) \leq 2$,for all x in X. $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership. Here $T_A(x)$ and $F_A(x)$ are dependent components and $I_A(x)$ is an independent components.

Definition 2.3 (8). **(Pythagorean neutrosophic sets with T and F are dependent neutrosophic components [PN])**[13] Let X be a non-empty set. A Pythagorean neutrosophic sets with T and F are dependent neutrosophic components (PN) $A = \{(X, T_A(x), I_A(x), F_A(x) : x \in X\}$ where $T_A : X \rightarrow [0, 1], I_A : X \rightarrow [0, 1]$ and $F_A : X \rightarrow [-1, 0]$ are the mappings such that $0 \leq T_A^2(x) + I_A^2(x) + F_A^2(x) \leq 2$ and $T_A(x)$ denote the membership degree,$I_A(x)$ denote the Indeterminacy and $F_A(x)$ denote the non-membership degree.Here T and F are dependent neutrosophic components and I is an independent components.

Definition 2.4 (8). Let $A = (T_A, I_A, F_A)$ and $\mu = (T_\mu, I_\mu, F_\mu)$ be two PNs,then their operations are defined as follows:
(1)$A \subseteq \mu$ if and if $T_A(x) \leq T_\mu(x), I_A(x) \geq I_\mu(x), F_A(x) \geq F_\mu(x)$
(2) $A = \mu$ if and only if $A \subseteq \mu$ and $\mu \subseteq A$
(3) $A \cup \mu = \{(x, max(T_A, T_\mu), min(I_A, I_\mu), min(F_A, T_\mu) : x \in X\}$
(4)$A \cap \mu = \{(x, min(T_A, T_\mu), max(I_A, I_\mu), max(F_A, T_\mu) : x \in X\}$
(5)$A^C = \{(x, F_A, I_A, F_A) : x \in X\}$.

3 Pythagorean Neutrosophic [PN] P-Spaces ,weak P-space and almost P-space(with T and F are dependent neutrosophic components)

Definition 3.1. A Pythagorean neutrosophic sets with T and F are dependent neutrosophic components topology (PNT in Short) on X is a family $PN\tau$ of PNs in X satisfying the following axioms:

(1) $0_X, 1_X \in PN\tau$

(2)$G_1 \cap G_2 \in PN\tau$, for any $G_1, G_2 \in PN\tau$

(3) $\cup G_i \in PN\tau$ for any family $\{G_i / i \in J\} \subseteq PN\tau$.

In this case the pair $(X, PN\tau)$ is called a Pythagorean neutrosophic sets with T and F are dependent neutrosophic components topological space (PNTS in Short) and any BPFTS in $PN\tau$ is known as a Pythagorean neutrosophic sets with T and F are dependent neutrosophic components open set (PNOS in Short) in X.

The Complement A^c of a PNOS A in a PNTS $(X, PN\tau)$ is called a Pythagorean neutrosophic sets with T and F are dependent neutrosophic components closed set (PNCS in Short) in X.

Definition 3.2. Let $(X, PN\tau)$ be a PNTS and be a PN in X. Then the Pythagorean neutrosophic sets with T and F are dependent neutrosophic components interior and closure of a Pythagorean neutrosophic sets with T and F are dependent neutrosophic components closure are defined by

$PNint(A) = \cup\{G / GisaPNOSinXandG \subseteq A\}$

$PNcl(A) = \cap\{K / KisaPNCSinXandA \subseteq K\}$.

Note that for any PN A in $(X, PN\tau)$, we have $(PNcl(A))^c = PNint(A^c)$ and $(PNint(A))^c = PNcl(A^c)$.

Definition 3.3. [16] A subset of A of a space $(X, PN\tau)$ is called:
(i) PN regular open if $A = PNint(PNcl(A))$.
(ii) π open if A is the union of PN regular open sets.

Definition 3.4. A PN A in a PN topological space $(X, PN\tau)$ is called PN dense if there exists no PN closed set μ in $(X, PN\tau)$ such that $A \subseteq \mu \subseteq 1$.

Definition 3.5. A PN A in a PN topological space $(X, PN\tau)$ is called PN nowhere dense if there exists no non-zero PN open set $\mu \subseteq cl(A)$.That is,$PNint(PNcl(A)) = 0_X$.

Definition 3.6. A PN A in a PN topological space $(X, PN\tau)$ is called PN F_σ-set in $(X, PN\tau)$ if $A = \bigcup_{i=1}^{\infty}(A_i)$ where $(A_i)^c \in PN\tau$ for $i \in I$.

Definition 3.7. A PN A in a PN topological space $(X, PN\tau)$ is called PN G_δ-set in $(X, PN\tau)$ if $A = \bigcap_{i=1}^{\infty}(A_i)$ where $A_i \in PN\tau$ for $i \in I$.

Definition 3.8. For a family $A = \{A_\alpha\}$ of PN of a PN topological space $(X, PN\tau)$, $\bigcup cl(A_\alpha) \subseteq cl(\bigcup(A_\alpha)$.In case A is a finite set, $\bigcup cl(A_\alpha) = cl(\bigcup(A_\alpha)$.Also $\bigcup PNint(A_\alpha)PNint(\bigcup(A_\alpha)$.

Definition 3.9. A PN topological space $(X, PN\tau)$ is called a PN P-space if countable PNintersection of PN open sets in $(X, PN\tau)$ is PN open.That is, every non-zero PN G_δ-set in $(X, PN\tau)$, is PN open in $(X, PN\tau)$.

Example 3.10. Let $X = \{a, b\}$.The PN sets A_1, A_2 and A_3 are defined on X as follows:

A_1={(x,(0.2,0.5),(0.4,0.7),(0.8,0.9))}
A_2={(x,(0.5,0.9),(0.1,0.4),(0.3,0.5))}
A_3={(x,(0.8,0.1),(0.5,0.6),(0.4,0.7))}

Then, $PN\tau$=$\{0, A_1, A_2, A_3, A_1 \cup A_2, A_1 \cup A_3, A_2 \cup A_3, A_1 \cap A_2, A_1 \cap A_3, A_2 \cap A_3, A_1 \cup (A_2 \cap A_3), A_3 \cup (A_1 \cap A_2), A_2 \cap (A_1 \cup A_3), A_1 \cup A_2 \cup A_3, 1\}$ is a PN topology on X.Now the PN sets $A_1 \cap A_3$=$\{A_1 \cap A_3 \cap (A_2 \cup A_3) \cap (A_1 \cup A_2)\}$ and $A_2 \cap (A_1 \cup A_3)$=$\{(A_1 \cup (A_2 \cap A_3)) \cap (A_3 \cup (A_1 \cap A_2))\}$ are PN G_δ-sets in $(X, PN\tau)$ and $A_1 \cap A_3$,$A_2 \cup (A_1 \cap A_3)$ are PN open in $(X, PN\tau)$.Hence $(X, PN\tau)$ is a PN P-space.

Proposition 3.11. If A is a non-zero PN $F_\sigma-$ set in a PN P-space $(X, PN\tau)$, then A is a PN closed set in $(X, PN\tau)$.

Proof. Since A is a non-zero PN F_σ-set in $(X, PN\tau)$, $A = \bigcup_{i=1}^{\infty} A_i$,where the PN sets A_i's are PN closed in $(X, PN\tau)$.Then $A^c = (\bigcup_{i=1}^{\infty}(A_i))^c = \bigcap_{i=1}^{\infty}(A_i)^c$.Now A_i's are PN closed in $(X, PN\tau)$,implies that $(A_i)^c$'s are PN open in $(X, PN\tau)$.Hence we have $A^c = \bigcap_{i=1}^{\infty}(A_i)^c$, where $(A_i)^c \in PN\tau$.Then A^c is a PN G_σ-set in $(X, PN\tau)$.
Since $(X, PN\tau)$ is a PN P-space, A^c is PN open in $(X, PN\tau)$.Therefore A is a PN closed set in $(X, PN\tau)$. □

Theorem 3.12. (i) The closure of a PN open set is a PN regular closed set, and
(ii) The PNinterior of a PN closed set is a PN regular open set.

Proof. We prove only (a).
Let A be a PN open set of a PN space X.
clearly, $PNint(PNcl(A)) \subseteq PNcl(A)$ implies that
$PNcl(PNint(PNcl(A)))) \subseteq PNcl(A)$.
Now, A is open implies that $A \subseteq PNint(PNcl(A))$ and
hence $PNcl(A) \subseteq PNcl(PNint(PNcl(A)))$.
Thus $PNcl(A)$ is a regular closed set. □

Proposition 3.13. If the PN topological space $(X, PN\tau)$ is a PN P-space, then $PNcl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} PNcl(A_i)$, where A_i's are non-zero PN closed sets in $(X, PN\tau)$.

Proof. Let A_i's be non-zero PN closed sets in a PN P-space $(X, PN\tau)$.
Then $A = \bigcup_{i=1}^{\infty} A_i$, is a non-zero PN F_σ-set in $(X, PN\tau)$.
By proposition 4.3, A is a PN closed set in $(X, PN\tau)$.Hence $PNcl(A) = A$,
which implies that $PNcl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} PNcl(A_i)$ [since A_i's are closed, $PNcl(A_i) = A_i$].
Therefore $PNcl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} PNcl(A_i)$, where A_i's are PN closed sets in $(X, PN\tau)$. □

Proposition 3.14. If A_i's are PN closed sets in a PN P-space $(X, PN\tau)$, then $PNcl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} A_i$.

Proof. Let A_i's be PN closed sets in a PN P-space $(X, PN\tau)$.Then A_i's are PN closed sets in $(X, PN\tau)$, which implies that $(A_i)^c$'s are PN open sets in $(X, PN\tau)$.Let $B = \bigcap_{i=1}^{\infty}(A_i)^c$.Then B is a non-zero PN G_σ-set in $(X, PN\tau)$. Since the PN topological space $(X, PN\tau)$ is a PN P-space, $PNintB = B$, which implies that $PNint(\bigcap_{i=1}^{\infty}(A_i)^c) = \bigcap_{i=1}^{\infty}(A_i)^c$.
Then $(PNcl(\bigcup_{i=1}^{\infty} A_i)^c = (\bigcup_{i=1}^{\infty}(A_i))^c$.
Hence we have $PNcl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} A_i$. □

Proposition 3.15. If the PN topological space $(X, PN\tau)$ is a PN P-space and if A is a PN first category set in $(X, PN\tau)$, then A is not a PN dense set in $(X, PN\tau)$.

Proof. Assume the contrary.Suppose that A is a PN first category set in $(X, PN\tau)$ such that $PNcl(A) = 1_X$.Then , $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are PN nowhere dense sets in $(X, PN\tau)$.Now $(PNcl(A_i))^c$ is a PN open set in $(X, PN\tau)$.
Let $B = \bigcap_{i=1}^{\infty} PNcl(A_i)^c$.Then B is a non-zero PN G_σ-set in $(X, PN\tau)$.Now we have $\bigcap_{i=1}^{\infty} PNcl(A_i)^c =$

$(\bigcup_{i=1}^{\infty}(PNint(A_i))^c \subseteq (\bigcup_{i=1}^{\infty}(A_i))^c = A^c$.Hence $B \subseteq A^c$.Then $PNint(B) \subseteq A^c = (PNcl(A))^c = 0_X$.That is, $PNint(B) = 0_{XPN\tau}$.Since $(X, PN\tau)$ is a PN P-space, $B = PNint(B)$ which implies that $B = 0_{XPN\tau}$, a contradiction to B being a non-zero PN G_σ-set in $(X, PN\tau)$.Hence $PNcl(A) \neq 1_X$. □

Proposition 3.16. If the PN topological space $(X, PN\tau)$ is a PN P-space and if A is a PN first category set in $(X, PN\tau)$, then A is not a PN nowhere dense set in $(X, PN\tau)$.

Proof. Let A be a PN first category set in a PN P-space $(X, PN\tau)$.Then, we have $A = \bigcup_{i=1}^{\infty}(A_i)$, where A_i's are PN nowhere dense sets in $(X, PN\tau)$.Now $PNint(PNcl(A)) = PNint(PNcl(\bigcup_{i=1}^{\infty} A_i)) \supseteq PNint(\bigcup_{i=1}^{\infty} PNcl(A_i)$ and $\bigcup_{i=1}^{\infty} PNcl(A_i)$ is a PN F_σ-set in $(X, PN\tau)$.Since $(X, PN\tau)$ is a PN P-space, by proposition 4.3, $(\bigcup_{i=1}^{\infty} PNcl(A_i)$ is a non-zero PN closed set in $(X, PN\tau)$.Also PNinterior of a PN closed is a PN regular open set,
$PNint(\bigcup_{i=1}^{\infty} PNcl(A_i))$ is a non-zero PN regular open set in $(X, PN\tau)$.
Hence we have $0_X \neq PNint(\bigcup_{i=1}^{\infty} PNcl(A_i)) \subseteq PNint(PNcl(A))$ implies that $PNint(PNcl(A)) \neq 0_X$.Therefore A is not a PN nowhere dense set in $(X, PN\tau)$. □

Proposition 3.17. If A is a PN first category set in a PN P-space $(X, PN\tau)$ such that $B \subseteq A^c$, where B is a non-zero dense PN G_δ-set in $(X, PN\tau)$, then A is a PN nowhere dense set in $(X, PN\tau)$.

Proof. Let A be a PN first category set in $(X, PN\tau)$.Then, $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are PN nowhere dense sets in $(X, PN\tau)$.Now $(PNcl(A_i))^c$ is a PN open set in $(X, PN\tau)$.Let $B = \bigcap_{i=1}^{\infty} PNcl(A_i)^c$.Then B is a non-zero PN G_δ-set in $(X, PN\tau)$.
Now we have $\bigcap_{i=1}^{\infty}(PNcl(A_i))^c = (\bigcup_{i=1}^{\infty}(PNint(A_i)))^c \subseteq (\bigcup_{i=1}^{\infty}(A_i))^c = A^c$.
Hence $B \subseteq A^c$.Then we have $A \subseteq B^c$.
Now $PNint(PNcl(A)) \subseteq PNint(PNcl(B^c))$,
which implies that $PNint(PNcl(A)) \subseteq (PNcl(PNint(B)))^c$.
Since $(X, PN\tau)$ is a PN P-space, the PN G_σ-set B is PN open in $(X, PN\tau)$ and $PNint(B) = B$.
Therefore $PNint(PNcl(A)) \subseteq (PNcl(B))^c = 0_X$(since B is PN dense).Then $PNint(PNcl(A)) = 0_X$ and hence A is a PN nowhere dense set in $(X, PN\tau)$. □

Theorem 3.18. Let $(X, PN\tau)$ be a PN topological space.Then the following are equivalent:

(i) $(X, PN\tau)$ is a PN Baire space
(ii) $PNint(A) = 0_X$ for every PN first category set A in $(X, PN\tau)$
(iii) $PNcl(B) = 1_X$ for every PN residual set B in $(X, PN\tau)$.

Proof. (i) $\Rightarrow$ (ii).Let A be a PN first category set in $(X, PN\tau)$.
Then $A = \bigcup_{i=1}^{\infty} A_i$ where A_i's PN nowhere dense sets in $(X, PN\tau)$.
Then,we have $PNint(A) = PNint(\bigcup_{i=1}^{\infty} A_i)$.
Since $(X, PN\tau)$ is a PN Baire space,$PNint(\bigcup_{i=1}^{\infty} A_i) = 0_X$.
Hence $PNint(A) = 0_X$ for any PN first category set in $(X, PN\tau)$.

(ii) $\Rightarrow$ (iii).Let B be a PN residual set in $(X, PN\tau)$.Then B^c is a PN first category set in $(X, PN\tau)$.By hypothesis,$PNint(B^c) = 0_X$.Then $(PNcl(A))^c = 0_X$.
Hence $PNcl(A) = 1_X$ for any PN residual set B in $(X, PN\tau)$.

(iii) $\Rightarrow$ (i).Let B be a PN first category set in $(X, PN\tau)$.Then $A = \bigcup_{i=1}^{\infty}(A_i)$ where A_i's are PN nowhere dense sets in $(X, PN\tau)$.Now A is a PN first category set in $(X, PN\tau)$ implies that A^c is a PN residual set in $(X, PN\tau)$.By hypothesis, we have $PNcl(A^c) = 1_X$.Then $(PNint(A))^c = 1_X$.Hence $PNint(A) = 0_X$.That is,$PNint(\bigcup_{i=1}^{\infty} A_i = 0_X$ where A_i's are PN nowhere dense sets in $(X, PN\tau)$.
Hence $(X, PN\tau)$ is a PN Baire space. □

Proposition 3.19. If A is a PN first category set in a PN P-space $(X, PN\tau)$ such that $B \subseteq A^c$, where B is a non-zero dense PN G_δ-set in $(X, PN\tau)$, then $(X, PN\tau)$ is a PN Baire space.

Proof. Let A be a PN first category set in $(X, PN\tau)$.As in proof 4.9,we have $PNint(PNcl(A)) = 0_X$.Then $PNint(A) \subseteq PNint(PNcl(A))$ implies that $PNint(A) = 0_X$ and hence by Theorem 4.10, $(X, PN\tau)$ is a PN Baire space. □

Proposition 3.20. If the PN topological space $(X, PN\tau)$ is a PN P-space and if A is a PN dense and fuzzy first category set in $(X, PN\tau)$,then there is no non-zero PN G_δ-set B in $(X, PN\tau)$ such that $B \subseteq A^c$.

Proof. Let A be a fuzzy first category set in $(X, PN\tau)$.As in proof 4.9,we have a PN G_δ-set B in $(X, PN\tau)$ such that $B \subseteq A^c$.Then $PNint(B) \subseteq PNint(A^c)$ implies that $PNint(B) \subseteq (PNcl(A))^c = 0_X$(since A is PN dense, $PNcl(A) = 1$).That is, $PNint(B) = 0_X$.Since $(X, PN\tau)$ is a PN P-space ,$PNint(B) = B$ and hence we have $B = 0_X$.Hence, if A is a PN dense and PN first category set in $(X, PN\tau)$,then there is no non-zero PN G_δ-set B in $(X, PN\tau)$ such that $B \subseteq A^c$. □

Definition 3.21. A PN topological space $(X, PN\tau)$ is called a weak PN P-space if the countable PN intersection PN regular open sets in $(X, PN\tau)$ is a PN regular open set in $(X, PN\tau)$.That is, $\bigcap_{i=1}^{\infty} A_i$ is PN regular open in $(X, PN\tau)$, where A_i's are PN regular open sets in $(X, PN\tau)$.

Example 3.22. Let $X = \{a, b\}$.The PN sets A_1, A_2 and A_3 are defined on X as follows:

A_1={(x,(0.1,0.3),(0.5,0.8),(0.8,0.9))}
A_2={(x,(0.2,0.4),(0.9,0.1),(0.7,0.6))}
A_3={(x,(0.5,0.1),(0.4,0.6),(0.7,0.8))}

Then, $PN\tau$=$\{0, A_1, A_2, A_3, A_1 \cup A_2, A_1 \cup A_3, A_2 \cup A_3, A_1 \cap A_2, A_1 \cap A_3, A_2 \cap A_3, A_1 \cup (A_2 \cap A_3), A_3 \cup (A_1 \cap A_2), A_2 \cup (A_1 \cap A_3), 1\}$ is a PN topology on X.Now the PN sets $A_1 \cap (A_1 \cup A_2), (A_1 \cap A_3), (A_1 \cap (A_2 \cup A_3)), A_2 \cup (A_1 \cap A_3), (A_3 \cap (A_1 \cup A_2)$ are PN regular open sets in $(X, PN\tau)$ and $\{A_1 \cap (A_1 \cup A_2) \cap (A_1 \cap A_3) \cap (A_1 \cap (A_2 \cup A_3)) \cap A_2 \cup (A_1 \cap A_3) \cap (A_3 \cap (A_1 \cup A_2))\}$=$A_1 \cap A_3$ is a PN regular open in $(X, PN\tau)$.Hence $(X, PN\tau)$ is a weak PN space.

Proposition 3.23. A PN topological space $(X, PN\tau)$ is a weak PN P-space if and only if $\bigcup_{i=1}^{\infty} B_i$, where B_i's are PN regular closed sets in $(X, PN\tau)$, is PN regular closed in $(X, PN\tau)$.

Proof. Let $(X, PN\tau)$ be a weak PN P-space.
Then $PNint(PNcl(\bigcap_{i=1}^{\infty} A_i)) = \bigcap_{i=1}^{\infty} A_i$, where A_i's are PN regular open sets in $(X, PN\tau)$.
Now $(PNint(PNcl(\bigcap_{i=1}^{\infty} A_i)))^c = (\bigcap_{i=1}^{\infty} A_i)^c$,
implies that $PNcl(PNint(\bigcup_{i=1}^{\infty}(A_i)^c)) = \bigcup_{i=1}^{\infty}(A_i)^c$.
Let $B_i = (A_i)^c$.Since A_i is a PN regular open set in $(X, PN\tau)$, $B_i)$ is a PN regular closed set in $(X, PN\tau)$.Then we have $PNcl(PNint(\bigcup_{i=1}^{\infty} B_i)) = \bigcup_{i=1}^{\infty} B_i$.
Hence $\bigcup_{i=1}^{\infty} B_i$ is a PN regular closed in $(X, PN\tau)$.

Conversely, suppose that
$PNcl(PNint(\bigcup_{i=1}^{\infty} B_i)) = \bigcup_{i=1}^{\infty} B_i$, where B_i's are PN regular closed sets in $(X, PN\tau)$.Then $(PNcl(PNint(\bigcup_{i=1}^{\infty} B_i))^c = (\bigcup_{i=1}^{\infty} B_i)^c$,
which implies that $PNint(PNcl(\bigcap_{i=1}^{\infty}(B_i)^c)) = \bigcap_{i=1}^{\infty}(B_i)^c$, where $(B_i)^c$'s are PN regular open sets in $(X, PN\tau)$.Therefore $(X, PN\tau)$ is a weak PN P-space. □

Proposition 3.24. If a PN topological space $(X, PN\tau)$ is a weak PN P-space, then $PNcl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} PNcl(A_i)$, where A_i's are non-zero PN open sets in $(X, PN\tau)$.

Proof. Let A_i's be PN open set in $(X, PN\tau)$.Then by Theorem 4.4,$PNcl(A_i)$'s are PN regular closed in $(X, PN\tau)$.Since $(X, PN\tau)$ is a weak PN P-space, by proposition 4.15, $\bigcup_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} PNcl(A_i)$, where $PNcl(A_i)$'s are non-zero PN regular closed set in $(X, PN\tau)$ is PN reguler closed in $(X, PN\tau)$.That is, $PNcl(PNint(\bigcup_{i=1}^{\infty} PNcl(A_i)) = \bigcup_{i=1}^{\infty} PNcl(A_i)$.
Then we have
$PNcl(PNint(\bigcup_{i=1}^{\infty} A_i)) \subseteq PNcl(PNint(\bigcup_{i=1}^{\infty} A_i)) = \bigcup_{i=1}^{\infty} PNcl(A_i)$.
That is,$PNcl(PNint(\bigcup_{i=1}^{\infty} A_i) \subseteq \bigcup_{i=1}^{\infty} PNcl(A_i)$.
Since $A_i \in PN\tau$,$\bigcup_{i=1}^{\infty} A_i \in PN\tau$ and $PNint(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} A_i$.
Hence $PNcl(\bigcup_{i=1}^{\infty} A_i) \subseteq \bigcup_{i=1}^{\infty} PNcl(A_i)$ (1).
But $\bigcup_{i=1}^{\infty} PNcl(A_i) \subseteq PNcl(\bigcup_{i=1}^{\infty} A_i$ (2).
From (1) and (2), we have $PNcl(\bigcup_{i=1}^{\infty} A_i)) = \bigcup_{i=1}^{\infty} PNcl(A_i)$, where A_i's are non-zero PN open sets in $(X, PN\tau)$. □

Definition 3.25. A PN topological space $(X, PN\tau)$ is called a PN almost Lindelof space if every PN open cover $\{A_\alpha\}_{\alpha \in \Delta}$ of $(X, PN\tau)$ admits a countable subcover $\{A_n\}_{n \in N}$ such that $\bigcup_{n \in N} PNcl(A_n)) = 1_X$.

Definition 3.26. A PN topological space $(X, PN\tau)$ is said to be PN weakly Lindelof space if every PN open cover $\{A_\alpha\}_{\alpha \in \Delta}$ of $(X, PN\tau)$ there exists a countable sub cover $\{A_n\}_{n \in N}$ such that $PNcl(\bigcup_{n \in N} A_n) = 1_X$.

Remark 3.27. Obviously every PN almost Lindelof space is a PN weakly Lindelof space.For, $\bigcup_{n\in N} PNcl(A_n) \subseteq PNcl(\bigcup_{n\in N} A_n)$ and $\bigcup_{n\in N} PNcl(A_n) = 1_X$, implies that $PNcl(\bigcup_{n\in N} A_n) = 1_X$.

Proposition 3.28. If the PN topological space $(X, PN\tau)$ is a weak PN P-space, then every PN weakly Lindelof space is a PN almost Lindelof space.

Proof. Let $(X, PN\tau)$ be a PN weakly Lindelof space and $\{A_\alpha\}_{\alpha\in\Delta}$ be a PN open cover of $(X, PN\tau)$.Then there exists a countable subcover $\{A_n\}_{n\in N}$ such that $PNcl(\bigcup_{n\in N} A_n) = 1_X$.Since $(X, PN\tau)$ is a weak PN P-space, $PNcl(\bigcup_{n\in N} A_n) = \bigcup_{n\in N} PNcl(A_n))$ where A_i's are non-zero PN open sets in $(X, PN\tau)$.Hence for the PN open cover $\{A_\alpha\}_{\alpha\in\Delta}$ of $(X, PN\tau)$,there exists a countable subcover $\{A_n\}_{n\in N}$ such that $\bigcup_{n\in N} PNcl(A_n) = 1_X$.Therefore $(X, PN\tau)$ is a PN almost Lindelof space. □

Proposition 3.29. If a PN topological space $(X, PN\tau)$ is a PN P-space, then $(X, PN\tau)$ is a weak PN P-space.

Proof. rm Let λ_i's be PN closed sets in $(X, PN\tau)$.Since $(X, PN\tau)$ is a PN P-space by Proposition 3.3, we have $cl(\bigcup_{i=1}^{\infty}(\lambda_i)) = \bigcup_{i=1}^{\infty}(\lambda_i)$.Now $clint(\bigcup_{i=1}^{\infty}(\lambda_i)) \subseteq cl(\bigcup_{i=1}^{\infty}(\lambda_i)) = \bigcup_{i=1}^{\infty}(\lambda_i)$.That is, $cl(int(\bigcup_{i=1}^{\infty}(\lambda_i)) \subseteq \bigcup_{i=1}^{\infty}(\lambda_i)$.....(1).Since λ_i's are PN regular closed sets in $(X, PN\tau)$,$clint(\lambda_i) = \lambda_i$.Then $\bigcup_{i=1}^{\infty} clint(\lambda_i) = \bigcup_{i=1}^{\infty}(\lambda_i)$, which implies that $\bigcup_{i=1}^{\infty}(\lambda_i) \subseteq clint(\bigcup_{i=1}^{\infty}(\lambda_i))$....(2).From (1) and (2), we have $clint(\bigcup_{i=1}^{\infty}(\lambda_i)) = \bigcup_{i=1}^{\infty}(\lambda_i)$.Hence by proposition 4.3, $(X, PN\tau)$ is a weak PN P-space. □

Remark 3.30. A weak PN P-sapce need not be a PN P-space.For, consider the following example:

Example 3.31. Let $X = \{a, b\}$.The PN sets A_1, A_2 and A_3 are defined on X as follows:

A_1={(x,(0.1,0.5),(0.9,0.2),(0.3,0.7))}
A_2={(x,(0.4,0.2),(0.5,0.3),(0.6,0.7))}
A_3={(x,(0.5,0.4),(0.8,0.9),(0.7,0.6))}

Then, $PN\tau$=$\{0_X, A_1, A_2, A_3, A_1 \cup A_2, A_1 \cup A_3, A_2 \cup A_3, A_1 \cap A_2, A_1 \cap A_3, A_2 \cap A_3, A_1 \cup (A_2 \cap A_3), A_3 \cup (A_1 \cap A_2), A_2 \cup (A_1 \cap A_3), A_1 \cap (A_2 \cup A_3), A_3 \cap (A_1 \cup A_2), A_2 \cap (A_1 \cup A_3), A_1 \cup A_2 \cup A_3, A_1 \cap A_2 \cap A_3, 1_X\}$ is a PN topology on X.Now the PN $A_1 \cup A_2, A_1 \cup A_3, A_2 \cup A_3, A_1 \cap A_2, A_1 \cap A_3, A_2 \cap A_3, A_1 \cup (A_2 \cap A_3), A_3 \cup (A_1 \cap A_2), A_2 \cup (A_1 \cap A_3), A_1 \cap (A_2 \cup A_3), A_3 \cap (A_1 \cup A_2), A_2 \cap (A_1 \cup A_3), A_1 \cup A_2 \cup A_3, A_1 \cap A_2 \cap A_3$ are PN regular open sets in $(X, PN\tau)$ and $\{(A_1 \cup A_2) \cap (A_1 \cup A_3) \cap (A_2 \cup A_3) \cap (A_1 \cap A_2) \cap (A_1 \cap A_3) \cap (A_2 \cap A_3) \cap (A_1 \cup (A_2 \cap A_3)) \cap (A_3 \cup (A_1 \cap A_2)) \cap (A_2 \cup (A_1 \cap A_3)) \cap (A_1 \cap (A_2 \cup A_3)) \cap (A_3 \cap (A_1 \cup A_2)) \cap (A_2 \cap (A_1 \cup A_3)) \cap (A_1 \cup A_2 \cup A_3) \cap (A_1 \cap A_2 \cap A_3)\}$=$A_1 \cap A_2 \cap A_3$ is a PN regular open in $(X, PN\tau)$.Hence $(X, PN\tau)$ is a weak PN space.But $(X, PN\tau)$ is not a P-space, since the PN G_δ-set $\{(A_1 \cup A_2) \cap (A_1 \cup A_3) \cap (A_2 \cup A_3)\}$ is not a PN open in $(X, PN\tau)$.

Definition 3.32. A PN topological space $(X, PN\tau)$ is called a PN almost P-space if for every non-zero PN G-set A in $(X, PN\tau)$,$PNint(A) \neq_{PN\tau}$ in $(X, PN\tau)$.

Example 3.33. Let $X = \{a, b\}$.The PN sets A_1, A_2 and A_3 are defined on X as follows:

A_1={(x,(0.1,0.8),(0.5,0.9),(0.9,0.1))}
A_2={(x,(0.2,0.5),(0.8,0.4),(0.3,0.7))}
A_3={(x,(0.3,0.5),(0.8,0.1),(0.4,0.6))}

Then, $PN\tau$=$\{0, A_1, A_2, A_3, A_1 \cup A_2, A_1 \cap A_2, 1\}$ is a PN topology on X.Now for the PN G_δ-sets $(A_1 \cap A_2 \cap A_3)$ and $\{A_1 \cup A_2) \cap (A_1 \cap A_2)\}$ in $(X, PN\tau)$, $PNint(A_1 \cap A_2 \cap A_3) = A_1 \cap A_2 \neq 0$ and $PNint((A_1 \cup A_2) \cap (A_1 \cap A_2)) = A_1 \cap A_2 \neq 0$.Hence $(X, PN\tau)$ is a PN almost P-space.

Remark 3.34. Clearly every PN P-space is a PN almost P-space, since for every non-zero PN G_δ-set δ in $(X, PN\tau)$, we have $int(\delta) = \delta \neq 0$.But the converse need not be true.For example 6.2, for every non-zero PN G_δ-set δ in $(X, PN\tau)$, we have $int(\delta) \neq 0$ in $(X, PN\tau)$.Hence $(X, PN\tau)$ is a PN almost P-space, but $(X, PN\tau)$ is not a PN P-space, since the PN G_δ-set $\{(A_1 \cup A_2) \cap (A_1 \cup A_3) \cap (A_2 \cup A_3)\}$ is not PN open in $(X, PN\tau)$.

Remark 3.35. A PN almost P-space need not be a weak PN P-space.For consider the following example:

Let $X = \{a, b\}$.The PN sets A_1, A_2 and A_3 are defined on X as follows:

A_1={(x,(0.8,0.7),(0.1,0.3),(0.2,0.5))}

A_2={(x,(0.6,0.7),(0.8,0.1),(0.2,0.3))}
A_3={(x,(0.6,0.8),(0.5,0.7),(0.6,0.1))}

Then, $PN\tau$=$\{0_X, A_1, A_2, A_3, A_1{\cup}A_2, A_1{\cup}A_3, A_2{\cup}A_3, A_1{\cap}A_2, A_1{\cap}A_3, A_2{\cap}A_3, A_1{\cup}(A_2{\cap}A_3), A_3{\cup}(A_1{\cap} A_2), A_2{\cup}(A_1{\cap}A_3), A_1{\cap}(A_2{\cup}A_3), A_3{\cap}(A_1{\cup}A_2), A_2{\cap}(A_1{\cup}A_3), A_1{\cup}A_2{\cup}A_3, A_1{\cap}A_2{\cap}A_3, 1_X\}$ is a PN topology on X.In $(X, PN\tau)$, for every non-zero PN G_{delta}-set $(A_1{\cup}A_2){\cap}(A_1{\cap}A_3){\cap}(A_1{\cup}A_3) = A_1{\cap}A_3$, we have $int(A_1 \cap A_3) = 0$.Hence $(X, PN\tau)$ is a not PN almost P-space.

Proposition 3.36. A weak PN P-space need not be a PN almost P-space.For consider the following example:

Example 3.37. Let $X = \{a, b\}$.The PN sets A_1, A_2 and A_3 are defined on X as follows:

A_1={(x,(0.1,0.2),(0.6,0.8),(0.3,0.4))}
A_2={(x,(0.6,0.5),(0.4,0.3),(0.8,0.6))}
A_3={(x,(0.3,0.2),(0.8,0.1),(0.4,0.9))}

Then, $PN\tau$=$\{0_X, A_1, A_2, A_3, A_1 \cup A_2, A_1 \cup A_3, A_2 \cup A_3, A_1 \cap A_2, A_1 \cap A_3, A_2 \cap A_3, A_1 \cup (A_2 \cap A_3), A_3 \cup (A_1{\cap}A_2), A_2{\cup}(A_1{\cap}A_3), A_1{\cap}(A_2{\cup}A_3), A_3{\cap}(A_1{\cup}A_2), A_2{\cap}(A_1{\cup}A_3), A_1{\cup}A_2{\cup}A_3, A_1{\cap}A_2{\cap}A_3, 1_X\}$ is a PN topology on X.Now $A_1 \cup A_3$, $A_1 \cup (A_2 \cap A_3)$ and $A_1 \cup A_2 \cup A_3$ are PN regular open sets in $(X, PN\tau)$ and $(A_1 \cup A_3) \cap (A_1 \cup (A_2 \cap A_3)) \cap (A_1 \cup A_2 \cup A_3) = A_1 \cup (A_2 \cap A_3)$ is a PN regular open sets in $(X, PN\tau)$ and Hence $(X, PN\tau)$ is a weak PN P-space.But $(X, PN\tau)$ is not a almost P-space, since for the non-zero PN G_δ-set $\{(A_1 \cap (A_2 \cup A_3)) \cap (A_2 \cap A_3, A_1 \cup (A_2 \cap A_3)) \cap A_3 \cap (A_1 \cup A_2)\}$, we have $int((A_1 \cap (A_2 \cup A_3)) \cap (A_2 \cap A_3, A_1 \cup (A_2 \cap A_3)) \cap A_3 \cap (A_1 \cup A_2)) = 0$.

Proposition 3.38. If A is a PN F_σ-set in a PN almost P-space $(X, PN\tau)$, then $PNcl(A) \neq 1_X$.

Proof. Let A be a PN F_σ-set in a PN almost P-space $(X, PN\tau)$.Then, A^c is a PN G_δ-set in $(X, PN\tau)$.Since $(X, PN\tau)$ is a PN almost P-space , for the PN G_δ-set A^c we have $PNint(A^c) \neq 0_X$. This implies that $(PNcl(A))^c \neq 0_X$ and hence we have $PNcl(A) \neq 1_X$. □

Proposition 3.39. If each non-zero PN G_δ-set is a PN regular closed set in a PN topological space $(X, PN\tau)$, then $(X, PN\tau)$ is a PN almost P-space.

Proof. Let A be a non-zero PN G_δ-set in $(X, PN\tau)$ such that $PNcl(PNint(A)) = A$. We claim that $PNint(A) \neq 0_X$. Assume the contrary.
Then $PNint(A) = 0_X$, will imply that $PNcl(PNint(A)) = PNcl(0_X) = 0_X$ and hence we will have $A = 0_X$, a contradiction to A being a non-zero PN G_δ-set in $(X, PN\tau)$.Hence we must have $PNint(A) \neq 0_X$, for a PN G_δ-set A in $(X, PN\tau)$ and therefore $(X, PN\tau)$ is a PN almost P-space. □

Proposition 3.40. If each non-zero PN G_δ-set is a PN semi-open set in a PN topological space $(X, PN\tau)$, then $(X, PN\tau)$ is a PN almost P-space.

Proof. Let A be a non-zero PN G_δ-set in $(X, PN\tau)$ such that $A \subseteq PNcl(PNint(A))$. We claim that $PNint(A) \neq 0_X$. Assume the contrary. Then $PNint(A) = 0_X$, will imply that $PNcl(PNint(A)) = PNcl(0_X) = 0_X$ and hence we will have $A = 0_X$, a contradiction to A being a non-zero PN G_δ-set in $(X, PN\tau)$.Hence we must have $PNint(A) \neq 0_X$, for a PN G_δ-set A in $(X, PN\tau)$ and therefore $(X, PN\tau)$ is a PN almost P-space. □

Remark 3.41. If a PN topological space $(X, PN\tau)$ has non-zero fuzzy nowhere dense PN G_δ-sets, then $(X, PN\tau)$ is not a PN almost P-space.For, consider the following proposition.

Proposition 3.42. If $A$is a non-zero PN nowhere dense PN G_δ-set in a PN topological space $(X, PN\tau)$, then $(X, PN\tau)$ is not a PN almost P-space.

Proof. Let A be a non-zero PN nowhere dense PN G_δ-set A in $(X, PN\tau)$.Then $PNint(A) \subseteq PNint(PNcl(A))$ and $PNint(PNcl(A)) = 0_X$, implies that $PNint(A) = 0_X$.Hence for the non-zero PN G_δ-set A in $(X, PN\tau)$, $PNint(A) = 0_X$ in $(X, PN\tau)$.Therefore $(X, PN\tau)$ is not a PN almost P-space. □

Theorem 3.43. For any PN topological space $(X, PN\tau)$, then the following are equivalent:

(i) X is PN basically disconnected.
(ii)For each PN closed G_δ-set A,$PNint(A)$ is PN closed.
(iii) For For each PN closed F_σ-set A,we have $PNcl(PNint(A^c) = (PNcl(A))^c$.
(iv) For a PN open F_σ-set A and for any PN B with $PNcl(A) = B^c$,we have $PNcl(B) = (PNcl(A))^c$.

Proof. (i) ⇒ (ii).Let A be any PN closed G_δ-set.Then A^c is a PN open F_σ-set.By assumption (i), $PNcl(A^c)$ is a PN open set.Now $PNcl(A^c) = PNint(A)$.
Hence $PNint(A)$ is a PN closed set.

(ii) ⇒ (iii).Let A be any PN open F_σ set.Then A^c is a PN closed G_δ-set.By assumption $PNint(A^c)$ is a PN closed set.
Consider $PNcl(PNint(A^c)) = PNint(A^c) = (PNcl(A))^c$.

(iii) ⇒ (iv).Let A be a PN open F_σ-set and for any PN B such that
$PNcl(A) = B^c$.By (iii),$PNcl(PNint(A^c)) = (PNcl(A))^c = PNint(A^c)$.
That is
$PNcl(B) = PNint(A^c)$.
(iv) ⇒ (i).Let A be any PN open F_σ-set.Let $(PNcl(A))^c = B$.
By (iv), it follows that $PNcl(B) = (PNcl(A))^c$.
That is,
$(PNcl(A))^c$ is a PN closed set.
This implies that $PNcl(A)$ is a PN open set.
Hence,$(X, PN\tau)$ is a PN basically disconnected space. □

Proposition 3.44. If A is a non-zero PN closed PN G_δ-set in a PN basically disconnected space $(X, PN\tau)$ and PN almost P-space, then $PNcl(PNint(A)) \neq 0_X$.

Proof. Let A be a non-zero PN closed G_δ-set A in $(X, PN\tau)$.Since $(X, PN\tau)$ is a PN basically disconnected space,by Theorem 4.28, $PNint(A)$ is PN closed.That is, $PNcl(PNint(A)) = PNint(A)$.Since $(X, PN\tau)$ is a PN almost P-space,
$PNint(B) \neq 0_X$.Hence we have $PNcl(PNint(A)) \neq 0_X$. □

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