A Contribution to Neutrosophic Groups

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Abstract

The objective of this paper is to define some new substructures (AH-substructures) in a neutrosophic group. Also, it deals with some elementary properties of AH-subgroups, AH-normality, AH-homomorphisms, AH-quotients and AH-direct products.

Keywords: Neutrosophic group, AH- subgroup, AH- homomorphism, AH-quotient.

1. Introduction

The fuzzy set and intuitionistic fuzzy set theory were adopted effectively from their initiation to solve optimization problems at vague and uncertain situation in our daily life activities. The intuitionistic fuzzy set theory introduced by Atanassov [5] deals with the degree of belongingness and the degree of non-belongingness of an object to a set simultaneously. Thus it is the more generalization concept than fuzzy set theory which can provide only the degree of belongingness of an object to a set. Both of the theories can only handle incomplete information not indeterminate. To access both incomplete and indeterminate information, Smarandache [8,9] generalized the intuitionistic fuzzy set to neutrosophic set (NS) where each proposition is estimated by three independent parameters namely truth-membership value (T) , indeterminacy membership value (I) and falsity-membership value (F) with $T,I,F \in]^{-}0,1^{+}[$ and $^{-}0 \leq \sup T + \sup I + \sup F \leq 3^{+}$. Smarandache used to practice the standard or nonstandard subsets of $]^{-}0,1^{+}[$ in philosophical ground. So, to incorporate this concept in real life scenario, Wang et al. [10] brought the concept of single valued neutrosophic set which takes the value from real standard subset of [0,1] only.

Also, Smarandache and Vasantha Kandasamy introduced the notion of neutrosophic group in [1]. Neutrosophic rings were introduced in [4]. The neutrosophic group in general does not have a group structure.

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In this paper, we shall continue the study of neutrosophic groups by introducing the notion of an AH-subgroup and develop some basic theory, we will use the idea of AH-concepts introduced in [1,2].

Neutrosophic AH-subgroup will be defined as a union of two subgroups $K \cup T$; K is a subgroup of G and T is a subgroup of the pure neutrosophic group GI.

2. Preliminaries

In this following section, we recall some important and useful definitions about neutrosophic groups.

Definition 2.1: [3]

Let (G,*) be a group . Then the neutrosophic group is generated by G and I under * denoted by

N(G)={$< G \cup I >$,$*$}.

I is called the indeterminate (neutrosophic element) with the property $I^2 = I$.

Definition 2.2: [3]

Let N(G) be a neutrosophic group and H be a neutrosophic subgroup, i.e., (H contains a proper subgroup of G). Then H is a neutrosophic normal subgroup of N(G) if $xH = Hx$ for all $x \in N(G)$.

Definition 2.3: [3]

Let N(G) be a neutrosophic group. Then the center of N(G) is denoted by C(N(G)), and defined

$C(N(G)) = \{x \in N(G); xy = yx \; \forall \, y \in N(G)\}$.

Definition 2.4: [3]

Let N(G) , N(H) be two neutrosophic groups, then $N(G) \times N(H) = \{(g,h); g \in N(G), h \in N(H)\}$.

Definition 2.5: [6]

Let N(G), N(H) be two neutrosophic groups and $\varphi: N(G) \to N(H)$ is called a neutrosophic homomorphism if it is a homomrphism between G , H and $\varphi(I) = I'$.

Where I' is the neutrosophic element of N(H).

If φ is a correspondence one-to-one it is called a neutrosophic isomorphism.

Example 2.6:

Let Z_7 = {0, 1, 2, …, 6} be the group of integers under addition modulo 7. N(G) = {⟨$Z_7 \cup$ I⟩, '+' modulo 7} is a neutrosophic group which is in fact a group. For N(G) = {a + bI ; a, b $\in Z_7$} is a group under '+' modulo 7. Thus this neutrosophic group is also a group.

Theorem 2.7: [3]

Let (G, *) be a group, N(G) = {⟨G $\cup$ I⟩, *} be the neutrosophic group. Then

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N(G) in general is not a group.
N(G) always contains a group.

Example 2.8:

Let N(G) be a neutrosophic group generated by $(Z_7^*, .)$, given by N(G) = {1, 2, 3, 4, 5, 6, I, 2I, 3I, 4I, 5I, 6I} be a neutrosophic group under multiplication modulo 7.

H = {1, 6, I, 6I} is a neutrosophic subgroup of N(G). For x∈ N(G), xH = Hx, so H is normal in N(G).

Definition 2.9: [11]

(a) Let G be any group. It is called meta abelian if it has a abelian derivative subgroup G'.

(b) Let G be a group. It is called nilpotent if it has a central series.

For the concept of central series, see [11].

Remark 2.10: [11]

S_3 is a solvable group, but it is not nilpotent.

D_4 is a meta abelian and nilpotent group.

The intersection, and the direct product of two meta abelian groups is meta abelian.

The intersection, and the direct product of two nilpotent groups is nilpotent.

AH-subgroups were firstly defined in n-refined neutrosophic groups. We recall some definitions.

Definition 2.11: [1]

Let $(G,*)$ be a group, we define the corresponding n-refined neutrosophic group $N_n(G)$ as follows:

$N_n(G) = (< G \cup \{I_1, \dots, I_n\} >, *) = \{(a_0, a_1 I_1, \dots, a_n I_n);\ a_i \in G\}$.

It is easy to see that $N_n(G)$ is closed under $*$, and it is a semigroup but not a group since I_i has no inverse with respect to $*$ in general.

Remark 2.12: [1]

If (G,+) is an additive abelian group, then addition on $N_n(G)$ can be described as follows:

Consider $x = (a_0, a_1 I_1, \dots, a_n I_n), y = (b_0, b_1 I_1, \dots, b_n I_n)$, we have

$x + y = (a_0 + b_0, [a_1 + b_1]I_1, \dots, [a_n + b_n]I_n)$. In this case $(N_n(G), +)$ is a classical abelian group.

The identity element is (0,0,…,0).

It is easy to see that $N_n(G) \cong G \times G \times .. \times G\ (n + 1\ times)$ in the case of abelian additive group G.

Remark 2.13: [1]

If G is a multiplicative group, then group product on $N_n(G)$ can be described as follows:

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Consider $x = (a_0, a_1I_1, \dots, a_nI_n), y = (b_0, b_1I_1, \dots, b_nI_n)$, we have

$xy = (t_0, t_1, \dots, t_n);\ t_s = \prod_{i,j=0}^{n}(a_ib_j)I_iI_j\ ;\ I_0 = e_G\ and\ I_iI_j = I_s.$

The identity element is $(e_G, e_GI_1, \dots, e_GI_n)$.

In this case $N_n(G)$ is not isomorphic to the direct product of n+1 copies of G, since it is not a classical group in this case.

The binary operation between the sub-indeterminacies is $I_i.I_j = I_{\min(i,j)}$.

3. Main discussion

Definition 3.1 :

Let $(G,*)$ be any group, the pure neutrosophic group GI is the set $\{x*I\ :for\ every\ x\ from\ G\}$.

Example 3.2 :

Let $G = (Z_5, +)$, then the corresponding pure neutrosophic group is

$\{0+I, 1+I, 2+I, 3+I, 4+I\}$, since (*) is considered as (+) between the elements of G.

Theorem 3.3 :

The set GI has a group structure under the binary operation (xI)(yI)=(xy)I with identity I and $f: G \to GI$;

$f(x) = xI$ is an isomorphism.

Proof :

It is easy to see that GI is a group under the previous operation with identity I.

We have $f(xy) = xyI = (xI)(yI) = f(x)f(y)$, so f is a homomorphism, and f is a one to one correspondence, thus f is an isomorphism clearly.

Remark 3.4 :

For every subgroup H of G, we can find an isomorphic subgroup with form HI of GI.

Definition 3.5 :

Let N(G) be a neutrosophic group and K be a subset of N(G), we say that K is an AH-subgroup if K=$H \cup T$ such that H is a subgroup of G and T is a subgroup of GI.

Example 3.6 :

Let $G = Z_4$, then K={$0, 2, I$} is an AH-subgroup of N(G) because {0, 2 } is a subgroup of G and {I} is a subgroup of the pure neutrosophic group GI under the operation defined in Theorem 3.3.

Definition 3.7 :

Let N(G) be a neutrosophic group, K be an AH-subgroup, we say that K is AHS-subgroup if T≅H.

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Definition 3.8 :

Let N(G) be a neutrosophic group, K be an AH-subgroup we say that K is an AH-normal subgroup if H is normal in G and T is normal in GI.

Definition 3.9 :

(a) Let N(G) be a neutrosophic group and K be an AHS-subgroup, we say that K is an AHS-normal subgroup if it is AH-normal.

(b) Let N(G) be a neutrosophic group, $K = K_1 \cup K_2$, $S = S_1 \cup S_2$ be two AH-subgroups. The intersection is defined as $K \cap S = (K_1 \cap S_1) \cup (K_2 \cap S_2)$.

Theorem 3.10 :

Let N(G) be a neutrosophic group, then

(a) If K, S are two AH-subgroups, $K \cap S$ is AH-subgroup.

(b) If K, S are two AHS-subgroups, $K \cap S$ is AHS-subgroup.

(c) If K, S are two AH-normal subgroups, $K \cap S$ is AH-normal subgroup.

(d) If K, S are two AHS-normal subgroups, $K \cap S$ is AHS-normal subgroup.

Proof :

(a) Suppose that $K = K_1 \cup K_2$, $S = S_1 \cup S_2$, then $K \cap S = (K_1 \cap S_1) \cup (K_2 \cap S_2)$ is an AH-subgroup, because

$K_1 \cap S_1$ is a subgroup of G and $K_2 \cap S_2$ is a subgroup of GI.

(b) The proof holds directly from (a).

(c) If K_1 , S_1 are normal subgroups of G and K_2 , S_2 are normal in GI, then $K_2 \cap S_2$ is normal in GI and $K_1 \cap S_1$ is normal in G, thus $K \cap S = (K_1 \cap S_1) \cup (K_2 \cap S_2)$ is AH-normal subgroup.

(d) It holds from (c).

Definition 3.11 :

Let N(G) be a neutrosophic subgroup with two AH-subgroups K=$K_1 \cup K_2$, S=$S_1 \cup S_2$.

We define KS = $K_1S_1 \cup K_2S_2$.

Theorem 3.12 :

Let N(G) be a neutrosophic subgroup with two AH-normal subgroups K=$K_1 \cup K_2$, S=$S_1 \cup S_2$, then KS is an AH-normal subgroup.

Proof :

Since K_1S_1 is normal subgroup of G and K_2S_2 is normal in GI. The proof is complete.

Definition 3.13:

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Let N(G) be a neutrosophic subgroup with an AH-normal subgroup K=$K_1 \cup K_2$. We define the AH-Quotient N(G)/K= $G/K_1 \cup GI/K_2$.

If K is an AHS-normal subgroup, then N(G)/K is called AHS-Quotient.

The AHS-Quotient N(G)/K must be understood as $G/K_1 \cup (G/K_1)I$ because $K_1 \cong K_2$.

Remark 3.14 :

If K=$K_1 \cup K_2$ and S=$S_1 \cup S_2$ are two AH-subgroups, we say that $K \cong S\ if\ and\ only\ if\ K_1 \cong S_1\ and\ K_2 \cong S_2$.

Theorem 3.15 :

Let N(G) be a neutrosophic group with an AH-normal subgroup K. Then

(a) If N(G) is abelian, then N(G)/K is abelian.

(b) If K is an AHS-normal subgroup, and $xK = yK$ for $x , y \in G$, then $xy^{-1} \in K_1$. Also, if $y = zI \in GI$, then $xz^{-1} \in K_1$.

(c) If G is finite and K is AHS-subgroup, o(K) will divide o(N(G)).

Proof :

(a) It can be proved as the classical case.

(b) Suppose that y$\in G$, then by the proposition we have $xK_1 \cup xIK_2 = yK_1 \cup yIK_2$, so $xK_1 = yK_1$, thus $xy^{-1} \in K_1$.

Now if $y = zI \in GI$, then $xK_1 \cup xIK_2 = zK_1 \cup zIK_2$, hence $xK_1 = zK_1$, thus $xz^{-1} \in K_1$.

(c) It holds directly from Lagrange's theorem.

Example 3.16 :

Let $G = Z_6$ be the group of integers modulo 6 with respect to addition, we have {0, 2, 4, I, 3+I} is an AH-normal subgroup, because {0,2,4} is normal in G and {I , 3+I} is normal in GI. The corresponding AH-quotient is

{ 1+{ 0 , 2, 4 } , {0 , 2, 4 } , {I, 3+I} , (1+I)+{I, 3+I} , (2+I)+{I, 3+I} }.

{0, 3 , I , 3+I } is an AHS-normal subgroup of N(G) and the related AHS-quotient is

{ {0,3} , 1+{0,3} , 2+{0,3} , {I, 3+I} , (1+I)+{I, 3+I} , (2+I)+{I, 3+I} }.

Theorem 3.17 :

Let N(G) be a neutrosophic group, K be an AH-normal subgroup, S be an AH-subgroup of N(G)/K, then there is an AH-subgroup T of N(G) such that S is contained in T as an AH-normal subgroup.

Proof :

Suppose that S is an AH-subgroup of N(G)/K, then $S = S_1 \cup S_2$ such S_1 is a subgroup of $G/K_1$and $S_2\ is\ a\ subgroup$ of GI/K_2, so that $S_1 = T_1/K_1\ , S_2 = T_2/K_2$ where T_1 , T_2 are two subgroups of G , GI respectively, and $K_1 \leq T_1 , K_2 \leq T_2$, we put $T = T_1 \cup T_2$, thus we get the proof.

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Definition 3.18 :

Let N(G) , N(H) be two neutrosophic groups with neutrosophic elements I , I' respectively, we define the AH-direct product $N(G) \times N(H)$ as a union

$(G \times H) \cup (G \times HI') \cup (GI \times H) \cup (GI \times HI')$.

For more comprehension of AH-structures we shall introduce the following definition.

Definition 3.19 :

Let N(G) , N(H) be two neutrosophic groups and $f_G: G \to H$ be a homomorphism, we define the corresponding AHS-homomorphism as follows:

$f: N(G) \to N(H); f(x + yI) = f_G(x) + f_G(y)I$.

We define $AH - Ker(f) = Ker(f_G) \cup Ker(f_{GI})$. It is easy to see that $Kerf_G \cong Kerf_{GI}$, so that $AH - Ker(f)$ is an AHS-subgroup of N(G).

We can understand the AH-Kernel as a union, $AH - ker(f) = Kerf_G \cup Kerf_G I$.

Theorem 3.20 :

Let f be an AHS-homomorphism between N(G) and N(H). We have

(a) $AH - Ker(f)$ is an AHS-normal subgroup of N(G).

(b) N(G)/$AH - Ker(f) \cong f_G(G) \cup f_G(G)I$. (The isomorphism here is taken by the concept of AHS-isomorphism).

The quotient in (b) is taken as AH-quotient.

Proof :

(a) Since $Ker(f_G)$ and $Ker(f_{GI})$ are normal in G , GI respectively, $AH - Ker(f)$ is AHS-normal.

(b) We have $G/Kerf_G \cong f_G(G)$ and $GI/Kerf_{GI} \cong f_{GI}(GI)$,

$f(N(G)) = f(G) \cup f(GI) \cong G/Ker(f_G) \cup GI/Ker(f_{GI}) =$ N(G)/$AH - Ker(f)$.

Theorem 3.21 :

Let N(G) be a neutrosophic group and $K = K_1 \cup K_2$, $H = H_1 \cup H_2$ be two neutrosophic AHS-normal subgroups with $H \leq K$. Then

$(N(G)/H)/(K/H) \cong N(G)/K$. (Quotients and isomorphisms are taken as AH-concept).

Proof :

We have $(N(G)/H)/(K/H) = (G \cup GI/H_1 \cup H_2) / (K_1 \cup K_2/H_1 \cup H_2) = G/H_1 \cup (GI/H_2)/(K_1/H_1) \cup (K_2/H_2)$.

So $(N(G)/H)/(K/H) \cong G/H_1/(K_1/H_1) \cup (GI/H_2)/(K_2/H_2) \cong G/K_1 \cup GI/K_2 = N(G)/K$.

Theorem 3.22:

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Let N(G) , N(H) be two neutrosophic groups with two AHS-normal subgroups $K = K_1 \cup K_2$, $S = S_1 \cup S_2$ respectively, Then

$N(G) \times N(H)/K \times S \cong N(G)/K \times N(H)/S$. (Direct products and isomorphisms are taken as AH-concepts).

Proof :

We have $N(G) \times N(H)/K \times S =$
$(G \times H) \cup (G \times HI') \cup (GI \times H) \cup (GI \times HI')/(K_1 \times S_1) \cup (K_1 \times S_2) \cup (K_2 \times S_1) \cup (K_2 \times S_2) =$
$(G \times H/K_1 \times S_1) \cup (G \times HI'/K_1 \times S_2) \cup (GI \times H/K_2 \times S_1) \cup (GI \times HI'/K_2 \times S_2) \cong N(G)/K \times N(H)/S$.

We will construct some examples to clarify the concepts defined previously.

Example 3.23:

Let $G = (Z, +), H = (Z_{12}, +)$ be two groups, $N(G), N(H)$ be their corresponding neutrosophic groups,

$f_G: Z \to Z_{12}; f(x) = x\ mod 12$ be a homomorphism, $Ker(f_G) = 12Z$, $GI = G + I = \{x + I; x \in Z\}$, since the considered operation in G is addition, $HI = H + I = \{x + I; x \in Z_{12}\}$, since the considered operation in H is addition modulo 12.

(a) $S_1 = 3Z, S_2 = 6Z + I = \{6x + I; x \in Z\}$ are two subgroups of G, G+I respectively. $K = S_1 \cup S_2$ is an AH-subgroup of N(G).

(b) $f: N(G) \to N(H); f(x + yI) = f_G(x) + f_G(y)I = (x\ mod 12) + (y\ mod 12)I$ is an AH-homomorphism.

(c) $AH - Ker(f) = Ker(f_G) \cup Ker(f_G)I = 12Z \cup (12Z + I)$ which is an AHS-subgroup.

(d) The AH-quotient
$N(G)/AH - Ker(f) = G/12Z \cup [G/12Z + I] \cong Z_{12} \cup [Z_{12} + I] = H \cup [H + I] = f_G(G) \cup f_G(G)I.$

(d) $f(K) = f_G(S_1) \cup f_G(S_2) = \{0,3,6,9\} \cup (\{0,6\} + I) = \{0,3,6,9, I, 6 + I\}$ which is an AH-subgroup of N(H).

(e) $K, f(K)$ are AH-normal, since commutavity implies normality in abelian groups.

(f) The AH-direct product of N(H) with itself is equal to

$N(H) \times N(H) = (H \times H) \cup (H \times HI) \cup (HI \times H) \cup (HI \times HI)$, according to Definition 3.18.

Example 3.24:

Let $G = Z_{12}$ be the group of integers modulo 6, $H = \{0,2,4,6,8,10\}$, $K = \{0,3,6,9\}, S = \{0,6\}$ are normal subgroups of G.

$M = H \cup KI = \{0,2,4,6,8,10\} \cup (\{0,3,6,9\} + I) = \{0,2,4,6,8,10, I, 3 + I, 6 + I, 9 + I\}$ is an AH-normal subgroup, $N = K \cup SI = \{0,3,6,9\} \cup (\{0,6\} + I) = \{0,3,6,9, I, 6 + I\}$ is another AH-normal subgroup, we clarify Theorem 3.12 as follows:

$MN = HK \cup KSI = \{h + k; h \in H, k \in K\} \cup (\{k + s; k \in K, s \in S\} + I)$=

$\{0,1,2,3,4,5,6,7,8,9,10,11\} \cup \{I, 3 + I, 6 + I, 9 + I\} = \{0,1,2,3,4,5,6,7,8,9,10,11, I, 3 + I, 6 + I, 9 + I\}$, which is an AH-normal subgroup.

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For a future research, we will show some definitions and new concepts related to AH-structures, such as AH-solvability, AH-nilpotency, and AH-cyclicity.

Definition 3.25:

Let $N(G)$ be a neutrosophic group, $N(H) = T \cup S; T \leq G, S \leq GI$ be an AH-subgroup, we say

(a) $N(H)$ is an AH-solvable subgroup if T, S are solvable.

(b) $N(H)$ is an AH-nilpotent subgroup if T, S are nilpotent.

(c) $N(H)$ is an AH-abelian subgroup if T, S are abelian.

(d) $N(H)$ is an AH-cyclic subgroup if T, S are cyclic.

Example 3.26:

Let $G = S_3, T =< (1\ 2) >, S =< (1\ 2\ 3) >$ be two subgroups of G, we have

(a) $SI = \{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} I, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} I, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} I\}$ is a subgroup of GI.

(b) $N(H) = T \cup S = \{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} I, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} I, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} I\}$ is an AH-subgroup of N(G).

(c) N(H) is AH-cyclic and AH-abelian subgroup, since T,S are cyclic and abelian.

(d) N(H) is AH-Nilpotent and AH-solvable, since T,S are nilpotent and solvable.

5. Conclusion

In this article, we have defined the concept of AH-homomorphism in a neutrosophic group for the first time. Also, we have introduced some corresponding notions such as AH-subgroup, AH-normailty, and AH-factors. Many examples and theorems were constructed to clarify the validity of these concepts.

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