Homomorphism and Isomorphism in strong neutrosophic graphs

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Abstract

The concept of strong neutrosophic graph is developed with example and some of their properties are investigated in this paper. Some definitions, homomorphism, isomorphism theorems and propositions in strong neutrosophic graphs are established. Basic operations(like union, intersection etc.,) and complement of strong neutrosophic graphs are also derived here.

Keywords: Strong neutrosophic graph, Complete neutrosophic graph, Complement of a neutrosophic graph, Cartesian product, Isomorphism.

1.Introduction

The concept of graph theory was first introduced by Euler in 1736[7]. It is an useful tool for solving problems in different mathematical areas such as algebra, operations research, optimization and computer science. Rosenfeld(1975) introduced the notion of fuzzy graph and several fuzzy analogs of theoretic concepts such as paths, cycles and connectedness[18]. Fuzzy models are very useful to reduce the difference between the traditional numerical models used in science and engineering and symbolic models used in expert system. Atanassov introduced the concept of intuitionistic fuzzy sets as a generalisation of fuzzy sets [2,3]. The fuzzy sets give the degree of membership of an element in a given set (and the non membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non membership which are more or less independent from each other with the condition that the sum of these two degrees is not greater than 1. Also many researchers established and studied about fuzzy graphs and intuitionistic fuzzy graphs in [4,8,9,10,11, 13,14,15,16]. Neutrosophic set proposed by Smarandache [20] is a powerful tool for dealing incomplete inconsistance, imprecision, uncertain, false and indeterminate problems in the real world whenever the fuzzy and intuitionistic fuzzy approaches fail in such type of situations and it is the generalization of classical sets, fuzzy set, intuitionistic fuzzy set and interval valued fuzzy set etc,. So Smarandache defined four main categories of neutrosophic graphs in [21, 22] to handle these type of situations. M.Mullai[23introduced the concept of domination

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in neutrosphic graphsIn this paper, strong neutrosophic graph is developed with examples. Also, some definitions, theorems and propositions related to opreations, homomorphism, isomorphism and complement of strong neutrosophic graphs are established.

2 Preliminaries

Basic definitions which are necessary to this article are reviewed in this section.

Definition 0.1 [5] A fuzzy graph $G:(\sigma,\mu)$ is a pair of functions $\sigma: S \to [0,1]$ and $\mu: S \times S \to [0,1]$, we have $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$ for all, x, y ∈ S, where ∧ stands for minimum and σ is a subset of a nonempty set S and μ is a fuzzy relation on σ .

Definition 0.2 [12] A fuzzy graph G is connected if $\mu^{\infty}(u,v) > 0$ for all , $u, v \in S$.

Definition 0.3 [12] A fuzzy graph G is said to be a strong fuzzy graph if $\mu(x,y) = \sigma(x) \wedge \sigma(y)$, for all $(x,y) \in S \times S$.

Definition 0.4 [12] A fuzzy graph G is said to be a complete fuzzy graph if $\mu(x,y) = \sigma(x) \wedge \sigma(y)$, for all (x,y) ∈ S.

Definition 0.5 [6] An arc (x,y) is said to be a strong arc if $\mu(x,y) \geq \mu^{\infty}(x,y)$. A node X is said to be an isolated node if $\mu(x,y) = 0$, for all, $y \neq x$.

Definition 0.6 [5] A fuzzy subset μ on a set X is a map $\mu: X \to [0,1]$. A map $\nu: X \times X \to [0,1]$ is called a fuzzy relation on X if $\nu(x,y) \leq \min(\mu(x),\mu(y))$ for all, $x, y \in X$. A fuzzy relation $\nu(x,y) = \nu(y,x)$ for all, $x, y \in X$.

Definition 0.7 [5] Let $G:(\sigma,\mu)$ be a fuzzy graph.The complement of G is defined as $\overline{G}:(\sigma,\overline{\mu})$ where $\overline{\mu}(x,y) = \sigma(x) \wedge \sigma(y) - \mu(x,y)$, for all $x, y \in S$. When G is a fuzzy graph, $\overline{G}:(\sigma,\overline{\mu})$ is also a fuzzy graph.

Definition 0.8 [17] A mappings $A = (\mu_A, \nu_A): X \to [0,1] \times [0,1]$ is called an intuitionistic fuzzy set in X, if $\mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$, where the mapping $\mu_A: X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\nu_A(x)$) of each element $x \in X$ to A , respectively.

Definition 0.9 [17] For every two intuitionistic fuzzy sets $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ in X , we define

$$(A \cap B)(x) = (\min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)))).$$

$$(A \cup B)(x) = (\max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)))).$$

Definition 0.10 [17] Let X be a non empty set. Then we call a mapping $A = (\mu_A, \nu_A): X \to [0,1] \times [0,1]$ an intuitionistic fuzzy relation on X if $A = (\mu_A(x,y) + \nu_A(x,y) \leq 1$ for all $(x,y) \in X \times X$.

Definition 0.11 [5] Let $A = (\mu_A, \nu_A)$ and $B = (\mu_A, \nu_B)$ be intuitionistic fuzzy sets on a set X. If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy relation on a set X, then $A = (\mu_A, \nu_A)$ is called an intuitionistic fuzzy relation on $B = (\mu_B, \nu_B)$ if

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$\mu_A(x,y) \leq \min(\mu_B(x), \mu_B(y))$ and $\nu_A(x,y) \geq \max(\nu_B(x), \nu_B(y))$ for all, $x, y \in X$. An intuitionistic fuzzy relation A on X is called symmetric if $\mu_A(x,y) = \mu_A(y,x)$ and $\nu_A(x,y) = \nu_A(y,x)$ for all, $x, y \in X$.

Definition 0.12 [19] Let X be a space of points (objects) with generic elements in X denoted by X , then the neutrosophic sets A (NS A) is an object having the form

$$A = \{< x: T_A(x), I_A(x), F_A(x) > 0\},\ x \in X,$$

where the functions $T, I, F: X \to]0^-, 1^+[$ define respectively the truth membership function, an indeterminacy membership function , and a falsity membership function of the element $x \in X$ the set A with condition

$$0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3 +$$

The function $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non standard subsets of $]0^-, 1[= 0$.

Definition 0.13 [19] Let X be a space of points (objects) with generic elements in X is denoted by X. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point x in X $T_A(x), F_A(x)$ and $I_A(x) \in [0,1]$. A SVNS A can be written as

$$A = \{< T_A(x), I_A(x), F_A x >, x \in X\}$$

Definition 0.14 [19] Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be single valued neutrosophic sets on a set x. If $A = (T_A, I_A, F_A)$ is a single valued neutrosophic relation on a set X, then $A = (T_A, I_A, F_A)$ is called a single valued neutrosophic relation on $B = (T_B, I_B, F_B)$ if

$$T_B(x,y) \leq \min(T_A(x), T_A(y))$$

$$I_B(x,y) \geq \max(I_A(x), I_A(y))$$

$$\text{and } F_B(x,y) \geq \max\big(F_A(x), F_A(y)\big) \text{ for all, } x, y \text{ in } X,$$

A single valued neutrosophic relation A on X is called symmetric if

$$T_A(x,y) = T_A(y,x), I_A(x,y) = I_A(y,x)$$

$$F_A(x,y) = F_A(y,x), \text{and}$$

$$T_B(x,y) = T_B(y,x), I_B(x,y) = I_B(y,x)$$

$$F_B(x,y) = F_B(y,x) \text{for all, } x, y \in X,$$

Definition 0.15 [19] A single valued neutrosophic graph (SVN - graph) with underlying set V is defined to be a pair $G = (A, B)$ where,

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(i) The functions $T_A: V \to [0,1], I_A: \to [0,1] and F_A: V \to [0,1]$ denote the degree of truth membership, degree of indeterminacy membership, and degree of falsity membership of the element $v_i \in V$, respectively and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$$

for all, $v_i \in V(i = 1,2, \dots . n)$ ii) The functions $T_B: E \subseteq V \times V \to [0,1], I_B: E \subseteq V \times V \to [0,1]$, and

$F_B: E \subseteq V \times V \to [0,1]$ are defined by

$$T_B(v_i, v_j) \leq \min(T_A(v_i), T_A(v_j)),$$

$$I_B(v_i, v_j) \geq \max(I_A(v_i), I_A(v_j), \text{and}$$

$$F_B(v_i, v_j) \geq \max(F_A(v_i), F_A(v_j).$$

denote the degree of truth membership , degree of indeterminacy membership ,and degree of falsity membership of the edge $(v_i), v_j) \in E$ respectively , where

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3,$$

for all, $(v_i), (v_j) \in E(i, j = 1,2, .. n)$.

Definition 0.16 [3] We say a fuzzy subgraph $H = (\sigma, \tau)$ is a full spanning fuzzy subgraph $G = (\sigma, \mu)$ on (V,X) if H is a spanning fuzzy subgraph of G and for all, $u, v \in V$ either $\tau(u, v) = 0$ or $\tau(u, v) = \mu(u, v)$.

3 Strong neutrosophic graphs

The concept of strong neutrosophic graph with examples, operations, properties, propositions, homomorphism and isomorphism theorems on strong neutrosophic graphs are introduced and discussed briefly here.

Definition 0.17 [19] An neutrosophic graph G = (A,B) is called strong neutrosophic graph if

$$T_B(xy) = \min(T_A(x), T_A(y)) and I_B(xy) = \max(I_A(x), I_A(y)), \text{and}$$

$$F_B(xy) = \max(F_A(x), F_A(y)) \text{ for all, } xy \in E.$$

Definition 0.18 [19]

Let $A = (T_A, I_A, F_A)$ and $A' = (T'_A, I'_A, F'_A)$ be neutrosophic fuzzy subsets of V_1 and V_2 and let $B = (T_B, I_B, F_B)$ and $B' = (T'_B, I'_B, F'_B)$ neutrosophic subsets of E_1 and E_2, respectively. The cartesian product of two strong neutrosophic graphs G_1^* and G_2^* is denoted by $G_1 \times G_2 = (A \times A', B \times B')$ and is defined as follows :

$$1. \begin{cases} ((T_A \times T'_A)(x_1, x_2) = \min((T_A(x_1, T'_A(x_2)) \\ ((I_A \times I'_A)(x_1, x_2) = \max((I_A(x_1, I'_A(x_2)) \quad \text{forall}, (x_1, x_2) \in V , \\ ((F_A \times F'_A)(x_1, x_2) = \max((F_A(x_1, F'_A(x_2)), \end{cases}$$

2. $\begin{cases} ((T_B \times T'_B)((x, x_2), (x, y_2)) = \min(T_A(x), T'_B(x_2, y_2)) \\ ((I_B \times I'_B)((x, x_2), (x, y_2)) = \max(I_A(x), I'_B(x_2, y_2)) \\ ((F_B \times F'_B)((x, x_2), (x, y_2)) = \max(F_A(x), F'_B(x_2, y_2)), \end{cases}$

for all, $x \in V_1 (x_1, x_2) \in E_2$

3. $\begin{cases} ((T_B \times T'_B)(x_1, z), (y_1, z)) = \min((T_B(x_1, y_1), T'_B(z)) \\ ((I_B \times I'_B)(x_1, z), (y_1, z)) = \max((I_B(x_1, y_1), I'_B(z)) \\ ((F_B \times I'_B)(x_1, z), (y_1, z)) = \max((F_B(x_1, y_1), F'_B(z)), \end{cases}$

for all, $z \in V_2$ for all, $x_1 y_2 \in E_1$.

Proposition 0.19 If G_1 and G_2 are the strong neurotrosophic graphs, then $G_1 \times G_2$ is a strong neutrosophic graph.

Proof:

It is a straight forward.

Proposition 0.20 If $G_1 \times G_2$ is strong neutrosophic graph, then at least G_1 or G_2 must be strong .

Proof:

Suppose that G_1 and G_2 are not strong neutrosophic graphs.

Then there exit $x_1 y_1 \in E_1$ and $x_2 y_2 \in E_2$ such that,

$$(T_{B_1}(x_1 y_1)) < min(T_{A_1}(x), T_{A_1}(y)), (T_{B_2}(x_1 y_1) < min(T_{A_2}(x), T_{A_2}(y))$$

$(I_{B_1}(x_1 y_1) > max(I_{A_1}(x), I_{A_1}(y)), (I_{B_2}(x_1 y_1) > max(I_{A_2}(x), I_{A_2}(y))$

$(F_{B_1}(x_1 y_1) > max(F_{A_1}(x), F_{A_1}(y)), (F_{B_2}(x_1 y_1)) > max(F_{A_2}(x), F_{A_2}(y))$

Assume that, $T_{B_2}(x_2 y_2) \leq T_{B_1}(x_1, y_1) < min(T_{A_1}(x_1), T_{A_1}(y_1)) \leq T_{A_1}(x_1)$ Let $E = (x, x_2), (x, y_2)/x_1 \in V_1, (x_2 y_2) \in E_2 \cup ((x_1, z), (y_1, z))/z \in V_2, \forall, x_1 y_1 \in E_1$.

Consider, $(x, x_2), (x, y_2) \in E$,

We have, $(T_{B_1} \times T_{B_2}((x, x_2), (x, y_2)) = \min(T_{A_1}(x), T_{B_2}(x_2 y_2))$

$< (T_{A_1}(x), T_{A_2}(x_2), T_{A_2}(y_2))$and

$(T_{A_1} \times T_{A_2})((x_1, x_2)) = \min(T_{A_1}(x_1), T_{A_2}(x_2), T_{A_1} \times T_{A_2})(x_1, y_2)$

$= \min(T_{A_1}(x_1), T_{A_2}(y_2))$

Therefore,

$$\min(T_{A_1} \times T_{A_2})(x, x_2), T_{A_1} \times T_{A_2}(x, y_2) = \min(T_{A_1}(x), T_{A_2})(x_2), T_{A_2}(y_2))$$

Hence $(T_{B_1} \times T_{B_2})((x, x_2), (x, y_2)) < min((T_{A_1} \times T_{A_2})(x, x_2), (T_{A_1} \times T_{A_2})(x, y_2))$

Similarly , we can easily show that

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$$(I_{B_1} \times I_{B_2})((x, x_2), (x, y_2)) > max(I_{A_1} \times I_{A_2}(x, x_2)), (I_{A_1} \times I_{A_2})(x, y_2))$$

$$(F_{B_1} \times F_{B_2})((x, x_2), (x, y_2)) > max(F_{A_1} \times F_{A_2})(x, x_2), (F_{A_1} \times F_{A_2})(x, y_2))$$

That is $G_1 \times G_2$ is not strong neutrosophic graph, a contradiction . Hence, this ends the proof .

Remark 0.21 If G_1 is strong and G_2 is not strong , $G_1 \times G_2$ may or may not be strong.

Proposition 0.22 Let G_1 be a strong neutrosophic graph of G_1^* . Then for every neutrosophic graph G_2 of G_2^* , $G_1 \times G_2$ is strong if and only if

$T_A(x_1) \le T_B(x_2y_2)$, $I_A(x_1) \le I_B(x_2y_2)$ and

$F_A(x_1) \le F_B(x_2y_2)$, for all, $x_1 \in V_1$ and $x_2y_2 \in E_2$.

Definition 0.23 [19]

Let $A = (T_A, I_A, F_A)$ and $A' = (T'_A, I'_A, F'_A)$ be neutrosophic subsets of V_1 and V_2 and let $B = (T_B, I_B, F_A)$and$B' = (T'_B, I'_B, F'_A)$ be neutrosophic subsets of E_1 and E_2 , respectively. The composition of two strong neutrosophic graphs G_1 and G_2 of the graphs G_1^* and G_2^* is denoted by $G_1[G_2] = (AoA', BoB')$ and is defined as follows :

1. $$\begin{cases} (T_A o T'_A)(x_1, x_2) = \min(T_A(x_1, T'_A(x_2)) \\ (I_A o I'_A)(x_1, x_2) = \max((I_A(x_1, I'_A(x_2)) \\ (F_A o F'_A)(x_1, x_2) = \max((F_A(x_1, F'_A(x_2)) \end{cases}$$

for all, $(x_1, x_2) \in V$,

2. $$\begin{cases} (T_B o T'_B)(x, x_2), (x, y_2) = \min(T_A(x), T'_B(x_2y_2)) \\ (I_B o I'_B)(x, x_2), (x, y_2) = \max((I_A(x), I'_B(x_2y_2)) \\ (F_B o F'_B)(x, x_2), (x, y_2) = \max(F_A(x), F'_B(x_2y_2)) \end{cases}$$

for all, $x \in V_1, \text{forall}, x_2y_2 \in E_2$

3. $$\begin{cases} (T_B o T'_B)(x_1, z), (y_1, z) = \min(T_B(x_1y_1), T'_A(z) \\ (I_B \times I'_B)(x_1, z), (y_1, z) = \max(I_B(x_1y_1), I'_A(z)) \\ (F_B \times F'_B)(x_1, z), (y_1, z) = \max(F_B(x_1y_1), F'_A(z)) \end{cases}$$

for all, $z \in V_2$, for all $x_1y_1 \in E_1$.

4. $$\begin{cases} (T_B o T'_B)((x_1, x_2), (y_1, y_2)) = \min(T'_A(x_2) T'_A(y_2), T_B(x_1y_1)), \\ ((I_B o I'_B)((x_1, x_2)(y_1, y_2)) = \max(I'_A(x_2), I'_A(y_2), I_B(x_1y_1)), \\ ((F_B o F'_B)((x_1, x_2)(y_1, y_2)) = \max(F'_A(x_2), F'_A(y_2), F_B(x_1y_1)), \end{cases}$$

for all, $((x_1, x_2)(y_1, y_2) \in E^0 - E$

We state the following propositions without proofs .

Proposition 0.24 If G_1 and G_2 are the strong neutrosophic graphs , then $G_1[G_2]$ is a strong neutrosophic graph.

Proposition 0.25$G_1[G_2]$ is strong neutrosopsic graph , then at least G_1 or G_2 must be strong.

Definition 0.26 [19]$A = (T_A, I_A, F_A)$ and $A' = (T'_A, I'_A, F'_A)$ be neutrosophic fuzzy subsets of V_1 and V_2 and let $B = (T_B, I_B, F_A)$and$B' = (T'_B, I'_B, F'_B)$ be neutrosophic subsets of E_1 and E_2 , respectively. The joint of two strong neutrosophic graphs G_1 and G_2 of the graphs G_1^* and G_2^* is denoted by $G_1[G_2] = (A + A', B + B')$ and is defined as follows :

1. $\begin{cases}(T_A + T'_A)(x) = (T_A + T'_A)(x)),\\ (I_A + I'_A)(x) = (I_A + I'_A)(x)),\\ (I_A + I'_A)(x) = (I_A + I'_A)(x)),\end{cases}$

if $x \in V_1 \cup V_2$,

2. $\begin{cases}(T_B + T'_B)(xy) = (T_B \cup T'_B)(xy) = T_B(xy),\\ (I_B + I'_B)(xy) = (I_B \cap I'_B)(xy) = I_B(xy),\\ (F_B + F'_B)(xy) = (F_B \cap F'_B)(xy) = F_B(xy),\end{cases}$

if $xy \in E_1 \cup E_2$,

3. $\begin{cases}(T_B + T'_B)(xy) = \min(T_A(x), T'_A)(y),\\ (I_B + I'_B)(xy) = \max(I_A(x), I'_A)(y),\\ (F_B + F'_B)(xy) = \max(F_A(x), F'_A)(y),\end{cases}$

if $xy \in E'$.

Proposition 0.27 If G_1 and G_2 are the strong neutrosophic graphs, then $G_1 + G_2$ is a strong neutrosophic graph.

Definition 0.28 [19]$A = (T_A, I_A, F_A)$ and $A' = (T'_A, I'_A, F'_A)$ be neutrosophic subsets of v_1 and V_2 and let $B = (T_B, I_B, F_A)$ and $B' = (T'_B, I'_B, F'_A)$ be neutrosophic subsets of E_1 and E_2 , respectively . The union of two strong neutrosophic graphs $G_1 \cup G_2$ of the graphs G_1^* and G_2^* is denoted by $G_1 \cup G_2 = (A \cup A', B \cup B')$ and is defined as follows :

1. $\{(T_A \cup T'_A)(x) = T_A(x), x \in V_1 \cup \overline{V}_2,(T_A \cup T'_A)(x) = T'_A(x), x \in V_2 \cup \overline{V}_2(T_A \cup T'_A)(x) = \max(T_A(x), T'_A(x)), x \in V_1 \cup V_2,$

2. $\{(I_A \cap I'_A)(x) = I_A(x), x \in V_1 \cap \overline{V}_2,(I_A \cap I'_A)(x) = I'_A(x), x \in V_2 \cap \overline{V}_2, (I_A \cap I'_A)(x) = \min(I_A(x), I'_A(x)), x \in V_1 \cap V_2,$

3. $\{(F_A \cap F'_A)(x) = F_A(x), x \in V_1 \cap \overline{v}_2,(F_A \cap F'_A)(x) = F'_A(x), x \in V_2 \cap \overline{V}_2, (F_A \cap F'_A)(x) = \min(F_A(x), F'_A(x)), x \in V_1 \cap V_2,$

4. $\{(T_A \cup T'_A)(xy) = T_B(xy), xy \in E_1 \cap \overline{E}_2,(T_A \cup T'_A)(xy) = T'_B(xy), xy \in E_2 \cap \overline{E}_1, (T_A \cup T'_A)(xy) = \max(T_B(x), T'_B(x)), x \in E_1 \cap E_2,$

5. $\{(I_B \cap I'_B)(xy) = I_B(xy), x \in E_1 \cap \overline{E}_2(I_B \cap I'_B)(xy) = I'_B(xy), x \in E_2 \cap \overline{E}_1(I_B \cap I'_B)(xy) = \max(I_B(xy), I'_B(xy)), xy \in E_1 \cup E_2,$

6. $\{(F_B \cap F'_B)(xy) = F_B(xy), x \in E_1 \cap \overline{E}_2,(F_B \cap F'_B)(xy) = F'_B(xy), x \in E_2 \cap \overline{E}_1, (F_B \cap F'_B)(xy) = \max(F_B(xy), F'_B(xy)), xy \in E_1 \cap E_2,$

Definition 0.29 [19] The complement of a strong neutrosophic graph $G = (A, B)$ of $G^* = (V, E)$ is a strong neutrosophic graph $\overline{G} = (\overline{A}, \overline{B})onG^*$, where $\overline{A} = (\overline{T}_A, \overline{I}_A, \overline{F}_B)$ and $\overline{B} = (\overline{T}_B, \overline{I}_B, \overline{F}_B)$ are defined by

(i) $\overline{V} = V$,

(ii) $\overline{T}_A(x) = T_A(x), \overline{I}_A(x) = I_A(x)\overline{F}_A(x) = F_A(x)$ for all, $x \in V$, (iii)

1. $\overline{T}_A(xy) = \begin{cases} 0 & \text{if} T_B(xy) > 0 \\ \min(T_A(x), T_A(y) & \text{if} T_B(xy) = 0 \end{cases}$

2. $\overline{I}_A(xy) = \begin{cases} 0 & \text{if} I_B(xy) > 0 \\ \max(I_A(x), I_A(y) & \text{if} I_B(xy) = 0 \end{cases}$

3. $\overline{F}_A(xy) = \begin{cases} 0 & \text{if} F_B(xy) > 0 \\ \max(F_A(x), F_A(y) & \text{if} F_B(xy) = 0 \end{cases}$

Remark 0.30 If $G = (A, B)$ is an neutrosophic graph of $G^* = (V, E)$ Then from definition 12 , it follows that

$\overline{\overline{G}}$ is given by the neutrosophic graph $\overline{\overline{G}} = (\overline{\overline{A}}, \overline{\overline{B}})$ on $G^* = (V, E)$

where $\overline{\overline{A}} = A$ and

$$\overline{\overline{T}}_B(xy) = \min(T_A(x), T_A(y))\overline{\overline{I}}_B(xy) = \max(I_A(x), I_A(y))\text{and}$$

$$\overline{\overline{F}}_B(xy) = \max(F_A(x), F_A(y)), \text{for all, } xy \in E.$$

Thus $\overline{\overline{T}}_B = T_B$ and $\overline{\overline{I}}_B = I_B, \overline{\overline{F}}_B = F_B$ on V where $B = (T_B, I_B, F_B)$ is the strongest neutrosophic graph relation on A.

For any neutrosophic graph $G, \overline{\overline{G}}$ is strong neutrosophic graph and $G \subseteq \overline{\overline{G}}$.

The following propositions are obvious.

Proposition 0.31$G = \overline{\overline{G}}$ if and only if G is a strong neutrosophic graph .

Proposition 0.32 Let $G = (A_i, B_i)$ be a strong neutrosophic graph of $G_i^* = (V_i, E_i)$ for i = 1,2.

Then the following are true :

(a) $G_i \subseteq \overline{\overline{G}}_i$.

(b) $\overline{G}_i = (\overline{\overline{G}}_i)$,

(c) If $G_1 \subseteq G_2$, then $\overline{\overline{G}}_1 \subseteq \overline{\overline{G}}_2$.

$\overline{\overline{G}}$ is the smaller strong neutrosophic graph that contains $G_i^* = (V, E)$.

Definition 0.33 [23] A strong neutrosophic graph G is called self complementary if $G \approx \overline{G}$.

Proposition 0.34 Let G be a self complementary strong neutrosophic graph. Then

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$$\sum_{x\neq y} T_B(xy) = \sum_{x\neq y} \min(T_A(x), T_A(y))$$

$\sum_{x\neq y} I_B(xy) = \sum_{x\neq y} \max(I_A(x), I_A(y))$.

$\sum_{x\neq y} F_B(xy) = \sum_{x\neq y} \max(F_A(x), F_A(y))$.

Proposition 0.35 Let G be a strong neutrosophic graph. $T_B(xy) = \min(T_A(x), T_A(y))$, for all, $x, y \in V$, then G is self complementary .

Proof:

Let G be a strong neutrosophic graph such that

$$T_B(xy) = \min(T_A(x), T_A(y))$$

$$I_B(xy) = \max(I_A(x), I_A(y))$$

$$F_B(xy) = \max(F_A(x) \text{for all}, \ x, y \in V.$$

Then $G \approx \overline{G}$ under the identity map $I: V \to V$.

Hence ,G is self complementary .

Proposition 0.36 Let G_1 and G_2 be strong neutrosophic graphs. Then $G_1 \approx G_2$ if and only if

$\overline{G}_1 \approx \overline{G}_2$.

Proof:

$T_{A_1}(x) = T_{A_2}(f(x)), I_{A_1}(x) = I_{A_2}(f(x)) \text{and} F_{A_1}(x) = F_{A_2}(f(x))$ for all, $x \in V_1$,

$T_{B_1}(xy) = T_{B_2}(f(x)f(y)), I_{B_1}(xy) = I_{B_2}(f(x)f(y))$, and

$F_{B_1}(xy) = F_{B_2}(f(x)f(y))$, for all, $xy \in E_1$.

By definition of complement , we have

$\overline{T}_{B_1}(xy) = \min(T_{A_1}(x), T_{A_1}(y)) = \min(T_{A_2}(f(x)), T_{A_2}(f(y))) = \overline{T}_{B_2}(f(x)f(y))$,

$\overline{I}_{B_1}(xy) = \max(I_{A_1}(x), I_{A_1}(y)) = \max(I_{A_2}(f(x)), I_{A_2}(f(y))) = \overline{I}_{B_2}(f(x)f(y))$,

$\overline{F}_{B_1}(xy) = \max(F_{A_1}(x), F_{A_1}(y)) = \max(F_{A_2}(f(x)), F_{A_2}(f(y))) = \overline{F}_{B_2}(f(x)f(y))$,

For all, $xy \in E_1$.

Hence, $\overline{G}_1 \approx \overline{G}_2$

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The proof of the converse part is straight forward. This completes the proof.

Definition 0.37 [23] An neutrosophic fuzzy graph G = (A,B) is called complete if

$T_B(xy) = \min(T_A(x), T_A(y))$ and , $I_B(xy) = \max(I_A(x), I_A(y))$

$F_B(xy) = \max(F_A(x), F_A(y))$ for all, $xy \in E$.

We use the notion $C_m(G)$ for a complete neutrosophic fuzzy graph where $|V| = m$.

Proposition 0.38 An neutrosophic graph $G = (A, B)$ is called bigraph if and only if there exists neutrosophic graphs $G_i = (A_i, B_i)$ for $i = 1,2$ of

G = (A,B) such that $G = (A, B)$ is the join $G_1 + G_2$

where $V_1 \cap V_2 = \emptyset$ and $E_1 \cap E_2 = \emptyset$.

An neutrosophic bigraph is said to be complete if and only if

$$T_B(xy) > 0, I_B(xy) > 0, F_B(xy) > 0 \forall xy \in E$$

We use the notion $C_{m,n}$ (G) for a bigraph, where $|V_1| = m$ and $|V_2| = n$.

Proposition 0.39 $C_{m,n}(G) = C_m(G_1) + C_n(G_2)$.

Proof:

It is a straight forward.

Definition 0.40 [19] Let G_1 and G_2 be the strong neutrosophic graphs . A homomorphism $f: V_1 \to V_2$ which satisfies the following conditions :

(a) $T_{A_1}(x_1) = T_{A_2}(f(x_1)), I_{A_1}(x_1) = I_{A_2}(f(x_1))$, and

$F_{A_1}(x_1) = F_{A_2}(f(x_1))$,

(b) $T_{B_1}(x_1y_1) = T_{B_2}(f(x_1)f(y_1), I_{B_1}(x_1y_1) = I_{B_2}(f(x_1)f(y_1)$,and

$F_{B_1}(x_1y_1) = F_{B_2}(f(x_1)f(y_1)$, for all $x_1 \in V_1, x_1y_1 \in E_1$

Definition 0.41 [19] Let G_1 and G_2 be strong neutrosophic graphs . isomorphism $f: G_1 \to G_2$ is bijective mapping $f: V_1 \to V_2$ which satisfies the following conditions :

(c) $T_{A_1}(x_1) = T_{A_2}(f(x_1)), I_{A_1}(x_1) = I_{A_2}(f(x_1))$, and

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$F_{A_1}(x_1) = F_{A_2}(f(x_1))$,

(d)$T_{B_1}(x_1y_1) = T_{B_2}(f(x_1)f(y_1), I_{B_1}(x_1y_1) = I_{B_2}(f(x_1)f(y_1)$ and

$F_{B_1}(x_1y_1) = F_{B_2}(f(x_1)f(y_1)$, for all, $x_1 \in V_1, x_1, y_1 \in E_1$.

Definition 0.42 [19] Let G_1 and G_2 be strong neutrosophic graphs . Then a weak isomorphism $f: G_1 \to G_2$ is bijective mapping $f: V_1 \to V_2$ which satisfies the following conditions :

(e) f is homomorphism ,

(f) $T_{A_1}(x_1) = T_{A_2}(f(x_1)), I_{A_1}(x_1) = I_{A_2}(f(x_1)), \text{and} F_{A_1}(x_1) = F_{A_2}(f(x_1))$, for all, $x_1 \in V_1$.

Thus , a weak isomorphism preserves the weights of the nodes but not necessarily the weights of the arcs.

Definition 0.43 [19] Let G_1 and G_2 be strong neutrosophic graphs . A co weak isomorphism $f: G_1 \to G_2$ is bijective mapping $f: V_1 \to V_2$ which satisfies the following conditions :

(g) f is homomorphism ,

(h)$T_{B_1}(x_1y_1) = T_{B_2}(f(x_1)f(Y_1)), I_{B_1}(x_1y_1) = I_{B_2}(f(x_1)f(y_1))$, and

$F_{B_1}(x_1y_1) = F_{B_2}(f(x_1)f(y_1))$, for all $x_1 \in V_1$.

Thus, a co weak isomorphism preserves the weights of the arcs but not necessarily the weights of the nodes.

Remark 0.44

1. If $G_1 = G_2 = G$, then the homomorphism f over itself is called an endomorphism . An isomorphism f over G is called an automorphism
2. Let $A = (T_A, I_A, F_A)$ be a strong neutrosophic graph with an underlying set V . Let Aut(G) be the set of all strong neutrosophic automorphism of G . Let $e: G \to G$ be a map defined by $e(x) = x$ for all $x \in V$.

   Clearly , $e \in \text{Aut}(G)$.
3. $G_1 = G_2$, then the weak and co weak isomorphism actually become isomorphic.
4. If $V_1 \to V_2$ is a bijective map , then $f^{-}: V_1 \to V_2$ is also a bijective map.

We state the following propositions without their proofs.

Proposition 0.45 Let G_1 and G_2 be strong neutrosophic graphs. If there is a weak isomorphism between G_1 and G_2 , then there is a weak isomorphism between $\overline{G}_1$ and $\overline{G}_2$.

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Proposition 0.46 Let G_1 and G_2 be strong neutrosophic graphs. If there is a weak isomorphism between G_1 and G_2 , then there is a weak isomorphism between $\overline{G}_1$ and $\overline{G}_2$.

4 Conclusion

A neutrosophic set is a generalization of the notion of a fuzzy set.neutrosophic models give more precision,flexibility and compatibility to the system as compared to the classic and fuzzy models.We have introduced the concepts of (i) strong neutrosophic graphs ,and have presented some of their properties in this paper.It is clear that the most of these results can be simply extented to (S,Y)-fuzzy graphs , whereS and T are given imaginable triangular norms.The obtained results can be applied in various areas of engineering,computer science : artificial intelligence,signal processing,pattern recognition,robotics, computer networks,expert systems,and medical diagnosis.Our future plan to extend our research of fuzzification to (1) Bipolar fuzzy hypergraphs;(2) neutrosophic hypergraphs;(3) Vague hypergraphs ; (4) Interval-valued hypergraphs;(5) Soft fuzzy hypergraphs.

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