Pre Separation Axioms In Neutrosophic Crisp Topological Spaces

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Abstract

In this paper, A new type of separation axioms in the neutrosophic crisp Topological space named neutrosophic crisp pre separation axioms is going to be defined , in which neutrosophic crisp pre open set and neutrosophic crisp point are to be depended on. Also, relations among them and the other type are going to be found.

Keywords: Neutrosophic crisp pre separation axiom, neutrosophic crisp separation axiom, neutrosophic crisp point, Neutrosophic crisp semi separation axiom.

1. Introduction

In 1995, F.Samarandache generalized the fuzzy logic concept into the neutrosophic logic which presents a more detaied and concise description than the fuzzy logic and classical logic; then several researches emerged in this logic, in all branches of mathematics, especially Topology.

In 2012 A. A Salama et al. defined the concept of the neutrosophic set. Also, in 2020 A. Al-Nafey, R. Al-Hamido and F. Smarandache define the neutrosophic crisp separation axioms[2]. Also, in 2020 R. K. Al-Hamido, L.A. Salha and T. Gharibah define neutrosophic crisp semi separation axioms[13].

Recently, the neutrosophic crisp set theory may have applications in image processing [3-4]and possible applications to database[6]. Also, neutrosophic sets [7] have applications in the medical field [8-11], [9], [10], [11] and in the field of geographic information systems[5].Many researchers studied topology, and they had many contributions to neutrosophic toplogy as [14], [15], [16], [17] and [18] and in neutrosophic bitopology in [19], [20], [21] and [22], and in neutrosophic algebra in [23], [24], [25], [26] and [27].

In this paper, neutrosophic crisp pre separation axioms via neutrosophic crisp pre open set and neutrosophic crisp point are going to be studied.

Lastly, the definition of separation axioms is as follows $T_i, i = 0,1,2$ and the relations among them.

2. Preliminaries

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In this paper, the symbol (χ,Ţ) means a neutrosophic crisp topological space (N_cTS), Also

N_c. OS (N_c. CS) means a neutrosophic crisp open(closed) sets and neutrosophic crisp pre open set in N_cTS mean a N_cP. OS .
Some important definitions to this paper will be shown.

Definition 2.1. [1]

Suppos that $X \neq \emptyset$ be a fixed set. A neutrosophic crisp set (N_c. S) U is an object with the U=< U_1, U_2, U_3> shape; U_1, U_2 and U_3 are subsets of X.

Definition 2.5. [12]

Suppos that χ be a non-empty set. And x,y,z∈ χ, then:

a. x_{N_1}= <{x},∅,∅ > is called a neutrosophic crisp point ($N_cP_{N_1}$) in χ.

b. y_{N_2}= <∅,{y},∅ > is called a neutrosophic crisp point ($N_cP_{N_2}$) in χ.

c. z_{N_3}= <∅,∅, {z} > is called a neutrosophic crisp point ($N_cP_{N_3}$) in χ .

The set of all neutrosophic crisp points ($N_cP_{N_1}$, $N_cP_{N_2}$, $N_cP_{N_3}$) is denoted by NCP_N.

Definition 2.6. [12]

Suppos that (χ,Ţ) be an $NcTS$. Then χ is called:

a. N_1Ţo-space for every two diffrant points from χ there exists neutrosophic crisp open set in χ containing one of them but not the other.

b. N_2Ţo-space for every two diffrant points from χ there exists neutrosophic crisp open set in χ containing one of them but not the other.

c. N_3Ţo-space for every two diffrant points from χ there exists neutrosophic crisp open set in χ containing one of them but not the other.

Definition 2.7. [12]

Suppos that (χ,Ţ) be an $NcTS$. Then χ is called:

a. N_1Ţ1-space for every two diffrant points from χ are x_{N_1}, y_{N_1} there exists two neutrosophic crisp open set M 1, M 2 in χ such that x_{N_1} ∈ M 1, y_{N_1} ∉ M 1 and x_{N_1} ∉ M2 , y_{N_1} ∈ M2.

b. N_2Ţ1-space for every two diffrant points from χ are x_{N_1}, y_{N_1} there exists two neutrosophic crisp open set M 1, M2 in χ such that x_{N_2} ∈ G1, y_{N_2} ∉ G1 and x_{N_2} ∉ G2 , y_{N_2} ∈ G2.

c. N_3Ţ1-space for every two diffrant points from χ are x_{N_1}, y_{N_1} there exists two neutrosophic crisp open set M 1, M2 in χ such that x_{N_3} ∈M1, y_{N_3} ∉M1 and x_{N_3} ∉ M2 , y_{N_3} ∈ M2.

Definition 2.8. [12]

Suppos that (χ,Ţ) be an $NcTS$. Then χ is called:

a. N_1Ţ2-space for every two diffrant points from χ are x_{N_1}, y_{N_1} there exists two neutrosophic crisp open set M 1, M2 in χ such that x_{N_1} ∈ M 1, y_{N_1} ∉ M 1 and x_{N_1} ∉ M2 , y_{N_1} ∈ M2. with M1∩M2= ∅.

b. N_2Ţ2-space for every two diffrant points from χ are x_{N_1}, y_{N_1} there exists two neutrosophic crisp open set M 1, M2 in χ such that x_{N_2} ∈ M1, y_{N_2} ∉ M1 and x_{N_2} ∉ M2 , y_{N_2} ∈ M2 with M1∩M2= ∅.

c. N_3Ţ2-space for every two diffrant points from χ are x_{N_1}, y_{N_1} there exists two neutrosophic crisp open set M1, M2 in χ such that x_{N_3} ∈ M1, y_{N_3} ∉ M1 and x_{N_3} ∉ M2, y_{N_3} ∈ M2 with M1∩ M2 = ∅.

Definition 2.9. [13]

Suppos that (χ,Ţ) be an $NcTS$. Then χ is called:

a. N_1semiŢo-space if for every x_{N_1}≠ y_{N_1} ∈χ there exists $NcS.OS$ M in χ containing one of them but not the other.

b. N_2semiŢo-space if ∀ x_{N_2}≠ y_{N_2} ∈$\mathcal{X}$ there exists $NcS.OS$ M in χ containing one of them but not the other.

c. N_3semiŢo-space if ∀ x_{N_3}≠ y_{N_3} ∈χ there exists $NcS.OS$ M in χ containing one of them but not the other.

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Definition 2.10. [13]

Suppos that (χ,Ţ) be an $NcTS$. Then χ is called:

a. N_1 semiŢ1-space if for every $x_{N_1} \neq y_{N_1} \in \chi$ there exist $NcS.OS$ M 1, M2 in χ such that $x_{N_1} \in$ M 1, $y_{N_1} \notin$ M 1 and $x_{N_1} \notin$ M2 , $y_{N_1} \in$ M2.

b. N_2semiŢ1-space if ∀ $x_{N_2} \neq y_{N_2} \in \chi$ there exist $NcS.OS$ M 1, M2 in χ such that $x_{N_2} \in$ G1, $y_{N_2} \notin$ G1 and $x_{N_2} \notin$ G2 , $y_{N_2} \in$ G2

c. N_3semiŢ1-space if ∀ $x_{N_3} \neq y_{N_3} \in \chi$ there exist $NcS.OS$ M 1, M2 in χ such that $x_{N_3} \in$M1, $y_{N_3} \notin$M1 and $x_{N_3} \notin$ M2 , $y_{N_3} \in$ M2

Definition 2.11. [13]

Suppos that (χ,Ţ) be an $NcTS$. Then χ is called:

a. N_1semiŢ2-space if for every $x_{N_1} \neq y_{N_1} \in \chi$ there exists $NcS.OS$ M 1, M2 in χ such that $x_{N_1} \in$ M 1, $y_{N_1} \notin$ M 1 and $x_{N_1} \notin$ M2 , $y_{N_1} \in$ M2. with M1∩M2= ∅.

b. N_2semiŢ2-space if ∀ $x_{N_2} \neq y_{N_2} \in \chi$ there exists $NcS.OS$ M 1, M2 in χ such that $x_{N_2} \in$ M1, $y_{N_2} \notin$ M1 and $x_{N_2} \notin$ M2 , $y_{N_2} \in$ M2 with M1∩M2= ∅.

c. N_3semiŢ2-space if ∀ $x_{N_3} \neq y_{N_3} \in \chi$ there exists $NcS.OS$ M1, M2 in χ such that $x_{N_3} \in$ M1, $y_{N_3} \notin$ M1 and $x_{N_3} \notin$ M2, $y_{N_3} \in$ M2 with M1∩ M2 = ∅.

3. Separation axioms via pre open sets

This section introuduces a new type of separation axioms in the neutrosophic crisp Topological space named neutrosophic crisp pre separation axioms.

Definition 3.1.

Suppos that (χ,Ţ) be an NcTS. Then χ is called:

d. N_1preŢo-space if for every $x_{N_1} \neq y_{N_1} \in \chi$ there exists NcP.OS M in χ containing one of them but not the other.

e. N_2preŢo-space if ∀ $x_{N_2} \neq y_{N_2} \in \mathcal{X}$ there exists NcP.OS M in χ containing one of them but not the other.

f. N_3preŢo-space if ∀ $x_{N_3} \neq y_{N_3} \in \chi$ there exists NcP.OS M in χ containing one of them but not the other.

Definition 3.2.

Suppos that (χ,Ţ) be an NcTS. Then χ is called:

d. N_1Ţ1-space if for every $x_{N_1} \neq y_{N_1} \in \chi$ there exist $NcP.OS$ M 1, M2 in χ such that $x_{N_1} \in$ M 1, $y_{N_1} \notin$ M 1 and $x_{N_1} \notin$ M2 , $y_{N_1} \in$ M2.

e. N_2preŢ1-space if ∀ $x_{N_2} \neq y_{N_2} \in \chi$ there exist $NcP.OS$ M 1, M2 in χ such that $x_{N_2} \in$ G1, $y_{N_2} \notin$ G1 and $x_{N_2} \notin$ G2 , $y_{N_2} \in$ G2

f. N_3preŢ1-space if ∀ $x_{N_3} \neq y_{N_3} \in \chi$ there exist $NcP.OS$ M 1, M2 in χ such that $x_{N_3} \in$M1, $y_{N_3} \notin$M1 and $x_{N_3} \notin$ M2 , $y_{N_3} \in$ M2

Definition 3.3.

Suppos that (χ,Ţ) be an NcTS. Then χ is called:

d. N_1preŢ2-space if for every $x_{N_1} \neq y_{N_1} \in \chi$ there exists $NcP.OS$ M 1, M2 in χ such that $x_{N_1} \in$ M 1, $y_{N_1} \notin$ M 1 and $x_{N_1} \notin$ M2 , $y_{N_1} \in$ M2. with M1∩M2= ∅.

e. N_2preŢ2-space if ∀ $x_{N_2} \neq y_{N_2} \in \chi$ there exists $NcP.OS$ M 1, M2 in χ such that $x_{N_2} \in$ M1, $y_{N_2} \notin$ M1 and $x_{N_2} \notin$ M2 , $y_{N_2} \in$ M2 with M1∩M2= ∅.

f. N_3preŢ2-space if ∀ $x_{N_3} \neq y_{N_3} \in \chi$ there exists $NcP.OS$ M1, M2 in χ such that $x_{N_3} \in$ M1, $y_{N_3} \notin$ M1 and $x_{N_3} \notin$ M2, $y_{N_3} \in$ M2 with M1∩ M2 = ∅.

Theorem 3.4.

Suppos that (χ,Ţ) be an NcTS, then :

1. Every N_1Ţ0-space is N_1preŢ0-space.
2. Every N_2Ţ0-space is N_2preŢ0-space.
3. Every N_3Ţ0-space is N_3preŢ0-space.

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Proof:

1. Suppose that (χ,Ţ) is an N_1Ţ0-space, therefore for every two $x_{N_1} \neq y_{N_1}$, there exists an N_c.OS Ϻ in χ containing one of them to which the other does not belong. So there exists an $NcP.OS$ Ϻ in χ containing one of them to which the other does not belong, therefore X is N_1preŢ0-space.
2. Suppose that (χ,Ţ) is an N_2Ţ0-space, therefore for every two $x_{N_2} \neq y_{N_2}$, there exists an N_c.OS Ϻ in χ containing one of them to which the other does not belong. So there exists an $NcP.OS$ Ϻ in χ containing one of them to which the other does not belong, therefore X is N_2preŢ0-space.
3. Suppose that (χ,Ţ) is an N_3Ţ0-space, therefore for every two $x_{N_3} \neq y_{N_3}$, there exists an N_c.OS Ϻ in χ containing one of them to which the other does not belong. So there exists an $NcP.OS$ Ϻ in χ containing one of them to which the other does not belong, therefore X is N_3preŢ0-space.

Remark 3.5.

The converse of theorem 3.4 is not true., as it is shown in the following examples.

Example 3.6.

Let $\chi = \{a, b, c\}$, $Ţ = \{\emptyset_N, X_N, A\}$, $A = \{< \{a\}, \emptyset, \emptyset >\}$

$N_cP.OS = Ţ \cup \{B = \{< \{a, c\}, \emptyset, \emptyset >\}, C = \{< \{a, b\}, \emptyset, \emptyset >\}\}$.

Let $x_{N_1} = \{< \{b\}, \emptyset, \emptyset >\} \neq y_{N_1} = \{< \{c\}, \emptyset, \emptyset >\} \in \chi$ there is no a N_c.OS M in χ containing one of them but not the other. Therefore (χ,Ţ) is not N_1Ţ0-space.

Let $x_{N_1} = \{< \{b\}, \emptyset, \emptyset >\} \neq y_{N_1} = \{< \{a\}, \emptyset, \emptyset >\} \in \chi$ there is a N_cP.OS B in χ containing one of them but not the other.

Let $x_{N_1} = \{< \{a\}, \emptyset, \emptyset >\} \neq y_{N_1} = \{< \{c\}, \emptyset, \emptyset >\} \in \chi$ there is a N_cP.OS A in χ containing one of them but not the other.

Let $x_{N_1} = \{< \{b\}, \emptyset, \emptyset >\} \neq y_{N_1} = \{< \{c\}, \emptyset, \emptyset >\} \in \chi$ there is a N_c.OS B in χ containing one of them but not the other. Therefore (χ,Ţ) $N_1 pre$Ţ0-space.

Then (χ,Ţ) $N_1 pre$Ţ0-space, But (χ,Ţ) is not N_1Ţ0-space.

Example 3.7.

Let $\chi = \{a, b, c\}$, $Ţ = \{\emptyset_N, X_N, A\}$, $A = \{< \{a\}, \emptyset, \emptyset >\}$

$N_cP.OS = Ţ \cup \{B = \{< \{a, c\}, \emptyset, \emptyset >\}, C = \{< \{a, b\}, \emptyset, \emptyset >\}\}$.

Let $x_{N_2} = \{< \emptyset, \{b\}, \emptyset >\} \neq y_{N_1} = \{< \emptyset, \{c\}, \emptyset >\} \in \chi$ there is no a N_c.OS M in χ containing one of them but not the other. Therefore (χ,Ţ) is not N_2Ţ0-space.

Let $x_{N_2} = \{< \emptyset, \{b\}, \emptyset >\} \neq y_{N_2} = \{< \emptyset, \{c\}, \emptyset >\} \in \chi$ there is a N_cP.OS B in χ containing one of them but not the other.

Let $x_{N_2} = \{< \emptyset, \{a\}, \emptyset >\} \neq y_{N_2} = \{< \emptyset, \{c\}, \emptyset >\} \in \chi$ there is a N_cP.OS A in χ containing one of them but not the other.

Let $x_{N_2} = \{< \emptyset, \{b\}, \emptyset >\} \neq y_{N_2} = \{< \emptyset, \{a\}, \emptyset >\} \in \chi$ there is a N_cP.OS A in χ containing one of them but not the other. Therefore (χ,Ţ) $N_2 pre$Ţ0-space.

Then (χ,Ţ) $N_2 pre$Ţ0-space, But (χ,Ţ) is not N_2Ţ0-space.

Example 3.8.

Let $\chi = \{a, b, c\}$, $Ţ = \{\emptyset_N, X_N, A\}$, $A = \{< \{a\}, \emptyset, \emptyset >\}$

$N_cP.OS = Ţ \cup \{B = \{< \{a, c\}, \emptyset, \emptyset >\}, C = \{< \{a, b\}, \emptyset, \emptyset >\}\}$.

Let $x_{N_3} = \{< \emptyset, \emptyset, \{b\} >\} \neq y_{N_3} = \{< \emptyset, \emptyset, \{c\} >\} \in \chi$ there is no a N_c.OS M in χ containing one of them but not the other. Therefore (χ,Ţ) is not N_3Ţ0-space.

Let $x_{N_3} = \{< \emptyset, \emptyset, \{b\} >\} \neq y_{N_3} = \{< \emptyset, \emptyset, \{c\} >\} \in \chi$ there is a N_cP.OS B in χ containing one of them but not the other.

Let $x_{N_3} = \{< \emptyset, \emptyset, \{a\} >\} \neq y_{N_3} = \{< \emptyset, \emptyset, \{c\} >\} \in \chi$ there is a N_cP.OS A in χ containing one of them but not the other.

Let $x_{N_3} = \{< \emptyset, \emptyset, \{b\} >\} \neq y_{N_3} = \{< \emptyset, \emptyset, \{a\} >\} \in \chi$ there is a N_c.OS A in χ containing one of them but not the other. Therefore (χ,Ţ) $N_3 pre$Ţ0-space.

Then (χ,Ţ) $N_3 pre$Ţ0-space, But (χ,Ţ) is not N_3Ţ0-space.

Theorem 3.9.

Let (χ,Ţ) be an N_cTS, then :

1. Every N_1Ţ1-space is N_1preŢ1-space.

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2. Every N_2Ţ1-space is N_2preŢ1-space.
3. Every N_3Ţ1-space is N_3preŢ1-space.

Proof:

1. Suppose that (χ,Ţ) is an N_1Ţ1-space , therefore for every two $x_{N_1} \neq y_{N_1}$, there exist an N_c.OS Ṃ 1, Ṃ2 in χ such that $x_{N_1} \in$ Ṃ 1, $y_{N_1} \notin$ Ṃ 1 and $x_{N_1} \notin$ Ṃ2 , $y_{N_1} \in$ Ṃ2. So there exists an $NcP.OS$ Ṃ 1, Ṃ2 in χ such that $x_{N_1} \in$ Ṃ 1, $y_{N_1} \notin$ Ṃ 1 and $x_{N_1} \notin$ Ṃ2 , $y_{N_1} \in$ Ṃ2.
Therefore X is N_1preŢ1-space.
2. Suppose that (χ,Ţ) is an N_2Ţ1-space , therefore for every two $x_{N_2} \neq y_{N_2}$, there exist an N_c.OS Ṃ 1, Ṃ2 in χ such that $x_{N_2} \in$ Ṃ 1, $y_{N_2} \notin$ Ṃ 1 and $x_{N_2} \notin$ Ṃ2 , $y_{N_2} \in$ Ṃ2. So there exists an $NcP.OS$ Ṃ 1, Ṃ2 in χ such that $x_{N_2} \in$ Ṃ 1, $y_{N_2} \notin$ Ṃ 1 and $x_{N_2} \notin$ Ṃ2 , $y_{N_2} \in$ Ṃ2.
Therefore X is N_2preŢ1-space.
3. Suppose that (χ,Ţ) is an N_3Ţ1-space , therefore for every two $x_{N_3} \neq y_{N_3}$, there exist an N_c.OS Ṃ 1, Ṃ2 in χ such that $x_{N_3} \in$ Ṃ 1, $y_{N_3} \notin$ Ṃ 1 and $x_{N_3} \notin$ Ṃ2 , $y_{N_3} \in$ Ṃ2. So there exists an $NcP.OS$ Ṃ 1, Ṃ2 in χ such that $x_{N_3} \in$ Ṃ 1, $y_{N_3} \notin$ Ṃ 1 and $x_{N_3} \notin$ Ṃ2 , $y_{N_3} \in$ Ṃ2.
Therefore X is N_3preŢ1-space.

Remark 3.10.
The converse of a theorem 3.9 is not true, as it is shown in the following example.

Example 3.11.
Let $\chi = \{a, b, c\}$, $Ţ = \{\emptyset_N, X_N, A, B\}, A = \{<\{a\}, \emptyset, \emptyset>\}, B = \{<\{b, c\}, \emptyset, \emptyset>\}$.
$N_cP.OS = Ţ \cup \{G = \{<\{b\}, \emptyset, \emptyset>\}, C = \{<\{c\}, \emptyset, \emptyset>\}, E = \{<\{a, b\}, \emptyset, \emptyset>\}, H = \{<\{a, c\}, \emptyset, \emptyset>\}\}$.
Let $x_{N_1} = \{<\{b\}, \emptyset, \emptyset>\} \neq y_{N_1} = \{<\{c\}, \emptyset, \emptyset>\} \in \chi$ there is no $Nc.OS$ Ṃ 1, Ṃ2 in χ such that $x_{N_1} \in$ Ṃ1, $y_{N_1} \notin$ Ṃ1 and $x_{N_1} \notin$ Ṃ2 , $y_{N_1} \in$ Ṃ2. Therefore (χ,Ţ) is not $Ɲ_1$Ţ1-space.

Then (χ,Ţ) $Ɲ_1 pre$Ţ1-space, but (χ,Ţ) is not $Ɲ_1$Ţ1-space.

Example 3.12.
Let $\chi = \{a, b, c\}$, $Ţ = \{\emptyset_N, X_N, A, B\}, A = \{<\emptyset, \{a\}, \emptyset>\}, B = \{<\emptyset, \{b, c\}, \emptyset>\}$.
$N_cS.OS = Ţ \cup \{G = \{<\emptyset, \{b\}, \emptyset>\}, C = \{<\emptyset, \{c\}, \emptyset>\}, E = \{<\emptyset, \{a, b\}, \emptyset>\}, H = \{<\emptyset, \{a, c\}, \emptyset>\}\}$.
Let $x_{N_2} = \{<\emptyset, \{b\}, \emptyset>\} \neq y_{N_2} = \{<\emptyset, \{c\}, \emptyset>\} \in \chi$ there is no $Nc.OS$ Ṃ 1, Ṃ2 in χ such that $x_{N_2} \in$ Ṃ1, $y_{N_2} \notin$ Ṃ1 and $x_{N_2} \notin$ Ṃ2 , $y_{N_2} \in$ Ṃ2. Therefore (χ,Ţ) is not $Ɲ_2$Ţ1-space.

Then (χ,Ţ) $Ɲ_2 pre$Ţ1-space, but (χ,Ţ) is not $Ɲ_2$Ţ1-space.

Example 3.13.
Let $\chi = \{a, b, c\}$, $Ţ = \{\emptyset_N, X_N, A, B\}, A = \{<\emptyset, \emptyset, \{a\}>\}, B = \{<\emptyset, \emptyset, \{b, c\}>\}$.
$N_cS.OS = Ţ \cup \{G = \{<\emptyset, \emptyset, \{b\}>\}, C = \{<\emptyset, \emptyset, \{c\}>\}, E = \{<\emptyset, \emptyset, \{a, b\}>\}, H = \{<\emptyset, \emptyset, \{a, c\}>\}\}$.
Let $x_{N_3} = \{<\emptyset, \emptyset, \{b\}>\} \neq y_{N_3} = \{<\emptyset, \emptyset, \{c\}>\} \in \chi$ there is no $Nc.OS$ Ṃ 1, Ṃ2 in χ such that $x_{N_3} \in$ Ṃ1, $y_{N_3} \notin$ Ṃ1 and $x_{N_3} \notin$ Ṃ2 , $y_{N_3} \in$ Ṃ2. Therefore (χ,Ţ) is not $Ɲ_3$Ţ1-space.

Then (χ,Ţ) $Ɲ_3 pre$Ţ1-space, but (χ,Ţ) is not $Ɲ_3$Ţ1-space.

Theorem 3.14.

Let (χ,Ţ) be an N_cTS, then :

1. Every N_1Ţ2-space is N_1preŢ2-space.
2. Every N_2Ţ2-space is N_2preŢ2-space.
3. Every N_3Ţ2-space is N_3preŢ2-space.

Proof:

1. Suppose that (χ,Ţ) is an N_1Ţ2-space , therefore for every two $x_{N_1} \neq y_{N_1}$, there exists an $Nc.OS$ Ṃ 1, Ṃ2 in χ such that $x_{N_1} \in$ Ṃ 1, $y_{N_1} \notin$ Ṃ 1 and $x_{N_1} \notin$ Ṃ2 , $y_{N_1} \in$ Ṃ2. with Ṃ1∩Ṃ2= ∅. So there exists $NcP.OS$ Ṃ 1, Ṃ2 in χ such that $x_{N_1} \in$ Ṃ 1, $y_{N_1} \notin$ Ṃ 1 and $x_{N_1} \notin$ Ṃ2 , $y_{N_1} \in$ Ṃ2. with Ṃ1∩Ṃ2= ∅.
Therefore X is N_1preŢ2-space.

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2. Suppose that $(\chi,Ţ)$ is an $N_2Ţ_2$-space , therefore for every two $x_{N_2} \neq y_{N_2}$, there exists an $Nc.OS$ M_1, M_2 in χ such that $x_{N_2} \in M_1$, $y_{N_2} \notin M_1$ and $x_{N_2} \notin M_2$, $y_{N_2} \in M_2$. with $M_1 \cap M_2 = \emptyset$. So there exists $NcP.OS$ M_1, M_2 in χ such that $x_{N_2} \in M_1$, $y_{N_2} \notin M_1$ and $x_{N_2} \notin M_2$, $y_{N_2} \in M_2$. with $M_1 \cap M_2 = \emptyset$.
   Therefore X is N_2preŢ$_2$-space.
3. Suppose that $(\chi,Ţ)$ is an $N_3Ţ_2$-space , therefore for every two $x_{N_3} \neq y_{N_3}$, there exists an $Nc.OS$ M_1, M_2 in χ such that $x_{N_3} \in M_1$, $y_{N_3} \notin M_1$ and $x_{N_3} \notin M_2$, $y_{N_3} \in M_2$. with $M_1 \cap M_2 = \emptyset$. So there exists $NcP.OS$ M_1, M_2 in χ such that $x_{N_3} \in M_1$, $y_{N_3} \notin M_1$ and $x_{N_3} \notin M_2$, $y_{N_3} \in M_2$. with $M_1 \cap M_2 = \emptyset$.
   Therefore X is N_3preŢ$_2$-space.
4.

Remark 3.15.
The converse of the Theorem 3.14 is not true., as it is shown in the following example.

Example 3.16.
In example 3.11. $(\chi,Ţ)$ $N_1preŢ_2$-space, but $(\chi,Ţ)$ is not $N_1Ţ_2$-space.

Example 3.17.
In example 3.12. $(\chi,Ţ)$ $N_2preŢ_2$-space, but $(\chi,Ţ)$ is not $N_2Ţ_2$-space.

Example 3.18.
In example 3.13. $(\chi,Ţ)$ $N_3preŢ_2$-space, but $(\chi,Ţ)$ is not $N_3Ţ_2$-space.

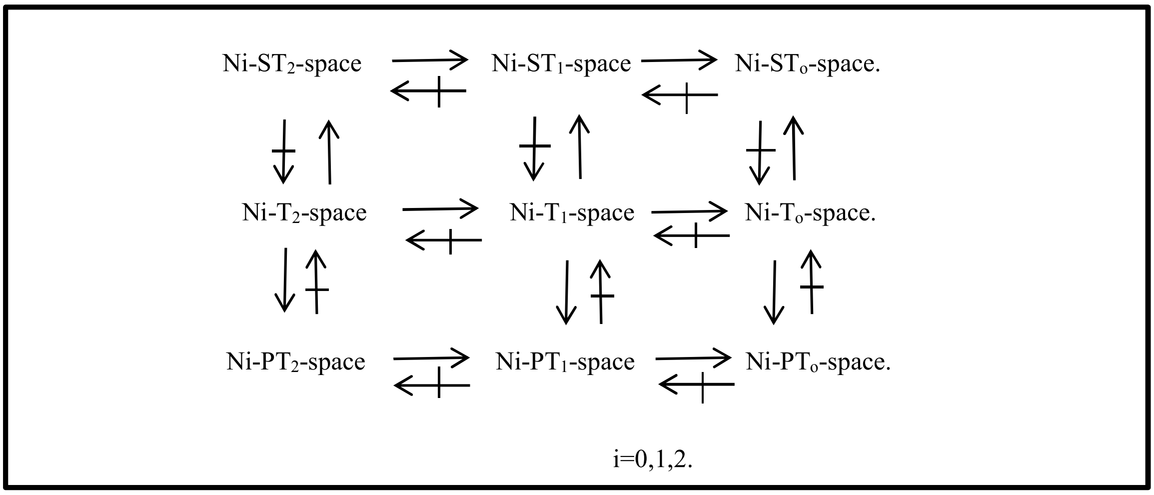
Theorem 3.19.

Let $(\chi,Ţ)$ be an N_cTS, then :

1. N_1preŢ$_2$-space $\Rightarrow$ N_1preŢ$_1$-space $\Rightarrow$ N_1preŢ$_0$-space.
2. N_2preŢ$_2$-space $\Rightarrow$ N_2preŢ$_1$-space $\Rightarrow$ N_2preŢ$_0$-space.
3. N_3preŢ$_2$-space $\Rightarrow$ N_3preŢ$_1$-space $\Rightarrow$ N_3preŢ$_0$-space.

The converse of the Theorem 3.19 is not true.

Remark. 3.21.
Relations among the different types of neutrosophic crisp separation axioms which were studied in this paper, appear in the following diagram.



Conclusion

In this paper, a new type of neutrosophic crisp separation axioms has been defined by using neutrosophic crisp pre open sets and certain point in the neutrosophic crisp topological spaces. Moreover, the connections between neutrosophic crisp pre separation axioms and the existing neutrosophic crisp separation axioms are studied. And many examples are presented to clear the concepts introduced. Also, proof their basic properties. Also, investigate their fundamental properties and characterizations.

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