



Weakly b-Closed Sets and Weakly b-Open Sets based of Fuzzy Neutrosophic bi-Topological Spaces

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Abstract

In this paper, we present and study some of the basic properties of the new class of sets called weakly b-closed sets and weakly b-open sets in fuzzy neutrosophic bi-topological spaces. We referred to some results related to the new definitions, which we took the case of equal in the definition of b-sets instead of subset. Then, we discussed the relations between the new defined sets by hand and others fuzzy neutrosophic sets which were studied before us on the other hand on fuzzy neutrosophic bi-topological spaces. Then, we have studied some of characteristics and some relations are compared with necessary examples.

Keywords: Fuzzy neutrosophic sets, Fuzzy neutrosophic bi-topology, Fuzzy neutrosophic weakly b-closed sets, Fuzzy neutrosophic weakly b-open sets.

1.Introduction

The notion of fuzzy set "FS" proved was show by L. Zadeh [1] where the membership of any element to this set "FS" be a single value between 0 and 1. After that K Atanassov [2-4] introduced the notion of intuitionistic fuzzy sets "IFS" which was generalization of "FS", where the elements have membership and non-membership value between the same interval 0 and 1. As proved and indulged F. Smarandache [5] introduced the concept of neutrosophic sets "NSs" where he added the independent value between the value of "IFS" also in the same value, 0 and 1, then in the next papers [6], studied the neutrosophic topological spaces "NTSs" on the non standard interval which made many important consequences and theorems so as so, foundation for all family of new mathematical theories generalizing both their fuzzy topology counterparts and old classical topology, A. A. Salama [7] studied the term of neutrosophic topology "NT". Finally, Y.Veereswari [8] gave an introduction of fuzzy neutrosophic topological spaces "FNTSs".

The concept of fuzzy neutrosophic weakly- α^m generalized closed set and fuzzy neutrosophic b-closed sets "FN-b-CS" was introduced and studied by F. Mohammed [9,10]. The term of bi-topological spaces was studied in neutrosophic topology by R. Al-Hamido [11-13], so in this paper we will show the idea of fuzzy neutrosophic weakly b-closed and fuzzy neutrosophic weakly b-closed sets also investigated some their properties on fuzzy neutrosophic bi-topological spaces and we getting some necessarily properties as generalized of many authors studying, for more details and information about the applications of neutrosophic theory in new trends see [14-21].

2. Preliminaries:

In this section of our study, we will refer to some basic definition and operations which are necessarily in our work.

Definition 2.1 [7]: "Let U_N be a non-empty fixed set. The fuzzy neutrosophic set (FNS) μ_N is an object having the form $\mu_N = \{ \langle u, \lambda_{\mu_N}(u), \gamma_{\mu_N}(u), V_{\mu_N}(u) \rangle : u \in U_N \}$ where the functions $\lambda_{\mu_N}(u), \gamma_{\mu_N}(u), V_{\mu_N}(u): U_N \rightarrow [0, 1]$ denote the degree of membership function (namely $\lambda_{\mu_N}(u)$), the degree of indeterminacy function (namely $\gamma_{\mu_N}(u)$) and the degree of non-membership function (namely $V_{\mu_N}(u)$) respectively of each element $u \in U_N$ to the set μ_N and $0 \leq \lambda_{\mu_N}(u) + \gamma_{\mu_N}(u) + V_{\mu_N}(u) \leq 3$, for each $u \in U_N$ ".

Remark 2.2: "FNS $\mu_N = \{ \langle u, \lambda_{\mu_N}(u), \sigma_{\mu_N}(u), V_{\mu_N}(u) \rangle : u \in U_N \}$ can be identified to an ordered triple $\langle u, \lambda_{\mu_N}, \sigma_{\mu_N}, V_{\mu_N} \rangle$ in $[0, 1]$ on U_N ".

Lemma 2.3 [8]: "Let U_N be a non-empty set and the FNSs μ_N and γ_N be in the form:

$\mu_N = \{ \langle u, \lambda_{\mu_N}, \sigma_{\mu_N}, V_{\mu_N} \rangle \}$ and $\gamma_N = \{ \langle u, \lambda_{\gamma_N}, \sigma_{\gamma_N}, V_{\gamma_N} \rangle \}$ on U_N . Then,

- (i) $\mu_N \subseteq \gamma_N$ iff $\lambda_{\mu_N} \leq \lambda_{\gamma_N}, \sigma_{\mu_N} \leq \sigma_{\gamma_N}$ and $V_{\mu_N} \geq V_{\gamma_N}$,
- (ii) $\mu_N = \gamma_N$ iff $\mu_N \subseteq \gamma_N$ and $\gamma_N \subseteq \mu_N$,
- (iii) $(\mu_N)^c = \{ \langle u, V_{\mu_N}, 1 - \sigma_{\mu_N}, \lambda_{\mu_N} \rangle \}$,
- (iv) $\mu_N \cup \gamma_N = \{ \langle u, \text{Mx}(\lambda_{\mu_N}, \lambda_{\gamma_N}), \text{Mx}(\sigma_{\mu_N}, \sigma_{\gamma_N}), \text{Mn}(V_{\mu_N}, V_{\gamma_N}) \rangle \}$,
- (v) $\mu_N \cap \gamma_N = \{ \langle u, \text{Mn}(\lambda_{\mu_N}, \lambda_{\gamma_N}), \text{Mn}(\sigma_{\mu_N}, \sigma_{\gamma_N}), \text{Mx}(V_{\mu_N}, V_{\gamma_N}) \rangle \}$,
- (vi) $0_N = \{ \langle u, 0, 0, 1 \rangle \}$ and $1_N = \{ \langle u, 1, 1, 0 \rangle \}$."

Definition 2.4 [8]: "A Fuzzy neutrosophic topology (For short, FNT) on a non-empty set U_N is a family T_N of fuzzy neutrosophic subset in U_N satisfying the following axioms.

- (i) $0_N, 1_N \in T_N$,
- (ii) $\mu_{N1} \cap \mu_{N2} \in T_N \forall \mu_{N1}, \mu_{N2} \in T_N$,
- (iii) $\cup \mu_{Nj} \in T_N, \forall \{ \mu_{Nj} : j \in J \} \subseteq T_N$.

In this case the pair (U_N, T_N) is called fuzzy neutrosophic topological space (for short, FNTS). The elements of T_N are called fuzzy neutrosophic-open sets (for short, FN-OS). The complement of FN-OS in the FNTS (U_N, T_N) is called fuzzy neutrosophic- closed set (for short, FN-CS)."

Definition 2.5 [8]: "Let (U_N, T_N) is FNTS and $\mu_N = \langle u, \lambda_{\mu_N}, \sigma_{\mu_N}, V_{\mu_N} \rangle$ is FNS in U_N . Then the fuzzy neutrosophic-closure (for short, $FN-CI$) and the fuzzy neutrosophic -interior (for short, $FN-In$) of μ_N are defined by:

$$FN-CI(\mu_N) = \cap \{ \gamma_N : \gamma_N \text{ is FN-CS set in } U \text{ and } \mu_N \subseteq \gamma_N \},$$

$$FN-In(\mu_N) = \cup \{ \gamma_N : \gamma_N \text{ is FN-OS set in } U \text{ and } \gamma_N \subseteq \mu_N \}.$$

Now, the $FN-CI(\mu_N)$ is FN-CS set and $FN-In(\mu_N)$ is FN-OS. set in U_N .

Further,

- (i) μ_N is FN-CS in U iff $FN-CI(\mu_N) = \mu_N$,

- (ii) μ_N is FN-OS in U iff $FN-In(\mu_N) = \mu_N$."

Definition 2.6 [9,10]: "The FNS λ_N in FNTS (U_N, T_N) is called:

- (i) Fuzzy neutrosophic regular-open set (FNR-OS) iff $\mu_N = FN-In(FN-CI(\mu_N))$,
(ii) Fuzzy neutrosophic regular-closed set (FNR-CS) iff $\mu_N = FN-CI(FN-In(\mu_N))$,
(iii) Fuzzy neutrosophic semi-open set (FNS-OS) iff $\mu_N \subseteq FN-CI(FN-In(\mu_N))$,
(iv) Fuzzy neutrosophic semi-closed set (FNS-CS) iff $FN-In(FN-CI(\mu_N)) \subseteq \mu_N$,
(v) Fuzzy neutrosophic pre-open set (FNP-OS) iff $\mu_N \subseteq FN-In(FN-CI(\mu_N))$,
(vi) Fuzzy neutrosophic pre-closed set (FNP-CS) iff $FN-CI(FN-In(\mu_N)) \subseteq \mu_N$,
(vii) Fuzzy neutrosophic α -open set (FN α -OS) iff $\mu_N \subseteq FN-In(FN-CI(FN-In(\mu_N)))$,
(viii) Fuzzy neutrosophic α -closed set (FN α -CS) iff $FN-CI(FN-In(FN-CI(\mu_N))) \subseteq \mu_N$,
(ix) Fuzzy neutrosophic β -open set (FN β -OS) iff $\mu_N \subseteq FN-CI(FN-In(FN-CI(\mu_N)))$,
(x) Fuzzy neutrosophic β -closed set (FN β -CS) iff $FN-In(FN-CI(FN-In(\mu_N))) \subseteq \mu_N$."

Definition 2.7 [7]: "A fuzzy neutrosophic set K in FNTS U_N is called fuzzy neutrosophic b-open set (for short, FNb-OS) if and only if $K \leq In(CI(K)) \vee CI(In(K))$."

Definition 2.8 [15]: "A fuzzy neutrosophic set K in FNTS U_N is called fuzzy neutrosophic b-closed (for short, FNb-CS) set iff $In(CI(K)) \vee CI(In(K)) \leq K$."

Definition 2.9 [11]: "Let U_N be a non-empty set and $(U, T_{N1}), (U, T_{N2})$ be two topological spaces then, the triple (U_N, T_{N1}, T_{N2}) is a fuzzy neutrosophic bi-topological space (for short, FN-bi-TS)."

Definition 2.10 [11]: "Let U_N be a non-empty set and T_{N1}, T_{N2} be two topologies on U_N . A subset A of U_N is called fuzzy neutrosophic bi-open set (for short, FN-OS) if $A \in T_{N1} \cup T_{N2}$. A is called fuzzy neutrosophic bi-closed set (for short, FN-CS) if I_N-A is FN-OS.

Definition 2.11 [18]: "A FNS K in FN-bi-TS (U, T_{N1}, T_{N2}) is called fuzzy neutrosophic nowhere dense set if there exists no FN-OS. set V such that $V \subseteq FN-CI(K)$. That is $FN-In(FN-CI(K)) = 0_N$."

Remark 2.12 [15]: "Let K be a FNS in FN-bi-TS (U, T_{N1}, T_{N2}) . If K is a fuzzy neutrosophic nowhere dense set in (U_N, T_{N1}, T_{N2}) , then, $FN-In(K) = 0_N$."

3. Weakly b.Closed Sets and Weakly b.open Sets in Fuzzy Neutrosophic bi-Topological Spaces

In this section, we introduce the concepts of weakly b-closed sets and weakly b-open sets and study some of their characteristics on fuzzy neutrosophic bi-topological spaces.

Definition 3.1: A FNS K in a FN-bi-TS (U_N, T_{N1}, T_{N2}) is said to be a fuzzy neutrosophic weakly b.closed set (for short, FNWb-CS) if $FN-In(FN-CI(K)) \cap FN-CI(FN-In(K)) = K$. The complement I_N-K of a FNWb-CS in a FN-bi-TS (U_N, T_{N1}, T_{N2}) is called a fuzzy neutrosophic weakly b.open set (for short, FNWb-OS) in U_N . The family of all FNWb-CS of a FN-bi-TS (U_N, T_{N1}, T_{N2}) is denoted by FNWb-CS(U_N).

Example 3.2: Let $U_N = \{a, b\}$ on $T_{N1} = \{0_N, E_1, I_N\}$ and $T_{N2} = \{0_N, I_N\}$ where $E_1 = \langle u, (0.4_a, 0.4_b), (0.4_a, 0.6_b), (0.2_a, 0.3_b) \rangle$ is a FN-bi-TS on U_N .

Let $K = \langle u, (0.4_a, 0.4_b), ((0.0_a, 0.0_b), (0.2_a, 0.3_b)) \rangle$.

Now, $FN-In(FN-CI(K)) \cap FN-CI(FN-In(K)) = E_1 \cap I_N-E_1 = E_1 = K$.

Then, K is a FNWb-CS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Remark 3.3: The following cases are independent to each other in general in a FN-bi-TS (U_N, T_{N1}, T_{N2}) .

- (i) FN-CS and FNWb-CS.
- (ii) FNR-CS and FNWb-CS.
- (iii) FNP-CS and FNWb-CS.
- (iv) $FN\alpha$ -CS and FNWb-CS.

Example 3.4: In Example 3.2, we have:

- (1) K is a FNWb-CS but not a FN-CS in U as $FN-Cl(K) = I_N - E_I \neq K$.
- (2) $K = \langle u, (0.4_a, 0.6_b), (0.5_a, 0.5_b), (0.4_a, 0.4_b) \rangle$ is a FN-CS as $FN-Cl(K) = I_N - E_I = K$, but not a FNWb-CS in U_N as $FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K)) = E_I \neq K$.
- (3) $K = \langle u, (0.4_a, 0.4_b), (0.5_a, 0.5_b), (0.4_a, 0.6_b) \rangle$ is a FNWb-CS in U_N , but not a FNR-CS in U_N as $FN-Cl(FN-In(K)) = I_N - E_I \neq K$.
- (4) $K = \langle u, (0.4_a, 0.6_b), (0.5_a, 0.5_b), (0.4_a, 0.4_b) \rangle$ is a FNR-CS in U_N as $FN-Cl(FN-In(K)) = I_N - E_I = K$, but not a FNWb-CS in U_N as $FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K)) \neq K$.
- (5) $K = \langle u, (0.4_a, 0.6_b), (0.5_a, 0.5_b), (0.4_a, 0.4_b) \rangle$ is a FNP-CS in U_N as $FN-Cl(FN-In(K)) = I_N - E_I \subseteq K$, but not a FNWb-CS in U_N as $FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K)) \neq K$.
- (6) $K = \langle u, (0.4_a, 0.4_b), (0.5_a, 0.5_b), (0.4_a, 0.6_b) \rangle$ is a FNWb-CS in U_N but not a FNP-CS in U_N as $FN-Cl(FN-In(K)) = I_N - E_I \not\subseteq K$.
- (7) $K = \langle u, (0.4_a, 0.6_b), (0.5_a, 0.5_b), (0.4_a, 0.4_b) \rangle$ is a FN- α clos. set in U_N as $FN-Cl(FN-In(FN-Cl(K))) = I_N - E_I \subseteq K$, but not a FNWb-CS in U_N as $FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K)) = E_I \neq K$.
- (8) $K = \langle u, (0.4_a, 0.4_b), (0.5_a, 0.5_b), (0.4_a, 0.6_b) \rangle$ is a FNWb-CS in U_N as $FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K)) = E_I = K$, but not a $FN\alpha$ -CS in U_N as $FN-Cl(FN-In(FN-Cl(K))) = I_N - E_I \not\subseteq K$.

Theorem 3.5 : Let (U_N, T_{N1}, T_{N2}) be a FN-bi-TS, then:

- (i) Every FNWb-CS is a FNB-CS.
- (ii) Every FNWb-CS is an FNS-CS.
- (iii) Every FNWb-CS is a $FN\beta$ -CS.

Proof : (i) Let K be a FNWb-CS in FN-bi-TS $((U_N, T_{N1}, T_{N2})$,

Then, $FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K)) = K$.

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Now, as $K \subseteq K$, we get $FN\text{-}In(FN\text{-}Cl(K)) \cap FN\text{-}Cl(FN\text{-}In(K)) \subseteq K$.

Therefore, K is a FNb-CS in (U_N, T_{N1}, T_{N2}) .

(ii): Let K be a FNWb-CS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $FN\text{-}In(FN\text{-}Cl(K)) \cap FN\text{-}Cl(FN\text{-}In(K)) = K$.

$$\begin{aligned} \text{Now, as } FN\text{-}In(FN\text{-}Cl(K)) &= FN\text{-}In(FN\text{-}Cl(FN\text{-}In(FN\text{-}Cl(K) \cap FN\text{-}Cl(FN\text{-}In(K)))) \\ &\subseteq FN\text{-}In(FN\text{-}Cl(FN\text{-}Cl(K) \cap FN\text{-}Cl(FN\text{-}In(K)))) \\ &\subseteq FN\text{-}In(FN\text{-}Cl(K)) \cap FN\text{-}Cl(FN\text{-}In(K)) = K. \end{aligned}$$

Hence, K is a FNS-CS in (U_N, T_{N1}, T_{N2}) .

(iii): Let K be a FNWb-CS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $FN\text{-}In(FN\text{-}Cl(K)) \cap FN\text{-}Cl(FN\text{-}In(K)) = K$.

$$\begin{aligned} \text{Now, } FN\text{-}In(FN\text{-}Cl(FN\text{-}In(K))) &= FN\text{-}In(FN\text{-}Cl(FN\text{-}In(K))) \cap FN\text{-}Cl(FN\text{-}In(K)) \\ &\subseteq FN\text{-}In(FN\text{-}Cl(K)) \cap FN\text{-}Cl(FN\text{-}In(K)) = K. \end{aligned}$$

Therefore, we have $FN\text{-}In(FN\text{-}Cl(FN\text{-}In(K))) \subseteq K$.

Hence, K is a FN β -CS in (U_N, T_{N1}, T_{N2}) .

Note: The converse of Theorem 3.5 is not true in general.

Example 3.6: In Example 3.2

(1) Let $K = \langle u, (0.4_a, 0.6_b), (0.5_a, 0.5_b), (0.4_a, 0.4_b) \rangle$.

Then, $FN\text{-}In(FN\text{-}Cl(K)) \cap FN\text{-}Cl(FN\text{-}In(K)) = E_I \neq K$.

Therefore, K is a FNb-CS but, not a FNWb-CS U_N as $FN\text{-}In(FN\text{-}Cl(K)) \cap FN\text{-}Cl(FN\text{-}In(K)) \neq K$.

(2) If $K = \langle u, (0.4_a, 0.6_b), (0.5_a, 0.5_b), (0.4_a, 0.4_b) \rangle$ is a FNS-CS in U_N as $FN\text{-}In(FN\text{-}Cl(K)) = E_I \subseteq K$, but not a FNWb-CS.

(3) If $K = \langle u, (0.4_a, 0.6_b), (0.5_a, 0.5_b), (0.4_a, 0.4_b) \rangle$ is a FN β -CS in U_N as

$$FN\text{-}In(FN\text{-}Cl(FN\text{-}In(K))) = E_I \subseteq I_N\text{-}E_I \text{ but, not a FNWb-CS in } U_N \text{ as } FN\text{-}In(FN\text{-}Cl(K)) \cap FN\text{-}Cl(FN\text{-}In(K)) \neq K.$$

Definition 3.7: Let (U_N, T_{N1}, T_{N2}) be a FN-bi-TS, then the subset K of U_N is called FN-clopen set (for shortly, FN-CLOP) iff K is FN-CS and FN-OS in the same time.

Remark 3.8 : Every FN-CLOP set is both FNR-OS and FNR-CS.

Proposition 3.9: Let (U_N, T_{N1}, T_{N2}) be a FN-bi-TS, then:

(i) If K is both a FNR-OS and a FNR-CS then, K is a FNWb-CS.

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(ii) If K is both a FN-CLOP set then, K is a FNWb-CS.

Proof: (i) Let K be both FNR-OS and FNR-CS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K)) = K \cap K = K \Rightarrow K$ is a FNWb-CS.

(ii) Let K be a FN-CLOP set in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K)) = FN-In(K) \cap FN-Cl(K) = FN-In(K) = K$.

Therefore, K is a FNWb-CS.

Theorem 3.10: If K is both a FNWb-CS and a FN-CS in FN-bi-TS (U_N, T_{N1}, T_{N2}) then, K is a FN-OS.

Proof: Let K be both a FNWb-CS and a FN-CS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $K = FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K))$.

Now, $K = FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K))$

$$= FN-In(K) \cap FN-Cl(FN-In(K)) = FN-In(K).$$

Therefore, K is a FN-OS in FN-bi-TS $((U_N, T_{N1}, T_{N2})$.

Definition 3.11: Let (U_N, T_{N1}, T_{N2}) is FNTS and $\mu_N = \langle u, \lambda_{\mu_N}, \sigma_{\mu_N}, \nu_{\mu_N} \rangle$ is FNS in U_N . Then the fuzzy neutrosophic semi-closure (for short, $FN-sCl$) and the fuzzy neutrosophic –semi interior (for short, $FN-sIn$) of μ_N are defined by:

$$FN-sCl(\mu_N) = \cap \{ \gamma_N : \gamma_N \text{ is FNS-CS in } U \text{ and } \mu_N \subseteq \gamma_N \} = \mu_N \cup FN-In(FN-Cl(\mu_N)),$$

$$FN-sIn(\mu_N) = \cup \{ \gamma_N : \gamma_N \text{ is FNS-OS in } U \text{ and } \gamma_N \subseteq \mu_N \} = .$$

Theorem 3.12: For any FNWb-CS K in a FN-bi-TS (U_N, T_{N1}, T_{N2}) , the following conditions hold:

(i) If K is a FNR-OS then, $FN-sCl(K)$ is a FNWb-CS.

(ii) If K is a FNR-CS then, $FN-sIn(K)$ is a FNWb-CS.

Proof:(i) Let K be a FNR-OS in FN-bi-TS (U_N, T_{N1}, T_{N2}) . Then, $FN-In(FN-Cl(K)) = K$.

By definition, we have $FN-sCl(K) = K \cup FN-In(FN-Cl(K)) = K \cup K = K$, by hypothesis.

Since, K is a FNWb-CS in (U_N, T_{N1}, T_{N2}) , $FN-sCl(K)$ is a FNWb-CS in U_N .

(ii) Let K be a FNR-CS in (U_N, T_{N1}, T_{N2}) , then, $FN-Cl(FN-In(K)) = K$.

By definition, we have $FN-sIn(K) = K \cap FN-Cl(FN-In(K)) = K \cap K = K$, by hypothesis.

Since, K is a FNWb-CS in (U_N, T_{N1}, T_{N2}) , $FN-sIn(K)$ is a FNWb-CS.

Theorem 3.13: For a FNS K in FN-bi-TS (U_N, T_{N1}, T_{N2}) , the following conditions are equivalent:

(i) K is both a FN-OS and a FNWb-CS.

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(ii) K is a FNR-OS.

Proof: (i) \Rightarrow (ii) Let K be a FN-OS and a FNWb-CS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $FN-Cl(FN-In(K)) \cap FN-In(FN-Cl(K)) = K$ and $FN-In(FN-Cl(K)) \cap FN-Cl(K) = K$,

Therefore, $K = FN-In(FN-Cl(K))$.

Hence, K is a FNR-OS in (U_N, T_{N1}, T_{N2}) .

(ii) \Rightarrow (i) Let K be a FNR-OS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Since every FNR-OS is a FN-OS, K is a FN-OS in (U_N, T_{N1}, T_{N2}) and $K = FN-In(FN-Cl(K))$.

Now, $FN-In(FN-Cl(K)) \cap FN-Cl(FN-In(K)) = K \cap FN-Cl(FN-In(K)) = K \cap FN-Cl(K) = K$.

Hence, K is a FNWb-CS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Definition 3.14: A FNS K in a FN-bi-TS (U_N, T_{N1}, T_{N2}) is said to be a fuzzy neutrosophic weakly b-open set (for short, FNWb-OS) iff $K = FN-In(FN-Cl(K)) \cup FN-Cl(FN-In(K))$.

The family of all FNWb-OS of a FN-bi-TS (U_N, T_{N1}, T_{N2}) is denoted by FNWb-OS(U_N).

Example 3.15: In Example 3.2,

Let $K = \langle u, (0.4_a, 0.6_b), (0.5_a, 0.5_b), (0.4_a, 0.4_b) \rangle$ be a FNS in (U_N, T_{N1}, T_{N2}) .

Now, $FN-In(FN-Cl(I_N-K)) \cap FN-Cl(FN-In(I_N-K)) = E_I \cap I_N-E_I = E_I = I_N-K \Rightarrow I_N-K$ is a FNWb-CS

$\Rightarrow I_N-K$ is a FNWb-CS in (U_N, T_{N1}, T_{N2}) .

Hence, K is a FNWb-OS in (U_N, T_{N1}, T_{N2}) .

Theorem 3.16: Every FNWb-OS are FNb-OS (FNS-OS., FN β -OS) but not conversely in general.

Proof: Straight forward.

Example 3.17: Obvious from Example 3.6 (1), (2), (3) by taking complement of K in the respective examples.

Theorem 3.18: Every FN-OS, FNR-OS, FNP-OS and FN α -OS are independent to FNWb-OS in FN-bi-TS (U_N, T_{N1}, T_{N2}) and vice versa in general.

Example 3.19: Obvious from Example 3.4 (1), (2), (3), (4), (5), (6), (7), (8), by taking the complement of A in the respective examples.

Theorem 3.20: If K is a FNWb-OS and fuzzy neutrosophic nowhere dense in FN-bi-TS $((U_N, T_{N1}, T_{N2})$, then:

(i) K is a FNR-CS.

(ii) K is a FNS-CS.

(iii) K is a FN α -CS.

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(iv) K is a $\text{FN}\beta$ -CS.

Proof: (i) Let K be a FNWb -OS and fuzzy neutrosophic nowhere dense in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $K = \text{FN-In}(\text{FN-Cl}(K)) \cup \text{FN-Cl}(\text{FN-In}(K)) = 0_N \cup \text{FN-Cl}(\text{FN-In}(K)) = \text{FN-Cl}(\text{FN-In}(K))$.

Therefore, $\text{FN-Cl}(\text{FN-In}(K)) = K$.

Hence, K is a FNR-CS in (U_N, T_{N1}, T_{N2}) .

(ii) Let K be a FNWb -OS and fuzzy neutrosophic nowhere dense in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $K \subseteq \text{FN-In}(\text{FN-Cl}(K)) \cup \text{FN-Cl}(\text{FN-In}(K)) = 0_N \cup \text{FN-Cl}(\text{FN-In}(K)) \subseteq \text{FN-Cl}(\text{FN-In}(K))$.

Therefore, $K \subseteq \text{FN-Cl}(\text{FN-In}(K))$.

Hence, K is a FNS-CS in (U_N, T_{N1}, T_{N2}) .

(iii) Let K be a FNWb -OS and fuzzy neutrosophic nowhere dense in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $\text{FN-In}(\text{FN-Cl}(K)) \cup \text{FN-In}(\text{FN-Cl}(\text{FN-In}(K))) \subseteq K$

Now, $0_N \cup \text{FN-In}(\text{FN-Cl}(\text{FN-In}(K))) \subseteq K$.

Therefore, $\text{FN-In}(\text{FN-Cl}(\text{FN-In}(K))) \subseteq K$.

Hence, K is a $\text{FN}\alpha$ -CS in (U_N, T_{N1}, T_{N2}) .

(iv) Let K be a FNWb -OS and fuzzy neutrosophic nowhere dense in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $\text{FN-In}(\text{FN-Cl}(K)) \cup \text{FN-In}(\text{FN-Cl}(\text{FN-In}(K))) \subseteq K$.

Now, $0_N \cup \text{FN-In}(\text{FN-Cl}(\text{FN-In}(K))) \subseteq K$.

Therefore, $\text{FN-In}(\text{FN-Cl}(\text{FN-In}(K))) \subseteq K$.

Hence, K is a $\text{FN}\beta$ -CS in (U_N, T_{N1}, T_{N2}) .

Theorem 3.21: If K is both a FNWb -OS and a FN-OS then, K is a FN-CS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Proof: Let K be both a FNWb -OS and a FN-OS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

Then, $K = \text{FN-In}(\text{FN-Cl}(K)) \cup \text{FN-Cl}(\text{FN-In}(K))$.

Now, $K = \text{FN-In}(\text{FN-Cl}(K)) \cup \text{FN-Cl}(\text{FN-In}(K)) = \text{FN-In}(\text{FN-Cl}(K)) \cup \text{FN-Cl}(K) = \text{FN-Cl}(K)$.

Hence, K is a FN-CS in (U_N, T_{N1}, T_{N2}) .

Theorem 3.22: Let K be a FNWb -OS in a FN-bi-TS (U_N, T_{N1}, T_{N2}) , such that $\text{FN-In}(K) = 0_N$, then, K is a FNP-OS .

Proof: Let K be a FNWb -OS then, K is a FNb-OS in FN-bi-TS $((U_N, T_{N1}, T_{N2}))$.

Now, $K \subseteq \text{FN-In}(\text{FN-Cl}(K)) \cup \text{FN-Cl}(\text{FN-In}(K)) \subseteq \text{FN-In}(\text{FN-Cl}(K)) \cup 0_N \subseteq \text{FN-In}(\text{FN-Cl}(K))$.

Hence, $K \subseteq \text{FN-In}(\text{FN-Cl}(K))$.

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So, K is a FNP-OS in FN-bi-TS (U_N, T_{N1}, T_{N2}) .

4. Conclusions

In this paper, we have introduced a new class of sets called fuzzy neutrosophic weakly b-closed sets and fuzzy neutrosophic weakly b-open sets via fuzzy neutrosophic topological spaces. Many results have been studied as generalization of classical fuzzy topology and intuitionistic fuzzy topology and applied in the field of fuzzy bi-neutrosophic topology. After giving some ideas to compare the already existing new sets with the others existing closed sets by definitions, theorems and propositions. Some interesting properties were investigated in addition, we have provided some examples where such properties failed to be preserved via fuzzy neutrosophic bi-topological spaces. We think, our studied class of sets belongs to the important class of fuzzy neutrosophic closed sets which is very useful not only in the deepening of our understanding of some special features of the already well-known notions of fuzzy neutrosophic topology but also proves useful in neutrosophic control theory.

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