



Concepts of Neutrosophic Complex Numbers

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Abstract

In this paper, concept of neutrosophic complex numbers and its properties were presented including the conjugate of neutrosophic complex number, division of neutrosophic complex numbers, the inverted neutrosophic complex number and the absolute value of a neutrosophic complex number. Theories related to the conjugate of neutrosophic complex numbers are proved, the product of a neutrosophic complex number by its conjugate equals the absolute value of number is also proved. This is an important introduction to define neutrosophic complex numbers in polar.

Keywords: Classical neutrosophic numbers, Neutrosophic complex numbers, Conjugate.

1. Introduction

As an alternative to the existing logics, Smarandache proposed the neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache [3][7]. He presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist, he defined the standard form of neutrosophic complex number, and found root index $n \geq 2$ of a neutrosophic real and complex number [2][4], studying the concept of the Neutrosophic probability [3][5], the Neutrosophic statistics [4][6], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1][8]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [9]. Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and right hand side represented with triangular neutrosophic numbers [10]. Chakraborty used pentagonal neutrosophic number in networking problem, and Shortest Path Problem [11][12].

This paper aims to study neutrosophic logic in the complex numbers by defining the conjugate of neutrosophic complex number, division of neutrosophic complex numbers, the inverted neutrosophic complex number, the absolute value of a neutrosophic complex number, I also have proven theories related to the conjugate of neutrosophic complex numbers, and finally we proved the product of a neutrosophic complex number by its conjugate equals the absolute value of number.

2. Preliminaries

2.1 Neutrosophic Real Number [4]

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a, b are real coefficients, and I represent indeterminacy, such $0.I = 0$ and $I^n = I$, for all positive integers n .

2.2 Neutrosophic Complex Number [4]

Suppose that z is a neutrosophic complex number, then it takes the following standard form: $z = a + cI + bi + diI$ where a, b, c, d are real coefficients, and I indeterminacy, such that $i^2 = -1 \Rightarrow i = \sqrt{-1}$.

Note: we can say that any real number can be considered a neutrosophic number.

For example: $2 = 2 + 0.I$, or: $2 = 2 + 0.I + 0.i + 0.i.I$

2.3 Division of neutrosophic real numbers [4]

Suppose that w_1, w_2 are two neutrosophic numbers, where

$$w_1 = a_1 + b_1I, \quad w_2 = a_2 + b_2I$$

To find $(a_1 + b_1I) \div (a_2 + b_2I)$, we can write:

$$\frac{a_1 + b_1I}{a_2 + b_2I} \equiv x + yI$$

where x and y are real unknowns.

$$a_1 + b_1I \equiv (a_2 + b_2I)(x + yI)$$

$$a_1 + b_1I \equiv a_2x + (b_2x + a_2y + b_2y)I$$

by identifying the coefficients, we get

$$a_1 = a_2x$$

$$b_1 = b_2x + (a_2 + b_2)y$$

We obtain unique one solution only, provided that:

$$\begin{vmatrix} a_2 & 0 \\ b_2 & a_2 + b_2 \end{vmatrix} \neq 0 \Rightarrow a_2(a_2 + b_2) \neq 0$$

Hence: $a_2 \neq 0$ and $a_2 \neq -b_2$ are the conditions for the division of two neutrosophic real numbers to exist.

Then:

$$\frac{a_1 + b_1I}{a_2 + b_2I} = \frac{a_1}{a_2} + \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}.I$$

2.4 Root index $n \geq 2$ of a neutrosophic real number [4]

1) Case: $n = 2$

Let $w = a + bI$ be a neutrosophic real number, then

$$\sqrt{a + bI} = x + y.I$$

$$a + bI \equiv (x + y.I)^2$$

$$a + bI \equiv x^2 + 2xy.I + y^2I$$

by identifying the coefficients, we get:

$$x^2 = a$$

$$y^2 + 2xy = b$$

$$\text{Hence } x = \pm\sqrt{a}$$

$$y^2 \pm 2\sqrt{a}y - b = 0$$

By solving the second equation in respect to y we find:

$$y = \frac{\mp 2\sqrt{a} \pm \sqrt{4a + 4b}}{2} = \mp\sqrt{a} \pm \sqrt{a + b}$$

Then we find four solutions of $\sqrt{a + bI}$:

$$\sqrt{a + bI} = \sqrt{a} + (-\sqrt{a} + \sqrt{a + b}).I$$

$$\text{Or: } \sqrt{a + bI} = \sqrt{a} - (-\sqrt{a} + \sqrt{a + b}).I$$

$$\text{Or: } \sqrt{a + bI} = -\sqrt{a} + (\sqrt{a} + \sqrt{a + b}).I$$

$$\text{Or: } \sqrt{a + bI} = -\sqrt{a} + (\sqrt{a} - \sqrt{a + b}).I$$

particular case: $\sqrt{I} = \pm I$

2) Case: $n > 2$

$$\sqrt[n]{a + bI} = x + y.I$$

$$a + bI \equiv (x + y.I)^n$$

$$a + bI \equiv x^n + \left(\sum_{k=0}^{n-1} C_n^k y^{n-k} x^k \right).I$$

$$x^n = a \Rightarrow x = \begin{cases} \sqrt[n]{a} & ; n \text{ odd} \\ \pm \sqrt[n]{a} & ; n \text{ even} \end{cases}$$

$$\sum_{k=0}^{n-1} C_n^k y^{n-k} a^{\frac{k}{n}} = b$$

Solve it in respect to y, we can distinguish two cases:

When the x and y solutions are real, we get neutrosophic real solutions,

When x and y solutions are complex, we get neutrosophic complex solutions.

2.5 Multiplying two neutrosophic complex numbers [2]

Let z_1, z_2 are two neutrosophic complex numbers, where

$$z_1 = a_1 + c_1I + b_1i + d_1iI, \quad z_2 = a_2 + c_2I + b_2i + d_2iI$$

Then:

$$\begin{aligned} z_1 \cdot z_2 &= (a_1 + c_1I + b_1i + d_1iI)(a_2 + c_2I + b_2i + d_2iI) \\ &= (a_1a_2 - b_1b_2) + (a_1c_2 + a_2c_1 + c_1c_2 - b_1d_2 - d_1b_2 - d_1d_2)I \\ &\quad + (a_1b_2 + a_2b_1)i + (a_1d_2 + c_1b_2 + c_1d_2 + b_1c_2 + a_2d_1 + d_1c_2)iI \end{aligned}$$

Example 2.1:

$$\begin{aligned} (3 + 5i + 2iI)(1 + 3iI) &= 3 + 9iI + 5i - 15I + 2iI - 6I \\ &= 3 - 21I + 5i + 11iI \end{aligned}$$

3. Conjugate of a neutrosophic complex number

Definition 3.1:

Suppose that z is a neutrosophic complex number, where $z = a + cI + bi + d.iI$. We denote the conjugate of a neutrosophic complex number by $\bar{z}$ and define it by the following form:

$$\bar{z} = a + cI - bi - d.iI$$

Example 3.1:

$$\begin{aligned} z = 4 + 5i - 7iI &\Rightarrow \bar{z} = 4 - 5i + 7iI \\ z = -2I - i + 8iI &\Rightarrow \bar{z} = -2I + i - 8iI \\ z = iI &\Rightarrow \bar{z} = -iI \end{aligned}$$

As consequences, we have:

1. the conjugate of neutrosophic complex number $\bar{z}$ is the same neutrosophic complex number z .

$$\overline{(\bar{z})} = z$$

2. If $z = a + cI + bi + d.iI$

then

$$z + \bar{z} = 2(a + cI) = 2\text{Re}(z) \quad \text{and} \quad z - \bar{z} = 2(b + d.I)i = 2\text{Im}(z)$$

where $\text{Re}(Z)$ is the real part of the complex number and $\text{Im}(Z)$ is the imagine

3. We conclude from this, that the neutrosophic complex number is real if and only if $z = \bar{z}$, and it is imaginary if and only if $z = -\bar{z}$.

Remark 3.1:

The conjugate of the sum of two neutrosophic complex numbers is equal to the sum of their two conjugates.

$$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$$

Proof:

Suppose that z_1, z_2 are two neutrosophic complex numbers, where

$$z_1 = a_1 + c_1I + b_1i + d_1iI, \quad z_2 = a_2 + c_2I + b_2i + d_2iI$$

Then:

$$\begin{aligned} z_1 + z_2 &= (a_1 + a_2) + (c_1I + c_2I) + (b_1 + b_2)i + (d_1 + d_2)iI \\ \overline{z_1 + z_2} &= (a_1 + a_2) + (c_1I + c_2I) - (b_1 + b_2)i - (d_1 + d_2)iI \\ &= a_1 + c_1I - b_1i - d_1iI + a_2 + c_2I - b_2i - d_2iI \\ &= \bar{z}_1 + \bar{z}_2 \end{aligned}$$

Theorem 3.1

The conjugate of multiplication two neutrosophic complex numbers is equal to the multiplication of their two conjugates.

$$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$$

Proof:

Suppose that z_1, z_2 are two neutrosophic complex numbers, where

$$z_1 = a_1 + c_1I + b_1i + d_1iI, \quad z_2 = a_2 + c_2I + b_2i + d_2iI$$

Then:

$$\begin{aligned} z_1 \cdot z_2 &= (a_1 + c_1I + b_1i + d_1iI)(a_2 + c_2I + b_2i + d_2iI) \\ &= (a_1a_2 - b_1b_2) + (a_1c_2 + a_2c_1 + c_1c_2 - b_1d_2 - d_1b_2 - d_1d_2)I \\ &\quad + (a_1b_2 + a_2b_1)i + (a_1d_2 + c_1b_2 + c_1d_2 + b_1c_2 + a_2d_1 + d_1c_2)iI \\ \overline{z_1 \cdot z_2} &= (a_1a_2 - b_1b_2) + (a_1c_2 + a_2c_1 + c_1c_2 - b_1d_2 - d_1b_2 - d_1d_2)I \\ &\quad - (a_1b_2 + a_2b_1)i - (a_1d_2 + c_1b_2 + c_1d_2 + b_1c_2 + a_2d_1 + d_1c_2)iI \\ \bar{z}_1 \cdot \bar{z}_2 &= (a_1 + c_1I - b_1i - d_1iI)(a_2 + c_2I - b_2i - d_2iI) \\ &= (a_1a_2 - b_1b_2) + (a_1c_2 + a_2c_1 + c_1c_2 - b_1d_2 - d_1b_2 - d_1d_2)I \\ &\quad - (a_1b_2 + a_2b_1)i - (a_1d_2 + c_1b_2 + c_1d_2 + b_1c_2 + a_2d_1 + d_1c_2)iI \end{aligned}$$

$$\Rightarrow \overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$$

4. Division of neutrosophic complex numbers

Suppose that z_1, z_2 are two neutrosophic complex numbers, where

$$z_1 = a_1 + c_1I + b_1i + d_1iI \quad , \quad z_2 = a_2 + c_2I + b_2i + d_2iI \quad ; \quad z_2 \neq 0$$

Then:

$$\frac{z_1}{z_2} = \frac{a_1 + c_1I + b_1i + d_1iI}{a_2 + c_2I + b_2i + d_2iI}$$

multiply the numerator and denominator by conjugate of z_2 we get:

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{(a_1 + c_1I + b_1i + d_1iI)(a_2 + c_2I - b_2i - d_2iI)}{(a_2 + c_2I + b_2i + d_2iI)(a_2 + c_2I - b_2i - d_2iI)} \\ &= \frac{(a_1 + c_1I + b_1i + d_1iI)(a_2 + c_2I - b_2i - d_2iI)}{(a_2 + c_2I)^2 + (b_2 + d_2I)^2} \\ &= \frac{(a_1a_2 + b_1b_2) + (a_1c_2 + a_2c_1 + c_1c_2 + b_1d_2 + d_1b_2 + d_1d_2)I}{(a_2 + c_2I)^2 + (b_2 + d_2I)^2} \\ &\quad + \frac{(a_2b_1 - a_1b_2)i + (b_1c_2 + a_2d_1 + d_1c_2 - a_1d_2 - c_1b_2 - c_1d_2)i \cdot I}{(a_2 + c_2I)^2 + (b_2 + d_2I)^2} \\ &= \frac{(a_1a_2 + b_1b_2) + (a_1c_2 + a_2c_1 + c_1c_2 + b_1d_2 + d_1b_2 + d_1d_2)I}{(a_2 + c_2I)^2 + (b_2 + d_2I)^2} \\ &\quad + \frac{(a_2b_1 - a_1b_2) + (b_1c_2 + a_2d_1 + d_1c_2 - a_1d_2 - c_1b_2 - c_1d_2) \cdot I}{(a_2 + c_2I)^2 + (b_2 + d_2I)^2} \cdot i \end{aligned}$$

Example4.1:

$$\frac{3 + 5i + 2iI}{1 + 3iI}$$

Solution:

multiply the numerator and denominator by conjugate of $(1 - 3iI)$ we get:

$$\begin{aligned} \frac{3 + 5i + 2iI}{1 + 3iI} &= \frac{(3 + 5i + 2iI)(1 - 3iI)}{(1 + 3iI)(1 - 3iI)} = \frac{3 - 21I + 5i + 11iI}{1 + 9I} \\ &= \frac{3 - 21I}{1 + 9I} + \frac{5 + 11I}{1 + 9I} i \quad (1) \end{aligned}$$

Let us find:

$$\frac{3 - 21I}{1 + 9I} \equiv x + yI$$

$$3 - 21I \equiv (1 + 9I)(x + yI)$$

$$3 - 21I \equiv x + 9xI + 10yI$$

$$3 - 21I \equiv x + (9x + 10y)I$$

$$\Rightarrow \begin{cases} x = 3 \\ 9x + 10y = -21 \end{cases}$$

$$\Rightarrow \begin{cases} x = 3 \\ 9(3) + 10y = -21 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = -\frac{48}{10} = -4.8 \end{cases}$$

$$\Rightarrow \frac{3 - 21I}{1 + 9I} \equiv 3 - 4.8I$$

Let us find:

$$\frac{5 + 11I}{1 + 9I} \equiv x + yI$$

$$5 + 11I \equiv (1 + 9I)(x + yI)$$

$$5 + 11I \equiv x + 9xI + 10yI$$

$$5 + 11I \equiv x + (9x + 10y)I$$

$$\Rightarrow \begin{cases} x = 5 \\ 9x + 10y = 11 \end{cases}$$

$$\Rightarrow \begin{cases} x = 5 \\ 9(5) + 10y = 11 \end{cases} \Rightarrow \begin{cases} x = 5 \\ y = -\frac{34}{10} = -3.4 \end{cases}$$

$$\Rightarrow \frac{5 + 11I}{1 + 9I} \equiv 5 - 3.4I$$

By substitution in (1):

$$\begin{aligned} \frac{3 + 5i + 2iI}{1 + 3iI} &= 3 - 4.8I + (5 - 3.4I)i \\ &= 3 - 4.8I + 5i - 3.4iI \end{aligned}$$

5. Inverted Neutrosophic complex number

Suppose that z is a neutrosophic complex number, where $z = a + cI + bi + d.iI$

Then:

$$\frac{1}{z} = \frac{1}{a + cI + bi + d.iI}$$

$$= \frac{a + cI}{(a + cI)^2 + (b + dI)^2} - \frac{b - dI}{(a + cI)^2 + (b + dI)^2} i$$

Example 5.1:

$$\begin{aligned} \frac{1}{1 - 2iI} &= \frac{1}{1 + 4I} + \frac{2I}{1 + 4I} i \\ &= 1 - \frac{4}{5}I + \frac{2}{5}I \cdot i \end{aligned}$$

6. The absolute value of a neutrosophic complex number

Suppose that $z = a + cI + bi + d \cdot iI$ is a neutrosophic complex number, the absolute value of a neutrosophic complex number defined by the following form:

$$|Z| = \sqrt{(a + cI)^2 + (b + dI)^2}$$

Example 6.1:

Let $z = 1 + 2I + iI$, find the absolute value of z .

Solution:

$$\begin{aligned} |Z| &= \sqrt{(a + cI)^2 + (b + dI)^2} \\ &= \sqrt{(1 + 2I)^2 + (I)^2} \\ &= \sqrt{1 + 4I + 4I + I} \\ &= \sqrt{1 + 10I} \\ \sqrt{1 + 10I} &\equiv x + yI \\ 1 + 10I &\equiv x^2 + 2xyI + y^2 \end{aligned}$$

by identifying we get:

$$\begin{cases} x^2 = 1 \\ y^2 + 2xy = 10 \end{cases}$$

Since the absolute value is positive, we take: $x = 1$

By substitution in the second equation:

$$y^2 + 2y = 10 \Rightarrow y^2 + 2y - 10 = 0$$

$$y = \frac{-2 + 2\sqrt{11}}{2} = -1 + \sqrt{11} \approx 2.3$$

Therefore,

$$|Z| = |1 + 2I + iI| = 1 + 2.3I$$

Theorem 6.1:

Suppose that $z = a + cI + bi + d.iI$ is a neutrosophic complex number, multiplication the absolute value of z by its conjugate equals to square of the absolute value of z .

$$z \cdot \bar{z} = |Z|^2$$

Proof:

$$z = a + cI + bi + d.iI \Rightarrow \bar{z} = a + cI - bi - d.iI$$

$$\begin{aligned} z \cdot \bar{z} &= (a + cI + bi + d.iI)(a + cI - bi - d.iI) \\ &= a^2 + acI - abi - adiI + acI + c^2I - bciI - cdiI + abi \\ &\quad + bciI + b^2 + bdiI + adiI + cdiI + bdiI + d^2I \\ &= (a^2 + 2acI + c^2I) + (b^2 + 2bdiI + d^2I) \\ &= (a + cI)^2 + (b + dI)^2 = |Z|^2 \end{aligned}$$

$$\Rightarrow z \cdot \bar{z} = |Z|^2$$

Example 6.2:

Let $z = 4 - I + 2i + 3iI$, find $z \cdot \bar{z}$.

Solution:

$$\begin{aligned} z \cdot \bar{z} &= |Z|^2 \\ &= (a + cI)^2 + (b + dI)^2 \\ &= (4 - I)^2 + (2 + 3I)^2 \\ &= 16 - 8I + I + 4 + 12I + 9I \\ &= 20 + 14I \end{aligned}$$

5. Conclusions

In this paper, conjugate of neutrosophic complex number was defined and used to find the division of neutrosophic complex numbers, the inverted neutrosophic complex number and the absolute value of a neutrosophic complex number. This research has proven theorems related to the conjugate of neutrosophic complex numbers. This approach can be applied to define the polar form and exponential form of the neutrosophic complex number.

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