A Note On Neutrosophic Soft Menger Topological Spaces

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Abstract

In this paper, the concept of neutrosophic soft Mengerness, neutrosophic soft near Mengerness and neutrosophic soft almost Mengerness are introduced and studied. Some characterizations of neutrosophic soft almost Mengerness in terms of neutrosophic soft regular open or neutrosophic soft regular closed are given.

Keywords: Neutrosophic soft sets, Mengerness on neutrosophic soft topological space, neutrosophic soft continuous.

1. Introduction

The concept of fuzzy set was introduced by Zadeh in his classic paper [20]. C.L.Chang [6] has defined fuzzy topological spaces. Atannasov [3] introduced the notion of intuitionistic fuzzy sets, Çoker [7] defined the intuitionistic fuzzy topological spaces. Soft sets theory was proposed by Molodtsov [12] in 1999, as a new mathematical tool for handling problems which contain uncertainties. Maji et al [10] gave the first practical application of soft sets in decision-making problems. Shabir and Naz [16] presented soft topological spaces and defined some concepts of soft sets on these spaces and separation axioms. The concept of neutrosophic set (NS) was first introduced by Smarandache [17,18,19] which is the generalization of classical sets, fuzzy set, intuitionistic fuzzy set etc. Following this concept Al-Omeri and Jafari defined and investigated Neutrosophic crisp sets via Neutrosophic crisp topological spaces [1,2]. The concept of connectedness and compactness on neutrosophic soft topological space was introduced by Bera and Mahapatra [4,5]. For more applications on neutrosophic logic the refrennces are suggested [21-23]

The investigation of covering properties of topological spaces has a long history going back to papers by Menger and Rothberger [11,14]. However more recently a new theory called Selection Principles was introduced by Scheepers [15]. The theory of Selection Principles has extra ordinary connections with numerous subareas of mathematics, for example, Set theory and General topology, Uniform structures, and Ditopological texture spaces [9].

In 1999, Kocinac defined and characterized the almost Menger property [9]. Following this concept, Aqsa, Moizud Din Khan defined and investigated nearly Menger and nearly star- Menger spaces [13]. For

In this paper we are concerned with the weaker forms of the fuzzy Mengerness in neutrosophic soft topological spaces.

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2. Preliminaries

In this section we now state certain useful definitions, theorems, and several existing results for neutrosophic soft topological spaces that we require in the next sections.

Definition 2.1. [17] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A is characterized by a truth-member function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x), I_A(x)$ and $F_A(x)$ are real Standard or non Standard subsets of $]^-0, 1^+[$. That is $T_A, I_A, F_A: X \to\ ]^-0, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$, $F_A(x)$ and so,
$^-0 \leq sup\ T_A(x) + sup\ I_A(x) + sup\ F_A(x) \leq 3^+$.

Definition 2.2. [12] Let U be an initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Then for $A \subseteq E$, a pair (F,A) is called a soft set over U, where F:A→P(U) is a mapping.

Definition 2.3. [15] Let U be an initial universe set and E be a set of parameters. Let NS(U) denote the set of neutrosophic sets (NSs) of U. Then for $A \subseteq E$, a pair (F,A) is called a neutrosophic soft set (NSS) over U, where F:A→NS(U) is a mapping.

Definition 2.4. [8] Let U be an initial universe set and E be a set of parameters. LetNS(U) denote the set of neutrosophic sets (NSs) of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function F_N representing a mapping $F_N: E \to NS(U)$ where F_N is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parametrized family of some elements of the set NS(U) and therefore it can be written as a set of ordered pairs,

$N = \{(e, \{< x, T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) >: x \in U\}): e \in E\}$where$T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) \in [0,1]$,

respectively the truth-membership, indeterminacy-membership, falsity-membership function obvious.

Definition 2.5. [8] The complement of a neutrosophic soft set N is denoted by N^c and is defined by
$N^c = \{(e, \{< x, F_{fN(e)}(x), 1 - I_{fN(e)}(x), T_{fN(e)}(x) >: x \in U\}): e \in E\}$,

Let N_1 and N_2 be two NSSs over the common universe (U,E). Then N_1 is said to be the neutrosophic soft subset of N_2 if for each $e \in E$ and for each $x \in U$,

$T_{fN1(e)}(x) \leq T_{fN2(e)}(x), I_{fN1(e)}(x) \geq I_{fN2(e)}(x), F_{fN1(e)}(x) \geq F_{fN2(e)}(x)$.

We write $N_1 \subseteq N_2$ and then N_2 is the is the neutrosophic soft superset of N_1.

Definition 2.6. [8] Let N_1 and N_2 be two NSSs over the common universe (U,E). Then their union is denoted by $N_1 \cup N_2 = N_3$ and is defined as:

$N_3 = \{(e, \{< x, T_{fN3(e)}(x), I_{fN3(e)}(x), F_{fN2(e)}(x) >: x \in U\}): e \in E\}$where
$T_{fN3(e)}(x) = T_{fN1(e)}(x) \diamond T_{fN2(e)}(x), I_{fN3(e)}(x) = I_{fN1(e)}(x) * I_{fN2(e)}(x), F_{fN3(e)}(x) = F_{fN1(e)}(x) * F_{fN2(e)}(x)$.

Their intersection is denoted by $N_1 \cap N_2 = N_4$ and is defined as:

$N_4 = \{(e, \{< x, T_{fN4(e)}(x), I_{fN4(e)}(x), F_{fN4(e)}(x) >: x \in U\}): e \in E\}$ where
$T_{fN4(e)}(x) = T_{fN1(e)}(x) * T_{fN2(e)}(x), I_{fN4(e)}(x) = I_{fN1(e)}(x) \diamond I_{fN2(e)}(x), F_{fN4(e)}(x) = F_{fN1(e)}(x) \diamond F_{fN2(e)}(x)$.

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Definition 2.7. [4] Let M and N be two NSSs over the common universe (U,E). Then M-N may be defined as, for each $e \in E$ and for each $x \in U$.

$$M - N = \left\{< x, T_{fM(e)}(x) * F_{fN(e)}(x), I_{fM(e)}(x) \diamond \left(1 - I_{fN(e)}(x)\right), F_{fM(e)}(x) \diamond T_{fN(e)}(x) >\right\};$$

A neutrosophic soft set N over (U,E) is said to be null neutrosophic soft set if $T_{fN(e)}(x) = 0, I_{fN(e)}(x) = 1, F_{fN(e)}(x)$=1 for each $e \in E$ and for each$x \in U$. It is denoted by Φ_u.

A neutrosophic soft set N over (U,E) is said to be absolute neutrosophic soft set if $T_{fN(e)}(x) = 1, I_{fN(e)}(x) = 0, F_{fN(e)}(x)$=0 for each $e \in E$ and for each $x \in U$. It is denoted by 1_u.

Clearly, $\Phi_u^c = 1_u, 1_u^c = \Phi_u$.

Definition 2.8. [4] Let NSS(U,E) be the family of all neutrosophic soft sets over U via parameters in E and $\tau_u \subseteq NSS(U,E)$. Then $\tau_u$is called neutrosophic soft topology on (U,E) if the following conditions are satisfied.

(i) $\Phi_u, 1_u \in \tau_u$,
(ii) The intersection of any finite number of members of τ_u also belongs to τ_u.
(iii) The union of any collection of members of $\tau_u$belongs toτ_u.

Then the triple (U,E, τ_u) is called a neutrosophic soft topological space. Every member of τ_u is called τ_u-open neutrosophic soft set. An NSS is called τ_u-closed iff its complement is τ_u-open.

Definition 2.9. [4] Let (U,E, τ_u) be a neutrosophic soft topological space over (U,E) and $M \in NSS(U,E)$ be arbitrary. Then the interior of M is denoted by M^0 or int(M) and is defined as:

$M^0 = \cup \{N_1 : N_1 is\ neutrosophic\ soft\ open\ and N_1 \subseteq M\}$.

Definition 2.10. [4] Let (U,E, τ_u) be a neutrosophic soft topological space over (U,E) and $A \in NSS(U,E)$ be arbitrary. Then the closure of A is denoted by $\overline{A}$ or cl(A) and is defined as:

$\bar{A} = \cap \{N_1 : N_1 is\ neutrosophic\ soft\ closed\ and\ A \subseteq N_1\}$.

Theorem 2.11. [4] Let (U,E, τ_u) be a neutrosophic soft topological space over (U,E) and $A \in NSS(U,E)$. Then, $(\bar{A})^C$=$(A^c)^0$ and $(A^0)^c = (A^c)^-$.

Proposition 2.12. [4] Let N_1 and N_2 be two neutrosophic soft sets over (U,E). Then,

(i) $(N_1 \cup N_2)^c = {N_1}^c \cap {N_2}^c$,
(ii) $(N_1 \cap N_2)^c = {N_1}^c \cup {N_2}^c$.

Definition 2.13. [4] Let (U,E, τ_u) be a neutrosophic soft topological space and $M \in \tau_u$. A family $\Omega = \{Q_i : i \in \Gamma\}$ of neutrosophic soft sets is said to be a cover of M if $M \subseteq \cup Q_i$.

If every member of that family which covers M is neutrosophic soft open then it is called open cover of M. A subfamily of Ω which also covers M is called a subcover of M.

Definition 2.14. [4] Let (U,E, τ_u) be a neutrosophic soft topological space and $M \in \tau_u$. Suppose Ω be a cover of M. If Ω has a finite subcover which also covers M then M is called neutrosophic soft compact.

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Definition 2.15. [4] Let $\varphi: U \to V$ and $\psi: E \to E$ be two functions where E is the parameter set each of the crisp sets U and V. Then the pair(φ, ψ) is called and NSS function from (U,E) to (V,E). We write, $(\varphi, \psi): (U, E) \to (V, E)$.

Definition 2.16. [4] Let (M,E) and (N,E) be two NSSs defined over U and V, respectively and (φ, ψ) be an NSS function from (U,E) to (V,E). Then,

(1) The image of (M,E) under (φ, ψ), denoted by $(\varphi, \psi)(M, E)$, is an NSS over V and is defined as:

$(\varphi, \psi)(M, E) = (\varphi(M), \psi(E)) = \{< \psi(a), f_{\varphi(M)}(\psi(a)) >: a \in E\}$ where for each $b \in \psi(E)$ and $y \in V$.

$$T_{\varphi(M)(b)}(y) = \begin{cases} \max\varphi(x)=y\max\psi(a)=b[Tf(M)(a)(x)], & \text{if } x\epsilon\varphi-1(y), \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{\varphi(M)(b)}(y) = \begin{cases} \min\varphi(x)=y \min \psi(a)=b[If(M)(a)(x)], & \text{if } x\epsilon\varphi-1(y), \\ 1, & \text{otherwise.} \end{cases}$$

$$F_{\varphi(M)(b)}(y) = \begin{cases} \min\varphi(x)=y \min\psi(a)=b[Ff(M)(a)(x)], & \text{if } x\epsilon\varphi-1(y), \\ 1, & \text{otherwise.} \end{cases}$$

(2) The pre-image of (N,E) under(φ, ψ), denoted by $(\varphi, \psi)^{-1}(N, E)$, is an NSS over U and is defined by:

$(\varphi, \psi)^{-1}(N, E) = (\varphi^{-1}(N), \psi^{-1}(E))$ where for each $a \in \psi^{-1}(E)$ and $x \in U$.

$T_{\varphi}^{-1}(N)(a)(x)=T_{fN(\psi(a))}(\varphi(x))$,

$I_{\varphi}^{-1}(N)(a)(x)=I_{fN(\psi(a))}(\varphi(x))$,

$F_{\varphi}^{-1}(N)(a)(x)=F_{fN(\psi(a))}(\varphi(x))$,

If ψ and φ are injective(surjective), then (φ, ψ) is injective (surjective).

Definition 2.17. [4] Let (U,E, τ_u) and (V,E, τ_v) be two neutrosophic soft topological spaces.
(φ, ψ): (U, E, τ_u) →(V,E, τ_v) is said to be a neutrosophic soft continuous mapping if for each $(N, E) \in T_v$, the inverse image $(\varphi, \psi)^{-1}(N, E) \in \tau_u$ i.e., the inverse image of each open NSS in (V,E,τ_v) is also open in (U,E,τ_u).

3. Neutrosophic Soft Mengerness

Here, the notion of Mengerness, almost Mengerness and near Mengerness on neutrosophic soft topological space is developed with some basic theorems.

Definition 3.1. (a) A neutrosophic soft topological space (U,E, τ_u)is called neutrosophic soft Menger iff every sequence $\{Q_n: n \in N\}$ of neutrosophic soft open covers of (U,E, τ_u), there exists a sequence $\{V_n: n \in N\}$ such that for every $n \in N, V_n$ is a finite subset of Q_n and $\cup_{n\in N} V_n = 1_u$.

(b) A neutrosophic soft topological space (U,E, τ_u) is called neutrosophic soft almost Menger iff every sequence $\{Q_n: n \in N\}$ of neutrosophic soft open covers of (U,E, τ_u), there exists a sequence $\{V_n: n \in N\}$ such that for every $n \in N, V_n$ is a finite subset of Q_n and $\cup_{n\in N} V_n^* = 1_u$, where $V_n^* = \{cl(V): V \subseteq V_n\}$.

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(c) A neutrosophic soft topological space (U,E,τ_u) is called neutrosophic soft nearly compact iff every sequence $\{Q_n: n \in N\}$ of neutrosophic soft open covers of (U,E,τ_u), there exists a sequence $\{V_n: n \in N\}$ such that for every $n \in N, V_n$ is a finite subset of Q_n and $\cup_{n\in N} V_n^* = 1_u$, where $V_n^* = \{int(cl(V)): V \subseteq V_n\}$.

It is clear that in neutrosophic soft topological spaces we have the following implications:

Neutrosophic soft Menger→neutrosophic soft nearly Menger→neutrosophic soft almost Menger.

Theorem 3.2. A neutrosophic soft topological space (U,E,τ_u) is called neutrosophic soft almost Menger iff for each family $\{Q_n: n \in N\}$ of neutrosophic soft open sets in (U,E,τ_u) having the finite intersection property we have $\cap_{n\in N} cl(Q_n) \neq \Phi_u$.

Proof. Let (U,E,τ_u) be a neutrosophic soft almost Menger topological space. Consider $\{Q_n: n \in N\}$ be a sequence of neutrosophic soft open sets in (U,E,τ_u) having the finite intersection property. Suppose the $\cap_{n\in N} cl(Q_n) = \Phi_u$. Then we have $\cup_{n\in N} [\mathrm{cl}(Q_n)]^c = \cap_{n\in N} int({Q_n}^c) = 1_u$. Since (U,E,$\tau_u$) neutrosophic soft almost Menger, for every $n \in N$, there exists a sequence $\{H_n: n \in N\}$ such that H_n is a finite subset of int(${Q_n}^c$) and $\cup_{n\in N} H_n^* = 1_u$, where $H_n^* = \{cl(H): H \subseteq H_n\}$. But from $H_n \subseteq int({Q_n}^c)$ and $Q_n \subseteq int(cl(Q_n))$, *we see that* $\cap_{n\in N} Q_n = \Phi_u$, which is a contradiction with the finite intersection property of $\{Q_n: n \in N\}$.

Conversely, let $\{Q_n: n \in N\}$ be a neutrosophic soft open cover. If $\cup_{n\in N} H_n^* \neq 1_u$, where $H_n^* = \{cl(H): H \subseteq H_n\}$ and H_n is a finite subset of Q_n, then $\{(H_n^*)^c: n \in N\}$ is an of neutrosophic soft open sequence with the finite intersection property. Hence, from the hypothesis it follows that

$\cap_{n\in N} cl((H_n^*)^c) \neq \Phi_u \Rightarrow \cup_{n\in N} [cl([cl(H_n^*)^c)]^c \neq 1_u$. Since $\cup_{n\in N} Q_n \subseteq \cup_{n\in N} [cl([cl(H_n^*)^c] \neq 1_u$, then $\cup_{n\in N} Q_n \neq 1_u$, which is a contradiction.

Definition 3.3. A neutrosophic soft set N_1 is called a neutrosophic soft regular open set iff N_1=int(cl(N_1)); a neutrosophic soft set N_2 is called a neutrosophic soft regular closed set iff N_2=cl(int(N_2)).

Theorem 3.4. In a neutrosophic soft topological space (U,E,τ_u) the following conditions are equivalent:

(i) $(\mathrm{U}, \mathrm{E}, \tau_u)$ is neutrosophic soft almost Menger.

(ii) For each sequence $\{Q_n: n \in N\}$ of neutrosophic soft regular closed sets such that $\cap_{n\in N} Q_n = \Phi_u$, there exists a sequence $\{V_n: n \in N\}$ such that for every $n \in N, V_n$ is a finite subset of Q_n and $\cap_{n\in N} V_n^* = \Phi_u$, where $V_n^* = \{int(V): V \subseteq V_n\}$.

(iii) $\cap_{n\in N} cl(Q_n) \neq \Phi_u$ holds for each sequence $\{Q_n: n \in N\}$ of neutrosophic soft regular open sets having the finite intersection property.

(iv) For each sequence $\{Q_n: n \in N\}$ of neutrosophic soft regular open covers of $(\mathrm{U}, \mathrm{E}, \tau_u)$, there exists a sequence $\{V_n: n \in N\}$ such that for every $n \in N$, V_n is a finite subset of Q_n and $\cup_{n\in N} V_n^* = \{cl(V): V \subseteq V_n\}$.

Proof. The proof of this theorem follows a similar pattern to Theorem 3.2.

Definition 3.5. Let $(\mathrm{U}, \mathrm{E}, \tau_u)$ and $(\mathrm{V}, \mathrm{E}, \tau_v)$ be two neutrosophic soft topological spaces. Then $(\varphi, \psi): (\mathrm{U}, \mathrm{E}, \tau_u) \to (\mathrm{V}, \mathrm{E}, \tau_v)$ is said to be a neutrosophic soft almost continuous mapping if for each (N,E) neutrosophic soft regular open set of $(\mathrm{V}, \mathrm{E}, \tau_v)$, the inverse image $(\varphi, \psi)^{-1}(N, E) \in \tau_u$. The inverse image of each neutrosophic soft regular open set in $(\mathrm{V}, \mathrm{E}, \tau_v)$ is neutrosophic soft open in $(\mathrm{U}, \mathrm{E}, \tau_u)$.

Theorem 3.6. Let $(\mathrm{U}, \mathrm{E}, \tau_u)$ and $(\mathrm{V}, \mathrm{E}, \tau_v)$ be two neutrosophic soft topological spaces and $(\varphi, \psi): (\mathrm{U}, \mathrm{E}, \tau_u) \to (\mathrm{V}, \mathrm{E}, \tau_v)$ a neutrosophic soft almost continuous surjection mapping. If (M,E) is neutrosophic soft almost Menger in $(\mathrm{U}, \mathrm{E}, \tau_u)$, then (φ, ψ)(M,E) is so in $(\mathrm{V}, \mathrm{E}, \tau_v)$.

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Proof. Let $\{Q_n, E\}: n \in N\}$ be a neutrosophic soft open cover of (φ,ψ)(M,E) i.e., C(M,E) $\subseteq \cup_{n\in N}(Q_n, E)$. Since (φ, ψ) is neutrosophic soft almost continuous, $\{(\varphi, \psi)^{-1} int(cl((Q_n, E))): n \in N\}$ is a neutrosophic soft open cover of (M,E) . Since (M,E) is almost Menger, there is a sequence $\{(H_n, E): n \in N\}$ such that H_n is a finite subset of $\{(\varphi, \psi)^{-1} int(cl((Q_n, E))): n \in N\}$ and $\cup_{n\in N} (H_n^*, E) = 1_u$, where $(H_n^*, E) = \{cl(H, E): H \subseteq H_n\}$. For every $n \in N$ and $H \subseteq H_n$ we can choose a member $(Q_H, E) \subseteq (Q_n, E)$ such that $(H, E) = (\varphi, \psi)^{-1}(Q_H, E)$. From the surjectivity of (φ, ψ) we have

$(M, E) \subseteq \cup_{n\in N}\, cl\left((\varphi, \psi)^{-1}\left(int\left(cl(Q_H, E)\right)\right)\right) = 1_u$.

Hence (φ, ψ)(M,E) $\subseteq (\varphi, \psi)[\,\cup_{n\in N}\, cl((\varphi, \psi)^{-1}(int(cl(Q_H, E))))]$= $\cup_{n\in N}(\varphi, \psi)[cl(\varphi, \psi)^{-1}(int(cl(Q_H, E)))] = f(1_u) = 1_v$. But from $int(cl(Q_H, E)) \subseteq cl(Q_H, E)$ and from the neutrosophic soft almost continuity of *f*,

$(\varphi, \psi)(\mathrm{cl}(\varphi, \psi)^{-1} int(cl((Q_H, E)))) \subseteq (\varphi, \psi)(\,(\varphi, \psi)^{-1} cl(Q_H, E)))) \subseteq cl(Q_H, E)$ for each $n \in N$, i.e., $\cup_{n\in N}\, cl(Q_H, E) = 1_v$. Hence (φ, ψ)(M,E) is neutrosophic soft almost Menger also.

Definition 3.7. Let $(\mathrm{U}, \mathrm{E}, \tau_u)$ and $(\mathrm{V}, \mathrm{E}, \tau_v)$ be two neutrosophic soft topological spaces. Then $(\varphi, \psi): (\mathrm{U}, \mathrm{E}, \tau_u) \rightarrow (\mathrm{V}, \mathrm{E}, \tau_v)$ is said to be a neutrosophic soft weakly continuous mapping if for each (N,E) neutrosophic soft regular open set of $(\mathrm{V}, \mathrm{E}, \tau_v)$, $(\varphi, \psi)^{-1}(N, E) \subseteq int\left((\varphi, \psi)^{-1}\left(cl(N, E)\right)\right)$.

Theorem 3.8. Let $(\mathrm{U}, \mathrm{E}, \tau_u)$ and $(\mathrm{V}, \mathrm{E}, \tau_v)$ be two neutrosophic soft topological spaces and $(\varphi, \psi): (\mathrm{U}, \mathrm{E}, \tau_u) \rightarrow (\mathrm{V}, \mathrm{E}, \tau_v)$ is said to be a neutrosophic soft weakly continuous surjection mapping. If (M,E) is neutrosophic soft Menger in $(\mathrm{U}, \mathrm{E}, \tau_u)$, then $(\varphi, \psi)(M, E)$ is neutrosophic soft almost Menger in $(\mathrm{V}, \mathrm{E}, \tau_v)$.

Proof. By using a similar technique of the proof of Theorem 3.6, the theorem holds.

Definition 3.9. Let $(\mathrm{U}, \mathrm{E}, \tau_u)$ and $(\mathrm{V}, \mathrm{E}, \tau_v)$ be two neutrosophic soft topological spaces. Then $(\varphi, \psi): (\mathrm{U}, \mathrm{E}, \tau_u) \rightarrow (\mathrm{V}, \mathrm{E}, \tau_v)$ is said to be a neutrosophic soft strongly continuous mapping if for each (M,E) neutrosophic soft set of $(\mathrm{V}, \mathrm{E}, \tau_v)$, $(\varphi, \psi)[cl(M, E)] \subseteq (\varphi, \psi)(M, E)$.

Theorem 3.10. Let $(\mathrm{U}, \mathrm{E}, \tau_u)$ and $(\mathrm{V}, \mathrm{E}, \tau_v)$ be two neutrosophic soft topological spaces and $(\varphi, \psi): (\mathrm{U}, \mathrm{E}, \tau_u) \rightarrow (\mathrm{V}, \mathrm{E}, \tau_v)$ a neutrosophic soft strongly continuous surjection mapping. If (M,E) is neutrosophic soft almost Menger in $(\mathrm{U}, \mathrm{E}, \tau_u)$, then $(\varphi, \psi)(M, E)$ is neutrosophic soft Menger in $(\mathrm{V}, \mathrm{E}, \tau_v)$.

Proof. By using a similar technique of the proof of Theorem 3.6, the theorem holds.

Corollary 3.11. Let $(\mathrm{U}, \mathrm{E}, \tau_u)$ and $(\mathrm{V}, \mathrm{E}, \tau_v)$ be two neutrosophic soft topological spaces and $(\varphi, \psi): (\mathrm{U}, \mathrm{E}, \tau_u) \rightarrow (\mathrm{V}, \mathrm{E}, \tau_v)$ a neutrosophic soft strongly continuous surjection mapping. If (M,E) is neutrosophic soft nearly Menger in $(\mathrm{U}, \mathrm{E}, \tau_u)$, then $(\varphi, \psi)(M, E)$ is neutrosophic soft Menger in $(\mathrm{V}, \mathrm{E}, \tau_v)$.

4. Conclusions

In this paper, the concepts of neutrosophic soft Menger topological spaces, Neutrosophic topological spaces, Neutrosophic Bitopological spaces and Neutrosophic crisp supra bitopological spaces were introduced and studied. Some interesting properties were also established. It would be interesting to study similar properties for neutrosophic soft weakly Menger topological spaces, Neutrosophic crisp supra bitopological spaces, Neutrosophic Bitopological Spaces and Neutrosophic Topological Spaces.

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References

[1] W.Al-Omeri, "Neutrosophic crisp Sets via Neutrosophic crisp Topological Spaces", Neutrosophic Sets and Systems, Vol 13, pp.96-104, 2016.
[2] W.Al-Omeri, S.Jafari, "On Generalized closed Sets and Generalized Pre-closed Sets in Neutrosophic Topological Spaces", Mathematics, Vol 7 (1), pp.1-12, 2019.
[3] K.Atanassov and S.Stoeva, "intuitionistic fuzzy sets", in:Polish Symp.on Interval and Fuzzy Mathematics, Poznan, pp.23-26, 1983.
[4] T.Bera and N.K.Mahapatatra, "On Neutrosophic Soft Topological Space", Neutrosophic Sets and Systems, Vol 19, pp.3-15, 2018.
[5] T.Bera and N.K.Mahapatatra, "Introduction to neutrosophic soft topological spaces", OPSEARCH, DOI 10. 1007/S12597-017-0308-7, (March, 2017).
[6] C.Chang, "Fuzzy topological spaces", J.Math.Anal.Appl., Vol 24, pp.182-190, 1968.
[7] D.Çoker, "An introduction to intuitionistic fuzzy topological spaces", Fuzzy Sets and Systems, Vol 88, pp.81-89, 1997.
[8] I.Deli and S.Broumi, "Neutrosophic Soft Matrices and NSM-decision Making", Journal of Intelligent and Fuzzy Systems, Vol 28 (5), pp.2233-2241, 2015.
[9] Lj.D.R.Kocinac, "Star-Menger and related spaces II", Filomat, Vol 13, pp.129-140, 1999.
[10] P.K.Maji, "Neutrosophic soft set", Annals of Fuzzy Mathematics and Informatics, Vol 5 (1), pp.157-168, 2013.
[11] K.Menger, "Einige Überdeckungssatze der Punktmengenlehre", Sitzungsberichte Abt.2a, Mathematik, Astronomie, Physik, Meteorologie und Mechanik. Wiener Akademie, Wien, Vol 133, pp.421-444, 1924.
[12] D.Molodtsov, "Soft set theory-First results", Computer and Mathematics with Applications, Vol 37, pp.19-31, 1999.
[13] A.Parvez and M.Khan, "On Nearly Menger and Nearly Star-Menger Spaces", Filomat, Vol 33 (19), pp.6219-6227, 2019.
[14] F.Rothberger, "Eine Verscharfung dar Eigenschaft G", Fund. Math., Vol 30, pp.50-55, 1938.
[15] M.Scheepers, "Combinatorics of open covers I: Ramsey Theory" , Topology Appl., Vol 73, pp.241-266, 1996.
[16] M.Shabir and M.Naz, "On soft topological spaces", Comput. Math. Appl., Vol 61, pp.1786-1799, 2011.
[17] F.Smarandache, "Neutrosophy, Neutrosophic Probability, Set and Logic," Amer.Res.Press, Rehoboth,USA., p105, 1998, http://fs.gallup.unm.edu/eBook-neutrosophics4.pdf (fourth version).
[18] F.Smarandache, "Neutrosophic set, a generalization of the intuitionistic fuzzy sets", Inter.J.Pure Appl.Math., Vol 24, pp287-297, 2005.
[19] F.Smarandache, S.Pramanik, Eds., "New Neutrosophic Sets via Neutrosophic Topological Spaces", In Neutrosophic Operational Research, Pons Editions: Brussel, Belgium, Volume 1, pp.189-209, 2017.
[20] L.A.Zadeh, "Fuzzy sets", Inf.Control, Vol 8, pp.338-353, 1965.
[21] M. Al- Tahan , Bijan Davvaz, "Neutrosophic ℵ–Ideals (ℵ-Subalgebras) of Subtraction Algebra", International Journal of Neutrosophic Science, Volume 3 , Issue 1, pp.44-53 , 2020.
[22] M. Songsaeng , A.Iampan, "Image and Inverse Image of Neutrosophic Cubic Sets in UP-Algebras under UP-Homomorphisms", International Journal of Neutrosophic Science, Volume 3 , Issue 2, pp. 89-107 , 2020
[23] T. Bera , Nirmal Kumar Mahapatra, "An Approach to Solve the Linear Programming Problem Using Single Valued Trapezoidal Neutrosophic Number", International Journal of Neutrosophic Science, Volume 3 , Issue 2, pp. 54-66 , 2020.

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