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FULL LENGTH ARTICLE

Generalization of Chatterjea Fixed Point Theorem in b-Metric Spaces

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ABSTRACT

In this paper, we generalize Chatterjea's fixed point theorem in b-metric spaces by using a subadditive altering distance function. We also present corollaries and examples as applications of the theorem.

Keywords: Fixed point theory ▪ b-metric space ▪ Subadditive altering distance function

1. INTRODUCTION

Banach's fixed point theorem is one of the principal tools used to study nonlinear equations. Fixed point theorems have applications in economics, engineering, biology, chemistry, physics, computer science, and other fields. They are also used to solve nonlinear and differential equations.

Chatterjea [1] introduced the following type of contraction.

Definition 1.1. *A mapping $T : X \to X$ on a metric space (X, d) is called a Chatterjea-type contraction if there exists $\alpha \in (0, \frac{1}{2})$ such that, for all $x, y \in X$,*

$$d(Tx, Ty) \leq \alpha \; d(x, Ty) + d(y, Tx) \; . \qquad (1.1)$$

Chatterjea proved that every Chatterjea-type contraction on a complete metric space has a unique fixed point.

The notion of a b-metric space was developed in [2, 3]. A function $d : X \times X \to [0, \infty)$, where $X \neq \varnothing$, is a b-metric with parameter $b \geq 1$ if, for all $x, y, z \in X$,

$$d(x,y) = 0 \iff x = y, \qquad d(x,y) = d(y,x),$$

and

$$d(x,z) \leq b \; d(x,y) + d(y,z) \; .$$

The pair (X, d) is then called a b-metric space with parameter b.

Many authors have generalized Banach's fixed point theorem in b-metric spaces [3, 4, 5, 6, 7, 8, 9, 10]. Boriceanu et al. [11] defined convergence, Cauchy sequences, and completeness in b-metric spaces. Other authors have extended Chatterjea's fixed point theorem [12, 13]. In 2025, Ampadu [14] established a common fixed point theorem for four self-mappings under Chatterjea contractive conditions in a cone pentagonal metric space. In 2025, Anghelina [15] established Matkowski-, Kannan-, and Chatterjea-type fixed point theorems in f-metric spaces, which generalize b-metric spaces.

This paper extends Chatterjea's result by using a subadditive altering distance function. The following definition is used throughout.

Definition 1.2 ([16]). *A subadditive altering distance function is a mapping $\phi : [0, \infty) \to [0, \infty)$ satisfying, for all $x, y \in [0, \infty)$:*

(i) ϕ is continuous and strictly increasing;

(ii) $\phi(x) = 0$ if and only if $x = 0$;

(iii) $\phi(\sigma x + \lambda y) \leq \sigma\phi(x) + \lambda\phi(y)$ for $\sigma, \lambda \in \mathbb{R}$.

2. MAIN RESULTS

Let ϕ be a subadditive altering distance function, and consider a b-metric space (X, d) with the topology generated by d. We first present the following fixed point theorem.

Theorem 2.1. *Let (X, d) be a complete b-metric space with parameter $b \geq 1$, and let $T : X \to X$ satisfy, for some $p \in (0, \frac{1}{b+2})$,*

$$\phi \; d(Tx, Ty) \; \leq p \; \phi \; d(x,y) \; + \phi \; d(x, Ty) \qquad (2.1)$$

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for all $x, y \in X$. Then T has a unique fixed point.

Proof. Choose $x_0 \in X$ and define a sequence by $x_n = Tx_{n-1}$ for $n = 1, 2, \ldots$. Applying (2.1) repeatedly gives

$$\begin{aligned}\phi\ d(x_n, x_{n-1}) &= \phi\ d(Tx_{n-1}, Tx_{n-2}) \\ &\le p\phi\ d(x_{n-1}, x_{n-2}) \\ &\le \cdots \le p^{n-1}\phi\ d(x_1, x_0)\ .\end{aligned}$$

It follows that, for every $n \ge 1$,

$$d(x_n, x_{n-1}) \le p^{n-1} d(x_1, x_0). \tag{2.2}$$

For $m > n$, the b-metric inequality yields

$$\begin{aligned}d(x_n, x_m) &\le \sum_{j=0}^{m-n-1} b^{j+1} d(x_{n+j}, x_{n+j+1}) \\ &\le p^{n-1}\ d(x_1, x_0) \sum_{j=1}^{m-n} (bp)^j.\end{aligned}$$

Since $bp < 1$, the parenthesized terms are bounded by a convergent geometric series, and the factor p^{n-1} tends to zero. Hence $\{x_n\}$ is Cauchy. Completeness gives some $x \in X$ such that $x_n \to x$.

Using the b-metric inequality, the contractive condition, and subadditivity, we obtain

$$\begin{aligned}\phi\ d(Tx, x) &\le bp\left[\phi\ d(x, x_n) + \phi\ d(x, Tx_n)\right] \\ &\quad + b\phi\ d(Tx_n, x)\ .\end{aligned}$$

Because $Tx_n = x_{n+1} \to x$, letting $n \to \infty$ gives $\phi(d(Tx, x)) = 0$. Thus $d(Tx, x) = 0$, so $Tx = x$.

Now let y be another fixed point, so $Ty = y$. By (2.1),

$$\phi\ d(x, y) = \phi\ d(Tx, Ty) \le 2p\phi\ d(x, y)\ .$$

Since $2p < \frac{2}{b+2} < 1$, it follows that $\phi(d(x, y)) = 0$, and hence $x = y$. □

Example 2.1. *Let $([0, 1], d)$ be the complete b-metric space with $d(x, y) = |x - y|$. Define $T : [0, 1] \to [0, 1]$ by*

$$Tx = \frac{1}{4x}, \qquad x \in [0, 1],$$

and let $\phi(x) = x$. The inequality in (2.1) becomes

$$\frac{1}{4x} - \frac{1}{4y} \le \frac{1}{4}\left|x - y| + x - \frac{1}{4y}\right|\ .$$

The example states that T has the unique fixed point [1].

On $X = [-1, 1]$, the map has the two fixed points $-\frac{1}{2}$ and $\frac{1}{2}$, but does not satisfy (2.1). For example, if $x = -\frac{1}{10}$ and $y = \frac{1}{10}$, no $p \le 1$ satisfies (2.1).

Corollary 2.1. *Let (X, d) be a complete b-metric space with parameter $b \ge 1$. If $T : X \to X$ satisfies, for some $p > 0$ with $p < \frac{1}{b+2}$,*

$$d(Tx, Ty) \le p\ \ d(x, y) + d(x, Ty)\ \ , \tag{2.3}$$

for all $x, y \in X$, then T has a unique fixed point.

Proof. Set $\phi(x) = x$ in Theorem 2.1. □

Corollary 2.2. *Let (X, d) be a complete b-metric space with parameter $b \ge 1$. Suppose $T : X \to X$ satisfies, for some $p > 0$ with $p < \frac{1}{b+2}$,*

$$\phi\ d(T^k x, T^k y) \le p\phi\ d(x, y) + \phi\ d(x, T^k y)\ . \tag{2.4}$$

Then T has a unique fixed point.

Proof. Apply Theorem 2.1 to $S = T^k$ to obtain a unique fixed point $x \in X$ of S. Thus $T^k x = Sx = x$. □

Theorem 2.2. *Let (X, d) be a complete b-metric space with parameter $b \ge 1$, and let $T : X \to X$ satisfy, for some $p \in (0, \frac{1}{b+2})$,*

$$\begin{aligned}\phi\ d(Tx, Ty) &\le p\ \ \phi\ d(x, y) + \phi\ d(Tx, y) \\ &\quad + \ \ \phi\ (d, x\ Ty)\end{aligned} \tag{2.5}$$

for all $x, y \in X$. Then T has a unique fixed point.

Proof. As in Theorem 2.1, set $x_n = Tx_{n-1}$. The same estimates give

$$d(x_n, x_{n-1}) \le p^{n-1} d(x_1, x_0),$$

so $\{x_n\}$ is Cauchy in the complete space X. Let $x_n \to x$. Applying (2.5) and the b-metric inequality gives an estimate whose limit is

$$\phi\ d(Tx, x) \le bp\phi\ d(Tx, x)\ .$$

Since $bp < 1$, we have $\phi(d(Tx, x)) = 0$, and therefore $Tx = x$. If y is another fixed point, (2.5) gives

$$\phi\ d(x, y) \le 3p\phi\ d(x, y)\ .$$

As $3p < \frac{3}{b+2} \le 1$, the strict upper bound on p implies $\phi(d(x, y)) = 0$ and hence $x = y$. □

Example 2.2. *Let $([-1, 1], d)$ be the complete b-metric space with $d(x, y) = |x - y|$. Define $T : [-1, 1] \to [-1, 1]$ by $Tx = \frac{1}{6}x$ for all $x \in [-1, 1]$, and take $\phi(x) = x$. In this case, condition (2.5) reads*

$$\frac{x}{6} - \frac{y}{6} \le \frac{1}{6}\left|x - y| + \frac{x}{6} - y + x - \frac{y}{6}\right|\ ,$$

and the unique fixed point is 0.

Theorem 2.3. *Let (X, d) be a complete b-metric space with parameter $b \ge 1$, and suppose $T : X \to X$ satisfies, for some p with $0 < bp < 1$,*

$$\phi\ d(Tx, Ty) \le p\ \ \phi\ d(Tx, y) + \phi\ d(x, Ty) \tag{2.6}$$

for all $x, y \in X$. Then T has a unique fixed point.

Proof. Since $p\phi(d(x,y)) \geq 0$, condition (2.6) implies

$$\phi\ d(Tx,Ty) \leq p\ \phi\ d(x,y) + \phi\ d(Tx,y) + \phi\ d(x,Ty) .$$

Applying Theorem 2.2 gives the result. □

In the special case where (X, d) is a complete metric space and $\phi(x) = x$, Theorem 2.3 coincides with the classical Chatterjea fixed point theorem.

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