Contradiction-Aware Neutrosophic–Fuzzy Sensor Fusion for Reliable Irrigation Scheduling under Missing and Conflicting Evidence

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Abstract

Smart irrigation decisions are often computed from multiple sensor and model channels whose evidence may be gradual, missing, or mutually contradictory. A conventional fuzzy controller compresses these conditions into a scalar membership $\mu \in [0,1]$, whereas a single-valued neutrosophic representation can retain support, indeterminacy, and opposition as (T, I, F). This paper proposes a contradiction-aware neutrosophic–fuzzy irrigation system (CNFIS) in which three evidence channels for criterion j are summarized by an availability ratio q_j, a robust fuzzy center $\widetilde{m}_j$, and a contradiction index c_j. The resulting state is

$$(T_j, I_j, F_j) = \big(q_j\widetilde{m}_j,\ 1 - q_j + q_jc_j,\ q_j(1 - \widetilde{m}_j)\big),$$

which satisfies $T_j + I_j + F_j = 1 + q_jc_j$ and therefore separates missingness from contradiction without interpreting the triple as a probability vector. A neutral-prior shrinkage rule $s_j = (1 - \alpha I_j)r_j + \alpha I_j/2$ is then fused through $\widehat{d} = \sum_j w_js_j$ to estimate normalized irrigation demand, while $U = \sum_j w_jI_j$ supplies an explicit re-sensing or abstention signal. The method is evaluated in a fully reproducible Monte Carlo benchmark containing 30 independent runs of $N = 5000$ cases under five evidence regimes; no field measurements are claimed. At moderate missingness/corruption $(p_m, p_c) = (0.10, 0.10)$, CNFIS obtains RMSE 0.03318 ± 0.00086 compared with 0.04192 ± 0.00079 for mean fuzzy fusion, a reduction of 20.84%. The reductions remain 12.63% and 10.57% under severe and extreme regimes, respectively, while the ordinary fuzzy mean retains a small advantage in clean evidence. At 80% selective coverage, the uncertainty score decreases CNFIS RMSE from 0.03320 to 0.02825 in the moderate regime. These results support the proposed representation as a mathematically transparent robustness layer for irrigation systems in which evidence quality, not only fuzzy demand, must be modeled explicitly.

Keywords: Neutrosophic systems; Fuzzy logic; Smart irrigation; Sensor fusion; Uncertainty; Contradiction; Missing data; Precision agriculture

1 Introduction

Irrigation scheduling can be expressed as a bounded decision map $f : \mathcal{X} \to [0,1]$ in which environmental measurements $\mathbf{x} = (x_1, \ldots, x_p)$ are converted into a normalized water-demand level $d = f(\mathbf{x})$. The mapping is rarely crisp because a soil-moisture value near a control threshold should not change an irrigation command discontinuously from $d = 0$ to $d = 1$. Fuzzy sets address this gradual transition through a membership function $\mu_A : X \to [0,1]$,[1] and smart-irrigation research has consequently used fuzzy inference to combine soil, climate, and temporal variables in closed-loop or decision-support configurations.[2]

The more difficult case is not vagueness alone but unreliable evidence. Let x_{jk} denote the kth channel reporting criterion j, for example a local sensor, a neighboring sensor, or a model-derived estimate. Even when

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every $x_{jk} \in [0,1]$ has already been fuzzified, the set $\{x_{j1}, x_{j2}, x_{j3}\}$ may contain missing values or strong disagreement. Existing fuzzy irrigation systems have demonstrated effective control and scheduling using sensor inputs,[3–9] but the scalar fusion $\bar{x}_j = n_j^{-1} \sum_k x_{jk}$ does not itself indicate whether $\bar{x}_j = 0.5$ was produced by consistent evidence near 0.5 or by contradictory values near 0 and 1.

Single-valued neutrosophic sets represent an observation by $N = \langle T, I, F \rangle \in [0,1]^3$ without requiring $T + I + F = 1$.[10] This additional freedom has been used in agricultural and resource-allocation settings, including energy–food–water optimization, biomass crop selection, agricultural water allocation, and uncertain crop-production planning.[11–14] Those studies show that neutrosophic mathematics is useful when uncertainty is not adequately represented by one fuzzy grade; however, their optimization-level constructions do not directly answer how contradictory sensor channels should be converted into (T, I, F) before an irrigation control decision is produced.

The present paper therefore considers the evidence-fusion problem $\mathcal{P} = (\mathbf{m}, \mathbf{a}, \mathbf{w})$, where $\mathbf{m}$ contains fuzzy demand observations, $\mathbf{a} \in \{0,1\}^{J \times K}$ indicates channel availability, and $\mathbf{w} \in \Delta^{J-1}$ is a convex criterion-weight vector. Its main contribution is a mapping $\Phi : (\mathbf{m}, \mathbf{a}) \mapsto (T, I, F)$ for which missingness transfers evidence mass into I, contradiction creates a measurable non-normalized excess $\kappa = T + I + F - 1$, and a subsequent fuzzy decision layer returns $\widehat{d} \in [0,1]$. The construction is deliberately algebraic and explainable: every increase in I can be traced to either reduced availability q or disagreement c, rather than to a learned latent parameter.

Four contributions follow from this formulation. First, CNFIS defines $I_j = 1 - q_j + q_j c_j$, yielding the identity $T_j + I_j + F_j = 1 + q_j c_j$ and hence the contradiction residual $\kappa_j = q_j c_j$. Second, it introduces an indeterminacy-controlled shrinkage $s_j = (1 - \alpha I_j) r_j + \alpha I_j \pi$ with neutral prior $\pi = 1/2$, which preserves $s_j \in [0,1]$. Third, it aggregates criterion uncertainty as $U = \sum_j w_j I_j$ and uses U to support selective decisions rather than forcing every irrigation command. Fourth, it evaluates the method under a reproducible stochastic model $x_{ijk} = \Pi_{[0,1]}(m_{ij} + \varepsilon_{ijk} + o_{ijk} b_{ijk})$ across controlled missingness and corruption regimes, allowing the robustness contribution to be separated from claims of field agronomy.

All literature used to motivate or position the article satisfies a temporal boundary $t_r \le \tau$, where $\tau =$ 31 May 2024; no post-May-2024 concept, dataset, result, or reference is used. The paper is organized around the methodological argument rather than a survey sequence. Section 2 establishes the research context and formulates the unresolved sensor-fusion question; Section 3 develops the complete CNFIS mathematical framework and its properties; Section 4 defines the reproducible computational experiment; Section 5 reports robustness, selective-control, and sensitivity results; Section 6 interprets the mechanism and deployment implications; Section 7 states validity boundaries and future validation requirements; and Section 8 concludes the study.

2 Research Context and Positioning

2.1 Fuzzy Irrigation as a Graded Control Map

Fuzzy irrigation systems treat agronomic states as graded evidence rather than crisp events. If z_j is a physical variable and $\mu_j(z_j) \in [0,1]$ is its membership in an irrigation-relevant concept, a generic rule ℓ has activation $\eta_\ell = \mathcal{T}(\mu_1, \ldots, \mu_p)$ and the controller maps the rule vector $\boldsymbol{\eta}$ to a normalized command $d = \mathcal{A}(\boldsymbol{\eta}) \in [0,1]$. Early sensor-connected implementations and decision-support systems demonstrated the value of this representation for irrigation and water conservation,[3,4] while later studies extended it to demand-side management, zoned IoT control, multi-input/multi-output irrigation, precision scheduling, and practically tested watering control.[5–9] The common mathematical strength is a smooth map $\mathbf{z} \mapsto \boldsymbol{\mu} \mapsto d$; the unresolved issue is that the map generally receives one scalar membership per criterion even when that scalar is reconstructed from evidence of unequal quality.

The distinction becomes important when criterion j is observed through replicated channels $x_{j1}, \ldots, x_{jK}$. A conventional fusion such as $\bar{x}_j = K^{-1} \sum_k x_{jk}$ or a robust center $\widetilde{m}_j = \mathrm{median}_k\, x_{jk}$ estimates location, but neither quantity alone identifies whether uncertainty came from unavailable channels or contradiction among

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available channels. Thus two evidence sets can satisfy $\bar{x}_j^{(1)} = \bar{x}_j^{(2)}$ while their dispersions $c_j^{(1)}$ and $c_j^{(2)}$ differ substantially. This paper treats that difference as information that should enter the control state before the final fuzzy aggregation.

2.2 Neutrosophic Representation of Evidence Quality

A single-valued neutrosophic state $N = \langle T, I, F \rangle \in [0,1]^3$ provides a natural representation when truth support, indeterminacy, and falsity support should remain separately observable.[10] In agricultural applications, neutrosophic formulations have primarily been used above the sensing layer: energy–food–water nexus optimization, biomass crop selection, and uncertain crop-production planning operate on alternatives, objectives, or uncertain coefficients;[11,12,14] related fuzzy optimization has also addressed agricultural water allocation under uncertain planning parameters.[13] In those formulations, uncertainty typically modifies a coefficient or decision alternative $\widetilde{a}$, whereas the present problem is local and pre-decisional: a channel set $\{x_{jk}, a_{jk}\}_{k=1}^{K}$ must first be transformed into a reliability-aware criterion state before d is computed.

The required mapping is therefore not a replacement for fuzzy irrigation logic but a reliability layer $\mathcal{G}$ placed in front of it. For each criterion, CNFIS seeks

$$\mathcal{G} : \{x_{jk}, a_{jk}\}_{k=1}^{K} \longmapsto (T_j, I_j, F_j, s_j), \qquad d = \sum_{j=1}^{J} w_j s_j, \tag{1}$$

where s_j remains a bounded fuzzy demand score and I_j exposes the quality of the evidence used to obtain it. This placement preserves existing fuzzy membership engineering $\mu_j(z_j)$ and final control logic while introducing a separate mathematical path for evidence reliability.

2.3 Research Gap and Testable Hypothesis

The specific gap is the absence, in the cited pre-June-2024 irrigation literature, of a sensor-fusion rule that decomposes indeterminacy into missingness and contradiction and then uses that quantity both to moderate the fuzzy action and to support selective re-sensing. Let $q_j \in [0,1]$ denote availability and $c_j \in [0,1]$ contradiction. The design objective is an indeterminacy function $I_j = I(q_j, c_j)$ satisfying $\partial I/\partial q \leq 0$ and $\partial I/\partial c \geq 0$, together with a decision score $s_j \in [0,1]$ whose deviation from the neutral prior decreases as I_j increases. These requirements lead to the construction developed in Section 3 rather than to another rule-base architecture.

The empirical hypothesis is intentionally conditional rather than universal. Define the robustness margin against ordinary fuzzy averaging as

$$\Delta_{\mathrm{R}}(p_m, p_c) = \mathrm{RMSE}_{\mathrm{FuzzyMean}}(p_m, p_c) - \mathrm{RMSE}_{\mathrm{CNFIS}}(p_m, p_c). \tag{2}$$

The study tests whether $\Delta_{\mathrm{R}} > 0$ once missingness p_m and gross corruption p_c become non-negligible, while allowing $\Delta_{\mathrm{R}} \leq 0$ in clean evidence. This formulation makes the contribution falsifiable: CNFIS is supported only if explicit $I(q, c)$ improves degraded-evidence robustness without requiring a claim of universal superiority.

3 CNFIS Mathematical Framework

3.1 Decision Model and Evidence Space

Let $i \in \{1, \ldots, N\}$ index an irrigation decision, $j \in \{1, \ldots, J\}$ a fuzzy demand criterion, and $k \in \{1, \ldots, K\}$ an evidence channel. A deployment may use $J = 4$ criteria such as soil-moisture deficit, heat stress, vapor-pressure stress, and absence of expected rainfall, with fuzzified values $m_{ij} \in [0,1]$. One possible soil-moisture

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transformation is $m_{i1} = \Pi_{[0,1]}[(\theta_{\mathrm{ref}} - \theta_i)/(\theta_{\mathrm{ref}} - \theta_{\mathrm{wp}})]$, while a temperature criterion may use $m_{i2} = [1 + \exp\{-\beta_T(T_i - T_0)\}]^{-1}$; the proposed fusion is agnostic to the specific membership functions as long as every criterion lies in $[0,1]$.

The ideal normalized irrigation demand is defined as a convex model

$$d_i^\star = \sum_{j=1}^{J} w_j m_{ij}, \qquad w_j \geq 0, \qquad \sum_{j=1}^{J} w_j = 1, \tag{3}$$

so $d_i^\star \in [0,1]$. In a site-specific controller, $d_i^\star$ may subsequently be mapped to a physical dose $y_i = Y_{\max} d_i^\star$, where $Y_{\max} > 0$ is an agronomically selected upper dose; the present study intentionally evaluates the normalized quantity d so that no unverified crop-specific value of $Y_{\max}$ is introduced.

Channel k returns a noisy and potentially corrupted observation $x_{ijk} \in [0,1]$ with availability flag $a_{ijk} \in \{0,1\}$. The available count and proportion are

$$n_{ij} = \sum_{k=1}^{K} a_{ijk}, \qquad q_{ij} = \frac{n_{ij}}{K} \in [0,1]. \tag{4}$$

The fusion problem is difficult because the same arithmetic mean $\bar{x}_{ij} = 0.5$ can result from $\{0.48, 0.50, 0.52\}$ or $\{0.05, 0.50, 0.95\}$; therefore the estimate must depend on both a center and a disagreement functional c_{ij}.

For $n_{ij} > 0$, define the robust center $\widetilde{m}_{ij} = \mathrm{median}\{x_{ijk} : a_{ijk} = 1\}$ and use the neutral convention $\widetilde{m}_{ij} = 1/2$ when $n_{ij} = 0$. The contradiction index is

$$c_{ij} = \begin{cases} \dfrac{1}{n_{ij}} \displaystyle\sum_{k:a_{ijk}=1} |x_{ijk} - \widetilde{m}_{ij}|, & n_{ij} \geq 2, \\ 0, & n_{ij} < 2, \end{cases} \tag{5}$$

which satisfies $0 \leq c_{ij} \leq 1$. Equation (5) is a bounded median-centered dispersion rather than a statistical variance; its purpose is to measure contradiction in already-normalized evidence without assuming a Gaussian distribution at deployment time.

3.2 Contradiction-Aware Neutrosophic Construction

For each (i,j), CNFIS maps $(q_{ij}, \widetilde{m}_{ij}, c_{ij})$ to the single-valued neutrosophic triple

$$T_{ij} = q_{ij}\widetilde{m}_{ij}, \qquad F_{ij} = q_{ij}(1 - \widetilde{m}_{ij}), \qquad I_{ij} = 1 - q_{ij} + q_{ij}c_{ij}. \tag{6}$$

The factors T_{ij} and F_{ij} contain the available evidence mass q_{ij}, while I_{ij} increases either when channels disappear ($q_{ij} \downarrow$) or when the available channels disagree ($c_{ij} \uparrow$). This construction deliberately leaves $T + I + F$ non-normalized under contradiction, consistent with the SVNS representation.[10]

The algebraic meaning of the construction follows immediately from Eq. (6):

$$T_{ij} + F_{ij} = q_{ij}, \qquad T_{ij} + I_{ij} + F_{ij} = 1 + q_{ij}c_{ij}, \qquad \kappa_{ij} = T_{ij} + I_{ij} + F_{ij} - 1 = q_{ij}c_{ij}. \tag{7}$$

Thus $\kappa_{ij} = 0$ when available evidence is internally consistent ($c_{ij} = 0$), even if some channels are absent, whereas $\kappa_{ij} > 0$ identifies contradiction among evidence that is actually present. Missingness and contradiction therefore have distinct signatures: missingness raises I by $1 - q$, whereas contradiction raises both I and the residual κ by qc.

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3.3 Reliability-Adjusted Fuzzy Decision

When $q_{ij} > 0$, the directional demand ratio embedded in (T, F) is $r_{ij} = T_{ij}/(T_{ij} + F_{ij}) = \widetilde{m}_{ij}$; when $q_{ij} = 0$, define $r_{ij} = 1/2$. CNFIS does not directly subtract I from T, because I represents unavailable or contradictory information rather than falsity. Instead, it shrinks r_{ij} toward a neutral prior $\pi = 1/2$:

$$s_{ij} = (1 - \alpha I_{ij})r_{ij} + \alpha I_{ij}\pi, \qquad 0 \le \alpha \le 1, \tag{8}$$

where α controls how strongly unreliable evidence is moderated. For $I_{ij} = 0$, Eq. (8) gives $s_{ij} = r_{ij}$; for increasing I_{ij}, s_{ij} moves toward $1/2$ rather than toward either "irrigate" or "do not irrigate."

The final normalized demand and aggregate uncertainty are

$$\widehat{d}_i = \sum_{j=1}^{J} w_j s_{ij}, \qquad U_i = \sum_{j=1}^{J} w_j I_{ij}, \tag{9}$$

with $\widehat{d}_i, U_i \in [0, 1]$. A conventional controller may use $\widehat{d}_i$ directly, whereas a safety-oriented controller can impose a selective rule $\delta_i(\tau) = \mathbf{1}\{U_i \le \tau\}$, so the command is issued only when U_i is below a threshold τ and otherwise a re-sensing or inspection action is requested.

3.4 Mathematical Properties

Proposition 1 (bounded neutrosophic coordinates). If $q, \widetilde{m}, c \in [0, 1]$, then the triple in Eq. (6) satisfies $(T, I, F) \in [0, 1]^3$ and $1 \le T + I + F \le 2$. Indeed, $T = q\widetilde{m} \in [0, 1]$ and $F = q(1 - \widetilde{m}) \in [0, 1]$, while $I = 1 - q(1 - c) \in [0, 1]$; Eq. (7) gives $T + I + F = 1 + qc \in [1, 2]$. The upper bound is tighter than the generic SVNS bound 3 because CNFIS reserves non-normalization specifically for observed contradiction.

Proposition 2 (separation of missingness and contradiction). For fixed c, $\partial I/\partial q = -(1 - c) \le 0$, so increasing availability weakly reduces indeterminacy; for fixed q, $\partial I/\partial c = q \ge 0$, so increasing contradiction raises indeterminacy. Meanwhile $\partial\kappa/\partial c = q$ and $\partial\kappa/\partial q = c$, which means $\kappa = qc$ can be positive only when contradictory evidence is actually observed. This dual response is the principal distinction from using only $I^{(m)} = 1 - q$, which recognizes missingness but not disagreement.

Proposition 3 (bounded and conservative fuzzy score). For $r, \pi, I, \alpha \in [0, 1]$, Eq. (8) is a convex combination because $0 \le \alpha I \le 1$, and therefore $s \in [0, 1]$. With $\pi = 1/2$, its derivative with respect to indeterminacy is $\partial s/\partial I = \alpha(1/2 - r)$, so uncertainty pulls an aggressive demand $r > 1/2$ downward and a low demand $r < 1/2$ upward. This is conservative in the epistemic sense that $|s - 1/2| = (1 - \alpha I)|r - 1/2|$ decreases monotonically with I.

Corollary 1 (bounded irrigation demand). Since $s_j \in [0, 1]$, $w_j \ge 0$, and $\sum_j w_j = 1$, Eq. (9) is a convex combination and hence $0 \le \widehat{d} \le 1$. Consequently, a physical mapping $y = Y_{\max}\widehat{d}$ automatically satisfies $0 \le y \le Y_{\max}$, although selection of $Y_{\max}$ must be performed from crop-, soil-, and site-specific agronomic information rather than from the controlled benchmark used here.

3.5 Algorithm and Computational Cost

For a decision matrix $X_i = [x_{ijk}] \in [0, 1]^{J \times K}$ and availability matrix $A_i = [a_{ijk}]$, the computational sequence is $X_i \to (q, \widetilde{m}, c) \to (T, I, F) \to (s, U) \to \widehat{d}$. The algorithm requires $O(JK)$ operations for medians when K is fixed and small; more generally sorting-based medians yield $O(JK \log K)$, while all remaining vector operations are $O(JK + J)$. Its storage is $O(JK)$ if channels are retained or $O(J)$ if summary statistics are streamed.

Algorithm 1: CNFIS for one irrigation decision. Given $X = [x_{jk}]$, $A = [a_{jk}]$, w, and α: (1) compute $n_j = \sum_k a_{jk}$ and $q_j = n_j/K$; (2) set $\widetilde{m}_j$ to the median of available x_{jk}, or $1/2$ if $n_j = 0$; (3) compute c_j from Eq. (5); (4) construct (T_j, I_j, F_j) from Eq. (6); (5) set $r_j = T_j/(T_j + F_j)$ when $q_j > 0$, otherwise $r_j = 1/2$; (6) calculate s_j by Eq. (8); and (7) return $\widehat{d} = \sum_j w_j s_j$, $U = \sum_j w_j I_j$, and the audit vector $\boldsymbol{\kappa} = (q_1c_1, \ldots, q_Jc_J)$.

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4 Reproducible Computational Experiment

4.1 Synthetic Evidence Generation

The experiments are designed to isolate the fusion mechanism rather than simulate a named crop. Four latent fuzzy criteria m_{ij} are generated independently from beta distributions $\mathrm{Beta}(a_j, b_j)$ with parameter matrix

$$[(a_j, b_j)]_{j=1}^{4} = ((2.2, 2.0), (2.0, 2.5), (2.0, 2.2), (1.8, 2.4)),$$

and the reference demand follows Eq. (3) with $\mathbf{w} = (0.45, 0.20, 0.20, 0.15)$. The larger $w_1 = 0.45$ represents an experimental design in which soil-moisture deficit is dominant; these weights are not claimed to be universal agronomic coefficients.

Each criterion is observed through $K = 3$ evidence channels according to

$$x_{ijk} = \Pi_{[0,1]}(m_{ij} + \varepsilon_{ijk} + o_{ijk}b_{ijk}), \tag{10}$$

where $\varepsilon_{ijk} \sim \mathcal{N}(0, \sigma^2)$ with $\sigma = 0.04$, $o_{ijk} \sim \mathrm{Bernoulli}(p_c)$ indicates gross corruption, and $|b_{ijk}| \sim \mathcal{U}(0.25, 0.55)$ with equiprobable sign. Channel availability is independently sampled as $a_{ijk} \sim \mathrm{Bernoulli}(1 - p_m)$, so p_m and p_c independently control missingness and contradictory outliers.

Five regimes are used: clean $(p_m, p_c) = (0, 0)$, mild $(0.05, 0.05)$, moderate $(0.10, 0.10)$, severe $(0.20, 0.20)$, and extreme $(0.30, 0.30)$. Every regime contains $R = 30$ seeded repetitions of $N = 5000$ decisions, giving $RN = 150{,}000$ evaluated cases per regime and 750,000 cases across the main experiment. The implementation file `simulation.py` supplied with the LaTeX package reproduces the generator, metrics, tables, and figures from fixed pseudo-random seeds.

4.2 Baselines and Fixed Parameterization

Four fusion rules are compared using the same $\mathbf{w}$ and the same observations. The *Fuzzy-mean* baseline uses $\bar{x}_{ij}$ over available channels and replaces an all-missing criterion by $1/2$; the *Robust-fuzzy* baseline uses $\widetilde{m}_{ij}$; the *Missingness-aware* baseline uses $s_{ij}^{(m)} = q_{ij}\widetilde{m}_{ij} + (1 - q_{ij})/2$; and CNFIS uses Eq. (8) with $I = 1 - q + qc$. This decomposition makes it possible to distinguish the effect of the median from the additional contribution of explicit contradiction c.

The principal CNFIS parameter is fixed at $\alpha = 0.40$ before the main comparison. A sensitivity experiment evaluates $\alpha \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$, so any regime-specific advantage at a larger α can be seen rather than hidden by re-optimizing the main table. For selective decisions, cases are ranked by U_i and the lowest-uncertainty $100\rho\%$ are retained at coverage $\rho \in \{1.0, 0.8, 0.6\}$; this implements τ through the empirical ρ-quantile of U and avoids selecting a threshold from the test error.

4.3 Evaluation Criteria and Reporting

For predictions $\widehat{d}_i$ and targets $d_i^\star$, the primary errors are

$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N} |\widehat{d}_i - d_i^\star|, \qquad \mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\widehat{d}_i - d_i^\star)^2}, \tag{11}$$

and goodness of fit is $R^2 = 1 - \sum_i(\widehat{d}_i - d_i^\star)^2 / \sum_i(d_i^\star - \bar{d})^2$. An auxiliary three-band decision uses $C(d) = 0$ for $d < 0.4$, $C(d) = 1$ for $0.4 \le d < 0.6$, and $C(d) = 2$ for $d \ge 0.6$, from which accuracy and macro-F_1 are computed. The continuous RMSE remains the primary metric because the thresholds 0.4 and 0.6 are experimental demand bands rather than crop-specific irrigation limits.

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5 Experimental Results and Robustness Analysis

5.1 Baseline Comparison across Evidence Regimes

Table 1 reports mean $\pm$ standard deviation across $R = 30$ repetitions. In clean evidence, Fuzzy-mean gives the lowest RMSE 0.01273 ± 0.00012, while CNFIS gives 0.01469 ± 0.00015; the difference is expected because median aggregation and uncertainty shrinkage trade a small amount of efficiency for robustness. Once corruption is introduced, the ordering changes: at $(p_m, p_c) = (0.10, 0.10)$, CNFIS reduces RMSE from 0.04192 to 0.03318, or 20.84%, and increases R^2 from 0.8793 to 0.9243.

Table 1: Main controlled benchmark. Values are mean $\pm$ standard deviation over 30 runs of $N = 5000$ cases; boldface marks the best mean for each metric within a regime.

Regime	Method	MAE	RMSE	R^2	Accuracy	Macro-F_1
Clean	Fuzzy-mean	$\mathbf{0.01015 \pm 0.00010}$	$\mathbf{0.01273 \pm 0.00012}$	$\mathbf{0.98883 \pm 0.00035}$	$\mathbf{0.95222 \pm 0.00296}$	$\mathbf{0.94857 \pm 0.00317}$
	Robust-fuzzy	0.01179 ± 0.00012	0.01479 ± 0.00015	0.98494 ± 0.00043	0.94437 ± 0.00322	0.94007 ± 0.00344
	Missingness-aware	0.01179 ± 0.00012	0.01479 ± 0.00015	0.98494 ± 0.00043	0.94437 ± 0.00322	0.94007 ± 0.00344
	CNFIS	0.01172 ± 0.00012	0.01469 ± 0.00015	0.98513 ± 0.00043	0.94457 ± 0.00309	0.94013 ± 0.00334
Mild	Fuzzy-mean	0.02094 ± 0.00023	0.02986 ± 0.00046	0.93798 ± 0.00229	0.89834 ± 0.00471	0.89073 ± 0.00514
	Robust-fuzzy	$\mathbf{0.01524 \pm 0.00022}$	0.02242 ± 0.00060	0.96503 ± 0.00210	$\mathbf{0.92710 \pm 0.00434}$	$\mathbf{0.92152 \pm 0.00473}$
	Missingness-aware	0.01851 ± 0.00029	0.02613 ± 0.00052	0.95249 ± 0.00222	0.91121 ± 0.00514	0.90268 ± 0.00581
	CNFIS	0.01561 ± 0.00023	$\mathbf{0.02223 \pm 0.00052}$	$\mathbf{0.96560 \pm 0.00183}$	0.92525 ± 0.00461	0.91862 ± 0.00525
Moderate	Fuzzy-mean	0.03019 ± 0.00052	0.04192 ± 0.00079	0.87926 ± 0.00489	0.85379 ± 0.00600	0.84357 ± 0.00643
	Robust-fuzzy	0.02222 ± 0.00047	0.03481 ± 0.00097	0.91668 ± 0.00486	0.89395 ± 0.00436	$\mathbf{0.88641 \pm 0.00467}$
	Missingness-aware	0.02654 ± 0.00043	0.03797 ± 0.00079	0.90094 ± 0.00407	0.87125 ± 0.00418	0.85739 ± 0.00456
	CNFIS	$\mathbf{0.02211 \pm 0.00042}$	$\mathbf{0.03318 \pm 0.00086}$	$\mathbf{0.92432 \pm 0.00401}$	$\mathbf{0.89396 \pm 0.00403}$	0.88437 ± 0.00454
Severe	Fuzzy-mean	0.04716 ± 0.00052	0.06323 ± 0.00072	0.72304 ± 0.00728	0.77515 ± 0.00607	0.76201 ± 0.00659
	Robust-fuzzy	0.04150 ± 0.00061	0.06031 ± 0.00088	0.74801 ± 0.00794	0.80651 ± 0.00655	0.79489 ± 0.00683
	Missingness-aware	0.04365 ± 0.00061	0.05913 ± 0.00076	0.75776 ± 0.00627	0.78746 ± 0.00748	0.75908 ± 0.00859
	CNFIS	$\mathbf{0.03887 \pm 0.00054}$	$\mathbf{0.05524 \pm 0.00076}$	$\mathbf{0.78858 \pm 0.00621}$	$\mathbf{0.81251 \pm 0.00618}$	$\mathbf{0.79550 \pm 0.00676}$
Extreme	Fuzzy-mean	0.06356 ± 0.00072	0.08355 ± 0.00102	0.51890 ± 0.01343	0.70355 ± 0.00543	0.68809 ± 0.00543
	Robust-fuzzy	0.06171 ± 0.00068	0.08365 ± 0.00098	0.51772 ± 0.01261	0.71615 ± 0.00601	0.70231 ± 0.00615
	Missingness-aware	0.05824 ± 0.00073	0.07579 ± 0.00083	0.60412 ± 0.00818	0.71543 ± 0.00665	0.66893 ± 0.00799
	CNFIS	$\mathbf{0.05563 \pm 0.00063}$	$\mathbf{0.07471 \pm 0.00087}$	$\mathbf{0.61529 \pm 0.00942}$	$\mathbf{0.73125 \pm 0.00651}$	$\mathbf{0.70740 \pm 0.00685}$

The severe and extreme regimes show that the advantage is not solely produced by median aggregation. Relative to Robust-fuzzy, CNFIS lowers RMSE by 8.40% in the severe regime and 10.69% in the extreme regime; relative to Missingness-aware, it lowers RMSE by 6.58% and 1.42%, respectively. The smaller extreme-regime gain over Missingness-aware indicates that when q is very low, missingness itself dominates $I = 1 - q + qc$, whereas at intermediate degradation the contradiction component qc carries more discriminative information.

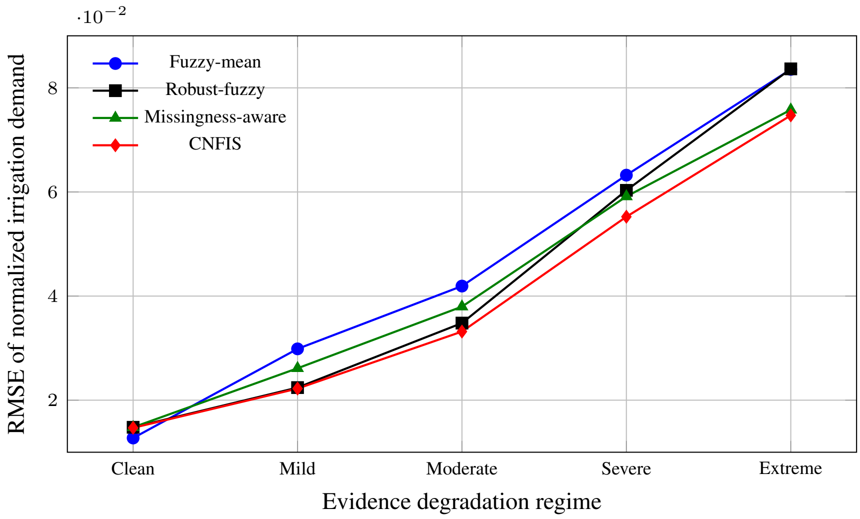

Figure 1: RMSE as evidence degradation increases from $(p_m, p_c) = (0, 0)$ to $(0.30, 0.30)$.

Figure 1 makes the robustness–efficiency trade-off explicit. Fuzzy-mean is statistically efficient when the three channels differ mainly by zero-mean noise $\varepsilon \sim \mathcal{N}(0, 0.04^2)$, whereas its error increases rapidly after the Bernoulli corruption term ob becomes non-negligible. CNFIS begins slightly above the clean fuzzy curve but grows more slowly because the median limits a corrupted channel and $I(q, c)$ further shrinks high-conflict evidence toward $\pi = 1/2$.

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5.2 Uncertainty-Guided Selective Control

The aggregate score $U_i = \sum_j w_j I_{ij}$ is useful only if it orders difficult cases. Table 2 tests this property by retaining the lowest-U fraction ρ of cases. Under moderate degradation, CNFIS RMSE falls from 0.03320 at $\rho = 1$ to 0.02825 at $\rho = 0.8$ and 0.02613 at $\rho = 0.6$; the monotone decrease means that cases rejected by the rule $U_i > \tau_\rho$ contain disproportionately large prediction errors.

Table 2: Selective RMSE when decisions are ranked by U; mean $\pm$ standard deviation over 30 runs.

Regime	**Method**	$\boldsymbol{\rho = 1.0}$	$\boldsymbol{\rho = 0.8}$	$\boldsymbol{\rho = 0.6}$
Moderate	CNFIS	0.03320 ± 0.00058	0.02825 ± 0.00053	0.02613 ± 0.00059
	Missingness-aware	0.03800 ± 0.00059	0.03176 ± 0.00058	0.02811 ± 0.00063
Severe	CNFIS	0.05507 ± 0.00076	0.04988 ± 0.00086	0.04617 ± 0.00107
	Missingness-aware	0.05904 ± 0.00089	0.05333 ± 0.00087	0.04951 ± 0.00101
Extreme	CNFIS	0.07427 ± 0.00093	0.06990 ± 0.00096	0.06635 ± 0.00134
	Missingness-aware	0.07544 ± 0.00093	0.07125 ± 0.00104	0.06806 ± 0.00131

At severe degradation, changing coverage from $\rho = 1.0$ to 0.8 reduces CNFIS RMSE by approximately 9.42%, while at extreme degradation the corresponding reduction is about 5.88%. The diminishing gain at very high (p_m, p_c) is consistent with a distribution in which most cases have elevated U, so rejecting only 20% cannot isolate all unreliable decisions. Operationally, ρ should therefore be chosen jointly with the cost of re-sensing rather than interpreted as a universal acceptance rate.

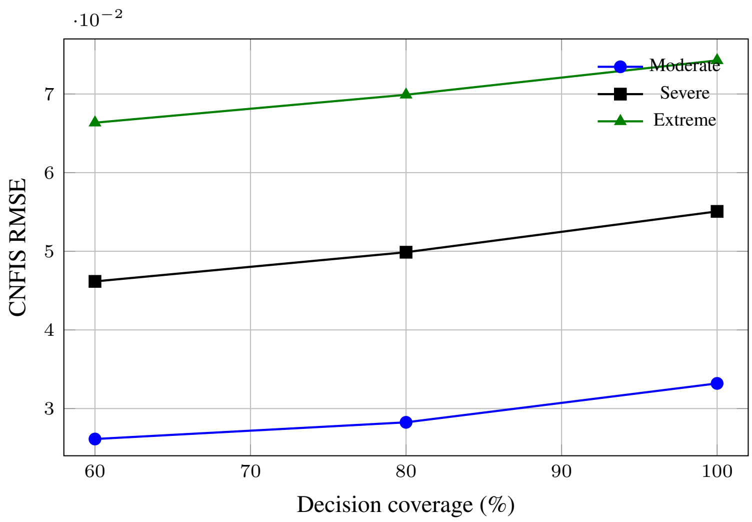

Figure 2: CNFIS risk–coverage behavior when cases with the largest U are deferred.

Figure 2 shows that $\mathrm{RMSE}(\rho)$ decreases as coverage decreases for all three tested regimes. This behavior gives U a second role beyond modifying s: the tuple $(\widehat{d}, U)$ can support a two-output controller in which $\widehat{d}$ is the demanded action and U is its evidence-quality indicator. Such a controller exposes epistemic limitations to downstream logic instead of hiding them in one scalar $\widehat{d}$.

5.3 Hyperparameter Stability

Table 3 evaluates α, where $\alpha = 0$ removes uncertainty shrinkage and reduces CNFIS to robust median fusion. The moderate regime has its lowest tested RMSE near $\alpha = 0.4$, the severe regime near $\alpha = 0.6$, and the extreme regime near $\alpha = 0.8$. Because the minimizing α increases with corruption, selecting a different α for every test regime would leak knowledge of degradation severity; the fixed value $\alpha = 0.4$ is therefore retained as a conservative balance in the main experiment.

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Table 3: CNFIS RMSE sensitivity to α (mean over 30 runs); boldface marks the lowest RMSE in each regime.

α	**Moderate**	**Severe**	**Extreme**
0.0	0.03469	0.06023	0.08317
0.2	0.03344	0.05709	0.07804
0.4	**0.03311**	0.05518	0.07435
0.6	0.03374	**0.05462**	0.07233
0.8	0.03527	0.05547	**0.07211**
1.0	0.03759	0.05765	0.07371

The sensitivity pattern is explained by $|s - 1/2| = (1 - \alpha I)|r - 1/2|$. When corruption is extreme, larger I is more likely to indicate genuinely unreliable r, so stronger shrinkage reduces squared error; when evidence is only moderately degraded, large α can over-shrink informative deviations from $1/2$. A deployment can therefore estimate α from a calibration set by minimizing $\mathcal{L}(\alpha) = n^{-1} \sum_i (\widehat{d_i}(\alpha) - d_i)^2$ subject to $0 \leq \alpha \leq 1$, but the current benchmark does not use such post hoc tuning.

6 Mechanistic Interpretation and Deployment Implications

6.1 Why Contradiction Awareness Changes the Decision

The numerical evidence supports the premise that fuzzy demand and evidence reliability are different mathematical objects. A membership $r \in [0, 1]$ expresses the direction and strength of irrigation demand, whereas $I \in [0, 1]$ expresses uncertainty in the evidence supporting that demand. Consequently, $(r, I) = (0.8, 0.05)$ and $(0.8, 0.70)$ should not produce identical operational confidence even though a conventional fuzzy controller receives the same nominal $r = 0.8$. Equation (8) resolves this ambiguity by enforcing $|s - 1/2| = (1 - \alpha I)|r - 1/2|$, so the action becomes progressively less extreme as indeterminacy grows.

The residual $\kappa = qc$ further separates contradiction from generic uncertainty. Equal indeterminacy can arise through different mechanisms: $(q, c) = (0.5, 0)$ gives $I = 0.5$ and $\kappa = 0$, while $(q, c) = (1, 0.5)$ also gives $I = 0.5$ but $\kappa = 0.5$. The decomposition $I = (1-q)+qc$ therefore provides two diagnostic coordinates, missing evidence $1 - q$ and observed contradiction qc, rather than one opaque reliability score. The corresponding operational responses can differ: the first case motivates re-acquisition, whereas the second motivates sensor validation or fault isolation.

6.2 Robustness–Efficiency Trade-off

The clean-regime result is important because it bounds the claim made for CNFIS. When $(p_m, p_c) = (0, 0)$, Fuzzy-mean achieves RMSE 0.01273 and CNFIS 0.01469, yielding $\Delta_{\mathrm{R}} < 0$ in Eq. (2). Once evidence degradation is introduced, the sign reverses: the RMSE improvement over Fuzzy-mean is 25.54% in the mild regime, 20.84% in moderate, 12.63% in severe, and 10.57% in extreme. CNFIS should therefore be interpreted as a robustness mechanism whose benefit is activated by unreliable evidence, not as a universally more efficient estimator under ideal measurements.

This trade-off also explains the sensitivity pattern for α. From $\partial s/\partial I = \alpha(1/2-r)$, larger α produces stronger contraction toward the neutral prior. Moderate degradation favors $\alpha \approx 0.4$ in the tested grid, whereas extreme degradation favors a larger value near 0.8 because high I more frequently identifies a corrupted direction r. A calibrated deployment can estimate α from historical validation data, but a fixed α remains preferable when degradation severity at test time is unknown.

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6.3 Integration with Existing Smart-Irrigation Architectures

The proposed layer can be inserted after physical measurements are fuzzified and before the final irrigation rule or weighted aggregator. If an existing controller uses $d_0 = \mathcal{A}(\mu_1, \ldots, \mu_J)$, a reliability-aware implementation replaces each raw criterion membership by $s_j = \mathcal{G}_j(\{x_{jk}, a_{jk}\})$ and evaluates $d = \mathcal{A}(s_1, \ldots, s_J)$ while exposing $U = \sum_j w_j I_j$ to a supervisory rule. This modularity permits fuzzy-IoT designs such as those investigated in[7–9] to retain their agronomic rule bases while changing how replicated or heterogeneous evidence is fused.

The pair $(\widehat{d}, U)$ also supports a control policy richer than a single irrigation command. For threshold τ, the controller can execute $\widehat{d}$ when $U \le \tau$ and defer the decision when $U > \tau$; equivalently, a risk–coverage operating point can be selected by the retained fraction ρ. The selective results show that lowering ρ reduces prediction risk under the benchmark, so U contains useful ordering information. In practice, the appropriate τ or ρ should be optimized jointly with the costs of delayed irrigation, communication, re-sensing, and manual inspection rather than adopted directly from the synthetic experiment.

7 Validity Boundaries and Future Validation

The principal limitation is that the evaluation is synthetic. The generator in Eq. (10) supplies known $d_i^\star$ and controlled (p_m, p_c), but real sensors can exhibit drift, spatial correlation, heteroscedastic error, calibration bias, packet bursts, and crop-dependent dynamics that are not represented by independent ε, o, and a. Consequently, the reported RMSE values quantify algorithmic robustness under the stated stochastic model and must not be interpreted as field water-saving percentages or agronomic yield improvements.

The second limitation concerns the experimental weights and membership distributions. The vector $\mathbf{w} = (0.45, 0.20, 0.20, 0.15)$ and beta parameters (a_j, b_j) define a reproducible benchmark, not a universal irrigation ontology. In a real farm, $\mathbf{w}$ should be estimated or elicited for a crop/soil/climate context, and physical measurements z_j should be transformed through validated functions $\mu_j(z_j)$. If $\mathbf{w}$ or μ_j is poorly specified, CNFIS can improve evidence robustness while still producing an agronomically biased demand $\widehat{d}$.

The third limitation is structural: c is a median absolute disagreement statistic and treats channels symmetrically. If channel reliabilities ρ_k are known, a natural extension is a weighted center $\widetilde{m}^{(\rho)}$ and contradiction $c^{(\rho)} = \sum_k a_k \rho_k |x_k - \widetilde{m}^{(\rho)}| / \sum_k a_k \rho_k$. Likewise, temporal dependence could replace the static state (T_t, I_t, F_t) by a recursion $N_t = \mathcal{H}(N_{t-1}, X_t)$ so persistent drift is separated from one-step conflict.

Finally, the selective rule $\delta_i(\tau) = \mathbf{1}\{U_i \le \tau\}$ demonstrates ranking utility but does not attach a calibrated probability to U. Future work should examine calibration functions $g(U) \approx \mathbb{E}[|\widehat{d} - d^\star| \mid U]$, real-world sensor-fault benchmarks, field trials, and comparisons with probabilistic robust fusion. Such validation would determine whether the algebraic transparency of $I = 1 - q + qc$ translates into operational advantages under actual irrigation constraints.

8 Conclusion

This paper introduced CNFIS, an original neutrosophic–fuzzy evidence-fusion method for irrigation scheduling under missing and conflicting channels. Its core mapping $T = q\widetilde{m}$, $F = q(1 - \widetilde{m})$, and $I = 1 - q + qc$ yields the interpretable identity $T + I + F = 1 + qc$, so missingness and contradiction are represented separately while the fuzzy action remains bounded through $s = (1 - \alpha I)r + \alpha I/2$ and $\widehat{d} = \sum_j w_j s_j$. The aggregate $U = \sum_j w_j I_j$ further supports selective re-sensing rather than compulsory action.

In 30×5000 controlled cases per regime, CNFIS reduced RMSE relative to Fuzzy-mean by 20.84% at $(p_m, p_c) = (0.10, 0.10)$, 12.63% at $(0.20, 0.20)$, and 10.57% at $(0.30, 0.30)$, although Fuzzy-mean remained better when $(p_m, p_c) = (0, 0)$. This trade-off is consistent with the intended use: CNFIS is a reliability layer for uncertain evidence, not an efficiency improvement when sensors are perfectly trustworthy. With real-world calibration of μ_j, $\mathbf{w}$, α, and channel reliabilities ρ_k, the framework provides a mathematically transparent basis for future field validation of contradiction-aware smart irrigation.

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Data and Code Availability

The study uses no external dataset. All numerical data in Tables 1–3 are generated by the supplied `simulation.py` under fixed random seeds, and the package contains the resulting CSV files so that $\widehat{d}$, RMSE, and the reported aggregate statistics can be regenerated without proprietary data.

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