Fuzzy and Neutrosophic Systems for Medical Diagnosis: A Mathematical Review of Uncertainty Representation and Decision Models

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Abstract

Medical diagnosis is an uncertainty-sensitive decision problem in which a patient state $\mathbf{x} = (x_1, \ldots, x_m)$ is mapped to a candidate disease $d_k \in \mathcal{C}$ using incomplete, gradual, and sometimes conflicting evidence. Fuzzy systems represent such evidence by a membership grade $\mu(x) \in [0, 1]$; intuitionistic fuzzy systems use independently assigned (μ, ν) with the derived hesitation $\pi = 1 - \mu - \nu$; and single-valued neutrosophic systems use an independently specified triple $(T, I, F) \in [0, 1]^3$. This article provides a structured mathematical review of these models in medical diagnosis using literature available no later than 31 May 2024. The literature is synthesized by representation space, constraints, distance and similarity measures, aggregation operators, temporal structure, and diagnostic decision rules. A common embedding Φ places fuzzy and intuitionistic fuzzy states inside the neutrosophic cube $\Omega = [0, 1]^3$, making the geometric relation among the three model families explicit. Within this space, a weighted diagnostic distance is written as

$$D_k^{(r)} = \left[\sum_{j=1}^{m} \frac{w_j}{3} \left(|T_{pj} - T_{kj}|^r + |I_{pj} - I_{kj}|^r + |F_{pj} - F_{kj}|^r \right) \right]^{1/r},$$

and the normalization residual $\kappa = T + I + F - 1$ is used to distinguish normalized states from underspecified or overlapping evidence without interpreting (T, I, F) as probabilities. The review further develops a synthesis-derived decision layer that combines distance, indeterminacy, and diagnostic margin through $\widetilde{S}_k = (1 + \widetilde{D}_k)^{-1}$, $C_k = \widetilde{S}_k(1 - \bar{I}_p)^{\gamma}$, and $\Delta_C = C_{(1)} - C_{(2)}$. The analysis indicates that fuzzy systems remain appropriate when uncertainty is predominantly gradual and rule interpretability is central, whereas neutrosophic systems are better justified when independent support, opposition, and indeterminacy must be retained. The resulting taxonomy clarifies when additional uncertainty dimensions are mathematically informative and identifies calibration, metric stability, explainability, temporal modeling, and clinical validation as the principal unresolved problems.

Keywords: Fuzzy systems; Neutrosophic sets; Medical diagnosis; Intuitionistic fuzzy sets; Clinical decision support; Similarity measures; Uncertainty modeling

1 Introduction

Clinical diagnosis is fundamentally an inference problem in which a vector of observations $\mathbf{x} = (x_1, x_2, \ldots, x_m) \in \mathbb{R}^m$ must be mapped to a disease class $d_k \in \mathcal{C}$. In an ideal deterministic model, one would

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construct a classifier $f : \mathbb{R}^m \to \mathcal{C}$ and determine $d^* = f(\mathbf{x})$; however, actual clinical information contains linguistic expressions such as "mild pain," "moderately elevated temperature," or "possible inflammation," for which a binary indicator $x_j \in \{0, 1\}$ discards clinically relevant gradation. Zadeh's fuzzy-set formulation[1] addresses this limitation by replacing binary inclusion with a membership function $\mu_A : X \to [0, 1]$, thereby permitting the proposition $x \in A$ to hold with degree $\mu_A(x)$.

Early medical applications of fuzzy logic recognized that diagnostic reasoning can be approximated by mappings of the form $R : S \times D \to [0, 1]$, where $S = \{s_1, \ldots, s_m\}$ denotes symptoms and $D = \{d_1, \ldots, d_n\}$ denotes diseases. Adlassnig,[2] for example, formalized diagnostic vagueness using fuzzy relations in which the association $R(s_j, d_k) = r_{jk}$ expresses the graded strength of a symptom–disease relation rather than a Boolean statement $r_{jk} \in \{0, 1\}$. Such models are particularly suitable when uncertainty originates from gradual boundaries, but a scalar μ alone cannot distinguish "weak evidence for" from "insufficient evidence to decide."

Atanassov's intuitionistic fuzzy set (IFS)[3] expands the state of an observation to $A(x) = (\mu_A(x), \nu_A(x))$, subject to $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, with hesitation

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

The triple (μ, π, ν) therefore separates positive evidence, uncertainty, and negative evidence while satisfying $\mu + \pi + \nu = 1$. In medical reasoning, this is mathematically important because two observations with equal support $\mu = 0.6$ may differ substantially when $(\nu, \pi) = (0.3, 0.1)$ versus $(0.1, 0.3)$, even though an ordinary fuzzy model assigns both the same membership grade.

Single-valued neutrosophic sets (SVNSs) generalize this representation by assigning

$$N(x) = \langle T_N(x), I_N(x), F_N(x) \rangle, \qquad T_N, I_N, F_N \in [0, 1],$$

without imposing $T_N + I_N + F_N = 1$.[4] Hence $0 \leq T_N + I_N + F_N \leq 3$, so incomplete and contradictory assessments can be retained instead of being normalized away. For instance, $(T, I, F) = (0.8, 0.4, 0.5)$ yields $T + I + F = 1.7$, which can represent simultaneously strong supporting evidence, substantial indeterminacy, and non-negligible opposing evidence; such a configuration cannot be represented by an IFS without forcing a redistribution of the original information.

The medical-diagnosis literature consequently contains several mathematical families: fuzzy relations R, fuzzy inference systems based on t-norms and s-norms, IFS distances $d(A, B)$, fuzzy-soft relations, SVNS similarity functions $S(A, B)$, neutrosophic recommender systems, probabilistic rough-neutrosophic structures, bipolar neutrosophic models, and hypersoft decision systems. These models are related but are often evaluated separately, even though their diagnostic decisions can generally be written as

$$d^* = \arg\max_{d_k \in D} S(\mathbf{p}, \mathbf{q}_k) = \arg\min_{d_k \in D} D(\mathbf{p}, \mathbf{q}_k),$$

where $\mathbf{p}$ is the patient representation and $\mathbf{q}_k$ is the reference profile of disease d_k.

The central contribution of this review is a mathematical synthesis in a common coordinate system rather than another isolated similarity coefficient. Specifically, fuzzy states are embedded as $\Phi_F(x) = (\mu, 0, 1 - \mu)$, intuitionistic fuzzy states as $\Phi_I(x) = (\mu, \pi, \nu)$, and neutrosophic states as $\Phi_N(x) = (T, I, F)$. The common normalization residual

$$\kappa(x) = T(x) + I(x) + F(x) - 1$$

then satisfies $\kappa = 0$ for the proposed fuzzy and IFS embeddings but may satisfy $-1 \leq \kappa \leq 2$ for a general SVNS. This quantity supplies a direct mathematical distinction between normalized uncertainty and explicitly non-normalized evidence.

The review additionally derives a unified diagnostic rule combining distance, indeterminacy, and decision margin. If $S_k = 1 - D_k$ denotes similarity to disease d_k, $\bar{I}_p = \sum_j w_j I_{pj}$ denotes patient-level indeterminacy, and $S_{(1)} \geq S_{(2)}$ are the two highest disease similarities, then the diagnostic margin $\Delta = S_{(1)} - S_{(2)}$ provides a natural mechanism for distinguishing confident classification from an ambiguous decision. A synthesis-derived abstention condition $\Delta < \theta$ or $\bar{I}_p > \rho$ is consequently proposed as a mathematically explicit alternative to forcing a diagnosis under weak evidence.

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The remainder of the paper is organized as a sequence $\mathcal{S} = \{S_2, \ldots, S_{10}\}$ of nine analytical sections. Section 2 defines the review boundary and bibliographic eligibility conditions using a publication cutoff $\tau = 31$ May 2024; Section 3 establishes the mathematical foundations; Section 4 reviews principal medical applications and summarizes them in a comparative table; Sections 5–7 develop the unified taxonomy and uncertainty geometry; Section 8 identifies unresolved mathematical and clinical issues; Section 9 discusses the cross-model implications; and Section 10 concludes the review.

2 Review Scope and Analytical Protocol

The review is bounded by $\tau = 31$ May 2024, and a source r is temporally eligible only when its verified publication or online-publication date satisfies $t_r \leq \tau$. The admissible evidence set is therefore

$$\mathcal{R}_\tau = \{r : t_r \leq \tau\},$$

so no method, numerical result, dataset description, or bibliographic item first made available after May 2024 is used. This temporal rule is applied consistently to foundational theory, journal articles, and conference or book chapters.

Substantive eligibility is represented by $\chi(r) = \chi_1(r)\chi_2(r)\chi_3(r)\chi_4(r)$, where $\chi_1 = 1$ for fuzzy, intuitionistic fuzzy, neutrosophic, or directly related hybrid uncertainty models; $\chi_2 = 1$ when the mathematical formulation identifies a state representation, relation, distance, similarity, aggregation, or inference operator; $\chi_3 = 1$ for medical diagnosis, clinical decision support, medical pattern recognition, or a mathematical foundation necessary for those applications; and $\chi_4 = 1$ when the bibliographic record can be independently verified. A study enters the analytical set only when $\chi(r) = 1$.

Literature identification was organized around the Cartesian product $\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3$, with

$$\begin{aligned}
\mathcal{Q}_1 &= \{\text{fuzzy}, \text{intuitionistic fuzzy}, \text{neutrosophic}\},\\
\mathcal{Q}_2 &= \{\text{medical diagnosis}, \text{clinical decision}, \text{pattern recognition}\},\\
\mathcal{Q}_3 &= \{\text{similarity}, \text{distance}, \text{entropy}, \text{aggregation}, \text{inference}\}.
\end{aligned}$$

For every retained source, authorship, title, venue, year, pagination or article number, and DOI when available were cross-checked against publisher or established scholarly records. This verification condition is denoted by $V(r) = 1$; records with unresolved bibliographic identity, $V(r) = 0$, were not used.

The article is a structured mathematical review rather than a bibliometric survey or statistical meta-analysis. Each retained study is encoded by

$$\Psi(r) = \big(\mathcal{S}_r, \mathcal{C}_r, \mathcal{M}_r, \mathcal{A}_r, \mathcal{L}_r\big),$$

where $\mathcal{S}_r$ is the uncertainty state space, $\mathcal{C}_r$ its constraint set, $\mathcal{M}_r$ the principal mathematical operator, $\mathcal{A}_r$ the diagnostic application, and $\mathcal{L}_r$ the main limitation. This representation supports cross-model comparison without pooling heterogeneous accuracy values obtained from different diseases, datasets, sample sizes, and validation protocols.

3 Mathematical Foundations of Diagnostic Uncertainty Models

3.1 Fuzzy Sets and Fuzzy Inference

For a universe X, a fuzzy set A is $A = \{(x, \mu_A(x)) : x \in X\}$ with $\mu_A(x) \in [0, 1]$.[1] If x denotes body temperature and A denotes the linguistic concept "high temperature," a triangular membership function can be written as

$$\mu_A(x) = \begin{cases} 0, & x \leq a, \\ \dfrac{x-a}{b-a}, & a < x \leq b, \\ \dfrac{c-x}{c-b}, & b < x < c, \\ 0, & x \geq c, \end{cases}$$

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so clinical thresholds become gradual rather than discontinuous and the crisp boundary $\mathbf{1}_{\{x\geq b\}}$ is replaced by a continuous degree of compatibility.

A fuzzy medical rule has the form $R_\ell:$ IF x_1 is $A_{\ell 1}\wedge\cdots\wedge x_m$ is $A_{\ell m}$ THEN d_k is B_ℓ. With the minimum t-norm, the firing strength is $\alpha_\ell=\min_{1\leq j\leq m}\mu_{A_{\ell j}}(x_j)$, while a product t-norm gives $\alpha_\ell=\prod_{j=1}^{m}\mu_{A_{\ell j}}(x_j)$. A Mamdani-type output can be aggregated as $\mu_B(y)=\max_\ell\min\{\alpha_\ell,\mu_{B_\ell}(y)\}$, after which a crisp diagnostic-risk estimate may be computed by centroid defuzzification $y^*=\int y\mu_B(y)\,dy/\int\mu_B(y)\,dy$.[2,5,6]

3.2 Intuitionistic Fuzzy Sets

An IFS A on X is characterized by $A=\{(x,\mu_A(x),\nu_A(x)):x\in X\}$ with $0\leq\mu_A(x)+\nu_A(x)\leq 1$ and $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$.[3] The quantity $\pi_A(x)$ is not simply $1-\mu_A(x)$; rather, it measures the uncommitted portion remaining after independent specification of membership and non-membership. Thus, a diagnostic state $A(x)=(0.6,0.2)$ has $\pi=0.2$, whereas $B(x)=(0.6,0.35)$ has $\pi=0.05$, even though both have the same positive support $\mu=0.6$.

For two IFSs A and B observed over m symptoms, a normalized Hamming-type distance can be written as

$$d_H(A,B)=\frac{1}{2m}\sum_{j=1}^{m}\left(|\mu_A(s_j)-\mu_B(s_j)|+|\nu_A(s_j)-\nu_B(s_j)|+|\pi_A(s_j)-\pi_B(s_j)|\right),$$

while an Euclidean counterpart is $d_E(A,B)=\left[(2m)^{-1}\sum_{j=1}^{m}(\Delta\mu_j^2+\Delta\nu_j^2+\Delta\pi_j^2)\right]^{1/2}$. The corresponding similarity may be defined by $S(A,B)=1-d(A,B)$ when $d\in[0,1]$; the importance of retaining the hesitation coordinate π in such metrics is central to the formulations of Szmidt and Kacprzyk[7,8] and to the diagnostic application of De et al.[9]

3.3 Fuzzy Soft Representations

When symptoms are parameterized by a set $E=\{e_1,\ldots,e_q\}$, a fuzzy soft set may be treated as a mapping $F:E\to\mathcal{F}(X)$, where $\mathcal{F}(X)$ denotes the family of fuzzy subsets of X. The information associated with patient p_i and parameter e_j is therefore $\mu_{F(e_j)}(p_i)\in[0,1]$, producing a matrix $M_F=[\mu_{ij}]_{n\times q}$. Çelik and Yamak[10] combined this parameterization with fuzzy arithmetic to support medical diagnosis, making the soft-set dimension useful when the diagnostic problem contains an explicit symptom/attribute structure that is not conveniently represented by a single membership vector.

A generalized weighted fuzzy-soft diagnostic score can be written as $S_i=\sum_{j=1}^{q}w_j g(\mu_{ij})$ with $\sum_{j=1}^{q}w_j=1$, where g may be an identity, threshold, or defuzzification function. The principal mathematical advantage is parameter decomposition through E, whereas the limitation remains that each μ_{ij} is typically scalar unless additional intuitionistic or neutrosophic components are introduced, in which case the state dimension changes from q to approximately $3q$.

3.4 Single-Valued Neutrosophic Sets

An SVNS N is represented by $N=\{\langle x,T_N(x),I_N(x),F_N(x)\rangle:x\in X\}$, where $0\leq T_N(x),I_N(x),F_N(x)\leq 1$ and no normalization equation such as $T+I+F=1$ is imposed.[4] In a diagnostic setting, T can represent evidence supporting a symptom or disease relation, F opposing evidence, and I evidence that remains indeterminate; importantly, $\sigma_N(x)=T_N(x)+I_N(x)+F_N(x)$ may satisfy $\sigma_N<1$, $\sigma_N=1$, or $\sigma_N>1$.

For two SVNS descriptions A and B, a normalized L_r distance can be defined by

$$D_r(A,B)=\left[\frac{1}{3m}\sum_{j=1}^{m}\left(|T_{Aj}-T_{Bj}|^r+|I_{Aj}-I_{Bj}|^r+|F_{Aj}-F_{Bj}|^r\right)\right]^{1/r},$$

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with $r = 1$ giving a Hamming-like measure and $r = 2$ giving an Euclidean-like measure. Similarity measures based on cosine, tangent, exponential, and other transformations have been developed because two alternatives may satisfy the same Euclidean distance while differing in directional alignment within the (T, I, F) space.[11–15]

A cosine-type neutrosophic similarity for symptom s_j can be written as

$$C_j(A, B) = \frac{T_{Aj}T_{Bj} + I_{Aj}I_{Bj} + F_{Aj}F_{Bj}}{\sqrt{T_{Aj}^2 + I_{Aj}^2 + F_{Aj}^2}\sqrt{T_{Bj}^2 + I_{Bj}^2 + F_{Bj}^2}},$$

and a weighted diagnostic similarity becomes $C_w(A, B) = \sum_{j=1}^{m} w_j C_j(A, B)$ with $\sum_{j=1}^{m} w_j = 1$. Ye[12] demonstrated how cosine-based formulations can be modified for simplified neutrosophic medical diagnosis, while later models extended similarity-based diagnosis to dynamic observations, disease recommendation, bipolar information, rough structures, and hypersoft decision settings.[13,16–21]

4 Related Work on Fuzzy and Neutrosophic Medical Diagnosis

The historical development of fuzzy medical diagnosis can be summarized as a progressive increase in state dimension. A conventional fuzzy state uses one degree μ, an IFS uses two explicitly assigned degrees (μ, ν) plus the derived hesitation π, and an SVNS uses three independently assignable degrees (T, I, F). If the effective representational dimension is denoted by d_R, then $d_R = 1$ for a standard fuzzy membership, $d_R = 2$ under the IFS constraint $\mu + \nu \leq 1$, and $d_R = 3$ for an unconstrained SVNS; this growth of d_R explains much of the literature's movement from fuzzy inference toward neutrosophic similarity and aggregation.

Table 1: Representative fuzzy, intuitionistic fuzzy, and neutrosophic studies in medical diagnosis.

Study	**Model**	**Mathematical mechanism**	**Application**	**Contribution / limitation**
Adlassnig (1986)[2]	Fuzzy set	$R(s, d) = \mu_{sd} \in [0, 1]$ and fuzzy diagnostic inference	General medical diagnosis	Established formal fuzzy reasoning for diagnosis; a scalar μ does not independently encode non-membership or indeterminacy.
De et al. (2001)[9]	IFS	(μ, π, ν) with $\pi = 1 - \mu - \nu$	Medical diagnosis	Extended fuzzy relation diagnosis by explicitly representing hesitation; the state remains normalized by $\mu + \nu + \pi = 1$.
Szmidt and Kacprzyk (2004)[8]	IFS	$S(A, B)$ using membership, nonmembership, and hesitation	Diagnostic reasoning	Demonstrated similarity-based IFS diagnosis; contradiction beyond the normalized simplex is not retained.
Torres and Nieto (2006)[5]	Fuzzy systems	Fuzzy hypercube $[0, 1]^m$ and approximate reasoning	Medicine and bioinformatics	Synthesized fuzzy medical applications; the principal state remains scalar at each attribute.
Khatibi and Montazer (2009)[22]	FS and IFS	Comparative similarity and distance functions	Medical pattern recognition	Demonstrated the value of IFS information; richer representation increases computational and elicitation burden.
Çelik and Yamak (2013)[10]	Fuzzy soft set	Parameterized relation $F : E \to \mathcal{F}(X)$	Medical diagnosis	Added parameter structure and fuzzy arithmetic; indeterminacy remains implicit unless the state is extended.
Ye (2015)[12]	Simplified NS	Improved cosine similarity $C(A, B)$	Medical diagnosis	Used (T, I, F) similarity and corrected weaknesses of conventional cosine formulations.
Ye and Fu (2016)[13]	SVNS	Tangent similarity plus temporal weights v_t	Multi-period diagnosis	Introduced longitudinal aggregation $\sum_t v_t S_t$; temporal weights require calibration.
Garg and Nancy (2017)[14]	SVNS	Biparametric distance $D_{\alpha,\beta}$	Pattern recognition and diagnosis	Increased flexibility of distance geometry; additional parameters complicate selection and sensitivity analysis.
Fu and Ye (2017)[15]	Simplified NS	Exponential similarity $S_{\exp}$	Benign prostatic hyperplasia (BPH)	Applied neutrosophic similarity to graded clinical symptoms; construction is disease-specific.
de Medeiros et al. (2017)[6]	Fuzzy inference	Rule activation α_ℓ and defuzzification	Real-time medical support	Preserved interpretable rule reasoning; scalar memberships restrict explicit contradiction modeling.

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Study	Model	Mathematical mechanism	Application	Contribution / limitation
Ali et al. (2018)[16]	Neutrosophic recommender	Algebraic similarity and prediction operators	Multiple medical datasets	Combined recommendation algebra and (T, I, F) information; model construction introduces many tunable quantities.
Zhang et al. (2018)[17]	Probabilistic rough SVNS multiset	Rough approximations and neutrosophic multiset operators	Medical diagnosis	Combined boundary-region uncertainty with neutrosophic grades; mathematical complexity is relatively high.
Liu et al. (2018)[23]	Neutrosophic set	Euclidean-distance-derived similarity	Medical diagnosis	Provided direct similarity measures; the chosen metric controls ranking sensitivity.
Abdel-Basset et al. (2019)[18]	Bipolar NS	Positive/negative cosine similarity	Bipolar disorder diagnosis	Represents positive and negative poles explicitly; state dimensionality exceeds that of an SVNS.
Chai et al. (2021)[19]	SVNS	New axiomatic distance and similarity measures	Pattern recognition and diagnosis	Addressed inconsistency of earlier similarity measures and emphasized axiomatic validity.
Rahman et al. (2022)[20]	Neutrosophic hypersoft set	Multi-attribute decision-making with possibility degree	Heart-disease diagnosis	Models sub-attributes and neutrosophic information jointly; parameter and rule spaces can grow rapidly.
Dutta and Borah (2024)[21]	SVNS	Advanced similarity measure $S_N(A, B)$	Medical decision-making and pattern recognition	Refines similarity geometry; practical performance remains dependent on elicitation of T, I, and F.
Present review	**Unified FS–IFS–SVNS geometry**	$\Phi(\cdot) \in [0,1]^3$, $\kappa = T + I + F - 1$, $\widetilde{D}_k$, C_k, and abstention	Cross-model clinical decision support	**Places the reviewed models in one constrained geometry, distinguishes indeterminacy from normalization residual, and derives a confidence-aware diagnostic layer rather than proposing only another similarity coefficient.**

The fuzzy branch of the literature can be written schematically as crisp data $\to \mu(x) \to \{\alpha_\ell\}_{\ell=1}^{L} \to y^*$, where L is the number of rules. Adlassnig[2] established an early formal basis for this path, Torres and Nieto[5] reviewed broader fuzzy applications in medicine, and de Medeiros et al.[6] demonstrated a real-time inference architecture. The strength of this family is interpretability through linguistic rules R_ℓ, while its central representational weakness is that uncertainty is encoded largely through μ and therefore does not explicitly distinguish lack of knowledge from opposing evidence.

The IFS branch changes the basic diagnostic object from a scalar to $\mathbf{z}_{IFS} = (\mu, \pi, \nu)$ with $\mu+\pi+\nu = 1$. De et al.[9] showed how this representation extends fuzzy diagnosis, while Szmidt and Kacprzyk[7,8] developed distance and similarity concepts using the full three-coordinate representation. Khatibi and Montazer[22] later compared fuzzy and intuitionistic fuzzy similarity measures empirically, illustrating that the additional coordinate π can improve discrimination when the evidence contains genuine hesitation.

The neutrosophic branch removes the normalization condition and uses $\mathbf{z}_N = (T, I, F)$ with $0 \le T + I + F \le 3$. This change permits a diagnostic system to retain simultaneous support and opposition, for example $T + F > 1$, as well as under-specified states such as $T + I + F < 1$, without forcing normalization. Ye[12] used improved cosine similarity for medical diagnosis, Ye and Fu[13] introduced temporal aggregation, Garg and Nancy[14] generalized the distance structure, and Fu and Ye[15] applied exponential similarity to BPH symptom assessment.

Subsequent neutrosophic research expanded the computational architecture rather than merely changing the similarity formula. Ali et al.[16] represented diagnosis as a recommender problem whose prediction depends on neutrosophic algebraic similarity, Zhang et al.[17] combined SVNS information with probabilistic rough multisets, and Abdel-Basset et al.[18] introduced bipolar neutrosophic similarity for bipolar disorder, where a state may be treated as a six-coordinate object $\mathbf{b} = (T^+, I^+, F^+, T^-, I^-, F^-)$ rather than only one (T, I, F) triple.

The more recent literature emphasizes axiomatic reliability and structured multi-attribute reasoning. Chai et al.[19] examined whether proposed measures satisfy appropriate distance and similarity properties, Rahman et al.[20] integrated neutrosophic hypersoft information into heart-disease diagnosis, and Dutta and Borah[21] continued the development of advanced SVNS similarity measures. Mathematically, this trend indicates that

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diagnostic quality depends not merely on using an expanded state (T, I, F), but also on whether the mapping $S : [0,1]^{3m} \times [0,1]^{3m} \to [0,1]$ preserves identity, symmetry, boundedness, and clinically sensible monotonic behavior.

5 Unified Mathematical Taxonomy

5.1 A Common Three-Coordinate Embedding

Let $\Omega = [0,1]^3$ be the unit neutrosophic cube with coordinates (T, I, F). A standard fuzzy observation with membership μ may be embedded as $\Phi_F(\mu) = (\mu, 0, 1-\mu)$, while an IFS observation is embedded as $\Phi_I(\mu, \nu) = (\mu, 1-\mu-\nu, \nu)$ and an SVNS observation is represented directly by $\Phi_N(T, I, F) = (T, I, F)$. This common map makes the representation dimension explicit while keeping every model in a single ambient space Ω.

Proposition 1. Under the mappings above, $\Phi_F([0,1]) \subset \Phi_I(\mathcal{I}) \subset \Omega$, where $\mathcal{I} = \{(\mu, \nu) : \mu, \nu \in [0,1],\ \mu + \nu \leq 1\}$.

Proof. For a fuzzy state $\Phi_F(\mu) = (\mu, 0, 1-\mu)$, define $\nu = 1-\mu$; then $\mu+\nu = 1$, so $(\mu, \nu) \in \mathcal{I}$ and $\Phi_I(\mu, \nu) = \Phi_F(\mu)$. Every IFS embedding satisfies each coordinate in $[0,1]$, so $\Phi_I(\mathcal{I}) \subset \Omega$; therefore $\Phi_F([0,1]) \subset \Phi_I(\mathcal{I}) \subset \Omega$. □

The geometric implication is that fuzzy states occupy the line $T + F = 1$ with $I = 0$, IFS states occupy the triangular plane $T + I + F = 1$, and unrestricted SVNS states occupy the full cube Ω. Thus, the increase in expressiveness can be quantified by moving from a one-dimensional manifold to a two-dimensional simplex and then to a three-dimensional cube, even though all three representations can be written using the common coordinate vector $\mathbf{z} = (T, I, F)$.

5.2 Normalization Residual and Evidence Overlap

Define the normalization residual $\kappa(\mathbf{z}) = T + I + F - 1$. For the proposed fuzzy embedding, $\kappa(\Phi_F) = \mu + 0 + (1-\mu) - 1 = 0$, and for an IFS, $\kappa(\Phi_I) = \mu + (1-\mu-\nu) + \nu - 1 = 0$; for an SVNS, however, $-1 \leq \kappa(\Phi_N) \leq 2$. The sign of κ therefore measures departure from the normalized plane without interpreting the neutrosophic coordinates as probabilities.

A value $\kappa < 0$ indicates a deficit relative to the unit-normalized plane, $\kappa = 0$ denotes a normalized state, and $\kappa > 0$ denotes overlap among the assigned grades; when T and F refer to support and opposition for the same proposition, such overlap may be interpreted as conflicting evidence. The magnitude $|\kappa|$ is consequently a scalar geometric distance from the plane $T + I + F = 1$ along the normal vector $(1, 1, 1)$ up to the factor $1/\sqrt{3}$, because the Euclidean point-to-plane distance is $|T + I + F - 1|/\sqrt{3} = |\kappa|/\sqrt{3}$.

5.3 Diagnostic Matrices

Let a diagnostic knowledge base contain n diseases and m symptoms. A unified disease matrix is $\mathcal{Q} = [\mathbf{q}_{kj}]_{n \times m}$ with $\mathbf{q}_{kj} = (T_{kj}, I_{kj}, F_{kj})$, while the patient is $\mathbf{P} = (\mathbf{p}_1, \ldots, \mathbf{p}_m)$ with $\mathbf{p}_j = (T_{pj}, I_{pj}, F_{pj})$. Fuzzy and IFS systems can use the same matrices after applying Φ_F or Φ_I, which allows different uncertainty models to be compared without changing the dimensional structure of the diagnostic problem.

For symptom weights $w_j \geq 0$ with $\sum_{j=1}^{m} w_j = 1$, define the weighted L_r disease distance

$$D_k^{(r)} = \left[\sum_{j=1}^{m} \frac{w_j}{3} \left(|T_{pj} - T_{kj}|^r + |I_{pj} - I_{kj}|^r + |F_{pj} - F_{kj}|^r \right) \right]^{1/r}.$$

When $r = 1$, the metric is linear in coordinate deviations; when $r = 2$, larger differences receive greater penalty; and as $r \to \infty$, $D_k^{(\infty)} = \max_{j,c} |\Delta z_{kjc}|$ for $c \in \{T, I, F\}$.

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5.4 Similarity and Ranking Families

Distance-based diagnosis uses $d^* = \arg\min_k D_k$, whereas similarity-based diagnosis uses $d^* = \arg\max_k S_k$. If $D_k \in [0,1]$, the transformation $S_k^{(D)} = 1 - D_k$ is sufficient, but many studies instead use angular, tangent, exponential, or related structures because those transformations alter sensitivity to relative vector orientation.[12–15,19,21,23]

A generalized cosine similarity can be expressed as

$$S_k^{(\cos)} = \sum_{j=1}^{m} w_j \frac{\mathbf{p}_j \cdot \mathbf{q}_{kj}}{\max\{\|\mathbf{p}_j\|_2 \|\mathbf{q}_{kj}\|_2, \varepsilon\}}, \qquad \varepsilon > 0,$$

where $\mathbf{p}_j \cdot \mathbf{q}_{kj} = T_{pj}T_{kj} + I_{pj}I_{kj} + F_{pj}F_{kj}$. The stabilizing constant ε prevents division by zero when one uncertainty vector is null. The distinction between $S_k^{(D)}$ and $S_k^{(\cos)}$ is mathematically important: the first measures absolute coordinate closeness, whereas the second emphasizes directional agreement in the uncertainty space.

6 Comparative Mathematical Analysis

Table 2: Mathematical capability of major diagnostic uncertainty models.

Model	State	Constraint	Hesitation	Contra-diction	Typical operator / relative state complexity
Crisp	$x \in \{0,1\}$	Binary	No	No	Rule/classifier; $O(m)$
Fuzzy set	$\mu \in [0,1]$	$0 \le \mu \le 1$	Implicit	No	Fuzzy inference or $d(\mu,\mu')$; $O(m)$
IFS	(μ,π,ν)	$\mu+\pi+\nu=1$	Yes, π	Limited	Distance/similarity; $O(3m)$ stored coordinates
Fuzzy soft set	μ_{ij}	Parameterized	Implicit	No	Relation/aggregation; $O(mq)$
SVNS	(T,I,F)	Each in $[0,1]$	Yes, I	Yes	Distance/similarity/multi-criteria decision; $O(3m)$
Bipolar NS	$(T^{\pm},I^{\pm},F^{\pm})$	Positive/negative domains	Yes	Yes, bipolar	Cosine/aggregation; $O(6m)$
Rough NS	Lower/upper (T,I,F)	Approximation regions	Yes	Yes	Rough approximation + similarity; $> O(3m)$
Neutrosophic hypersoft	(T,I,F) over sub-attributes	Multi-parameter	Yes	Yes	MADM aggregation; $O(3mq)$ or higher

The first major trade-off is representational capacity versus elicitation burden. Let p_M denote the number of primary membership coordinates required per symptom; then a fuzzy model has approximately $p_F = 1$, an IFS has $p_I = 2$ independent values because π is derived, and an SVNS has $p_N = 3$. If elicitation cost is approximated as $C_E(M) = mp_M$, then $C_E(N) > C_E(I) > C_E(F)$ even before additional parameters for weights, similarity exponents, temporal coefficients, or rule bases are considered.

The increased cost is justified only when the additional dimensions contain clinically meaningful information. Suppose two evidence states have the same fuzzy support $T = 0.7$: a fuzzy system treats them identically, whereas an SVNS can distinguish $\mathbf{z}_1 = (0.7, 0.1, 0.2)$ from $\mathbf{z}_2 = (0.7, 0.6, 0.5)$. For $\mathbf{z}_1$, $\kappa_1 = 0$; for $\mathbf{z}_2$, $\kappa_2 = 0.8$, so the second state carries substantially more indeterminacy and overlapping evidence even though the truth coordinate is unchanged.

The second trade-off concerns interpretability. A fuzzy inference system with L rules can be inspected through $\mathcal{R} = \{R_1, \ldots, R_L\}$, and each decision can be traced to firing strengths $\alpha_1, \ldots, \alpha_L$. By contrast, a similarity-based neutrosophic classifier may be computationally concise, $d^* = \arg\max_k S(\mathbf{P}, \mathbf{Q}_k)$, but the clinical meaning of a small change ΔS_k is not always transparent unless the system also reports contributions from T, I, and F.

The third trade-off concerns metric stability. Let S_a and S_b be two admissible similarity measures, and let $r_a = \mathrm{rank}_{S_a}(D)$ and $r_b = \mathrm{rank}_{S_b}(D)$. If $r_a \neq r_b$, the diagnosis depends on the geometry chosen by

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the model rather than only on the underlying patient information, which motivates the axiomatic analysis emphasized by Chai et al.[19] and the continued development of similarity functions.[21] A clinically robust system should therefore test ranking stability under a family $\mathcal{S} = \{S_1, \ldots, S_q\}$ rather than report the outcome of one coefficient alone.

The fourth trade-off concerns time. A single-period diagnosis uses $S_k = S(\mathbf{P}, \mathbf{Q}_k)$, whereas the multi-period structure of Ye and Fu[13] can be expressed as $S_k^{(T)} = \sum_{t=1}^{h} v_t S_{kt}$ with $v_t \geq 0$ and $\sum_{t=1}^{h} v_t = 1$. When $v_h > v_1$, recent observations dominate; when $v_t = 1/h$, all periods are equally weighted. The mathematical advantage is longitudinal evidence fusion, but the inferred disease can change if the temporal vector $\mathbf{v} = (v_1, \ldots, v_h)$ is poorly calibrated.

The fifth trade-off concerns parameter granularity. Hypersoft and rough-neutrosophic systems increase structure by replacing an attribute e_j with a family of sub-attributes $E_j = \{e_{j1}, \ldots, e_{jq_j}\}$ or by creating lower and upper approximations $\underline{R}(A)$ and $\overline{R}(A)$. If $Q = \sum_{j=1}^{m} q_j$, the effective information dimension can increase from $3m$ to approximately $3Q$, which permits finer representation but also increases elicitation, storage, and validation requirements.[17,20]

7 Synthesis-Derived Diagnostic Uncertainty Geometry

7.1 Motivation

The reviewed studies point to three quantities that should be distinguished at the decision layer: disease proximity, unresolved evidence, and departure from the normalized uncertainty plane. They are represented by a similarity S_k, patient indeterminacy $\bar{I}_p$, and the normalization residual κ. The synthesis therefore evaluates a candidate diagnosis through $\Gamma_k = (S_k, \bar{I}_p, K_k)$ rather than through S_k alone, where K_k measures the residual mismatch between patient and disease profiles.

7.2 Normalization-Residual-Aware Diagnostic Distance

For patient symptom s_j, define $\kappa_{pj} = T_{pj} + I_{pj} + F_{pj} - 1$ and $\kappa_{kj} = T_{kj} + I_{kj} + F_{kj} - 1$. The weighted normalization-residual mismatch between the patient and disease d_k is $K_k = \sum_{j=1}^{m} w_j |\kappa_{pj} - \kappa_{kj}|$, and the residual-aware distance is

$$\widetilde{D}_k = \lambda D_k + (1 - \lambda) K_k, \qquad 0 \leq \lambda \leq 1.$$

When $\lambda = 1$, the formulation reduces to ordinary coordinate distance; when $0 < \lambda < 1$, two disease profiles that are equally close in (T, I, F) can still be separated according to how well their normalization residuals match that of the patient evidence.

The corresponding bounded similarity can be written as $\widetilde{S}_k = (1 + \widetilde{D}_k)^{-1}$, which satisfies $0 < \widetilde{S}_k \leq 1$. Unlike $1 - \widetilde{D}_k$, this transformation remains non-negative even if an extended distance or penalty makes $\widetilde{D}_k > 1$, while preserving monotonicity because $d\widetilde{S}_k / d\widetilde{D}_k = -(1 + \widetilde{D}_k)^{-2} < 0$.

7.3 Indeterminacy-Adjusted Confidence

A high disease similarity should not automatically imply a confident diagnosis when the input evidence itself is strongly indeterminate. Define patient-level indeterminacy as $\bar{I}_p = \sum_{j=1}^{m} w_j I_{pj}$ and a confidence-adjusted disease score as

$$C_k = \widetilde{S}_k (1 - \bar{I}_p)^{\gamma}, \qquad \gamma \geq 0.$$

For $\gamma = 0$, $C_k = \widetilde{S}_k$; for $\gamma > 0$, high indeterminacy reduces decision confidence while leaving the relative disease similarity visible.

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If two diseases satisfy $\widetilde{S}_a > \widetilde{S}_b$ and the same patient uncertainty $\bar{I}_p$ is used for both, then $C_a - C_b = (1 - \bar{I}_p)^\gamma(\widetilde{S}_a - \widetilde{S}_b) > 0$. The adjustment therefore preserves disease ordering while scaling the absolute confidence level, which is desirable because evidence uncertainty should reduce confidence without arbitrarily reversing the ranking generated by the selected similarity model.

7.4 Diagnostic Margin and Abstention

Let $C_{(1)} = \max_k C_k$ and let $C_{(2)}$ denote the second-highest confidence. Define the diagnostic margin $\Delta_C = C_{(1)} - C_{(2)}$. A forced decision returns $d^* = \arg\max_k C_k$, but a safer soft-computing system can return an indeterminate diagnostic outcome whenever $\Delta_C < \theta$ or $\bar{I}_p > \rho$, where θ is a minimum discrimination margin and ρ is a maximum tolerated indeterminacy.

The abstention rule gives I an operational role rather than treating it merely as a coordinate inside a similarity equation. In particular, if $I_{pj} \to 1$ for most heavily weighted symptoms, then $\bar{I}_p \to 1$ and $C_k \to 0$ for every k when $\gamma > 0$; a system that nevertheless returns only $\arg\max_k S_k$ would suppress the practical meaning of high indeterminacy.

7.5 Model Selection Criterion

The review suggests that a more expressive model is not automatically a better clinical model. Let E_M denote representational expressiveness, I_M interpretability, V_M empirical validation quality, and C_M complexity for model M; a conceptual selection utility can be written as $U(M) = \omega_E E_M + \omega_I I_M + \omega_V V_M - \omega_C C_M$ with $\omega_E, \omega_I, \omega_V, \omega_C \geq 0$. A fuzzy system may maximize $U(M)$ when I_M is heavily weighted and the clinical uncertainty is predominantly gradual, whereas an SVNS or hybrid neutrosophic model may maximize $U(M)$ when contradiction and incomplete evidence substantially increase E_M.

Table 3: Suggested correspondence between clinical uncertainty and mathematical model.

Clinical uncertainty structure	**Mathematical signature**	**Preferred model family**
Gradual symptom severity	$\mu(x) \in (0, 1)$	Fuzzy set / fuzzy inference
Positive evidence plus explicit hesitation	$\mu + \nu < 1, \pi > 0$	Intuitionistic fuzzy set
Under-specified or overlapping evidence	$T + I + F \neq 1$	Single-valued neutrosophic set
Positive and negative semantic poles	$(T^+, I^+, F^+; T^-, I^-, F^-)$	Bipolar neutrosophic set
Temporal symptom evolution	$\sum_t v_t S_t$	Dynamic / multi-period fuzzy-neutrosophic model
Multi-level symptom parameters	$E_j = \{e_{j1}, \ldots, e_{jq_j}\}$	Soft / hypersoft model
Boundary-region ambiguity	$\underline{R}(A) \neq \overline{R}(A)$	Rough-neutrosophic model

8 Research Gaps Identified by the Review

8.1 Calibration of Membership and Neutrosophic Degrees

A recurring mathematical weakness is that the values μ, ν, T, I, and F are frequently assigned through expert judgment, heuristic transformations, or fixed linguistic scales. If a raw clinical variable is x, a complete diagnostic model requires calibrated transformations $T = f_T(x)$, $I = f_I(x)$, and $F = f_F(x)$, yet the reviewed literature provides no universally accepted statistical calibration rule for these mappings. Future development should therefore connect soft membership functions with empirical likelihood, reliability, and inter-expert variation without incorrectly imposing the probability identity $T + I + F = 1$.

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8.2 Stability of Similarity-Based Diagnosis

The proliferation of measures means that the same patient may produce disease rankings $r^{(1)}, r^{(2)}, \ldots, r^{(q)}$ under q different similarity functions. A useful robustness statistic is the pairwise rank stability

$$R_s = \frac{2}{q(q-1)} \sum_{a<b} \rho\Big(r^{(a)}, r^{(b)}\Big),$$

where ρ may be Spearman's rank correlation. A high $R_s \approx 1$ indicates that the diagnosis is insensitive to the metric family, whereas a low value reveals that the decision geometry requires further justification.

8.3 Longitudinal and Dynamic Evidence

Most early fuzzy and IFS diagnostic models operate on one observation vector $\mathbf{x}$, while clinical practice often provides a sequence $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(h)}$. The multi-period formulation[13] represents an important step, but a general dynamic model should permit time-dependent membership states $\mathbf{z}_j(t) = (T_j(t), I_j(t), F_j(t))$ and time-dependent weights v_t. Research is therefore needed on whether v_t should be learned from data, derived from symptom half-life, or determined by disease-specific clinical knowledge.

8.4 Missing Data and Indeterminacy

Missingness should not always be encoded as an arbitrary midpoint such as $\mu = 0.5$. In a neutrosophic representation, an unavailable observation may instead be modeled by elevated indeterminacy, for example $(T, I, F) = (0, 1, 0)$, although such a convention must be distinguished from genuine clinical ambiguity. A mathematically principled model should separate missing-data indeterminacy I_M from epistemic diagnostic indeterminacy I_E, perhaps through $I = 1 - (1 - I_M)(1 - I_E)$ so that distinct uncertainty sources are not collapsed into one coordinate.

8.5 Explainability

Similarity-based systems often produce only a disease score S_k, while clinicians require an explanation of which symptoms contributed to the result. For the weighted L_1 distance, symptom-level contribution is naturally

$$c_{kj} = \frac{w_j}{3} \left(|T_{pj} - T_{kj}| + |I_{pj} - I_{kj}| + |F_{pj} - F_{kj}|\right), \qquad D_k = \sum_{j=1}^{m} c_{kj}.$$

Reporting the largest c_{kj} values makes the mathematical diagnosis more transparent by showing which symptoms most strongly separate the patient from each disease profile.

8.6 Clinical Validation Versus Numerical Illustration

Many soft-computing medical studies establish a method using a numerical example in which $d^* = \arg\max_k S_k$ matches an expected disease. Such an example demonstrates computational consistency but does not establish sensitivity, specificity, calibration, or external generalization. A stronger validation design requires a dataset $\mathcal{D} = \{(\mathbf{x}_i, y_i)\}_{i=1}^{N}$ and performance estimates computed on unseen observations, preferably with uncertainty intervals and comparisons against probabilistic or machine-learning baselines.

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8.7 Computational Complexity

For n diseases, m symptoms, and three neutrosophic coordinates, a direct distance-based classifier requires approximately $O(3nm) = O(nm)$ coordinate comparisons per patient. When h time periods or q sub-attributes are introduced, the cost can increase to $O(nmh)$ or $O(nmq)$; although these complexities remain manageable for many clinical systems, very large rule bases or hypersoft parameterizations can suffer combinatorial growth, making dimensionality reduction and sparse expert models important research directions.

9 Discussion

The mathematical trajectory of fuzzy and neutrosophic medical diagnosis can be interpreted as an expansion of the admissible uncertainty state space. Fuzzy diagnosis uses the interval $\Omega_F = [0, 1]$, IFS diagnosis uses the simplex $\Omega_I = \{(T, I, F) \in [0, 1]^3 : T + I + F = 1\}$, and SVNS diagnosis uses $\Omega_N = [0, 1]^3$. Therefore Ω_F can be embedded into Ω_I and $\Omega_I \subset \Omega_N$, but the additional volume of Ω_N is useful only if the system can meaningfully estimate the extra degrees of freedom.

The review also shows that the main innovation in medical neutrosophic systems has shifted from representation to information measurement. Once (T, I, F) became established, much of the literature concentrated on defining $S : \Omega_N^m \times \Omega_N^m \to [0, 1]$ or $D : \Omega_N^m \times \Omega_N^m \to \mathbb{R}_{\geq 0}$. Cosine, tangent, exponential, Euclidean, biparametric, algebraic, rough, and other measures differ in sensitivity, scale, and axiomatic properties, so measure selection should be considered part of the diagnostic model rather than a neutral implementation choice.[11–14, 19, 21, 23]

A second major conclusion is that indeterminacy should have an operational role. If I is represented but the final classifier is always $d^* = \arg\max_k S_k$, then the system can still issue a strong categorical diagnosis when all candidate similarities are nearly equal and the patient evidence is highly indeterminate. The confidence-adjusted score $C_k = \widetilde{S}_k(1 - \bar{I}_p)^\gamma$ and the abstention conditions $\Delta_C < \theta$ or $\bar{I}_p > \rho$ demonstrate one mathematically simple way of allowing I to influence the decision layer.

A third conclusion concerns model selection. For a clinical problem in which uncertainty is almost entirely due to gradual symptom boundaries, the additional dimensions of an SVNS may not improve the utility $U(M) = \omega_E E_M + \omega_I I_M + \omega_V V_M - \omega_C C_M$. In such cases, a transparent fuzzy rule base may be preferable; conversely, if experts genuinely report supporting, opposing, and unresolved evidence simultaneously, forcing the information onto the normalized IFS plane $T + I + F = 1$ can remove distinctions that a neutrosophic representation preserves.

Finally, the mathematical maturity of the field increasingly depends on validation of the entire mapping $\mathcal{X} \xrightarrow{\text{uncertainty encoding}} \Omega \xrightarrow{\text{aggregation}} \Omega' \xrightarrow{\text{similarity/ranking}} D \xrightarrow{\text{decision}} \mathcal{C}$ rather than only proposing another operator inside the chain. Future medical systems should therefore evaluate calibration of the first mapping, axiomatic behavior of aggregation and similarity, diagnostic discrimination of the ranking stage, and clinical validity of the final decision.

9.1 Limitations of the Review

The review is intentionally selective in mathematical depth rather than exhaustive in bibliometric coverage. If $\mathcal{U}_\tau$ denotes all potentially relevant publications available by the cutoff, the analyzed set satisfies $\mathcal{R}_\tau \subseteq \mathcal{U}_\tau$ and was constructed to represent major model families and diagnostically relevant operators, not to estimate publication frequencies or effect sizes. Consequently, absence from Table 1 should not be interpreted as evidence that a model family was nonexistent or unimportant.

The synthesis-derived quantities $\widetilde{D}_k$, C_k, Δ_C, and the thresholds $(\lambda, \gamma, \theta, \rho)$ are analytical devices for comparing decision architectures; they are not claimed to be clinically validated diagnostic parameters. Before deployment, these quantities require calibration on an external dataset $\mathcal{D}_{\text{ext}} = \{(\mathbf{x}_i, y_i)\}_{i=1}^{N_{\text{ext}}}$ and evaluation with discrimination, calibration, and decision-utility measures. This distinction prevents the mathematical synthesis from being interpreted as evidence of clinical efficacy.

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10 Conclusion

This review examined fuzzy, intuitionistic fuzzy, and neutrosophic systems for medical diagnosis under the strict literature boundary $t \leq 31$ May 2024. The mathematical comparison shows that conventional fuzzy diagnosis represents uncertainty through $\mu \in [0,1]$, IFS diagnosis extends this state to (μ, π, ν) with $\mu + \pi + \nu = 1$, and SVNS diagnosis uses the independent triple $(T, I, F) \in [0,1]^3$. These models can therefore be interpreted within one unified coordinate space in which fuzzy and intuitionistic fuzzy systems occupy constrained subsets of the neutrosophic cube.

The literature demonstrates that medical applications have progressed from fuzzy rule systems and IFS similarity measures toward neutrosophic distance functions, temporal aggregation, recommender systems, rough-neutrosophic structures, bipolar representations, and hypersoft decision models. Despite this progression, the core diagnostic operation usually retains the form $d^* = \arg\max_k S(\mathbf{P}, \mathbf{Q}_k)$ or its equivalent distance minimization, so representational richness alone does not guarantee a better diagnostic system; the reliability of S, the calibration of (T, I, F), and the interpretability of the resulting ranking remain equally important.

To consolidate these observations, the paper introduced the normalization residual $\kappa = T + I + F - 1$, a residual-aware distance $\widetilde{D}_k$, an indeterminacy-adjusted confidence $C_k = \widetilde{S}_k(1-\bar{I}_p)^\gamma$, and a diagnostic margin $\Delta_C = C_{(1)} - C_{(2)}$. These quantities provide a common mathematical language for comparing fuzzy and neutrosophic diagnostic systems and for preventing high-indeterminacy inputs from being treated as ordinary confident classifications.

The principal implication is that the choice between fuzzy and neutrosophic systems should depend on the structure of uncertainty rather than on model novelty. If ambiguity is primarily gradual, $\mu(x)$ and interpretable fuzzy rules may be sufficient; if uncertainty contains independent truth, falsity, incompleteness, or contradiction, the larger state space (T, I, F) becomes mathematically justified. The most promising direction is therefore not unrestricted growth in uncertainty dimensions, but calibrated, explainable, and clinically validated models in which every added coordinate contributes measurable diagnostic information.

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