



# Generating Neutrosophic Random Variables Based on Generalized Gamma Distribution

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## ABSTRACT

In practice, we encounter many systems that cannot be studied directly, either due to high costs or because some of these systems are not directly detectable. Therefore, we resort to simulation, which involves applying the study to systems similar to real-world systems and then projecting the results if they are suitable for the real system. The simulation process requires a thorough understanding of probability distributions and the methods used to transform random numbers following a regular distribution on  $[0, 1]$  into random variables that follow it. This allows us to maximize the benefits of the simulation process and obtain more accurate results for all emerging conditions. The generalized gamma distribution is a family of three parameters characterized by high flexibility. It includes several important distributions as special cases, including the gamma, Weibull, exponential, and Rayleigh distributions, making it exceptionally valuable in engineering and reliability analysis. In previous research, we presented a neutrosophic view of the process of generating random numbers and some techniques used to generate random variables. In this research, we present a neutrosophic study for generating neutrosophic random variables following the generalized gamma distribution, a distribution widely used in engineering applications. The neutrosophic approach takes into account the uncertainty and indeterminacy of the parameters, resulting in random intervals for the variables rather than specific values, and thus provides more accurate simulation results that adapt to all the conditions that the system in operation may encounter.

**Keywords:** Simulation ▪ Random number generation ▪ Neutrosophic logic ▪ Generalized gamma distribution ▪ Neutrosophic random variable generation ▪ Accept–reject technique

## 1. INTRODUCTION

Operations research is the applied branch of mathematics. Since its inception, it has contributed to improving the performance of many systems that utilize its methods in their operations. Among the methods of operations research is simulation. The importance of simulation in all branches of science lies in the significant difficulty we may encounter when studying the operation of any real-world system, either due to high costs or the impossibility of studying some systems directly. The simulation process relies on generating

a series of random numbers that follow a uniform probability distribution in the range  $[0, 1]$ , and then transforming these random numbers into random variables that follow the probability distribution of the system being simulated.

Researchers interested in simulation have focused on providing scientific methods that facilitate the transformation of random numbers into random variables that follow probability distributions more commonly used in practical applications. To respond to the major advances brought about by neutrosophic logic in all fields of science, many researchers have presented studies aimed at offering a neutrosophic perspec-

tive on various scientific disciplines. Among these studies is the neutrosophic study of Monte Carlo simulations for generating neutrosophic random numbers, which are then transformed into neutrosophic random variables that follow probability distributions applicable to many real-world systems [1, 2, 3, 4, 5, 6, 7].

In this research, we present a neutrosophic study to generate neutrosophic random variables following the generalized gamma distribution (GGD), a three-parameter distribution that is more flexible than the standard two-parameter gamma distribution and is widely used in engineering applications, reliability analysis, and survival studies. By introducing indeterminacy into all three parameters of the GGD, we extend the traditional accept–reject technique to include the full neutrosophic framework, which enables us to obtain more accurate and reliable simulation results under all conditions to which the work environment may be exposed.

## 2. DISCUSSION

### 2.1 The Principle Behind the Simulation Process

The principle is to generate a series of random numbers that follow a uniform distribution over  $[0, 1]$ , and then convert these random numbers into random variables that follow the distribution according to which the system to be simulated operates. There are many techniques used to generate these random numbers, such as the mid-square, mid-product, and Fibonacci methods, among others. To generate neutrosophic random numbers, a study using the mid-square method was presented in [5], reaching the following results.

The mid-square method for generating a series of random numbers is defined by

$$R_{i+1} = \text{Mid}[R_i^2], \quad i = 0, 1, 2, 3, \dots,$$

where Mid denotes the middle four digits of  $R_i^2$ , and  $R_i$  is chosen as a four-digit fractional seed that does not contain zero in any of its four digits.

We convert these random numbers into neutrosophic random numbers. We distinguish three states of the field  $[0, 1]$  with a margin of indeterminacy, where  $\varepsilon \in [0, n]$  and  $0 < n < 1$ :

**Case 1:**  $[0 + \varepsilon, 1]$ —indeterminacy at the minimum of the range. We use

$$R_{iN} = \frac{R_i - \varepsilon}{1 - \varepsilon}.$$

**Case 2:**  $[0, 1 + \varepsilon]$ —indeterminacy at the upper limit. We use

$$R_{iN} = \frac{R_i}{1 + \varepsilon}.$$

**Case 3:**  $[0 + \varepsilon, 1 + \varepsilon]$ —indeterminacy in both limits. We use

$$R_{iN} = R_i - \varepsilon.$$

### 2.2 Generating Neutrosophic Random Variables

To generate neutrosophic random variables that follow the probability distribution according to which the system to be simulated operates, we distinguish three cases:

**Case 1:** Neutrosophic random numbers and a probability distribution in classical form.

**Case 2:** Classical random numbers and a probability distribution in neutrosophic form.

**Case 3:** Neutrosophic random numbers and a probability distribution in neutrosophic form.

### 2.3 Generating Neutrosophic Random Variables Based on the Generalized Gamma Distribution

The generalized gamma distribution is one of the important distributions used in engineering applications. Given this importance, and so that we can provide more accurate simulation results suitable for all conditions, we present a neutrosophic study of the process of generating random variables that follow the generalized gamma distribution, based on the classical and neutrosophic studies presented in [5, 7].

#### 2.3.1 Classical Formula for the Generalized Gamma Distribution

The generalized gamma distribution, introduced by Stacy [8], is a three-parameter family whose probability density function is

$$f(x; a, d, p) = \frac{p}{a^d \Gamma(d/p)} x^{d-1} \exp\left[-\left(\frac{x}{a}\right)^p\right],$$

where  $x \geq 0$ ,  $a > 0$ ,  $d > 0$ , and  $p > 0$ . Here,  $a$  is the scale parameter, while  $d$  and  $p$  are shape parameters. The gamma function is

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt.$$

The mode of the GGD, for  $d > 1$ , is obtained by differentiating  $f(x)$  with respect to  $x$  and setting the result to zero:

$$x_{\text{mode}} = a \left( \frac{d-1}{p} \right)^{1/p}. \quad (1)$$

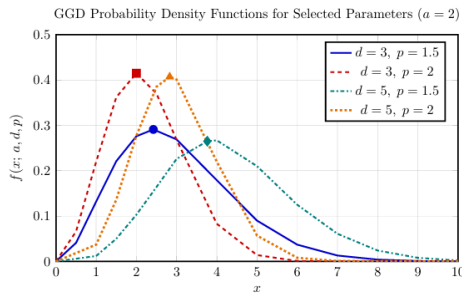
Special cases of the generalized gamma distribution are summarized in Table 1.

**Table 1.** Special cases of the generalized gamma distribution.

| Scale $a$ | Shape $d$              | Shape $p$         | Resulting distribution                  |
|-----------|------------------------|-------------------|-----------------------------------------|
| $a$       | $d$                    | $p = 1$           | Standard gamma (shape $d$ , scale $a$ ) |
| $a$       | $d = p$                | $p$               | Weibull ( $a, p$ )                      |
| $a$       | $d = 1$                | $p = 1$           | Exponential with mean $a$               |
| $a$       | $d = 2$                | $p = 2$           | Rayleigh ( $a/\sqrt{2}$ )               |
| $a$       | $d = 1$                | $p = 2$           | Half-normal                             |
| $a$       | $d \rightarrow \infty$ | $p$               | Normal (approximately)                  |
| $a$       | $d$                    | $p \rightarrow 0$ | Log-normal                              |

#### 2.3.2 Neutrosophic Formula for the Generalized Gamma Distribution

If we take one or more of  $a$ ,  $d$ , and  $p$  as a neutrosophic number, where a neutrosophic number is written in the standard form  $N = \lambda + \gamma I$  ( $\lambda$  is the definite part and  $\gamma I$  is the indefinite part), and it is possible that the indeterminate component is represented by  $[\lambda_1, \lambda_2]$ ,  $\{\lambda_1, \lambda_2\}$ , or any set close to the real value  $\lambda$ , we obtain the neutrosophic generalized gamma distribution. We distinguish the following cases:



**Figure 1.** GGD probability density functions for the four parameter combinations used in the example (all with  $a = 2$ ). Filled circles mark the mode of each curve.

**Case 1:**  $d_n$  is neutrosophic, while  $a$  and  $p$  are classical:

$$f_n(x) = \frac{p}{a^{d_n} \Gamma(d_n/p)} x^{d_n-1} \times \exp\left[-\left(\frac{x}{a}\right)^p\right], \quad x \geq 0, d_n > 0.$$

**Case 2:**  $p_n$  is neutrosophic, while  $a$  and  $d$  are classical:

$$f_n(x) = \frac{p_n}{a^d \Gamma(d/p_n)} x^{d-1} \times \exp\left[-\left(\frac{x}{a}\right)^{p_n}\right], \quad x \geq 0, p_n > 0.$$

**Case 3:**  $a_n$  is neutrosophic, while  $d$  and  $p$  are classical:

$$f_n(x) = \frac{p}{a_n^d \Gamma(d/p)} x^{d-1} \times \exp\left[-\left(\frac{x}{a_n}\right)^p\right], \quad x \geq 0, a_n > 0.$$

**Case 4:** All three parameters  $a_n$ ,  $d_n$ , and  $p_n$  are neutrosophic:

$$f_n(x) = \frac{p_n}{a_n^{d_n} \Gamma(d_n/p_n)} x^{d_n-1} \times \exp\left[-\left(\frac{x}{a_n}\right)^{p_n}\right], \quad x \geq 0. \quad (2)$$

Here,  $a_n = a \pm \eta$ ,  $d_n = d \pm \delta$ , and  $p_n = p \pm \gamma$ , where  $\eta$ ,  $\delta$ , and  $\gamma$  represent the indeterminacies of the parameters  $a$ ,  $d$ , and  $p$ , respectively.

Thus, we obtain the following special cases of the neutrosophic GGD:

- When  $p_n = 1$ , the neutrosophic GGD reduces to the neutrosophic standard gamma distribution [7].
- When  $d_n = p_n$ , the neutrosophic GGD reduces to the neutrosophic Weibull distribution.
- When  $d_n = p_n = 1$ , the neutrosophic GGD reduces to the neutrosophic exponential distribution [9].
- As  $d_n \rightarrow \infty$ , the neutrosophic GGD approaches the neutrosophic normal distribution.

### 2.3.3 Generating Neutrosophic Random Variables Based on the Generalized Gamma Distribution

From the classical studies in [10, 11] and the neutrosophic studies in [5, 7], the general formulation of the technique

used to generate neutrosophic random variables following a generalized gamma distribution is as follows.

We search for the largest value of the function  $f_n(x)$  on its domain by calculating the first derivative and setting it equal to zero. For the neutrosophic GGD with  $d_n > 1$ , we obtain the following neutrosophic mode:

$$x_{n,\text{mode}} = a_n \left( \frac{d_n - 1}{p_n} \right)^{1/p_n}. \quad (3)$$

We substitute this value into  $f_n(x)$  to obtain  $M_n$ , the largest value of the function corresponding to the neutrosophic mode. We then follow these steps:

- Generate two random numbers,  $R_1$  and  $R_2$ , from  $[0, 1]$  using the mid-square method.
- Convert these two numbers into neutrosophic random numbers,  $R_{n1}$  and  $R_{n2}$ , using the appropriate case from Section 2.1.
- Form a new variable  $x_n = 1/R_{n1}$ , so that the domain of  $x_n$  becomes  $[0, +\infty)$ , identical to the support of  $f_n(x)$ .
- Compare  $R_{n2}$  with  $f_n(1/R_{n1})/M_n$ .
- If  $R_{n2} > f_n(1/R_{n1})/M_n$ , accept  $x_n = 1/R_{n1}$  as a neutrosophic random variable subject to the generalized gamma distribution.
- If  $R_{n2} \leq f_n(1/R_{n1})/M_n$ , reject  $R_{n1}$  and  $R_{n2}$  and return to step (a).

## 3. EXAMPLE

We explain the above through an example that corresponds to the third case of neutrosophic random numbers (indeterminacy in both the upper and lower bounds of  $[0, 1]$ ), with the generalized gamma distribution in fully neutrosophic form.

Find a random variable that follows the generalized gamma distribution defined by

$$f_n(x) = \frac{p_n}{a_n^{d_n} \Gamma(d_n/p_n)} x^{d_n-1} \exp\left[-\left(\frac{x}{a_n}\right)^{p_n}\right],$$

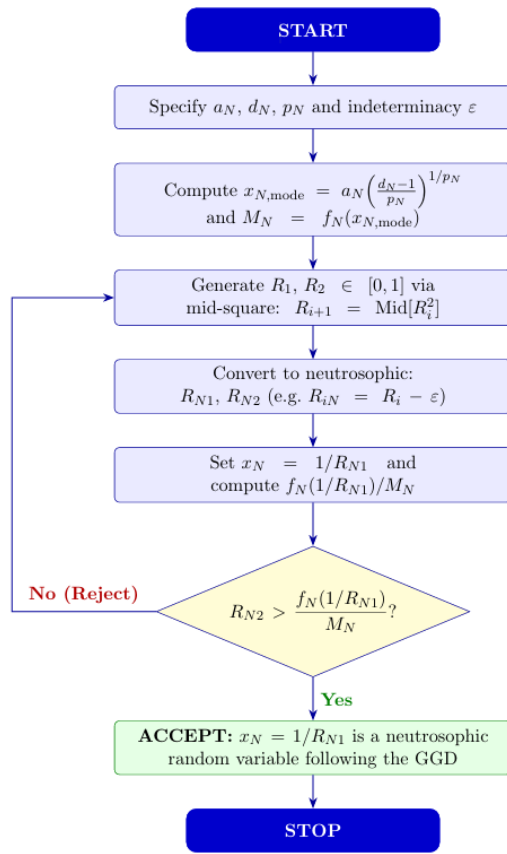
where  $x \geq 0$ ,  $a_n \in [2, 2.5]$ ,  $d_n \in \{3, 5\}$ , and  $p_n \in [1.5, 2]$ . Based on these data, the study is divided into two parts.

### 3.1 First Part

The data are  $x \geq 0$ ,  $d_n = 3$ ,  $a_n \in [2, 2.5]$ , and  $p_n \in [1.5, 2]$ . First, we calculate  $M_n$  from (3):

$$\begin{aligned} x_{n,\text{mode}} &= a_n \left( \frac{d_n - 1}{p_n} \right)^{1/p_n} \\ &= [2, 2.5] \left( \frac{3 - 1}{[1.5, 2]} \right)^{1/[1.5, 2]} \\ &= [2, 2.5] \left( \frac{2}{[1.5, 2]} \right)^{[0.5, 0.667]} \\ &\in [2.000, 3.028]. \end{aligned}$$

Substituting into  $f_n(x)$  at the mode gives the maximum density  $M_n \in [0.232, 0.415]$ .



**Figure 2.** Flowchart of the neutrosophic rejection and acceptance algorithm for generating neutrosophic random variables based on the GGD.

**a. Random-number generation.** We generate two random numbers using the mid-square method with seed  $R_0 = 0.3176$ , as shown in Table 2.

**Table 2.** Random-number generation using the mid-square method (seed  $R_0 = 0.3176$ ).

| $i$ | $R_i$  | $R_i^2$    | $R_i^2$ (8 digits) | $R_{i+1} = \text{Mid}[R_i^2]$ |
|-----|--------|------------|--------------------|-------------------------------|
| 0   | 0.3176 | 0.10086976 | 10086976           | 0.0869                        |
| 1   | 0.0869 | 0.00755161 | 00755161           | 0.7551                        |
| 2   | 0.7551 | 0.57017601 | 57017601           | 0.0176                        |

From this we obtain  $R_1 = 0.0869$  and  $R_2 = 0.7551$ .

**b. Conversion to neutrosophic numbers.** We convert the two numbers using the third-case relation  $R_{iN} = R_i - \varepsilon$ , with  $\varepsilon \in [0, 0.03]$ , as shown in Table 3.

**Table 3.** Conversion to neutrosophic random numbers for  $\varepsilon \in [0, 0.03]$ .

| Classical number | Relation             | Neutrosophic number           |
|------------------|----------------------|-------------------------------|
| $R_1 = 0.0869$   | $0.0869 - [0, 0.03]$ | $R_{n1} \in [0.0569, 0.0869]$ |
| $R_2 = 0.7551$   | $0.7551 - [0, 0.03]$ | $R_{n2} \in [0.7251, 0.7551]$ |

### c. Formation of the new variable.

$$x_n = \frac{1}{R_{n1}} = \frac{1}{[0.0569, 0.0869]} \in [11.5075, 17.5746].$$

**d. Acceptance test.** We calculate  $f_n(1/R_{n1})$  by substituting  $x_n \in [11.5075, 17.5746]$ ,  $d_n = 3$ ,  $a_n \in [2, 2.5]$ , and  $p_n \in [1.5, 2]$  into (2):

$$f_n([11.5075, 17.5746]) \in [0, 6.55 \times 10^{-4}].$$

We then calculate the ratio

$$\frac{f_n(1/R_{n1})}{M_n} = \frac{[0, 6.55 \times 10^{-4}]}{[0.232, 0.415]} \in [0, 2.82 \times 10^{-3}].$$

We have  $R_{n2} \in [0.7251, 0.7551]$  and  $f_n(1/R_{n1})/M_n \in [0, 2.82 \times 10^{-3}]$ . We note that

$$[0.7251, 0.7551] \gg [0, 2.82 \times 10^{-3}] \implies R_{n2} > \frac{f_n(1/R_{n1})}{M_n}.$$

The acceptance criterion is satisfied. We therefore consider

$$x_n = \frac{1}{R_{n1}} \in [11.5075, 17.5746]$$

to be a neutrosophic random variable subject to the generalized gamma distribution with  $d_n = 3$ . Table 4 summarizes all computed values for Part 1.

**Table 4.** Summary of computed values—Part 1 ( $d_n = 3$ ,  $a_n \in [2, 2.5]$ ,  $p_n \in [1.5, 2]$ ).

| Quantity                  | Formula                 | Neutrosophic value            |
|---------------------------|-------------------------|-------------------------------|
| Mode $x_{n,mode}$         | Eq. (3)                 | [2.000, 3.028]                |
| Maximum density $M_n$     | $f_n(mode)$             | [0.232, 0.415]                |
| $R_{n1}$                  | $R_1 - \varepsilon$     | [0.0569, 0.0869]              |
| $R_{n2}$                  | $R_2 - \varepsilon$     | [0.7251, 0.7551]              |
| Variable $x_n = 1/R_{n1}$ | —                       | [11.5075, 17.5746]            |
| $f_n(1/R_{n1})$           | Eq. (2)                 | [0, 6.55 × 10 <sup>-4</sup> ] |
| Ratio $f_n(x_n)/M_n$      | —                       | [0, 2.82 × 10 <sup>-3</sup> ] |
| Decision                  | $[0.73] \gg [0, 0.003]$ | ACCEPT ✓                      |

### 3.2 Second Part

The data are  $x \geq 0$ ,  $d_n = 5$ ,  $a_n \in [2, 2.5]$ , and  $p_n \in [1.5, 2]$ . We calculate  $M_n$  from (3):

$$\begin{aligned}
 x_{n,mode} &= [2, 2.5] \left( \frac{5-1}{[1.5, 2]} \right)^{1/[1.5, 2]} \\
 &= [2, 2.5] \left( \frac{4}{[1.5, 2]} \right)^{[0.5, 0.667]} \\
 &\in [2.828, 4.705].
 \end{aligned}$$

Substituting at the mode gives  $M_n \in [0.213, 0.407]$ .

**a. Random-number generation.** We use the same seed  $R_0 = 0.3176$  (Table 2), obtaining  $R_1 = 0.0869$  and  $R_2 = 0.7551$ .

**b. Conversion to neutrosophic numbers.** With  $\varepsilon \in [0, 0.03]$ , as in Part 1,

$$R_{n1} \in [0.0569, 0.0869], \quad R_{n2} \in [0.7251, 0.7551].$$

### c. Formation of the new variable.

$$x_n = \frac{1}{R_{n1}} = \frac{1}{[0.0569, 0.0869]} \in [11.5075, 17.5746].$$

### d. Acceptance test.

With  $d_n = 5$ ,

$$f_n([11.5075, 17.5746]) \in [0, 5.18 \times 10^{-3}],$$

and

$$\frac{f_n(1/R_{n1})}{M_n} = \frac{[0, 5.18 \times 10^{-3}]}{[0.213, 0.407]} \in [0, 2.43 \times 10^{-2}].$$

We have  $R_{n2} \in [0.7251, 0.7551]$  and  $f_n(1/R_{n1})/M_n \in [0, 2.43 \times 10^{-2}]$ . We note that

$$[0.7251, 0.7551] \gg [0, 2.43 \times 10^{-2}] \Rightarrow R_{n2} > \frac{f_n(1/R_{n1})}{M_n}.$$

The acceptance criterion is satisfied. We therefore consider  $x_n = 1/R_{n1} \in [11.5075, 17.5746]$  to be a neutrosophic random variable subject to the generalized gamma distribution with  $d_n = 5$ . Table 5 summarizes all computed values for Part 2.

**Table 5.** Summary of computed values—Part 2 ( $d_n = 5$ ,  $a_n \in [2, 2.5]$ ,  $p_n \in [1.5, 2]$ ).

| Quantity                  | Formula              | Neutrosophic value            |
|---------------------------|----------------------|-------------------------------|
| Mode $x_{n,\text{mode}}$  | Eq. (3)              | [2.828, 4.705]                |
| Maximum density $M_n$     | $f_n(\text{mode})$   | [0.213, 0.407]                |
| $R_{n1}$                  | $R_1 - \varepsilon$  | [0.0569, 0.0869]              |
| $R_{n2}$                  | $R_2 - \varepsilon$  | [0.7251, 0.7551]              |
| Variable $x_n = 1/R_{n1}$ | —                    | [11.5075, 17.5746]            |
| $f_n(1/R_{n1})$           | Eq. (2)              | [0, 5.18 × 10 <sup>-3</sup> ] |
| Ratio $f_n(x_n)/M_n$      | —                    | [0, 2.43 × 10 <sup>-2</sup> ] |
| Decision                  | [0.73] >> [0, 0.024] | ACCEPT ✓                      |

From the results of the first and second parts, we note that  $x_n = 1/R_{n1} \in [11.5075, 17.5746]$  can be considered a neutrosophic random variable that follows a generalized gamma distribution defined by (2), where  $x \geq 0$ ,  $a_n \in [2, 2.5]$ ,  $d_n \in \{3, 5\}$ , and  $p_n \in [1.5, 2]$ .

### 3.3 Classical Wording of the Example

Find a random variable that follows the generalized gamma distribution defined by

$$f(x; a, d, p) = \frac{p}{a^d \Gamma(d/p)} x^{d-1} \exp\left[-\left(\frac{x}{a}\right)^p\right],$$

where  $x \geq 0$ ,  $a = 2$ ,  $d = 3$ , and  $p = 1.5$ .

First, we calculate  $M$  from the mode formula:

$$x_{\text{mode}} = a \left(\frac{d-1}{p}\right)^{1/p} = 2 \left(\frac{2}{1.5}\right)^{1/1.5} = 2 \times 1.211 = 2.422.$$

Then,

$$M = f(2.422) = \frac{1.5}{8 \times 1} (2.422)^2 e^{-(2.422/2)^{1.5}} = 0.2905.$$

**a.** We generate two random numbers using the mid-square method with seed  $R_0 = 0.3176$ , obtaining  $R_1 = 0.0869$  and  $R_2 = 0.7551$ .

### b. We form a new variable:

$$x = \frac{1}{R_1} = \frac{1}{0.0869} = 11.5075.$$

### c. To compare $R_2$ with $f(1/R_1)/M$ , we calculate

$$f(11.5075) = \frac{1.5}{8 \times 1} (11.5075)^2 e^{-(11.5075/2)^{1.5}} \approx 2.46 \times 10^{-5},$$

and

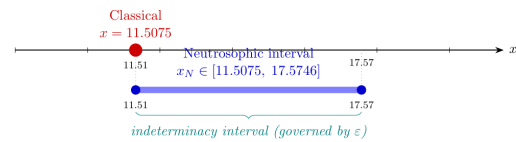
$$\frac{f(1/R_1)}{M} = \frac{2.46 \times 10^{-5}}{0.2905} \approx 8.47 \times 10^{-5}.$$

We have  $R_2 = 0.7551$  and  $f(1/R_1)/M = 8.47 \times 10^{-5}$ . Since  $0.7551 > 8.47 \times 10^{-5}$ , the acceptance criterion is satisfied. We therefore consider  $x = 1/R_1 = 11.5075$  to be a classical random variable subject to the generalized gamma distribution with  $a = 2$ ,  $d = 3$ , and  $p = 1.5$ .

The classical example is treated in the same way if the data are  $x \geq 0$ ,  $a = 2$ ,  $d = 5$ , and  $p = 1.5$ . Table 6 compares all classical and neutrosophic results side by side.

**Table 6.** Comparison of classical and neutrosophic results.

| Quantity              | Classical<br>( $d = 3$ ) | Neutrosophic<br>( $d_n = 3$ ) | Neutrosophic<br>( $d_n = 5$ ) |
|-----------------------|--------------------------|-------------------------------|-------------------------------|
| Mode                  | 2.422                    | [2.000, 3.028]                | [2.828, 4.705]                |
| $M$ (maximum density) | 0.2905                   | [0.232, 0.415]                | [0.213, 0.407]                |
| $R_1 / R_{n1}$        | 0.0869                   | [0.0569, 0.0869]              | [0.0569, 0.0869]              |
| $R_2 / R_{n2}$        | 0.7551                   | [0.7251, 0.7551]              | [0.7251, 0.7551]              |
| $x = 1/R_1$           | 11.5075                  | [11.5075, 17.5746]            | [11.5075, 17.5746]            |
| $f(x)$                | $2.46 \times 10^{-5}$    | [0, 6.55 × 10 <sup>-4</sup> ] | [0, 5.18 × 10 <sup>-3</sup> ] |
| $f(x)/M$              | $8.47 \times 10^{-5}$    | [0, 2.82 × 10 <sup>-3</sup> ] | [0, 2.43 × 10 <sup>-2</sup> ] |
| Decision              | Accept                   | Accept                        | Accept                        |



**Figure 3.** Classical (crisp) result versus the neutrosophic interval. The classical approach yields the single value  $x = 11.5075$ , while the neutrosophic approach yields the interval  $x_n \in [11.5075, 17.5746]$ , capturing the full extent of parameter uncertainty.

## 4. CONCLUSION

In this research, we presented a neutrosophic study whose purpose is to obtain neutrosophic random variables that follow the generalized gamma distribution, a distribution that has many uses in engineering applications. We found that:

- The neutrosophic values of the required random variables have a margin of freedom and take into account all the conditions through which the system's operating environment passes. The system intended to be simulated operates according to the generalized gamma distribution, while the classical study gives a specific value for the random variable that is appropriate only for conditions similar to those in which such data were adopted.
- The new neutrosophic mode formula  $x_{n,\text{mode}} = a_n[(d_n -$

- 1)/ $p_n]^{1/p_n}$  generalizes the classical gamma mode  $(\alpha - 1)/\beta$  and reduces to it when  $p_n = 1$ .
- The three-parameter structure of the GGD, combined with neutrosophic indeterminacy in all three parameters, provides a significantly higher degree of modeling flexibility than the standard two-parameter gamma distribution studied in [7].
  - The results confirm these findings: the classical study yields the single value  $x = 11.5075$  (for  $a = 2$ ,  $d = 3$ , and  $p = 1.5$ ), while the neutrosophic study yields the interval  $x_n \in [11.5075, 17.5746]$  for  $a_n \in [2, 2.5]$ ,  $d_n \in \{3, 5\}$ , and  $p_n \in [1.5, 2]$ .

Therefore, to obtain more accurate simulation results that take into account all conditions, we recommend using the neutrosophic study of the generalized gamma distribution.

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