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FULL LENGTH ARTICLE

A Note on q-Fractional Neutrosophic Sets and Graphs

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ABSTRACT

A *neutrosophic set* assigns to each element three (generally independent) degrees: truth, indeterminacy, and falsity. A *q-fractional fuzzy set* assigns membership and nonmembership degrees in $[0,1]$ to each element, constrained by $(\mu+\nu)/q \leq 1$ for $q \geq 2$. A *q-fractional fuzzy graph* is a graph whose vertices and edges carry q-fractional fuzzy degrees, with edge membership bounded by endpoints and edge nonmembership dominating them. In this paper, we introduce *q-fractional neutrosophic sets and graphs* as an extension of q-fractional fuzzy sets and q-fractional fuzzy graphs, and we investigate their fundamental properties.

Keywords: Neutrosophic Graph ▪ q-fractional fuzzy set ▪ q-fractional fuzzy graph

1. INTRODUCTION

Classical (crisp) set theory encodes membership in a binary way and, by design, does not directly accommodate vagueness, partial truth, or inconsistent evidence. To model graded information, Zadeh introduced *fuzzy sets* in 1965 [1]. Atanassov subsequently proposed *intuitionistic fuzzy sets* [2, 3], in which membership and non-membership degrees are specified separately and the residual part is interpreted as hesitation. Smarandache further extended this line of research to *neutrosophic sets* [4], assigning to each element three typically independent degrees of truth, indeterminacy, and falsity, thereby providing a flexible framework for representing incomplete, indeterminate, and contradictory information. Several extensions and related variants of neutrosophic sets are also known, including *Bipolar Neutrosophic Sets*[5, 6], *HyperNeutrosophic Sets*[7], and *m-polar Neutrosophic Sets*. Because of their theoretical and practical significance, *neutrosophic sets* and their extensions have been extensively studied in numerous recent publications.

Uncertainty-aware semantics have also been incorporated into network models. *Fuzzy graphs* attach values in $[0,1]$ to vertices and edges [8, 9], while *intuitionistic fuzzy graphs* enrich this framework by adding explicit non-membership information [10, 11]. *Neutrosophic graphs* further include an indeterminacy component, enabling a finer representation of ambiguity in relational data [12, 13, 14]. Neutrosophic graphs have been actively studied in various recent works [15, 16]. More recently, *plithogenic graphs* have been proposed to quantify contradiction effects across multiple attributes [17]. For clarity, Table 1 summarizes the main differences between fuzzy graphs and neutrosophic graphs.

Along a different axis of generalization, *q-fractional* models relax the usual normalization constraints by introducing a parameter $q \geq 2$. A *q-fractional fuzzy set* assigns membership and non-membership degrees in $[0,1]$ to each element, subject to the scaled constraint $(\mu+\nu)/q \leq 1$[18, 19, 20]. Analogously, a *q-fractional fuzzy graph* is a graph whose vertices and edges carry q-fractional fuzzy degrees, where edge membership is bounded by its endpoints and edge non-

Table 1. Comparison of fuzzy graphs and neutrosophic graphs.

Feature	Fuzzy Graph	Neutrosophic Graph
Vertex information	Membership degree $\mu_A(v) \in [0,1]$	Truth, indeterminacy, and falsity degrees $(T_A(v), I_A(v), F_A(v))$
Edge information	Membership degree $\mu_B(uv) \in [0,1]$	Truth, indeterminacy, and falsity degrees $(T_B(uv), I_B(uv), F_B(uv))$
Edge compatibility	$\mu_B(uv) \leq \min\{\mu_A(u), \mu_A(v)\}$	$T_B(uv) \leq \min\{T_A(u), T_A(v)\}$, $I_B(uv) \leq \min\{I_A(u), I_A(v)\}$, and $F_B(uv) \geq \max\{F_A(u), F_A(v)\}$
Uncertainty representation	Graded membership	Truth, indeterminacy, and falsity
Expressive scope	Suitable for graded or vague relations	More explicitly represents incomplete, indeterminate, and conflicting information

membership dominates them[21]. Owing to their theoretical and practical significance, q-fractional fuzzy sets, q-fractional fuzzy graphs, and their related concepts have been actively investigated in numerous recent studies. For reference, a comparison of fuzzy graphs and q-fractional fuzzy graphs is presented in Table 2.

Although the theories of fuzzy, intuitionistic fuzzy, and neutrosophic graphs are well developed, a systematic *neutrosophic* counterpart of q-fractional fuzzy graphs does not yet appear to be standard in the literature. The purpose of this paper is to fill this gap. Specifically, we introduce *q-fractional neutrosophic sets and graphs* as natural extensions of q-fractional fuzzy sets and q-fractional fuzzy graphs, and we investigate their fundamental properties. For clarity, Table 3 summarizes the main differences between q-fractional fuzzy graphs and q-fractional neutrosophic graphs.

2. PRELIMINARIES

We begin by reviewing the basic terminology and notation used throughout this paper.

2.1 Neutrosophic Graphs

A *neutrosophic set* assigns to each element three (generally independent) degrees: truth, indeterminacy, and falsity, each in $[0,1]$ [4]. Neutrosophic graphs and hypergraphs label vertices and/or incidences by such triples.

Definition 2.1 (Single-Valued Neutrosophic Graph)**.** [22] Let $G^* = (V,E)$ be a crisp (classical) graph, where V is the vertex set and $E \subseteq V \times V$ the edge set. A *single-valued neutrosophic graph* (SVNG) on G^* is defined as a pair

$$G = (A,B),$$

where

- $A = \{\langle v, T_A(v), I_A(v), F_A(v)\rangle : v \in V\}$ is the *single-valued neutrosophic vertex set*, with

  $$T_A, I_A, F_A : V \to [0,1],$$

  denoting respectively the *truth-membership*, *indeterminacy-membership*, and *falsity-membership* functions of vertices, such that for every $v \in V$,

  $$0 \leq T_A(v) + I_A(v) + F_A(v) \leq 3.$$

- $B = \{\langle uv, T_B(uv), I_B(uv), F_B(uv)\rangle : uv \in E\}$ is the *single-valued neutrosophic edge set*, with

  $$T_B, I_B, F_B : E \to [0,1],$$

  satisfying for all $u, v \in V$ with $uv \in E$:

  $$T_B(uv) \leq \min\{T_A(u), T_A(v)\},$$
  $$I_B(uv) \leq \min\{I_A(u), I_A(v)\},$$
  $$F_B(uv) \geq \max\{F_A(u), F_A(v)\}.$$

If B is symmetric, $G = (A,B)$ is called an *undirected SVNG*; otherwise, it is a *directed SVNG*.

2.2 q-fractional fuzzy graph

A *q-fractional fuzzy set* assigns membership and nonmembership degrees in $[0,1]$ to each element, constrained by $(\mu+\nu)/q \leq 1$ for $q \geq 2$ [20]. A *q-fractional fuzzy graph* is a graph whose vertices and edges carry q-fractional fuzzy degrees, with edge membership bounded by endpoints and edge nonmembership dominating them [21].

Definition 2.2 (q-fractional fuzzy set)**.** [20] Let X be a nonempty set and let $q \geq 2$. A *q-fractional fuzzy set* (briefly, *q-FFS*) on X is an ordered pair

$$A = (\mu_A, \nu_A),$$

where $\mu_A, \nu_A : X \to [0,1]$ are respectively called the *membership* and *non-membership* functions, such that for every $x \in X$,

$$0 \leq \frac{1}{q}\mu_A(x) + \frac{1}{q}\nu_A(x) \leq 1. \tag{1}$$

Equivalently, $\mu_A(x) + \nu_A(x) \leq q$ for all $x \in X$. We may write

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle_q : x \in X\}.$$

Remark 2.3 (On an alternative convention)**.** Some authors impose instead the constraint $\mu_A(x)^q + \nu_A(x)^q \leq 1$ (with $q \geq 2$). All definitions below remain valid *mutatis mutandis* by replacing (1) and (2) with that alternative q-power constraint.

Definition 2.4 (q-fractional fuzzy graph)**.** [21] Let $G^* = (V,E)$ be a simple undirected graph, where $V \neq \emptyset$ and $E \subseteq \binom{V}{2}$. Fix $q \geq 2$. A *q-fractional fuzzy graph* (briefly, *q-FFG*)

Table 2. Comparison of fuzzy graphs and q-fractional fuzzy graphs.

Feature	Fuzzy Graph	q-Fractional Fuzzy Graph
Vertex information	A membership degree $\mu_A(v) \in [0,1]$	Membership and non-membership degrees $\mu_A(v), \nu_A(v) \in [0,1]$
Edge information	A membership degree $\mu_B(uv) \in [0,1]$	Membership and non-membership degrees $\mu_B(uv), \nu_B(uv) \in [0,1]$
Vertex constraint	No separate non-membership constraint	$\dfrac{\mu_A(v)+\nu_A(v)}{q} \le 1$, where $q \ge 2$
Edge compatibility	$\mu_B(uv) \le \min\{\mu_A(u), \mu_A(v)\}$	$\mu_B(uv) \le \min\{\mu_A(u), \mu_A(v)\}$ and $\nu_B(uv) \ge \max\{\nu_A(u), \nu_A(v)\}$
Main characteristic	Represents graded membership in vertices and edges	Represents both graded membership and non-membership under a q-fractional framework

Table 3. Comparison of q-fractional fuzzy graphs and q-fractional neutrosophic graphs.

Feature	q-Fractional Fuzzy Graph	q-Fractional Neutrosophic Graph
Vertex information	Membership and non-membership degrees (μ_A, ν_A)	Truth, indeterminacy, and falsity degrees (T_A, I_A, F_A)
Edge information	Membership and non-membership degrees (μ_B, ν_B)	Truth, indeterminacy, and falsity degrees (T_B, I_B, F_B)
q-fractional constraint	$\mu + \nu \le q$	$T + I + F \le q$
Edge compatibility	$\mu_B(uv) \le \min\{\mu_A(u), \mu_A(v)\}$, $\nu_B(uv) \ge \max\{\nu_A(u), \nu_A(v)\}$	$T_B(uv) \le \min\{T_A(u), T_A(v)\}$, $I_B(uv) \le \min\{I_A(u), I_A(v)\}$, $F_B(uv) \ge \max\{F_A(u), F_A(v)\}$
Uncertainty representation	Membership and non-membership	Truth, indeterminacy, and falsity
Relationship	Recovered from a q-FNG when the indeterminacy degrees are identically zero	Generalizes the corresponding q-fractional fuzzy graph framework

on G^* is a pair

$$G_q = (A, B),$$

where:

(i) $A = (\mu_A, \nu_A)$ is a q-FFS on V (Definition 2.2), i.e., $\mu_A, \nu_A \colon V \to [0,1]$ satisfy $\frac{1}{q}\mu_A(v) + \frac{1}{q}\nu_A(v) \le 1$ for all $v \in V$;

(ii) $B = (\mu_B, \nu_B)$ assigns to each edge $uv \in E$ a pair $\big(\mu_B(uv), \nu_B(uv)\big) \in [0,1]^2$, i.e., $\mu_B, \nu_B \colon E \to [0,1]$, such that for every $uv \in E$,

$$0 \le \frac{1}{q}\mu_B(uv) + \frac{1}{q}\nu_B(uv) \le 1, \tag{2}$$

and the edge degrees are *compatible* with the endpoint vertex degrees:

$$\begin{aligned} \mu_B(uv) &\le \min\{\mu_A(u), \mu_A(v)\}, \\ \nu_B(uv) &\ge \max\{\nu_A(u), \nu_A(v)\}. \end{aligned} \tag{3}$$

Example 2.5 (A 2-fractional fuzzy graph)**.** Let

$$G^* = (V, E)$$

be the simple undirected graph with

$$V = \{v_1, v_2, v_3\}, \qquad E = \{v_1v_2, v_2v_3, v_1v_3\},$$

and let $q = 2$.
Define the vertex membership and non-membership degrees by

$$\begin{aligned} (\mu_A(v_1), \nu_A(v_1)) &= (0.8, 0.3), \\ (\mu_A(v_2), \nu_A(v_2)) &= (0.7, 0.5), \\ (\mu_A(v_3), \nu_A(v_3)) &= (0.6, 0.4). \end{aligned}$$

Then

$$\frac{\mu_A(v_1)+\nu_A(v_1)}{2} = \frac{0.8+0.3}{2} = 0.55 \le 1,$$

$$\frac{\mu_A(v_2)+\nu_A(v_2)}{2} = \frac{0.7+0.5}{2} = 0.60 \le 1,$$

and

$$\frac{\mu_A(v_3)+\nu_A(v_3)}{2} = \frac{0.6+0.4}{2} = 0.50 \le 1.$$

Hence $A = (\mu_A, \nu_A)$ is a 2-fractional fuzzy set on V.
Next, define the edge degrees by

$$\begin{aligned} (\mu_B(v_1v_2), \nu_B(v_1v_2)) &= (0.6, 0.6), \\ (\mu_B(v_2v_3), \nu_B(v_2v_3)) &= (0.5, 0.7), \\ (\mu_B(v_1v_3), \nu_B(v_1v_3)) &= (0.4, 0.5). \end{aligned}$$

The 2-fractional conditions on the edges are satisfied because

$$\frac{0.6+0.6}{2} = 0.60 \le 1,$$
$$\frac{0.5+0.7}{2} = 0.60 \le 1,$$
$$\frac{0.4+0.5}{2} = 0.45 \le 1.$$

Moreover, for the edge v_1v_2,

$$\mu_B(v_1v_2) = 0.6 \le \min\{0.8, 0.7\} = 0.7,$$

and

$$\nu_B(v_1v_2) = 0.6 \ge \max\{0.3, 0.5\} = 0.5.$$

For the edge v_2v_3,

$$\mu_B(v_2v_3) = 0.5 \le \min\{0.7, 0.6\} = 0.6,$$

and

$$\nu_B(v_2v_3) = 0.7 \ge \max\{0.5, 0.4\} = 0.5.$$

Finally, for the edge v_1v_3,

$$\mu_B(v_1v_3) = 0.4 \le \min\{0.8, 0.6\} = 0.6,$$

and

$$\nu_B(v_1v_3) = 0.5 \ge \max\{0.3, 0.4\} = 0.4.$$

Therefore,

$$G_2 = (A, B)$$

is a 2-fractional fuzzy graph.

Figure 1 illustrates this graph. Each vertex is labeled by its pair

$$(\mu_A, \nu_A),$$

whereas each edge is labeled by its pair

$$(\mu_B, \nu_B).$$

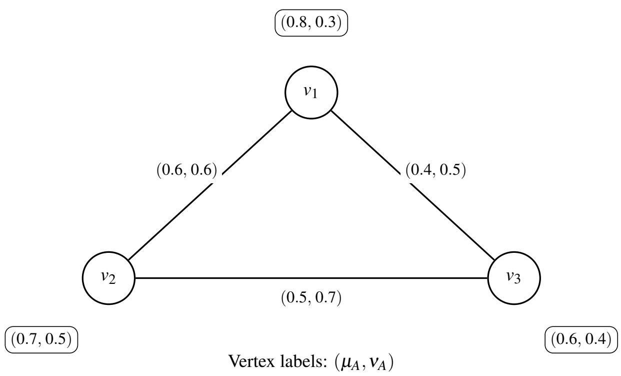

Figure 1. An example of a 2-fractional fuzzy graph.

3. RESULTS

In this section, we present the main results of this paper.

3.1 q-fractional neutrosophic sets

q-fractional neutrosophic sets assign truth, indeterminacy, and falsity degrees in $[0,1]$ per element, satisfying $T + I + F \le q$.

Definition 3.1 (q-fractional (single-valued) neutrosophic set)**.** Let X be a nonempty set and let $q \ge 2$. A *q-fractional neutrosophic set* (briefly, *q-FNS*) on X is a triple

$$N = (T_N, I_N, F_N),$$

where

$$T_N,\ I_N,\ F_N : X \to [0,1]$$

are respectively called the *truth-membership*, *indeterminacy-membership*, and *falsity-membership* functions, such that for every $x \in X$,

$$0 \le \frac{T_N(x) + I_N(x) + F_N(x)}{q} \le 1. \tag{4}$$

Equivalently, $T_N(x) + I_N(x) + F_N(x) \le q$ for all $x \in X$. We may write

$$N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle_q : x \in X\}.$$

Example 3.2 (A 2-fractional neutrosophic set for product evaluation)**.** Let

$$X = \{p_1, p_2, p_3\}$$

be a set of three products under evaluation, and let $q = 2$. Suppose that the truth-membership, indeterminacy-membership, and falsity-membership degrees are given by

Product	T_N	I_N	F_N
p_1	0.8	0.3	0.4
p_2	0.6	0.5	0.7
p_3	0.4	0.2	0.6

where $T_N(p_i)$ represents the degree to which product p_i is regarded as satisfactory, $I_N(p_i)$ represents the degree of uncertainty in the evaluation, and $F_N(p_i)$ represents the degree to which it is regarded as unsatisfactory.

For p_1, we have

$$\frac{T_N(p_1) + I_N(p_1) + F_N(p_1)}{2} = \frac{0.8 + 0.3 + 0.4}{2} = 0.75 \le 1.$$

Similarly,

$$\frac{T_N(p_2) + I_N(p_2) + F_N(p_2)}{2} = \frac{0.6 + 0.5 + 0.7}{2} = 0.90 \le 1,$$

and

$$\frac{T_N(p_3) + I_N(p_3) + F_N(p_3)}{2} = \frac{0.4 + 0.2 + 0.6}{2} = 0.60 \le 1.$$

Hence all elements satisfy the 2-fractional neutrosophic constraint.

Therefore,

$$N = \{\langle p_1, 0.8, 0.3, 0.4\rangle_2, \\ \langle p_2, 0.6, 0.5, 0.7\rangle_2, \\ \langle p_3, 0.4, 0.2, 0.6\rangle_2\}$$

is a 2-fractional neutrosophic set on X.

Remark 3.3 (Recall: single-valued neutrosophic set). A (classical) *single-valued neutrosophic set* (SVNS) on X is a triple (T, I, F) with $T, I, F : X \to [0,1]$ satisfying $0 \le T(x) + I(x) + F(x) \le 3$ for all $x \in X$.

Theorem 3.4 (Generalization of neutrosophic sets and q-fractional fuzzy sets)**.** *Let X be a nonempty set.*

(a) *A triple (T, I, F) is a single-valued neutrosophic set on X if and only if it is a 3-fractional neutrosophic set on X.*

(b) *Every q-fractional fuzzy set $A = (\mu_A, \nu_A)$ on X canoni-*

cally induces a q-fractional neutrosophic set

$$\Phi(A) := (T, I, F), \quad T := \mu_A, \quad I := 0, \quad F := \nu_A.$$

Conversely, if $N = (T_N, I_N, F_N)$ *is a q-fractional neutrosophic set with* $I_N \equiv 0$*, then*

$$\Psi(N) := (\mu, \nu) := (T_N, F_N)$$

is a q-fractional fuzzy set. Hence q-fractional fuzzy sets are exactly the $I \equiv 0$ *subclass of q-fractional neutrosophic sets.*

Proof. (a) Assume (T, I, F) is a single-valued neutrosophic set. Then for each $x \in X$,

$$0 \le T(x) + I(x) + F(x) \le 3 \implies 0 \le \frac{T(x) + I(x) + F(x)}{3} \le 1,$$

so (4) holds with $q = 3$; hence (T, I, F) is a 3-FNS.

Conversely, assume (T, I, F) is a 3-FNS. Then for each $x \in X$,

$$0 \le \frac{T(x) + I(x) + F(x)}{3} \le 1 \implies 0 \le T(x) + I(x) + F(x) \le 3,$$

so (T, I, F) is a single-valued neutrosophic set. This proves the equivalence.

(b) Let $A = (\mu_A, \nu_A)$ be a q-FFS. Then for each $x \in X$ we have $\mu_A(x), \nu_A(x) \in [0, 1]$ and $\mu_A(x) + \nu_A(x) \le q$. Define $T := \mu_A$, $I := 0$, $F := \nu_A$. Clearly $T(x), I(x), F(x) \in [0, 1]$ and

$$T(x) + I(x) + F(x) = \mu_A(x) + 0 + \nu_A(x) \le q,$$

which is equivalent to (4). Hence $\Phi(A)$ is a q-FNS.

Conversely, let $N = (T_N, I_N, F_N)$ be a q-FNS with $I_N \equiv 0$. Then $T_N, F_N : X \to [0, 1]$ and for each $x \in X$,

$$T_N(x) + F_N(x) = T_N(x) + I_N(x) + F_N(x) \le q.$$

Thus $(\mu, \nu) := (T_N, F_N)$ satisfies the q-FFS constraint, so $\Psi(N)$ is a q-FFS. Therefore the q-FFSs coincide with the subclass of q-FNSs with $I \equiv 0$.

□

3.2 q-fractional neutrosophic graph

A *q-fractional neutrosophic graph* labels vertices and edges by q-fractional neutrosophic triples, respecting endpoint min/max constraints and $T + I + F \le q$.

Definition 3.5 (q-fractional neutrosophic graph). Let $G^* = (V, E)$ be a crisp (simple undirected) graph, where $V \ne \emptyset$ and $E \subseteq \binom{V}{2}$. Fix $q \ge 2$. A *q-fractional neutrosophic graph* (briefly, *q-FNG*) on G^* is a pair

$$G_q = (A, B),$$

where

- $A = \{\langle v, T_A(v), I_A(v), F_A(v)\rangle : v \in V\}$ is a q-fractional neutrosophic set on V, i.e. $T_A, I_A, F_A : V \to [0, 1]$ satisfy

$$T_A(v) + I_A(v) + F_A(v) \le q \qquad (v \in V);$$

- $B = \{\langle uv, T_B(uv), I_B(uv), F_B(uv)\rangle : uv \in E\}$ assigns to each edge a triple in $[0, 1]^3$, i.e. $T_B, I_B, F_B : E \to [0, 1]$, such that for every $uv \in E$,

$$T_B(uv) + I_B(uv) + F_B(uv) \le q. \tag{5}$$

$$T_B(uv) \le \min\{T_A(u), T_A(v)\}, \quad I_B(uv) \le \min\{I_A(u), I_A(v)\}. \tag{6}$$

$$F_B(uv) \ge \max\{F_A(u), F_A(v)\}. \tag{7}$$

If B is symmetric (i.e. the same value is attached to uv and vu when one views E as unordered pairs), then G_q is called an *undirected q-FNG*.

Example 3.6 (A 2-fractional neutrosophic graph)**.** Let

$$G^* = (V, E)$$

be the simple undirected graph with

$$V = \{v_1, v_2, v_3\}, \qquad E = \{v_1v_2, v_2v_3, v_1v_3\},$$

and let $q = 2$.

Define the vertex degrees by

$$(T_A(v_1), I_A(v_1), F_A(v_1)) = (0.8, 0.4, 0.3),$$
$$(T_A(v_2), I_A(v_2), F_A(v_2)) = (0.7, 0.3, 0.5),$$
$$(T_A(v_3), I_A(v_3), F_A(v_3)) = (0.6, 0.5, 0.4).$$

Then

$$0.8 + 0.4 + 0.3 = 1.5 \le 2,$$

$$0.7 + 0.3 + 0.5 = 1.5 \le 2,$$

and

$$0.6 + 0.5 + 0.4 = 1.5 \le 2.$$

Hence the vertex assignment A is a 2-fractional neutrosophic set on V.

Next, define the edge degrees by

$$(T_B(v_1v_2), I_B(v_1v_2), F_B(v_1v_2)) = (0.6, 0.2, 0.6),$$
$$(T_B(v_2v_3), I_B(v_2v_3), F_B(v_2v_3)) = (0.5, 0.2, 0.7),$$
$$(T_B(v_1v_3), I_B(v_1v_3), F_B(v_1v_3)) = (0.4, 0.3, 0.5).$$

First, the edge-sum conditions are satisfied:

$$0.6 + 0.2 + 0.6 = 1.4 \le 2,$$

$$0.5 + 0.2 + 0.7 = 1.4 \le 2,$$

and

$$0.4 + 0.3 + 0.5 = 1.2 \le 2.$$

For the edge v_1v_2, we have

$$T_B(v_1v_2) = 0.6 \le \min\{0.8, 0.7\} = 0.7,$$

$$I_B(v_1v_2) = 0.2 \le \min\{0.4, 0.3\} = 0.3,$$

and

$$F_B(v_1v_2) = 0.6 \geq \max\{0.3, 0.5\} = 0.5.$$

Similarly, for v_2v_3,

$$0.5 \leq \min\{0.7, 0.6\} = 0.6, \qquad 0.2 \leq \min\{0.3, 0.5\} = 0.3,$$

and

$$0.7 \geq \max\{0.5, 0.4\} = 0.5.$$

Finally, for v_1v_3,

$$0.4 \leq \min\{0.8, 0.6\} = 0.6, \qquad 0.3 \leq \min\{0.4, 0.5\} = 0.4,$$

and

$$0.5 \geq \max\{0.3, 0.4\} = 0.4.$$

Therefore,

$$G_2 = (A, B)$$

is a 2-fractional neutrosophic graph.

Figure 2 illustrates this graph. Each vertex is labeled by its triple

$$(T_A, I_A, F_A),$$

and each edge is labeled by its triple

$$(T_B, I_B, F_B).$$

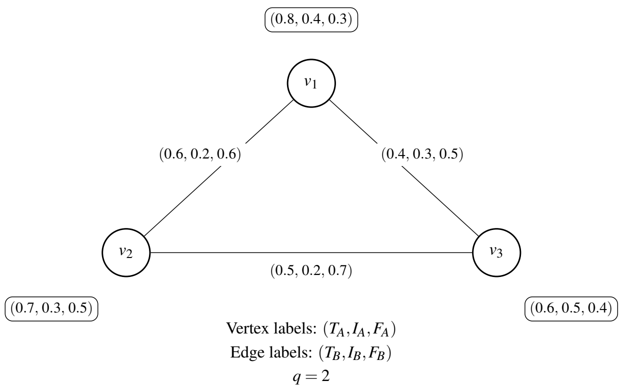

Figure 2. An example of a 2-fractional neutrosophic graph.

Theorem 3.7 (Generalization of neutrosophic graphs and q-fractional fuzzy graphs)**.** *Let $G^* = (V, E)$ be a crisp graph.*

(a) *A pair (A, B) is a (single-valued) neutrosophic graph in the usual sense (with constraints $T + I + F \leq 3$ on vertices/edges and (6)–(7)) if and only if it is a 3-fractional neutrosophic graph.*

(b) *Every q-fractional fuzzy graph $G_q^{(f)} = (\mu_V, \nu_V; \mu_E, \nu_E)$ on G^* canonically induces a q-fractional neutrosophic graph*

$$\Phi(G_q^{(f)}) = (A, B)$$

by setting, for vertices $v \in V$ and edges $e \in E$,

$$T_A(v) := \mu_V(v), \quad I_A(v) := 0, \quad F_A(v) := \nu_V(v),$$
$$T_B(e) := \mu_E(e), \quad I_B(e) := 0, \quad F_B(e) := \nu_E(e).$$

Conversely, if $G_q = (A, B)$ is a q-fractional neutrosophic graph with $I_A \equiv 0$ and $I_B \equiv 0$, then it determines a q-fractional fuzzy graph by

$$\Psi(G_q) := (\mu_V, \nu_V; \mu_E, \nu_E),$$
$$\mu_V := T_A, \quad \nu_V := F_A,$$
$$\mu_E := T_B, \quad \nu_E := F_B.$$

Hence q-fractional fuzzy graphs coincide exactly with the subclass of q-fractional neutrosophic graphs having identically zero indeterminacy on vertices and edges.

Proof. (a) Assume (A, B) is a usual (single-valued) neutrosophic graph. Then for every vertex $v \in V$, $0 \leq T_A(v) + I_A(v) + F_A(v) \leq 3$, and for every edge $e \in E$, $0 \leq T_B(e) + I_B(e) + F_B(e) \leq 3$, together with the compatibility conditions (6)–(7). Dividing the sum constraints by 3, so (A, B) is a 3-FNG.

Conversely, if (A, B) is a 3-FNG, then $0 \leq (T_A + I_A + F_A)/3 \leq 1$ and $0 \leq (T_B + I_B + F_B)/3 \leq 1$ imply $0 \leq T_A + I_A + F_A \leq 3$ and $0 \leq T_B + I_B + F_B \leq 3$, and the same compatibility inequalities hold. Hence (A, B) is a usual neutrosophic graph.

(b) Let $G_q^{(f)} = (\mu_V, \nu_V; \mu_E, \nu_E)$ be a q-fractional fuzzy graph. By definition, for all $v \in V$ and $e \in E$,

$$\mu_V(v) + \nu_V(v) \leq q, \qquad \mu_E(e) + \nu_E(e) \leq q,$$

and for each edge $uv \in E$,

$$\mu_E(uv) \leq \min\{\mu_V(u), \mu_V(v)\},$$
$$\nu_E(uv) \geq \max\{\nu_V(u), \nu_V(v)\}.$$

Define $\Phi(G_q^{(f)}) = (A, B)$ by setting $T := \mu$, $I := 0$, $F := \nu$ on both vertices and edges. Then the vertex and edge sum constraints become

$$T_A(v) + I_A(v) + F_A(v) = \mu_V(v) + 0 + \nu_V(v) \leq q,$$
$$T_B(e) + I_B(e) + F_B(e) = \mu_E(e) + 0 + \nu_E(e) \leq q,$$

and the compatibility conditions (6)–(7) reduce exactly to the fuzzy-graph inequalities above (with the I-inequality trivial). Thus $\Phi(G_q^{(f)})$ is a q-FNG.

Conversely, let $G_q = (A, B)$ be a q-FNG with $I_A \equiv 0$ and $I_B \equiv 0$. Then for each $v \in V$,

$$T_A(v) + F_A(v) = T_A(v) + I_A(v) + F_A(v) \leq q,$$

and similarly for each $e \in E$,

$$T_B(e) + F_B(e) = T_B(e) + I_B(e) + F_B(e) \leq q.$$

Moreover, (6)–(7) become

$$T_B(uv) \leq \min\{T_A(u), T_A(v)\},$$
$$F_B(uv) \geq \max\{F_A(u), F_A(v)\}.$$

Therefore $\Psi(G_q)$ satisfies exactly the axioms of a q-fractional fuzzy graph. This proves the equivalence with the $I \equiv 0$ subclass.

□

4. CONCLUSION

In this paper, we introduced *q-fractional neutrosophic sets and graphs* as natural extensions of q-fractional fuzzy sets and q-fractional fuzzy graphs, respectively. The proposed framework incorporates truth-membership, indeterminacy-membership, and falsity-membership degrees under a q-fractional constraint, thereby providing a more expressive representation of uncertain and indeterminate information. We also established fundamental relationships among these structures, showing in particular that classical single-valued neutrosophic sets and graphs arise as special cases for an appropriate choice of q, while q-fractional fuzzy sets and graphs are recovered when the indeterminacy component is identically zero.

Future work may investigate additional structural properties, operations, and graph-theoretic parameters for q-fractional neutrosophic graphs, as well as computational methods and practical applications. It would also be worthwhile to develop higher-order extensions based on hypergraphs [23, 24] and SuperHyperGraphs [25], together with corresponding algorithms and application-oriented models.

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