Neutrosophic Crisp Semi Separation Axioms In Neutrosophic Crisp Topological Spaces

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Abstract

The main goal of this paper is to propose a new type of separation axioms via neutrosophic crisp semi open sets and neutrosophic crisp points in neutrosophic crisp topological spaces, namely neutrosophic crisp semi separation axioms. Finally, we examine the relationship between them in details. And also includes the study of the connections between these neutrosophic crisp semi separation axioms and the existing neutrosophic crisp separation axioms. Moreover, many examples are presented, to illustrate the concepts introduced in this paper. and investigate their fundamental properties, relationships and characterizations.

Keywords: Neutrosophic crisp semi separation axiom, neutrosophic crisp separation axiom, neutrosophic crisp point.

1. Introduction

After F.Samarandache established the concept of neutrosophy in 1980 the neutrosophy in 1980 as a new logic which generalizes the fuzzy logic, many of the pure mathematical concepts, especially in topology, were found according to this new logic. One of the most important topological developments according to this logic is finding out and defining the neutrosophic crisp topological space [1] in 2014 by A. Salama and et al. Since the elements of the neutrosophic crisp sets[1] are neutrosophic crisp points, A. Salama defined the concept of neutrosophic crisp points [1,2] in 2014. Recently, the neutrosophic crisp set theory may have applications in image processing [3],[4], the field of geographic information systems[5] and possible applications to database[6]. Also, neutrosophic sets [7] have applications in the medical field [8], [9], [10], [11]. We can't use neutrosophic crisp points were defined in [1,2] for defining the neutrosophic crisp separation axioms and this encouraged A. Al-nafey, R. Al-Hamido and F. Smarandache to think of presenting another new concept of the neutrosophic crisp points [12] in 2018, which enabled them to define separation axioms in the neutrosophic crisp space for the first time in [12] . Moreover, neutrosophic crisp semi open sets were first defined and investigated by A. Salama [5] in 2015. Since the separation

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axioms are considered one of the very important useful topics and one of the newly studied in topology, we thought of developing and generalizing the neutrosophic crisp separation axioms to neutrosophic crisp semi separation axioms. Finally, we study the relations between them on the one hand and between the separation neutrosophic crisps in [12] on the other hand. Many researchers studied topology, and they had many contributions to neutrosophic toplogy as [13], [14], [15], [16] and [17] and in neutrosophic bitopology in [18], [19], [20] and [21], and in neutrosophic algebra in [22], [23], [24], [25] and [26].

In this paper, Section 2 focuses on the related definitions. Section 3 presents new separation axioms in the neutrosophic crisp topological spaces, the relationship among these new separation axioms and the neutrosophic crisp separation axioms is determined.

2. Preliminaries

Throughout the paper, (χ, Υ) means neutrosophic crisp topological space (N_cTS).

N_c.OS (N_c.CS) means a neutrosophic crisp open(closed) sets and N_cS.OS means a neutrosophic crisp semi open set in N_cTS.
Now, we recall some definitions which are useful in this paper.

Definition 2.1. [1] Let $X \neq \emptyset$ be a fixed set. A neutrosophic crisp set (N_c.S) U is an object with the $U=< U_1, U_2, U_3>$ shape ; U_1, U_2 *and* $U_3$are subsets of X.

Definition 2.2. [1]

$\emptyset_N$ can be defined in four ways, as below :

1. $\emptyset_N =<\emptyset, \emptyset, \emptyset>$.
2. $\emptyset_N =<\emptyset, X, \emptyset>$.
3. $\emptyset_N =<\emptyset, X, X>$.
4. $\emptyset_N =<\emptyset, \emptyset, \emptyset>$.

X_N can be defined in four ways, as below :

1. $X_N =<X, \emptyset, \emptyset>$.
2. $X_N =<X, X, \emptyset>$.
3. $\emptyset_N =<X, \emptyset, X>$.
4. $X_N =<X, X, X>$.

Definition 2.3. [1]

Let $\chi \neq \emptyset$ be a fixed set, and $U =< U_1, U_2, U_3 >, V =< V_1, V_2, V_3 >$ are two neutrosophic crisp sets, then:

$U \cup V$ can be defined as two ways, as below :

1. $U \cup V =< U_1 \cup V_1, U_2 \cup V_2, U_3 \cap V_3 >$.
2. $U \cup V =< U_1 \cup V_1, U_2 \cap V_2, U_3 \cap V_3 >$.

$U \cap V$ can be defined as two ways, as below :

1. $U \cap V =< U_1 \cap V_1, U_2 \cap V_2, U_3 \cup V_3 >$.
2. $U \cap V =< U_1 \cap V_1, U_2 \cup V_2, U_3 \cup V_3 >$.

Definition 2.4. [1]

A neutrosophic crisp topology (NCT) on a non-empty set χ is a family Υ of neutrosophic crisp subsets in χ may be satisiying the following axioms:

1. X_N and $\emptyset_N$ belong to Υ.

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2. T is closed under finite intersection.

3. T is closed under arbitrary union.

The pair $(\chi,Ţ)$is neutrosophic crisp topological space (NCTS) in Ţ. Moreover, the elements in Ţ are said to be neutrosophic crisp open sets (NCOS). A neutrosophic crisp set F is closed (NCCS) if and only if its complement F^c is neutrosophic crisp open set.

Definition 2.5. [12]

Let χ be a non-empty set. And $x,y,z\in \chi$, then:

a. $x_{N_1}=<\{x\},\emptyset,\emptyset>$ is called a neutrosophic crisp point (NCP_{N_1}) in χ .

b. $y_{N_2}=<\emptyset,\{y\},\emptyset>$ is called a neutrosophic crisp point (NCP_{N_2}) in χ.

c. $z_{N_3}=<\emptyset,\emptyset,\{z\}>$ is called a neutrosophic crisp point (NCP_{N_3}) in χ .

The set of all neutrosophic crisp points ($NCP_{N_1}, NCP_{N_2}, NCP_{N_3}$) is denoted by NCP_N.

Definition 2.6. [12]

Let $(\chi,Ţ)$ be an N_cTS. Then χ is called:

a. $N_1Ţ_o$-space if for every $x_{N_1}\neq y_{N_1}\in\chi$ there exists N_c.OS M in χ containing one of them but not the other.

b. $N_2Ţ_o$-space if $\forall\ x_{N_2}\neq y_{N_2}\in\mathcal{X}$ there exists N_c.OS M in χ containing one of them but not the other.

c. $N_3Ţ_o$-space if $\forall\ x_{N_3}\neq y_{N_3}\in\chi$ there exists N_c.OS M in χ containing one of them but not the other.

d. $N_1Ţ_1$-space if for every $x_{N_1}\neq y_{N_1}\in\chi$ there exists N_c.OS M_1, M_2 in χ such that $x_{N_1}\in M_1$, $y_{N_1}\notin M_1$ and $x_{N_1}\notin M_2$, $y_{N_1}\in M_2$.

e. $N_2Ţ_1$-space if $\forall\ x_{N_2}\neq y_{N_2}\in\chi$ there exist N_c.OS M_1, M_2 in χ such that $x_{N_2}\in G_1$, $y_{N_2}\notin G_1$ and $x_{N_2}\notin G_2$, $y_{N_2}\in G_2$.

f. $N_3Ţ_1$-space if $\forall\ x_{N_3}\neq y_{N_3}\in\chi$ there exist N_c.OS M_1, M_2 in χ such that $x_{N_3}\in M_1$, $y_{N_3}\notin M_1$ and $x_{N_3}\notin M_2$, $y_{N_3}\in M_2$.

g. $N_1Ţ_2$-space if for every $x_{N_3}\neq y_{N_3}\in\chi$ there exist N_c.OS M_1, M_2 in χ such that $x_{N_1}\in M_1$, $y_{N_1}\notin M_1$ and $x_{N_1}\notin M_2$, $y_{N_1}\in M_2$. with $M_1\cap M_2=\emptyset$.

h. $N_2Ţ_2$-space if $\forall\ x_{N_2}\neq y_{N_2}\in\chi$ there exist N_c.OS M_1, M_2 in χ such that $x_{N_2}\in M_1$, $y_{N_2}\notin M_1$ and $x_{N_2}\notin M_2$, $y_{N_2}\in M_2$ with $M_1\cap M_2=\emptyset$.

i. $N_3Ţ_2$-space if $\forall\ x_{N_3}\neq y_{N_3}\in\chi$ there exist N_c.OS M_1, M_2 in χ such that $x_{N_3}\in M_1$, $y_{N_3}\notin M_1$ and $x_{N_3}\notin M_2$, $y_{N_3}\in M_2$ with $M_1\cap M_2=\emptyset$.

3. Separation axioms in a neutrosophic crisp topological space

Definition 3.1.

Let $(\chi,Ţ)$ be an NcTS. Then χ is called:

a. N_1semiŢo-space if for every $x_{N_1}\neq y_{N_1}\in\chi$ there exists NcS.OS M in χ containing one of them but not the other.

b. N_2semiŢo-space if $\forall\ x_{N_2}\neq y_{N_2}\in\mathcal{X}$ there exists NcS.OS M in χ containing one of them but not the other.

c. N_3semiŢo-space if $\forall\ x_{N_3}\neq y_{N_3}\in\chi$ there exists NcS.OS M in χ containing one of them but not the other.

Definition 3.2.

Let $(\chi,Ţ)$ be an N_cTS. Then χ is called:

a. N_1semi$Ţ_1$-space if for every $x_{N_1}\neq y_{N_1}\in\chi$ there exist N_cS.OS M_1, M_2 in χ such that $x_{N_1}\in M_1$, $y_{N_1}\notin M_1$ and $x_{N_1}\notin M_2$, $y_{N_1}\in M_2$.

b. N_2semi$Ţ_1$-space if $\forall\ x_{N_2}\neq y_{N_2}\in\chi$ there exist N_cS.OS M_1, M_2 in χ such that $x_{N_2}\in G_1$, $y_{N_2}\notin G_1$ and $x_{N_2}\notin G_2$, $y_{N_2}\in G_2$

c. N_3semi$Ţ_1$-space if $\forall\ x_{N_3}\neq y_{N_3}\in\chi$ there exist N_cS.OS M_1, M_2 in χ such that $x_{N_3}\in M_1$, $y_{N_3}\notin M_1$ and $x_{N_3}\notin M_2$, $y_{N_3}\in M_2$

Definition 3.3.

Let $(\chi,Ţ)$ be an N_cTS. Then χ is called:

a. N_1semi$Ţ_2$-space if for every $x_{N_1}\neq y_{N_1}\in\chi$ there exists N_cS.OS M_1, M_2 in χ such that $x_{N_1}\in M_1$, $y_{N_1}\notin M_1$ and $x_{N_1}\notin M_2$, $y_{N_1}\in M_2$. with $M_1\cap M_2=\emptyset$.

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b. N_2semiT_2-space if $\forall$ $x_{N_2} \neq y_{N_2} \in \chi$ there exists N_cS.OS M_1, M_2 in χ such that $x_{N_2} \in M_1$, $y_{N_2} \notin M_1$ and $x_{N_2} \notin M_2$, $y_{N_2} \in M_2$ with $M_1 \cap M_2 = \emptyset$.

c. N_3semiT_2-space if $\forall$ $x_{N_3} \neq y_{N_3} \in \chi$ there exists N_cS.OS M_1, M_2 in χ such that $x_{N_3} \in M_1$, $y_{N_3} \notin M_1$ and $x_{N_3} \notin M_2$, $y_{N_3} \in M_2$ with $M_1 \cap M_2 = \emptyset$.

Theorem 3.4.

Let (χ,T) be an N_cTS, then :

1. Every $\mathrm{N}_1\mathrm{T}_0$-space is N_1semiT_0-space.
2. Every $\mathrm{N}_2\mathrm{T}_0$-space is N_2semiT_0-space.
3. Every $\mathrm{N}_3\mathrm{T}_0$-space is N_3semiT_0-space.

Proof:

1. Suppose that (χ,T) is an $\mathrm{N}_1\mathrm{T}_0$-space , therefore for every two $x_{N_1} \neq y_{N_1}$, there exists an N_c.OS M in χ containing one of them to which the other does not belong. So there exists an N_cS.OS M in χ containing one of them to which the other does not belong, therefore X is N_1semiT_0-space.
2. Similar to Proof 1.
3. Similar to Proof 1.

Remark 3.5.

The converse of theorem 3.4 is not true., as it is shown in the following examples.

Example 3.6.

Let $\chi = \{a,b,c\}, \mathrm{T} = \{\emptyset_N, X_N, A\}, A = \{<\{a\},\emptyset,\emptyset>\}$.
$N_cS.OS = \mathrm{T} \cup \{C = \{<\{a,b\},\emptyset,\emptyset>\}, B = \{<\{a,c\},\emptyset,\emptyset>\}$.
Let $x_{N_1} = \{<\{b\},\emptyset,\emptyset>\} \neq y_{N_1} = \{<\{c\},\emptyset,\emptyset>\} \in \chi$ there is no a N_c.OS M in χ containing one of them but not the other. Therefore (χ,T) is not $\mathrm{N}_1\mathrm{T}_0$-space.
Then (χ,T) $\mathrm{N}_1 semi\mathrm{T}_0$-space, But (χ,T) is not $\mathrm{N}_1\mathrm{T}_0$-space.

Example 3.7.

Let $\chi = \{a,b,c\}, \mathrm{T} = \{\emptyset_N, X_N, A\}, A = \{<\emptyset,\{a\},\emptyset>\}$.
$N_cS.OS = \mathrm{T} \cup \{C = \{<\emptyset,\{a,b\},\emptyset>\}, B = \{<\emptyset,\{a,c\},\emptyset>\}$.
Let $x_{N_2} = \{<\emptyset,\{b\},\emptyset>\} \neq y_{N_2} = \{<\emptyset,\{c\},\emptyset>\} \in \chi$ there is no a N_c.OS M in χ containing one of them but not the other. Therefore (χ,T) is not $\mathrm{N}_2\mathrm{T}_0$-space.
Then (χ,T) $\mathrm{N}_2 semi\mathrm{T}_0$-space, But (χ,T) is not $\mathrm{N}_2\mathrm{T}_0$-space.

Example 3.8.

Let $\chi = \{a,b,c\}, \mathrm{T} = \{\emptyset_N, X_N, A\}, A = \{<\emptyset,\{a\},\emptyset>\}$.
$N_cS.OS = \mathrm{T} \cup \{C = \{<\emptyset,\emptyset,\{a,b\}>\}, B = \{<\emptyset,\emptyset,\{a,c\},>\}$.
Let $x_{N_3} = \{<\emptyset,\emptyset,\{b\}>\} \neq y_{N_3} = \{<\emptyset,\emptyset,\{c\}>\} \in \chi$ there is no a N_c.OS M in χ containing one of them but not the other. Therefore (χ,T) is not $\mathrm{N}_3\mathrm{T}_0$-space.
Then (χ,T) $\mathrm{N}_3 semi\mathrm{T}_0$-space, But (χ,T) is not $\mathrm{N}_3\mathrm{T}_0$-space.

Theorem 3.9.

Let (χ,T) be an N_cTS, then :

1. Every $\mathrm{N}_1\mathrm{T}_1$-space is N_1semiT_1-space.
2. Every $\mathrm{N}_2\mathrm{T}_1$-space is N_2semiT_1-space.
3. Every $\mathrm{N}_3\mathrm{T}_1$-space is N_3semiT_1-space.

Proof:

1. Suppose that (χ,T) is an $\mathrm{N}_1\mathrm{T}_1$-space , therefore for every two $x_{N_1} \neq y_{N_1}$, there exist an N_c.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$. So there exists an N_cS.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$.
   Therefore X is N_1semiT_1-space.
2. Similar to Proof 1.
3. Similar to Proof 1.

Remark 3.10.

The converse of a theorem 3.9 is not true, as it is shown in the following example.

Example 3.11.

Let $\chi = \{a,b,c\}, \mathrm{T} = \{\emptyset_N, X_N, A, B, C\}, A = \{<\{a\},\emptyset,\emptyset>\}, B = \{<\{b\},\emptyset,\emptyset>\}, C = \{<\{a,b\},\emptyset,\emptyset>\}$.

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$N_cS.OS = Ţ \cup \{G = \{< \{a, c\}, \emptyset, \emptyset >\}, H = \{< \{b, c\}, \emptyset, \emptyset >\}$.
Let $x_{N_1} = \{< \{b\}, \emptyset, \emptyset >\} \neq y_{N_1} = \{< \{c\}, \emptyset, \emptyset >\} \in \chi$ there is no N_c.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$. Therefore $(\chi, Ţ)$ is not $Ɲ_1Ţ_1$-space.

Then $(\chi, Ţ)$ $Ɲ_1 semiŢ_1$-space, But $(\chi, Ţ)$ is not $Ɲ_1Ţ_1$-space.
Also $(\chi, Ţ)$ $Ɲ_1 semiŢ_2$-space, But $(\chi, Ţ)$ is not $Ɲ_1Ţ_2$-space.

Example 3.12.
Let $\chi = \{a, b, c\}, Ţ = \{\emptyset_N, X_N, A, B, C\}, A = \{< \emptyset, \{a\}, \emptyset >\}, B = \{< \emptyset, \{b\}, \emptyset >\}, C = \{< \emptyset, \{a, b\}, \emptyset >\}$.
$N_cS.OS = Ţ \cup \{G = \{< \emptyset, \{a, c\}, \emptyset >\}, H = \{< \emptyset, \{b, c\}, \emptyset >\}$.
Let $x_{N_1} = \{< \emptyset, \{b\}, \emptyset >\} \neq y_{N_1} = \{\emptyset, < \{c\}, \emptyset >\} \in \chi$ there are no a N_c.OS M_1, M_2 in χ such that $x_{N_2} \in M_1$, $y_{N_2} \notin M_1$ and $x_{N_2} \notin M_2$, $y_{N_2} \in M_2$. Therefore $(\chi, Ţ)$ is not $Ɲ_2Ţ_1$-space.

Then $(\chi, Ţ)$ $Ɲ_2 semiŢ_1$-space, But $(\chi, Ţ)$ is not $Ɲ_2Ţ_1$-space.

Also $(\chi, Ţ)$ $Ɲ_2 semiŢ_2$-space, But $(\chi, Ţ)$ is not $Ɲ_2Ţ_2$-space.

Example 3.13.
Let $\chi = \{a, b, c\}, Ţ = \{\emptyset_N, X_N, A, B, C\}, A = \{< \emptyset, \emptyset, \{a\} >\}, B = \{< \emptyset, \emptyset, \{b\} >\}, C = \{< \emptyset, \emptyset, \{a, b\} >\}$.
$N_cS.OS = Ţ \cup \{G = \{< \emptyset, \emptyset, \{a, c\} >\}, H = \{< \emptyset, \emptyset, \{b, c\} >\}$.
Let $x_{N_3} = \{< \emptyset, \emptyset, \{b\} >\} \neq y_{N_3} = \{< \emptyset, \emptyset, \{c\} >\} \in \chi$ there are no N_c.OS M_1, M_2 in χ such that $x_{N_3} \in M_1$, $y_{N_3} \notin M_1$ and $x_{N_3} \notin M_2$, $y_{N_3} \in M_2$. Therefore $(\chi, Ţ)$ is not $Ɲ_3Ţ_1$-space.

Then $(\chi, Ţ)$ $Ɲ_3 semiŢ_1$-space, But $(\chi, Ţ)$ is not $Ɲ_3Ţ_1$-space.
Also $(\chi, Ţ)$ $Ɲ_3 semiŢ_2$-space, But $(\chi, Ţ)$ is not $Ɲ_3Ţ_2$-space.

Theorem 3.14.

Let $(\chi, Ţ)$ be an N_cTS, then :

1. Every $Ɲ_1Ţ_2$-space is $Ɲ_1$semi$Ţ_2$-space.
2. Every $Ɲ_2Ţ_2$-space is $Ɲ_2$semi$Ţ_2$-space.
3. Every $Ɲ_3Ţ_2$-space is $Ɲ_3$semi$Ţ_2$-space.

Proof:

1. Suppose that $(\chi, Ţ)$ is an $Ɲ_1Ţ_2$-space , therefore for every two $x_{N_1} \neq y_{N_1}$, there exists an N_c.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$. with $M_1 \cap M_2 = \emptyset$. So there exists N_cS.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$. with $M_1 \cap M_2 = \emptyset$.
   Therefore X is $Ɲ_1$semi$Ţ_2$-space.
2. Similar to Proof 1.
3. Similar to Proof 1.

Remark 3.15.
The converse of the Theorem 3.14 is not true., as it is shown in the following example.
Example 3.16.
Let $\chi = \{a, b, c\}, Ţ = \{\emptyset_N, X_N, A, B, C\}, A = \{< \{a\}, \emptyset, \emptyset >\}, B = \{< \{b\}, \emptyset, \emptyset >\}, C = \{< \{a, b\}, \emptyset, \emptyset >\}$.
$N_cS.OS = Ţ \cup \{G = \{< \{a, c\}, \emptyset, \emptyset >\}, H = \{< \{b, c\}, \emptyset, \emptyset >\}$.
Let $x_{N_1} = \{< \{b\}, \emptyset, \emptyset >\} \neq y_{N_1} = \{< \{c\}, \emptyset, \emptyset >\} \in \chi$ there are no a N_c.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$. Therefore $(\chi, Ţ)$ is not $Ɲ_1Ţ_1$-space.

Then $(\chi, Ţ)$ $Ɲ_1 semiŢ_1$-space, but $(\chi, Ţ)$ is not $Ɲ_1Ţ_1$-space.
Also $(\chi, Ţ)$ $Ɲ_1 semiŢ_2$-space, but $(\chi, Ţ)$ is not $Ɲ_1Ţ_2$-space.

Example 3.17.
Let $\chi = \{a, b, c\}, Ţ = \{\emptyset_N, X_N, A, B, C\}, A = \{< \emptyset, \{a\}, \emptyset >\}, B = \{< \emptyset, \{b\}, \emptyset >\}, C = \{< \emptyset, \{a, b\}, \emptyset >\}$.
$N_cS.OS = Ţ \cup \{G = \{< \emptyset, \{a, c\}, \emptyset >\}, H = \{< \emptyset, \{b, c\}, \emptyset >\}$.

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Let $x_{N_1} = \{<\emptyset, \{b\}, \emptyset>\} \neq y_{N_1} = \{\emptyset, <\{c\}, \emptyset>\} \in \chi$ there are no N_c.OS M_1, M_2 in χ such that $x_{N_2} \in M_1$, $y_{N_2} \notin M_1$ and $x_{N_2} \notin M_2$, $y_{N_2} \in M_2$. Therefore $(\chi,Ƭ)$ is not $Ɲ_2Ƭ_1$-space.

Then $(\chi,Ƭ)$ $Ɲ_2 semiƬ_1$-space, but $(\chi,Ƭ)$ is not $Ɲ_2Ƭ_1$-space.

Also $(\chi,Ƭ)$ $Ɲ_2 semiƬ_2$-space, but $(\chi,Ƭ)$ is not $Ɲ_2Ƭ_2$-space.

Example 3.18.
Let $\chi = \{a,b,c\}, Ƭ = \{\emptyset_N, X_N, A, B, C\}, A = \{<\emptyset, \emptyset, \{a\}>\}, B = \{<\emptyset, \emptyset, \{b\}>\}, C = \{<\emptyset, \emptyset, \{a,b\}>\}$.
$N_cS.OS = Ƭ \cup \{G = \{<\emptyset, \emptyset, \{a,c\}>\}, H = \{<\emptyset, \emptyset, \{b,c\}>\}$.
Let $x_{N_3} = \{<\emptyset, \emptyset, \{b\}>\} \neq y_{N_3} = \{<\emptyset, \emptyset, \{c\}>\} \in \chi$ there are no N_c.OS M_1, M_2 in χ such that $x_{N_3} \in M_1$, $y_{N_3} \notin M_1$ and $x_{N_3} \notin M_2$, $y_{N_3} \in M_2$. Therefore $(\chi,Ƭ)$ is not $Ɲ_3Ƭ_1$-space.

Then $(\chi,Ƭ)$ $Ɲ_3 semiƬ_1$-space, But $(\chi,Ƭ)$ is not $Ɲ_3Ƭ_1$-space.
Also $(\chi,Ƭ)$ $Ɲ_3 semiƬ_2$-space, But $(\chi,Ƭ)$ is not $Ɲ_3Ƭ_2$-space.

Theorem 3.19.

Let $(\chi,Ƭ)$ be an N_cTS, then :

1. $Ɲ_1$semi$Ƭ_2$-space $\Rightarrow$ $Ɲ_1$semi$Ƭ_1$-space $\Rightarrow$ $Ɲ_1$semi$Ƭ_0$-space.
2. $Ɲ_2$semi$Ƭ_2$-space $\Rightarrow$ $Ɲ_2$semi$Ƭ_1$-space $\Rightarrow$ $Ɲ_2$semi$Ƭ_0$-space.
3. $Ɲ_3$semi$Ƭ_2$-space $\Rightarrow$ $Ɲ_3$semi$Ƭ_1$-space $\Rightarrow$ $Ɲ_3$semi$Ƭ_0$-space.

The converse of the Theorem 3.19 is not true.

Example 3.20.
Let $X = \{a,b,c\}, S^{NC} = \{\emptyset_N, X_N, A\}, A = \{<\{a\}, \emptyset, \emptyset>\}$.
$N_cS.OS = \{A = \{<\{a\}, \emptyset, \emptyset>\}, C = \{<\{a,b\}, \emptyset, \emptyset>\}, B = \{<\{a,c\}, \emptyset, \emptyset>\}$.
Then $(\chi,Ƭ)$ $Ɲ_1 semiƬ_0$-space, but $(\chi,Ƭ)$ is not $Ɲ_1 semiƬ_1$-space. So $(\chi,Ƭ)$ $Ɲ_1 semiƬ_0$-space, But $(\chi,Ƭ)$ is not $Ɲ_1 semiƬ_2$-space.

Remark. 3.21.
Relations among the different types of neutrosophic crisp separation axioms which were studied in this paper, appear in the following diagram.

$$\begin{array}{ccccc}
\text{Ni-T}_2\text{-space} & \Rightarrow & \text{Ni-T}_1\text{-space} & \Rightarrow & \text{Ni-T}_o\text{-space.} \\
 & \nLeftarrow & & \nLeftarrow & \\
\Downarrow \not\Uparrow & & \Downarrow \not\Uparrow & & \Downarrow \not\Uparrow \\
\text{Ni-ST}_2\text{-space} & \Rightarrow & \text{Ni-ST}_1\text{-space} & \Rightarrow & \text{Ni-ST}_o\text{-space.} \\
 & \nLeftarrow & & \nLeftarrow & \\
 & & & \text{i=0,1,2.} &
\end{array}$$

Definition 3.22.
An N_cTS $(\chi,Ƭ)$ is called:
a. $Ɲ semiƬ_0$-space if $(\mathcal{X},\mathcal{T})$ is $Ɲ_1 semiƬ_0$-space and $Ɲ_2 semiƬ_0$-space and $Ɲ_3 semiƬ_0$-space.
b. $Ɲ semiƬ_1$-space if $(\mathcal{X},\mathcal{T})$ is $Ɲ_1 semiƬ_1$-space and $Ɲ_2 semiƬ_1$-space and $Ɲ_3 semiƬ_1$-space.
c. $Ɲ semiƬ_2$-space if $(\mathcal{X},\mathcal{T})$ is $Ɲ_1 semiƬ_2$-space and $Ɲ_2 semiƬ_2$-space and $Ɲ_3 semiƬ_2$-space.

Remark 3.23.
For an N_cTS $(\chi,Ƭ)$
1. Every $Ɲ semiƬ_0$-space if $(\mathcal{X},\mathcal{T})$ is $Ɲ_1 semiƬ_0$-space.

2. Every $\mathrm{N}semi\mathrm{T}_0$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_2semi\mathrm{T}_0$-space.
3. Every $\mathrm{N}semi\mathrm{T}_0$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_3semi\mathrm{T}_0$-space.
The converse is not true as it is shown in the below example.

Example 3.24.
Assume that $\mathrm{T} = \{x, y\}$, $\mathrm{T}_1 = \{\mathcal{X}_N, \emptyset_N, A\}$,$\mathrm{T}_2 = \{\mathcal{X}_N, \emptyset_N, B\}$, $\mathrm{T}_3 = \{\mathcal{X}_N, \emptyset_N, G\}$.
A =<{x},∅,∅>
B =<∅,{y},∅>
G =<∅,∅,{x}>
Then: (χ,Ţ) is $\mathrm{N}_1semi\mathrm{T}_0$-space but not $\mathrm{N}semi\mathrm{T}_0$-space.
(χ,Ţ) is $\mathrm{N}_2semi\mathrm{T}_0$-space but not $\mathrm{N}semi\mathrm{T}_0$-space.
(χ,Ţ) is $\mathrm{N}_3semi\mathrm{T}_0$-space but not $\mathrm{N}semi\mathrm{T}_0$-space.

Remark 3.25.
For an N_cTS (χ,Ţ)
1. Every $\mathrm{N}semi\mathrm{T}_1$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_1semi\mathrm{T}_1$-space.
2. Every $\mathrm{N}semi\mathrm{T}_1$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_2semi\mathrm{T}_1$-space.
3. Every $\mathrm{N}semi\mathrm{T}_1$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_3semi\mathrm{T}_1$-space.
The converse is not true as it is shown in the following example.

Example 3.26.
Assume that = {x, y} ,$\mathcal{T}_1 = \{\mathcal{X}_N , \emptyset_N , A, B\}$, $\mathcal{T}_2 = \{\mathcal{X}_N, \emptyset_N, G , F\}$.
A =<{x},{y},∅>
B =<{y},{x},∅>
G =<∅,∅,{x}>
F =<∅,∅,{y}>
Then: (χ,Ţ₁)is $\mathrm{N}_1semi\mathrm{T}_1$-space but not $\mathrm{N}semi\mathrm{T}_1$.
(χ,T_1)is $\mathrm{N}_2semi\mathrm{T}_1$-space but not $\mathrm{N}semi\mathrm{T}_1$.
(χ,T_2)is $\mathrm{N}_3semi\mathrm{T}_1$-space but not $\mathrm{N}semi\mathrm{T}_1$.

Remark 3.27.
For an N_cTS (χ,Ţ)
1. Every $\mathrm{N}semi\mathrm{T}_2$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_1semi\mathrm{T}_2$-space.
2. Every $\mathrm{N}semi\mathrm{T}_2$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_2semi\mathrm{T}_2$-space.
3. Every $\mathrm{N}semi\mathrm{T}_2$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_3semi\mathrm{T}_2$-space.
The converse is not true as it is shown in the example.

Remark 3.28.
For an neutrosophic crisp topological space $(\mathcal{X},\mathcal{T})$
1. Every $\mathrm{N}semi\mathrm{T}_1$-space but not $\mathrm{N}semi\mathrm{T}_0$-space.
2. Every $\mathrm{N}semi\mathrm{T}_2$-space but not $\mathrm{N}semi\mathrm{T}_1$-space.
The converse is not true as it is shown in the following example :

Example 3.29.
Assume that = {x, y} , $\mathcal{T} = \{\mathcal{X}_N , \emptyset_N , A, B, G\}$.

A =<{x},∅,∅>
B =<∅,{y},∅>
G =<∅,∅,{x}>
Then: $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}semi\mathrm{T}_0$-space but not $\mathrm{N}semi\mathrm{T}_1$-space.

Conclusion

In this paper, we have defined a new type of neutrosophic crisp separation axioms by using neutrosophic crisp semi open sets and certain point in the neutrosophic crisp topological spaces. Moreover,we study the connections between neutrosophic crisp semi separation axioms and the existing neutrosophic crisp separation axioms. And

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many examples are presented, to illustrate the concepts introduced in this paper. Also, investigate their fundamental properties and characterizations.

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