A Short Note on Interval-Valued Bipolar Fuzzy SuperHyperGraphs

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Abstract

Hypergraphs extend classical graphs by allowing *hyperedges* to connect arbitrary nonempty subsets of vertices, thereby capturing higher-order, group-level interactions. Superhypergraphs further broaden this setting by iterating the powerset construction, which yields layered supervertices and supports multi-level relational structure. An interval-valued bipolar fuzzy graph assigns positive and negative membership intervals to vertices and edges while satisfying bipolar consistency constraints. In this paper, we extend interval-valued bipolar fuzzy graphs to the settings of hypergraphs and superhypergraphs.

Keywords: SuperHyperGraph; HyperGraph; Fuzzy SuperHyperGraph; Interval-valued bipolar fuzzy graph

1 Preliminaries

This section establishes notation and summarizes the foundational objects used in the sequel. Unless explicitly stated, all graphs and hypergraphs considered in this paper are finite.

1.1 Hypergraphs and SuperHyperGraphs

A classical graph represents a system by a vertex set together with *binary* edges, and this abstraction is often sufficient when interactions are naturally pairwise.[1] In many applications, however, a single relation may involve three or more entities simultaneously. Hypergraphs capture such higher-order interactions by allowing an edge—called a *hyperedge*—to be any nonempty subset of the vertex set.[2–5]

Beyond higher-order relations, many real systems exhibit *hierarchical* or *multi-level* organization. To model such nested structure, SuperHyperGraphs extend the hypergraph formalism by iterating the powerset operator: vertices may themselves be set-valued objects (supervertices) drawn from higher powerset layers, and edges describe incidence among these higher-level objects.[6] We therefore recall the required set-theoretic operators and then state the incidence-form definition used here.

Definition 1.1 (Base set)**.** A *base set* is a nonempty set S selected as the primitive domain from which the subsequent set-valued constructions (such as powersets and iterated powersets) are generated.

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Definition 1.2 (Powerset)**.** (cf.[7]) For any set S, the *powerset* of S is

$$\mathcal{P}(S) \;=\; \{\, A \mid A \subseteq S \,\},$$

the family of all subsets of S (including $\emptyset$ and S itself).

Definition 1.3 (n-th powerset and nonempty n-th powerset)**.** [8] Let H be a set. The iterated powersets are defined recursively by

$$\mathcal{P}^0(H) = H, \qquad \mathcal{P}^{k+1}(H) = \mathcal{P}\big(\mathcal{P}^k(H)\big) \quad (k \geq 0).$$

Their *nonempty* counterparts are defined by

$$\mathcal{P}^{*0}(H) = H, \qquad \mathcal{P}^{*(k+1)}(H) = \mathcal{P}^*\big(\mathcal{P}^{*k}(H)\big) \quad (k \geq 0),$$

where $\mathcal{P}^*(X) = \mathcal{P}(X) \setminus \{\emptyset\}$.

Definition 1.4 (Hypergraph)**.** [5,9] A (finite) *hypergraph* is a pair $H = (V, E)$, where V is a nonempty finite set of vertices and E is a finite family of hyperedges such that each $e \in E$ satisfies $e \subseteq V$ and $e \neq \emptyset$.

Definition 1.5 (Level-n SuperHyperGraph (incidence form))**.** (cf.[6]) Fix a finite base set V_0 and an integer $n \geq 0$. Let $V_n \subseteq \mathcal{P}^n(V_0)$ be a finite set; its elements are called *n-supervertices*. A *level-n SuperHyperGraph* is a pair

$$\mathrm{SHG}^{(n)} = \big(V_n, \mathcal{E}\big), \qquad \emptyset \neq \mathcal{E} \subseteq \mathcal{P}(V_n) \setminus \{\emptyset\}.$$

Thus each *n-superedge* $E \in \mathcal{E}$ is a nonempty subset of the vertex set V_n. When $n = 0$, this construction coincides with an ordinary finite hypergraph; if, in addition, $|E| = 2$ holds for all $E \in \mathcal{E}$, it reduces to a graph.

Remark 1.6 (Incidence form and scope)**.** In this paper, we adopt the *incidence form* of an n-SuperHyperGraph. Accordingly, we do *not* encode or track the internal hierarchical *type* information of supervertices (e.g., their precise level-wise construction in $\mathcal{P}^k(V_0)$). Instead, we treat each supervertex as a set-valued vertex object and focus on *higher-order relations* represented by hyperedges among such set-valued vertices.

1.2 Fuzzy n-SuperHyperGraphs

A fuzzy set assigns to each element of a universe a membership degree in $[0, 1]$.[10] Fuzzy graphs and fuzzy hypergraphs incorporate such grades on vertices and (hyper)edges to represent uncertainty, partial participation, or graded affinity.[11–14] Following the same philosophy, a fuzzy n-SuperHyperGraph equips supervertices and superedges with membership functions, thereby enabling graded modeling of higher-order relations among set-valued (layered) objects (cf.[15]).

Definition 1.7 (Fuzzy n-SuperHyperGraph)**.** [6] Let $\mathrm{SHG}^{(n)} = (V, E)$ be an n-SuperHyperGraph whose superedge family contains no empty edge, i.e.,

$$E \subseteq \mathcal{P}^*(V) \;\; \mathcal{P}(V) \setminus \{\emptyset\}.$$

A *fuzzy n-SuperHyperGraph* is a quadruple

$$\big(V, E, \sigma, \mu\big),$$

where $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ satisfy the *appurtenance constraint*

$$\mu(e) \;\leq\; \min_{v \in e} \sigma(v) \qquad \text{for every } e \in E.$$

Remark 1.8. The condition $E \subseteq \mathcal{P}^*(V)$ guarantees that the quantity $\min_{v \in e} \sigma(v)$ is well-defined, since every superedge e is nonempty. If one wishes to admit an empty superedge $e = \emptyset$, then an explicit convention (for example, $\min \emptyset = 1$) must be specified; in this paper, empty superedges are excluded.

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1.3 Interval-Valued Bipolar Fuzzy Graphs

Interval-valued bipolar fuzzy sets assign positive and negative membership intervals to each element, modeling supportive and opposing evidence simultaneously.[16,17] An interval-valued bipolar fuzzy graph assigns positive and negative membership intervals to vertices and edges, respecting bipolar consistency constraints.[18]

Definition 1.9 (Interval families and endpoint order)**.** Let

$$[I]\big\{[a,b]\subseteq[0,1]\ :\ 0\le a\le b\le 1\big\},\qquad [-I]\big\{[a,b]\subseteq[-1,0]\ :\ -1\le a\le b\le 0\big\}.$$

For intervals $J_1=[a_1,b_1]$ and $J_2=[a_2,b_2]$ in $[I]$ (or in $[-I]$), write

$$J_1\preceq J_2 \iff a_1\le a_2 \text{ and } b_1\le b_2.$$

For a finite nonempty family $\{J_i=[a_i,b_i]\}_{i\in\Gamma}$ of intervals in $[I]$ (or $[-I]$), define

$$\bigwedge_{i\in\Gamma} J_i\Big[\min_{i\in\Gamma} a_i,\ \min_{i\in\Gamma} b_i\Big],\qquad \bigvee_{i\in\Gamma} J_i\Big[\max_{i\in\Gamma} a_i,\ \max_{i\in\Gamma} b_i\Big].$$

Definition 1.10 (Interval-valued bipolar fuzzy graph)**.** Let $G^*=(V,E)$ be a (simple) undirected graph. An *interval-valued bipolar fuzzy graph* (*IVBG*) of G^* is a pair $G=(A,B)$ such that

$$A=(A^P,A^N):V\to[I]\times[-I],\qquad B=(B^P,B^N):E\to[I]\times[-I],$$

where $[I]$ (resp. $[-I]$) denotes the family of closed subintervals of $I=[0,1]$ (resp. $-I=[-1,0]$). Equivalently, for each $x\in V$ and $e\in E$,

$$A^P(x)=[A^{P,-}(x),A^{P,+}(x)]\subseteq[0,1],\quad A^N(x)=[A^{N,-}(x),A^{N,+}(x)]\subseteq[-1,0],$$

and similarly $B^P(e)=[B^{P,-}(e),B^{P,+}(e)]$, $B^N(e)=[B^{N,-}(e),B^{N,+}(e)]$. These satisfy, for every edge $xy\in E$,

$$B^P(xy)\le A^P(x)\wedge A^P(y),\qquad B^N(xy)\ge A^N(x)\vee A^N(y),$$

where $\wedge,\vee$ and $\le,\ge$ are understood componentwise on interval endpoints. In this case, A is called the *interval-valued bipolar fuzzy vertex set* and B the *interval-valued bipolar fuzzy edge set*.

2 Main Results

This section presents the main results of this paper.

2.1 Interval-Valued Bipolar Fuzzy HyperGraphs

An interval-valued bipolar fuzzy hypergraph assigns positive/negative membership intervals to vertices and hyperedges, subject to endpointwise meet/join consistency constraints.

Definition 2.1 (Interval-valued bipolar fuzzy hypergraph)**.** Let $H^*=(V,\mathcal{E})$ be a (finite) hypergraph with $\emptyset\notin\mathcal{E}$. An *interval-valued bipolar fuzzy hypergraph* (*IVBFHG*) on H^* is a pair $H=(A,B)$ where

$$A=(A^P,A^N):V\to[I]\times[-I],\qquad B=(B^P,B^N):\mathcal{E}\to[I]\times[-I],$$

such that for every hyperedge $e\in\mathcal{E}$,

$$B^P(e)\ \preceq\ \bigwedge_{x\in e} A^P(x),\qquad B^N(e)\ \succeq\ \bigvee_{x\in e} A^N(x),$$

where $\preceq,\succeq$ and $\wedge,\vee$ are the endpoint order and meet/join from Definition 1.9. In this case, A is called the *interval-valued bipolar fuzzy vertex set* and B the *interval-valued bipolar fuzzy hyperedge set*.

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Remark 2.2. Equivalently, writing $A^P(x) = [A^{P,-}(x), A^{P,+}(x)]$, $A^N(x) = [A^{N,-}(x), A^{N,+}(x)]$ and similarly for B, the constraints in Definition 2.1 mean

$$B^{P,-}(e) \le \min_{x\in e} A^{P,-}(x), \quad B^{P,+}(e) \le \min_{x\in e} A^{P,+}(x),$$

and

$$B^{N,-}(e) \ge \max_{x\in e} A^{N,-}(x), \quad B^{N,+}(e) \ge \max_{x\in e} A^{N,+}(x),$$

for every $e \in \mathcal{E}$.

Theorem 2.3 (IVBFHG generalizes IVBFG)**.** *Let $G^* = (V, E)$ be a (simple) undirected graph, viewed as a hypergraph $H^* = (V, \mathcal{E})$ by setting $\mathcal{E} = E \subseteq \binom{V}{2}$. If $H = (A, B)$ is an interval-valued bipolar fuzzy hypergraph on H^* in the sense of Definition 2.1, then H is exactly an interval-valued bipolar fuzzy graph on G^* in the sense of Definition 1.10. Conversely, every interval-valued bipolar fuzzy graph on G^* is an interval-valued bipolar fuzzy hypergraph on H^*. Hence, interval-valued bipolar fuzzy hypergraphs strictly generalize interval-valued bipolar fuzzy graphs.*

Proof. Regard $G^* = (V, E)$ as the hypergraph $H^* = (V, \mathcal{E})$ with $\mathcal{E} = E$ and each hyperedge $e \in \mathcal{E}$ having the form $e = \{x, y\}$ with $x \neq y$.

Assume first that $H = (A, B)$ is an IVBFHG on H^*. Fix an edge $xy \in E$ and write $e = \{x, y\} \in \mathcal{E}$. By Definition 1.9, the meet/join over the two-element set e satisfy

$$\bigwedge_{z\in e} A^P(z) = A^P(x) \wedge A^P(y), \qquad \bigvee_{z\in e} A^N(z) = A^N(x) \vee A^N(y).$$

Substituting these identities into the IVBFHG constraints

$$B^P(e) \preceq \bigwedge_{z\in e} A^P(z), \qquad B^N(e) \succeq \bigvee_{z\in e} A^N(z),$$

yields, for every edge $xy \in E$,

$$B^P(xy) \preceq A^P(x) \wedge A^P(y), \qquad B^N(xy) \succeq A^N(x) \vee A^N(y),$$

which is precisely the bipolar consistency condition in Definition 1.10 (after identifying B on $\mathcal{E} = E$ with B on E). Thus H is an interval-valued bipolar fuzzy graph on G^*.

Conversely, suppose $G = (A, B)$ is an interval-valued bipolar fuzzy graph on G^*. For any hyperedge $e \in \mathcal{E} = E$, we again have $e = \{x, y\}$, and therefore

$$\bigwedge_{z\in e} A^P(z) = A^P(x) \wedge A^P(y), \qquad \bigvee_{z\in e} A^N(z) = A^N(x) \vee A^N(y).$$

The IVBFG inequalities $B^P(xy) \preceq A^P(x) \wedge A^P(y)$ and $B^N(xy) \succeq A^N(x) \vee A^N(y)$ then rewrite exactly as

$$B^P(e) \preceq \bigwedge_{z\in e} A^P(z), \qquad B^N(e) \succeq \bigvee_{z\in e} A^N(z),$$

which is Definition 2.1. Hence G is also an IVBFHG on H^*.

Finally, the inclusion is strict because hypergraphs allow edges of size ≥ 3, whereas graphs restrict to edges of size 2. □

2.2 Interval-Valued Bipolar Fuzzy SuperHyperGraphs

An Interval-valued bipolar fuzzy n-SuperHyperGraph assigns such intervals to n-supervertices and n-superedges across layered powerset levels, preserving consistency.

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Definition 2.4 (Interval-valued bipolar fuzzy n-SuperHyperGraph)**.** Let $\mathcal{H}^{(n)} = (V_n, \mathcal{E}_n)$ be a level-n SuperHyperGraph (Definition 1.5) and assume $\emptyset \notin \mathcal{E}_n$. An *interval-valued bipolar fuzzy n-SuperHyperGraph (IVBF n-SHG)* on $\mathcal{H}^{(n)}$ is a pair $\mathcal{G}^{(n)} = (A, B)$ such that

$$A = (A^P, A^N) : V_n \to [I] \times [-I], \qquad B = (B^P, B^N) : \mathcal{E}_n \to [I] \times [-I],$$

and for every n-superedge $E \in \mathcal{E}_n$,

$$B^P(E) \preceq \bigwedge_{X \in E} A^P(X), \qquad B^N(E) \succeq \bigvee_{X \in E} A^N(X),$$

where $\preceq, \succeq$ and $\wedge, \vee$ are as in Definition 1.9.

Example 2.5 (A concrete IVBF 2-SuperHyperGraph)**.** Let the base set be

$$V_0 = \{1, 2, 3\}, \qquad n = 2.$$

Consider the following two 2-supervertices in $\mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}(V_0))$:

$$X_1\{\{1\}, \{1, 2\}\}, \qquad X_2\{\{2\}, \{2, 3\}\}.$$

Set

$$V_2\{X_1, X_2\}, \qquad \mathcal{E}_2\{E_1\}, \qquad E_1\{X_1, X_2\} \subseteq V_2,$$

so $\mathcal{H}^{(2)} = (V_2, \mathcal{E}_2)$ is a level-2 SuperHyperGraph.

Define the interval-valued bipolar fuzzy vertex map $A = (A^P, A^N) : V_2 \to [I] \times [-I]$ by

$$A^P(X_1) = [0.60, 0.80], \quad A^N(X_1) = [-0.40, -0.20],$$

$$A^P(X_2) = [0.50, 0.70], \quad A^N(X_2) = [-0.50, -0.30].$$

Using the endpointwise meet/join from Definition 1.9, we have

$$A^P(X_1) \wedge A^P(X_2) = \big[\min(0.60, 0.50), \min(0.80, 0.70)\big] = [0.50, 0.70],$$

$$A^N(X_1) \vee A^N(X_2) = \big[\max(-0.40, -0.50), \max(-0.20, -0.30)\big] = [-0.40, -0.20].$$

Now define the interval-valued bipolar fuzzy superedge map $B = (B^P, B^N) : \mathcal{E}_2 \to [I] \times [-I]$ by

$$B^P(E_1) = [0.45, 0.65], \qquad B^N(E_1) = [-0.35, -0.20].$$

Then

$$B^P(E_1) \preceq [0.50, 0.70] = \bigwedge_{X \in E_1} A^P(X),$$

$$B^N(E_1) \succeq [-0.40, -0.20] = \bigvee_{X \in E_1} A^N(X),$$

since $0.45 \leq 0.50$, $0.65 \leq 0.70$, and $-0.35 \geq -0.40$, $-0.20 \geq -0.20$. Therefore $\mathcal{G}^{(2)} = (A, B)$ is an interval-valued bipolar fuzzy 2-SuperHyperGraph on $\mathcal{H}^{(2)}$.

Theorem 2.6 (Generalization of IVBFG and IVBFHG)**.** *Let $\mathcal{G}^{(n)} = (A, B)$ be an IVBF n-SHG on $\mathcal{H}^{(n)} = (V_n, \mathcal{E}_n)$.*

(i) *If $n = 0$, then $\mathcal{G}^{(0)}$ is exactly an interval-valued bipolar fuzzy hypergraph on the underlying hypergraph $\mathcal{H}^{(0)} = (V_0, \mathcal{E}_0)$.*

(ii) *If $n = 0$ and $\mathcal{E}_0 \subseteq \{\{x, y\} \subseteq V_0 : x \neq y\}$ (i.e., the underlying hypergraph is a graph), then $\mathcal{G}^{(0)}$ is exactly an interval-valued bipolar fuzzy graph on $G^* = (V_0, \mathcal{E}_0)$ in the sense of Definition 1.10.*

Hence, interval-valued bipolar fuzzy n-SuperHyperGraphs simultaneously generalize interval-valued bipolar fuzzy graphs and interval-valued bipolar fuzzy hypergraphs.

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Proof. *(i)* If $n = 0$, then by Definition 1.5 we have $V_0 \subseteq \mathcal{P}^0(V_0) = V_0$ and $\mathcal{E}_0 \subseteq \mathcal{P}(V_0) \setminus \{\emptyset\}$, so $\mathcal{H}^{(0)} = (V_0, \mathcal{E}_0)$ is an ordinary hypergraph. Definition 2.4 then states precisely that $A : V_0 \to [I] \times [-I]$ and $B : \mathcal{E}_0 \to [I] \times [-I]$ satisfy

$$B^P(e) \preceq \bigwedge_{x \in e} A^P(x), \qquad B^N(e) \succeq \bigvee_{x \in e} A^N(x) \quad (\forall e \in \mathcal{E}_0),$$

which is exactly Definition 2.1. Thus $\mathcal{G}^{(0)}$ is an IVBFHG.

(ii) Assume $n = 0$ and every $e \in \mathcal{E}_0$ has the form $e = \{x, y\}$ with $x \neq y$. Then, for such an edge $e = \{x, y\}$,

$$\bigwedge_{z \in e} A^P(z) = A^P(x) \wedge A^P(y), \qquad \bigvee_{z \in e} A^N(z) = A^N(x) \vee A^N(y),$$

where $\wedge, \vee$ on the right-hand side are the binary meet/join from Definition 1.9. Substituting these into the constraints of Definition 2.4 yields, for every edge $xy \in \mathcal{E}_0$,

$$B^P(xy) \preceq A^P(x) \wedge A^P(y), \qquad B^N(xy) \succeq A^N(x) \vee A^N(y),$$

which is exactly the bipolar consistency condition in Definition 1.10 (up to the notational identification $E = \mathcal{E}_0$). Hence $\mathcal{G}^{(0)}$ is an IVBFG.

The final statement follows immediately from (i) and (ii). □

3 Conclusion

In this paper, we extended interval-valued bipolar fuzzy graphs to the settings of hypergraphs and superhypergraphs. In future work, we hope to develop further extensions based on Neutrosophic graphs[19,20] and Plithogenic graphs.[21] We also hope that quantitative analyses using computational experiments will be carried out and that studies on algorithm design will be pursued.

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Conflicts of Interest

The authors declare that they have no conflicts of interest related to this work.

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Data Availability

No empirical data were generated or analyzed in this theoretical study. We encourage future work that applies and evaluates the proposed concepts in practical settings.

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Research Integrity

The authors confirm that this manuscript is original, has not been published previously, and is not under consideration for publication elsewhere.

Use of Computational Tools

All proofs and derivations were carried out manually. No computational software (e.g., Mathematica, SageMath, Coq) was used.

Code Availability

No code or software was developed for this study.

Ethical Approval

This study did not involve human participants or animals and therefore did not require ethical approval.

Use of Generative AI and AI-Assisted Tools

The authors used generative AI and AI-assisted tools only for tasks such as English grammar checking and language polishing. These tools were not used to generate scientific results, and their use did not violate ethical standards.

Supplementary Information

No supplementary materials are associated with this paper.

Disclaimer

The ideas presented in this paper are theoretical and have not yet been validated by empirical testing. While we have made every effort to ensure accuracy and proper attribution, inadvertent errors may remain. Readers are encouraged to verify cited material independently. The views expressed are those of the authors and do not necessarily represent those of their affiliated institutions.

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