On S_G-Fréchet Space and S_G-Hausdorff Space in Soft Group Topological Spaces and Neutrosophic Soft Group Sets

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Abstract

In this paper, we introduce some concepts : soft group point, soft group set, soft group topology ,define soft group Fréchet space and soft group Hausdorff space in soft group topological spaces**,** study a relation between F_G - topological space and soft group topological space with examples. Finally we introduce a new generalized definition called *NSG*- sets study the relations between it and the related sets.

Keywords: Soft group set; Soft group topology; Group topology; S_G-Fréchet space ; S_G-Hausdorff space ; F_G-point ; F_G-Fréchet space ; F_G-Hausdorff space; *NSG*- sets

1. Introduction

Soft set theory was introduced by Molodtsov 1999 [5]. Aktas and Cagman 2007 [1] give the notion of a soft group using Molodtsov soft set definition which is a generalization of fuzzy sets to deal with uncertain problems. And then several authors like Aktas and Ozlu, 2014 [1]; Aygun and Aygun, 2009[2]; Muhammad and Saqib, 2012[4]; Nazmul and Syamal, 2012[6]; Ray and Goldar, 2017[7] have generalized the idea of soft group depending on Aktas and Cagman definition. In this search, to making examples on these types of spaces, we introduce new definitions and structures different from the previous studies, we define the soft group element to construct the soft group set then generate soft group topology and define soft group Fréchet space and soft group Hausdorff space, fuzzy group Fréchet space and fuzzy group Hausdorff space with a comparison between them and with some theorems and examples. Finally define *NSG*- sets depend on [3], study the relations between it and the related sets.

2. On S_G - Sets and S_G - topology

We will introduce soft group points, soft group set (S_G-set), and operations between them. In each example, the soft group sets are generated by the same group and the same set X.

Definition 2.1

Let X any set ; G is a group , G is called soft group set (S_G-set), if $\forall a$ in G $\exists$ membership function $M_G(a)$ where $M_G(a_\lambda)$: $a_\lambda \to$ P(X), by symbols $\tilde{A} = \{\tilde{a}: \tilde{a} = a_\lambda^{M_G(a_\lambda)}, M_G(a_\lambda) \in P(X), \forall a_\lambda \in G, \lambda \in \omega\}$.

Definitions 2.2

Let G be a group, X be any set:

1- The S_G-null soft set $\widetilde{\emptyset}$ = the set which contains no soft group elements.

2- The S_G-universal soft set $\tilde{U} = \{\tilde{a}: \tilde{a} = a_\lambda^{M_G(a_\lambda)}, M_G(a_\lambda) \in P(X), \forall a_\lambda \in G, \lambda \in \omega\}$.

3- $\tilde{a}$ is S_G-element of S_G-set $\tilde{A} \leftrightarrow \tilde{a} \,\widetilde{\in}\, \tilde{A}$.

4- Two S_G-elements $\tilde{a} = a_\lambda^{M_G(a_\lambda)}$, $\tilde{e} = e_\lambda^{M_G(e_\lambda)}$ are S_G-equal $\leftrightarrow a_\lambda = e_\lambda$ and $M_G(a_\lambda) = M_G(e_\lambda)$.

DOI: https://doi.org/10.54216/IJNS.270239

5- Let $\tilde{A}$ and $\tilde{Y}$ are S_G-sets generated by group G and a set X,
$\tilde{A}$ is S_G-subset of $\tilde{Y}$ ($\tilde{A} \tilde{\subseteq} \tilde{Y}$) ↔ ∀ S_G-element $\tilde{a} \in \tilde{A}$ then $\tilde{a} \in \tilde{Y}$.
6- For two S_G-sets $\tilde{A}$, $\tilde{Y}$ over the common universe X and a common group G, which defined by:
$\tilde{A} = \{\tilde{a}_\lambda : \tilde{a}_\lambda = a_\lambda^{M_G(a_\lambda)}, M_G(a_\lambda) \in P(X), \forall a_\lambda \in G, \lambda \in \omega\}$,
$\tilde{Y} = \{\tilde{a}_\sigma : \tilde{a}_\sigma = a_\sigma^{M_G(a_\sigma)}, \forall M_G(a_\sigma) \in P(X), \forall a_\sigma \in G, \sigma \in \omega\}$

Then the S_G-union and the S_G-intersection is defined as follows:

$\tilde{N} = \tilde{A} \tilde{\cup} \tilde{Y}$ = The set of all soft group elements in both $\tilde{A}$ *and* $\tilde{Y}$.
$\tilde{N} = \tilde{A} \tilde{\cap} \tilde{Y}$ = The set of all common soft group elements in both $\tilde{A}$ *and* $\tilde{Y}$.

Examples 2.3

1- Let G = (G, .) = {*1*, *-1*, *i*, *-i*}and let X = [0,1]. Define

$$M_G(x) = \begin{cases} 0.76 & x = 1, \\ 0.44 & x = -1, \\ 0.5 & x = \mp i. \end{cases}$$

S_G - set $\tilde{A} = \{1^{0.76}, -1^{0.44}, i^{0.5}, -i^{0.5}\}$.

2- G = (G, .) = {*1*, *-1*, *i* , *-i* }and let X = {*m*, *b*, *d*}. Define

$$M_G(x) = \begin{cases} m & x = 1, \\ b & x = -1, \\ d & x = \mp i. \end{cases}$$

Then $\tilde{A} = \{1^m, -1^b, i^d, -i^d\}$ is S_G-set.

Definition 2.4

Let X any set ; G is a group. Then (G, ℑ) is called soft group topological space (simply S_G -topological space) on $\tilde{V}$ if ℑ satisfy:

(1) The S_G-intersection of any finite S_G-set in ℑ is belongs to ℑ.

(2) $\tilde{\emptyset}$, $\tilde{V}$ belongs to the relation ℑ.

(3) The S_G-union of any of S_G-set in ℑ belongs to the relation ℑ.

The resulting sets in of ℑ will be S_G-open sets *.

Remark 2.5

Let $\tilde{U}$ be an S_G-set , $\tilde{A}$ contained in $\tilde{U}$, if $\tilde{A} \tilde{\in}$ ℑ, the S_G-complement of

$\tilde{A} = \{\tilde{a}: \tilde{a} = a_\lambda^{M_G(a_\lambda)}, M_G(a_\lambda) \in P(X), \forall a_\lambda \in G, \lambda \in \omega\}$ is defined by: $\tilde{A}^c = \tilde{U} / \tilde{A}$.

Examples 2.6

1. Let G = (G, ·) = {*- 1*, *1*}, X = { *1* , *0* }
   $\tilde{A} = \{1^0, -1^1\}$, $\tilde{U} = \{1^0, -1^0, 1^1, -1^1\}$, $\tilde{A}^c = \{-1^0, 1^1\}$.

   Then ℑ = {$\tilde{\emptyset}$, $\tilde{A}$, $\tilde{A}^c$, $\tilde{U}$} is soft group topology on $\tilde{U}$.

2. Let G = (G,.) = {*1*, *-1*}, X = {0,1} and S_G-sets are defined as follows:

$\tilde{A} = \{1^0\}$, $\tilde{U} = \{1^0, -1^0, 1^1, -1^1\}$, $\tilde{V} = \{1^0, -1^0\}$

Then ℑ = {$\tilde{\emptyset}$, $\tilde{A}$, $\tilde{V}$, $\tilde{U}$} is soft group topology on $\tilde{U}$.

Definition 2.7

S_G-set $\tilde{A}$ in (G, ℑ) is S_G-neighborhood of $\tilde{a}$ in (G, ℑ) ↔ ∃ S_G - set $\tilde{V}$ in ℑ such that $\tilde{a} \tilde{\in} \tilde{V} \tilde{\subseteq} \tilde{A}$.

Definition 2.8

Let X arbitrary set , G be a group, (G, ℑ) be an S_G-space

1. S_G-*cl*($\tilde{A}$) = $\tilde{\cap}$\{$\tilde{Y}$: $\tilde{Y}$ *is* S_G-closed set in ℑ, $\tilde{A} \tilde{\subseteq} \tilde{Y}$\}.
2. S_G-*int*($\tilde{A}$) = $\tilde{\cup}$\{ $\tilde{Y}$: $\tilde{Y}$ *is* S_G-open set in ℑ, $\tilde{Y} \tilde{\subseteq} \tilde{A}$\}.

DOI: https://doi.org/10.54216/IJNS.270239

Example 2.9

From example 2.6 (1), S_G - cl $(\tilde{A})$ = $\tilde{\cap}$**{** $\tilde{A}$,$\tilde{U}$ } = $\tilde{A}$

S_G - $int(\tilde{A})$= $\tilde{\cup}${ $\tilde{\emptyset}$, $\tilde{A}$ **}** = $\tilde{A}$**,**

S_G - $cl(\tilde{U})$ = $\tilde{\cap}${ $\tilde{U}$ } = $\tilde{U}$**,**

S_G - $int(\tilde{U})$= $\tilde{\cup}${ $\tilde{\emptyset}$, $\tilde{A}$**,** $\tilde{U}$**}** = $\tilde{U}$**.**

Definition 2.10

The function $f : (G_1, \Im) \rightarrow (G_2, \mathscr{L})$ between two S_G - spaces is said to be S_G - continuous if for each S_G - open set $\tilde{A}$ in $\mathscr{L}$; $f^{-1}(\tilde{A})$ is S_G - open in space $\Im$.

Definition 2.11

An S_G - homeomorphism is an S_G - bijective and S_G -continuous function between S_G -topological spaces that has an S_G -continuous inverse function.

Definition 2.12

An S_G -topological invariant is proper class of S_G -topological spaces closed under S_G -homeomorphisms.

Definition 2.13

A property of topological space is hereditary property [if every sub spaces of the given space be a topological space], [if Y ⊆ X, then the collection ℑy = {G ∩ Y: G ∈ ℑx} is a topology on Y].

3. S_G -Fréchet Space ($\Im_1$-space)

Definition 3.1

Let $(G, \Im)$ be soft group topological spaces then $(G, \Im)$ is $\Im_1$-space (S_G-*Fréchet Space*) if for each two different S_G-elements $\tilde{a} = a_\lambda^{M_G(a_\lambda)}$, $\tilde{e} = e_\lambda^{M_G(e_\lambda)}$ there exist S_G-open set containing one of them but not the other.

Example 3.2

From example 2.6 (1), $\tilde{A}$, $\tilde{A}^c$ are S_G- open sets of each of their points. Then $1^0 \neq 1^1$, $1^0 \tilde{\in} \tilde{A}$ but

$1^1 \tilde{\notin} \tilde{A}$ similarly for other points. So $\Im = \{\tilde{\emptyset}, \tilde{A}, \tilde{A}^c, \tilde{U}\}$ is $\Im_1$-space.

Example 3.3

From example 2.6 (2) $\tilde{A}$, $\tilde{V}$ are S_G- open sets of each of their points. Then $1^0 \neq -1^0$, $1^0 \tilde{\in} \tilde{A}$ and

$-1^0 \tilde{\in} \tilde{V}$ but $1^0 \tilde{\in} \tilde{V}$. So $\Im = \{\tilde{\emptyset}, \tilde{A}, \tilde{A}^c, \tilde{U}\}$ is not $\Im_1$-space.

Example 3.4

Let G = (G, +) = $\begin{pmatrix} 6 & 7 \\ 3 & 2 \end{pmatrix}$ be a 2×2 matrix group, let X = $\{ki\}_{i=1}^{4}$,

$\tilde{M} = \begin{pmatrix} 6^{K1} & 7^{K2} \\ 3^{K3} & 2^{K4} \end{pmatrix}$ be S_G-set

$\tilde{\emptyset} = \begin{pmatrix} \emptyset & \emptyset \\ \emptyset & \emptyset \end{pmatrix}$, $\tilde{N}_1 = \begin{pmatrix} 6^{K1} & \emptyset \\ \emptyset & \emptyset \end{pmatrix}$

$\tilde{N}_2 = \begin{pmatrix} \emptyset & 7^{K2} \\ \emptyset & \emptyset \end{pmatrix}$, $\tilde{N}_3 = \begin{pmatrix} 6^{K1} & 7^{K2} \\ \emptyset & \emptyset \end{pmatrix}$

Then $\Im = \{\tilde{\emptyset}, \tilde{M}, \tilde{N}_i\}$, i = 1,2,3, is a $\Im_1$ -space .

Theorem 3.5

Let $(G, \Im)$ be S_G -space. Then $(G, \Im)$ is $\Im_1$-space iff every singleton S_G-set is S_G-closed.

Proof: Let $(G, \Im)$ be $\Im_1$-space, and let $\tilde{a} = a_\lambda^{M_G(a_\lambda)} \tilde{\in}$ $\Im_1$-space, to prove $\{\tilde{a}\}$ is S_G-closed which is equivalent to prove $\{\tilde{a}\}^c$ is S_G-open. Let $\tilde{e} \tilde{\in} \{ \tilde{a} \}^c$, $\tilde{e} = e_\lambda^{M_G(e_\lambda)}$, $\tilde{a} \neq \tilde{e}$, assume there exist S_G-open set $\hat{W}$ containing $\tilde{e}$ *not* $\tilde{a}$. If $\tilde{e} \tilde{\in} \hat{W}$, $\tilde{a} \tilde{\notin} \hat{W}$, then $\tilde{e} \tilde{\in} \hat{W} \tilde{\subseteq} \{\tilde{a}\}^c$. So $\{\tilde{a}\}^c$ is S_G-open and $\{\tilde{a}\}$ is S_G-closed.

Conversely let $\tilde{a} = a_\lambda^{M_G(a_\lambda)}$ and $\tilde{e} = e_\lambda^{M_G(e_\lambda)}$ be an S_G-elements, for all $\lambda \in \omega$, $\tilde{a} \neq \tilde{e}$. And suppose that every singleton $\{\tilde{a}\}$, $\{\tilde{e}\}$ is S_G-closed set. Hence $\{\tilde{a}\}^c$ and $\{\tilde{e}\}^c$ are S_G-open sets and we get $\tilde{e}$ in $\{ \tilde{a} \}^c$

DOI: https://doi.org/10.54216/IJNS.270239

$\tilde{a}$ not in{ $\tilde{a}$ }^{c} and $\tilde{a}$ belong to { $\tilde{e}$ }^{c}, $\tilde{e}$ not in{ $\tilde{e}$ }^{c} so (G, $\Im$) is $\Im_1$-space.

Example 3.6

Let G = (G,.) = {1, -1}, X = {$0,0.5$} and S_G-sets are defined as follows:

$\tilde{A} = \{1^0\}$, $\tilde{\tilde{E}} = \{-1^{0.5}\}$, $\tilde{V} = \{1^0, -1^{0.5}\}$

Then $\Im = \{\tilde{\emptyset}, \tilde{A}, \tilde{\tilde{E}}, \tilde{V}\}$ is soft group topology on $\tilde{V}$.

$\tilde{A}$ and $\tilde{\tilde{E}}$ are both singleton S_G - open sets and complement each other so they are singleton S_G - closed sets .

Theorem 3.7

If $\partial : (G_1, \Im) \rightarrow (G_2, \mathscr{L})$ is an S_G - bijective S_G - continuous function and (G_1, $\Im$) is $\Im_1$-space, then (G_2, $\mathscr{L}$) is $\mathscr{L}_1$ - space**.**

Proof: Let $\tilde{a} \neq \tilde{e}$. be distinct S_G - elements in (G_2, $\mathscr{L}$) , such that $\tilde{a} \tilde{\in} \tilde{A}$ and $\tilde{e} \tilde{\in} \tilde{V}$

where $\tilde{A}$, $\tilde{V}$ are S_G - open sets in $\mathscr{L}$**,**

so $\partial^{-1}(\tilde{A})$, $\partial^{-1}(\tilde{V})$ are S_G - open sets in $\Im$ containing $\partial^{-1}(\tilde{a})$

and $\partial^{-1}(\tilde{e})$ respectively. Since (G_1, $\Im$) is a $\Im_1$-space, then

$\tilde{a}_x = \partial^{-1}(\tilde{a}) \tilde{\in} \partial^{-1}(\tilde{A})$, $\tilde{e}_x = \partial^{-1}(\tilde{e}) \tilde{\notin} \partial^{-1}(\tilde{A})$

and $\tilde{e}_x = \partial^{-1}(\tilde{e}) \tilde{\in} \partial^{-1}(\tilde{V})$; $\tilde{a}_x = \partial^{-1}(\tilde{a}) \tilde{\notin} \partial^{-1}(\tilde{V})$

Then, $\partial(\tilde{a}_x) \tilde{\in} \tilde{A}$, $\partial(\tilde{e}_x) \tilde{\notin} \tilde{A}$; $\partial(\tilde{e}_x) \tilde{\in} \tilde{V}$, $\partial(\tilde{a}_x) \tilde{\notin} \tilde{V}$

$\therefore \tilde{a} \tilde{\in} \tilde{A}$, $\tilde{e} \tilde{\notin} \tilde{A}$ and $\tilde{e} \tilde{\in} \tilde{V}$, $\tilde{a} \tilde{\notin} \tilde{V}$. Therefore, ($G_2$, $\mathscr{L}$) is an $\mathscr{L}_1$ - space.

Corollary 3.8

S_G-Fréchet space is an S_G-topological invariant.

Proof : From (theorem 3.7) , S_G- *Fréchet* space is closed under *a* bijective S_G-continuous function so it is closed under S_G - homeomorphisms and from (definition 2.12) S_G-*Fréchet space* is an S_G-topological invariant.

4. On S_G -Hausdorff Space ($\Im_2$-space)

Definition 4.1

Let (G, $\Im$) be S_G - topological spaces , (G, $\Im$) is $\Im_2$-space (S_G - *Hausdorff Space*) if for each two different S_G-elements $\tilde{a} = a_\lambda^{M_G(a_\lambda)}$, $\tilde{e} = e_\lambda^{M_G(e_\lambda)}$ there exist two S_G - open neighborhoods $\tilde{A}$, $\tilde{V}$ such that $\tilde{a} \tilde{\in} \tilde{A}$, $\tilde{e} \tilde{\in} \tilde{V}$, $\tilde{A} \tilde{\cap} \tilde{V} = \tilde{\emptyset}$.

Example 4.2

In example 2.6 (1), $\tilde{A}^c$, $\tilde{A}$ are S_G -open neighborhoods of each of their points. Then $1^0 \neq -1^1$ cannot be separated by S_G - open sets. So $\Im = \{\tilde{\emptyset}, \tilde{A}, \tilde{A}^c, \tilde{U}\}$ is not $\Im_2$ -space.

Example 4.3

For X = {read ,blue} , G = (G,.) = {-1, 1} , $\tilde{U} = \{1^{\text{read}}, -1^{\text{blue}}\}$, $\tilde{A} = \{1^{\text{read}}\}$, $\tilde{A}^c = \{-1^{\text{blue}}\}$are S_G - sets

Then $\Im = \{\tilde{\emptyset}, \tilde{A}, \tilde{A}^c, \tilde{U}\}$ is soft group topology on $\tilde{U}$.

It is also a $\Im_2$ -space .

Since $\tilde{A}$, $\tilde{A}^c$ are S_G -open neighborhoods of each of their points. Then $1^{\text{read}} \neq -1^{\text{blue}}$, can be separated by S_G - open sets .

Theorem 4.4

Let (G, $\Im$) be S_G - space. Then, each $\Im_2$- space is $\Im_1$- space.

Proof: Directly from definitions of $\Im_2$- space and $\Im_1$- space.

Proposition 4.5

Let (G, $\Im$) be S_G - space, if (G, $\Im$) is $\Im_2$ - space then every singleton S_G - set is S_G - closed.

Proof: from theorem 4.4 and corollary 4.5.

DOI: https://doi.org/10.54216/IJNS.270239

Example 4.6

Let G = (G,.) = {*1*, *-1*}, X = {2 ,3} and S_G - sets are defined as follows:

$\tilde{U} = \{1^2, -1^3\}, \tilde{A} = \{1^2\}, \tilde{A}^c = \{-1^3\}$.

Then $\Im = \{\tilde{\emptyset}, \tilde{A}, \tilde{A}^c, \tilde{U}\}$ is soft group topology on $\tilde{U}$.

It is also an $\Im_2$ - space.

Since $\tilde{A}, \tilde{A}^c$ are S_G -open neighborhoods of each of their points.

Example 4.7

Let G = (G, +) = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ be a 2×2 matrix group, let X = $\{S_i\}_{i=1}^{n}$,

$\tilde{M} = \begin{pmatrix} 1^{S1} & 2^{S2} \\ 3^{S3} & 4^{S4} \end{pmatrix}$ be S_G-set**;** $\tilde{\emptyset} = \begin{pmatrix} \emptyset & \emptyset \\ \emptyset & \emptyset \end{pmatrix}$**;** $\tilde{N}_1 = \begin{pmatrix} 1^{S1} & \emptyset \\ \emptyset & \emptyset \end{pmatrix}$

$\tilde{N}_2 = \begin{pmatrix} \emptyset & 2^{S2} \\ \emptyset & \emptyset \end{pmatrix}$; $\tilde{N}_3 = \begin{pmatrix} \emptyset & \emptyset \\ 3^{S3} & \emptyset \end{pmatrix}$; $\tilde{N}_4 = \begin{pmatrix} \emptyset & \emptyset \\ \emptyset & 4^{S4} \end{pmatrix}$;

$\tilde{N}_5 = \begin{pmatrix} 1^{S1} & 2^{S2} \\ \emptyset & \emptyset \end{pmatrix}$; $\tilde{N}_6 = \begin{pmatrix} 1^{S1} & \emptyset \\ 3^{S3} & \emptyset \end{pmatrix}$; $\tilde{N}_7 = \begin{pmatrix} 1^{S1} & \emptyset \\ \emptyset & 4^{S4} \end{pmatrix}$; $\tilde{N}_8 = \begin{pmatrix} \emptyset & 2^{S2} \\ 3^{S3} & \emptyset \end{pmatrix}$

$\tilde{N}_9 = \begin{pmatrix} \emptyset & 2^{S2} \\ \emptyset & 4^{S4} \end{pmatrix}$, $\tilde{N}_{10} = \begin{pmatrix} \emptyset & \emptyset \\ 3^{S3} & 4^{S4} \end{pmatrix}$

Then $\Im = \{\tilde{\emptyset}, \tilde{M}, \tilde{N}_i\}_{i=1,2,\ldots,10}$, is $\Im_2$-space, since

Each two distant S_G - elements are contained in different S_G - open neighborhoods it is also an $\Im_1$ -space .

Theorem 4.8

If $\sigma : (G_1, \Im) \to (G_2, \mathscr{L})$ is a S_G - bijective S_G - continuous function and $(G_1, \Im)$ is $\Im_2$-space, then $(G_2, \mathscr{L})$ is $\mathscr{L}_2$ - space**.**

Proof: Let $\tilde{a} \neq \tilde{e}$. be distinct S_G - elements in $(G_2, \mathscr{L})$, since d is S_G - bijective, Then $\sigma(\tilde{a}_x) = \tilde{a}$ and $\sigma(\tilde{e}_x) = \tilde{e}$ and $\tilde{a}_x \neq \tilde{e}_x$ in $(G_1, \Im)$, Such that $\tilde{a} \tilde{\in} \tilde{A}$ and $\tilde{e} \tilde{\in} \tilde{V}$, where $\tilde{A}, \tilde{V}$ are S_G - open sets in $\mathscr{L}$ since σ is S_G - continuous function , then $\sigma^{-1}(\tilde{A})$, $\sigma^{-1}(\tilde{V})$ are S_G - open sets in $\Im$ containing $\sigma^{-1}(\tilde{a})$ and $\sigma^{-1}(\tilde{e})$ respectively.

Since $(G_1, \Im)$ is a $\Im_2$-space, then

$\tilde{a}_x = \sigma^{-1}(\tilde{a}) \tilde{\in} \sigma^{-1}(\tilde{A})$,

$\tilde{e}_x = \sigma^{-1}(\tilde{e}) \tilde{\in} \sigma^{-1}(\tilde{V})$,

$\sigma^{-1}(\tilde{A}) \tilde{\cap} \sigma^{-1}(\tilde{V}) = \tilde{\emptyset}$

Then, $\sigma(\tilde{a}_x) \tilde{\in} \tilde{A}$, $\sigma(\tilde{e}_x) \tilde{\in} \tilde{V}$

$\sigma(\sigma^{-1}(\tilde{A}) \tilde{\cap} \sigma^{-1}(\tilde{V}))$

$= \sigma(\sigma^{-1}(\tilde{A})) \tilde{\cap} \sigma(\sigma^{-1}(\tilde{V}))$

$= \tilde{A} \tilde{\cap} \tilde{V} = \sigma(\tilde{\emptyset}) = \tilde{\emptyset}$

So $\tilde{a} \tilde{\in} \tilde{A}$, $\tilde{e} \tilde{\in} \tilde{V}$ and $\tilde{A} \tilde{\cap} \tilde{V} = \tilde{\emptyset}$ .Therefore, $(G_2, \mathscr{L})$ is an $\mathscr{L}_2$ - space.

Corollary 4.9

S_G- Hausdorff space is an S_G-topological invariant.

Proof : From (theorem 4.8) , S_G- Hausdorff space is closed under a bijective S_G - continuous function so it is closed under S_G - homeomorphisms and from (definition 2.12) S_G- Hausdorff space is an S_G-topological invariant.

5. F_G - topological space and its relation with a soft group topological space.

Remark 5.1

Each fuzzy set is soft set and the converse not necessary true.

DOI: https://doi.org/10.54216/IJNS.270239

Definition 5.2

Let X be any set, G be a group then G is fuzzy group set (F_G - set), if $\forall$ a in G $\exists$ a membership $M_G(a)$,

$M_G(a_\lambda) : a_\lambda \rightarrow$ P([0,1]), by symbols

$\tilde{A} = \{\tilde{a}: \tilde{a} = a_\lambda^{M_G(a_\lambda)}, M_G(a_\lambda) \in$ P([0,1]), $\forall$ $a_\lambda \in G$, $\lambda \in \omega$ }.

Remark 5.3

1- Each F_G - set is S_G - set .

2- Each F_G - topology is S_G - topology .

3- Each F_G - $\Im_1$ -space is S_G -$\Im_1$ -space .

4- Each F_G - $\Im_2$ -space is S_G -$\Im_2$ -space .

5- The converse of (1,2,3,4)is not necessary true .

Preposition 5.4

Let (G, $\Im$) be F_G - space. Then each F_G - $\Im_2$- space is F_G - $\Im_1$- space .

Proof: By theorem 4.4 and remark 5.1.

Proposition 5.5

Let (G, $\Im$) be F_G - space, if (G, $\Im$) is $\Im_2$-space then every singleton F_G - set is F_G - closed.

Proof: By theorem 4.5 and remark 4.8.

Example 5.6

Let G = (G, +) = $\begin{pmatrix} 7 & 6 \\ 7 & 5 \end{pmatrix}$ be a 2×2 matrix group, let X = { 0,1, 0.3 , 0.7 }

$\tilde{B} = \begin{pmatrix} 7^{h1} & 6^{h2} \\ 7^{h3} & 5^{h4} \end{pmatrix}$ be F_G-set

$\tilde{\emptyset} = \begin{pmatrix} \Phi & \Phi \\ \Phi & \Phi \end{pmatrix}$, $\tilde{N}_1 = \begin{pmatrix} 7^0 & \Phi \\ \Phi & \Phi \end{pmatrix}$, $\tilde{N}_2 = \begin{pmatrix} \Phi & 6^1 \\ \Phi & \Phi \end{pmatrix}$, $\tilde{N}_3 = \begin{pmatrix} \Phi & \Phi \\ 7^{0.3} & \Phi \end{pmatrix}$

$\tilde{N}_4 = \begin{pmatrix} \Phi & \Phi \\ \Phi & 5^{0.7} \end{pmatrix}$, $\tilde{N}_5 = \begin{pmatrix} 7^0 & 6^1 \\ \Phi & \Phi \end{pmatrix}$, $\tilde{N}_6 = \begin{pmatrix} 7^0 & \Phi \\ 7^{0.3} & \Phi \end{pmatrix}$, $\tilde{N}_7 = \begin{pmatrix} 7^0 & \Phi \\ \Phi & 5^{0.7} \end{pmatrix}$

$\tilde{N}_8 = \begin{pmatrix} \Phi & 6^1 \\ 7^{0.3} & \Phi \end{pmatrix}$, $\tilde{N}_9 = \begin{pmatrix} \Phi & 6^1 \\ \Phi & 5^{0.7} \end{pmatrix}$, $\tilde{N}_{10} = \begin{pmatrix} \Phi & \Phi \\ 7^{0.3} & 5^{0.7} \end{pmatrix}$

Then $\Im = \{\tilde{\emptyset}, \tilde{B}, \tilde{N}_i\}_{i = 1,2,..,10}$, is F_G - $\Im_2$ -space, since

Each two distant F_G - elements are contained in different F_G - open neighborhoods it is also an F_G -$\Im_1$ -space .

6. *NS*-set and *NS$_G$*-set

In [5] Molodtsov D. define a soft set , in [3] Maji P.K. define neutrosophic soft set (*NS* -set)((as special case of soft se)) t.

Remarks 6.1

1- From definition [5] , if each element in U associated to membership between [0,1] then the result set is called fuzzy soft set.

2- From Maji P.K. [3] , if each element in U associated to a three memberships between $]0^-,1^+[$then the result set is called neutrosophic set.

3- From previous remarks 1and 2; neutrosophic soft set is a (specific generalized set of fuzzy soft set and soft set) and fuzzy soft set is not neutrosophic set , see next example .

Example 6.2

D = {a_1, a_2} is the universal set , ρ = {w_1, w_2} be a parameters set
F(w_1) ={(a_1, 0.5, 0.6, 0.3), (a_2, 0.4, 0.76, 0.6)},
F(w_2) = {(a_1, 0.6, 0.3, 0.5), (a_2, 0.77, 0.4, 0.3)},
F : $\rho \rightarrow$ P(D) , P(D) power neutrosophic sets of D
The *NS* -set (F, A) = { F (w_1) , F (w_2) }

DOI: https://doi.org/10.54216/IJNS.270239

= {(a_1, 0.5, 0.6, 0.3), (a_2, 0.4, 0.76, 0.6)}, {(a_1, 0.6, 0.3, 0.5), (a_2, 0.77, 0.4, 0.3)}
(F, A) is a soft set and a generalized state of fuzzy soft set
But F " :ρ → P(I),
(F", A") = { F (w_1) , F (w_2) } = {(a_1, 0.55), (a_2, 0.49)}, {(a_1, 0.62), (a_2, 0.73)} is fuzzy soft set but *NS* -set.

Definition 6.3

Let X any set ; *G* is a group , *G* is called neutrosophic soft group set (NS_G-set), if ∀ a_λ in *G* ∃ membership M_G (a_λ): a_λ → P(X), by symbols $\tilde{A}$ = {$\tilde{a}$: $\tilde{a} = a_\lambda^{M_G(a_\lambda)}$, $M_G(a_\lambda)$ = α, β, γ : α, β, γ ∈ P(I*),

I* =]$^-$0,1$^+$ [,∀ $a_\lambda \in G$, $\lambda \in \omega$, α » degree of membership of a_λ , β » degree of indeterminacy of a_λ ,

γ » degree of non-membership of a_λ }

Definition 6.4

For arbitrary group *G*, *G* is called fuzzy soft group set (FS_G-set), if ∀ a_λ in *G* ∃ membership

M_G (a_λ): a_λ → P(I), by symbols $\tilde{A}$ = {$\tilde{a}$: $\tilde{a} = a_\lambda^{M_G(a_\lambda)}$, $M_G(a_\lambda)$ = χ ∈ P(I), I = [0,1] , ∀ $a_\lambda \in G$, $\lambda \in \omega$ } .

Corollary 6.5

Neutrosophic soft group set is specific generalized state of fuzzy soft group set.

Proof: Since by remark 6.1(3) each neutrosophic soft set is a specific generalized state of fuzzy soft set .

Example 6.6

Consider the group *G* = (G, .) = {*1*, *-1*, *i*, *-i*} and let X =]$^-$0,1$^+$ [. Define

$$M_G(x) = \begin{cases} 0.51\,, \quad 0.11\,, \quad 0,76 & x = 1, \\ 0.1\,, \quad 0.7\,, \quad 0,1 & x = -1, \\ 0.33\,, \quad 0.33\,, \quad 0,79 & x = \mp i. \end{cases}$$

$\tilde{B}$= {$1^{0.51\,,0.11\,,0,76}$, $-1^{0.1\,,0.7\,,0,1\ .4}$, $i^{0.3\,,0.3\,,0,79}$, $-i^{0.33\,,0.33\,,0,79}$ } is NS_G - set.

Remark 6.7

Fuzzy soft group set is not neutrosophic soft group set.

Example 6.8

Let *G* = (G, +) = $\begin{pmatrix} 6 & 7 \\ 3 & 2 \end{pmatrix}$ be a 2×2 matrix group, let ρ = [0,1] ,

$\tilde{M}$ = $\begin{pmatrix} 6^{0.12} & 7^{0.22} \\ 3^{0.31} & 2^{0.53} \end{pmatrix}$ be FS_G-set but not neutrosophic soft group set.

Remarks 6.9

Since elements in general case are not necessary being elements in a group then:

1- Neutrosophic soft set is not necessary neutrosophic soft group set.
2- Fuzzy soft set is not necessary fuzzy soft group set.

7. Conclusion

In this work, we introduce some concept: soft group set, soft group topology and define S_G -Fréchet space and S_G -Hausdorff space in soft group topological spaces, study a relation between F_G -topological space and soft group topological space also introduce a study the relation between these separation axioms with examples as seen in the next diagram.

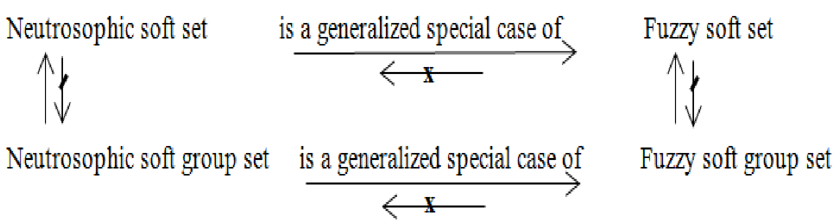

Figure 1. A study the relation between these separation axioms with examples.

DOI: https://doi.org/10.54216/IJNS.270239

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