On Complex Fuzzy Soft Graph With Operations

Sana Abu-Ghurra[1], **Ghada Alafifi**[2], **Eman A. AbuHijleh**[3,*], **Firas Safi**[4]
[1]Mathematics Department, Faculty of Science, Applied Science Private University, Amman, Jordan
[2]Mathematics Department, Science College, Philadelphia University, Amman, Jordan
[3]Department of Basic Sciences, Al-Zarqa University College, Al-Balqa Applied University, Zarqa 313, Jordan
[4]Department of Mathematics, Faculty of Science, Zarqa University, Zarqa, Jordan
Emails: s_abugurrah@asu.edu.jo; gafifi@philadelphia.edu.jo; emanhijleh@bau.edu.jo; fsafi@zu.edu.jo

Abstract

The goal of this paper is to study complex fuzzy soft graph (CFSG). We introduce the concept of complex fuzzy soft graph from apply complex fuzzy set on fuzzy soft graph. The notations and definitions of some operations on two complex fuzzy soft graphs presented such as union, cartesian product, tensor product, normal product and composition of two complex fuzzy soft graphs. Also, a decision-making (DM) problem on supply chain management is discussed.

Keywords: Complex fuzzy soft graph; Union; cartesian product; Tensor product; Normal product and composition of two complex fuzzy soft graphs

1 Introduction

Soft set theory was first proposed in 1990 by Molodtsov,[1] as a new mathematical technique for dealing with uncertainty. Soft set theory has attracted a lot of interest since its conception and a great deal of a study has been done on it worldwide. For example, several new operations in soft set theory were proposed by Ali et.al.,[2] in 2009.

One of the main areas of research now is the integration of soft set theory with other mathematical frameworks. A more generalized idea that combines fuzzy sets and soft sets is fuzzy soft sets which were created by Maji et.al.,[3] in 2001. Numerous academics have investigated the theory's applicability because of this notion of fuzzy soft sets, for instance see.[4] In 2002, Ramot et.al. produced complex fuzzy set (CFS).[5] After that, many papers appeared like as Thirunavukaras et.al.[6] (2017) presented a complex fuzzy soft set with applications (CFSS).

Real-time systems where information levels change across different levels of precision are increasingly being modelled using fuzzy graph theory. Because they may bridge the gap between traditional numerical models used in the sciences and engineering with symbolic models used in expert systems, fuzzy models have become increasingly popular. Kaufmann's (1973)[7] presented the foundation of a fuzzy graph. Then Rosenfeld in 1975

DOI: https://doi.org/10.54216/IJNS.260331

discussed graph-theoretical structure ideas, see.[8] Later in 1987, Bhattacharya[9] offered more information on fuzzy graphs and Mordeson with Peng (1994)[10] presented operations on fuzzy graphs. The definition of a fuzzy graph's complement was changed in 2002 to guarantee that the original fuzzy graph is returned by the complement of the complement which is consistent with the behavior of crisp graphs, see.[11] Akram et.al. presented a novel idea, they proposed a combination between soft set and fuzzy graphs through 2015-2016, see[12] and.[13]

The paper organized after this introduction, preliminaries in section two. The concept of complex fuzzy soft graphs produced in section three. We presented and discussed some operations on complex fuzzy soft graphs in section four. A decision-making (DM) problem on supply chain management is discussed in section five. Finally, section six, we summarized the contribution of this paper and proposed some suggested further works.

2 Preliminaries

A soft set (SS) was defined by Molodtsov in 1999.[1]

Definition 2.1. A pair of $(\gamma, \mathcal{A})$ is called soft set over U, where $\mathcal{A} \subseteq \Lambda$ is a parameter set and $\gamma : \mathcal{A} \to \mathbb{P}(U)$ is a set-valued mapping, called the approximate function of the soft set $(\gamma, \mathcal{A})$. In other words, a soft set over U is a parameterized family of subsets of U. For any $\alpha \in \mathcal{A}$, $\gamma(\alpha)$ may be considered as set of α-approximate elements of soft set $(\gamma, \mathcal{A})$.

In other words, a soft set over U is a parameterized family of subsets of universe U. Thus, a soft set over U can be written in a set of ordered pairs

$$(\gamma, \mathcal{A}) = \{(\alpha, \gamma(\alpha)) : \ \alpha \in \mathcal{A}, \ \gamma(\alpha) \in \mathbb{P}(U)\}$$

A fuzzy soft set (FSS) was defined by Maji et.al. in 2001,[3] which is an improvement of soft set to be fuzzy set too.

Definition 2.2. Let U be an initial universe, Λ be the set of all parameters and $A \subseteq \Lambda$ and $\mathbb{P}(U)$ the collection of all fuzzy subsets of U. Then $(\Psi, \mathcal{A})$ is called fuzzy soft set (FSS), where $\Psi : \mathcal{A} \to \mathbb{P}(U)$ is a mapping, called fuzzy approximate function of the fuzzy soft set $(\Psi, \mathcal{A})$.

As an example of FSS we have a FSS $(\Psi, \mathcal{A})$ over vertex set V and a FSS $(\Phi, \mathcal{A})$ over edge set E. see Example 2.5.

Later in 2017, Thirunavukarasu et.al.,[6] defined(proposed) a complex fuzzy soft set (CFSS).

Definition 2.3. Let U be an initial universe, Λ be the set of all parameters, $A \subseteq \Lambda$ and $\mathbb{P}(U)$ the collection of all complex fuzzy subsets of U. Then $(\Psi, \mathcal{A})$ is called complex fuzzy soft set (CFSS), where $\Psi : \mathcal{A} \to \mathbb{P}(U)$ is a mapping which called complex fuzzy approximate function of the complex fuzzy soft set $(\Psi, \mathcal{A})$.

As an example of CFSS we have a CFSS $(\Psi, \mathcal{A})$ over vertex set V and a CFSS $(\Phi, \mathcal{A})$ over edge set E. see Example 3.4.

In 2015, Akram and Nawaz[12] presented and defined a fuzzy soft graph (FSGr).

Definition 2.4. A fuzzy soft graph (FSGr) $\mathcal{G} = (\tilde{G}, \Psi, \Phi, \mathcal{A})$ ia a 4-tuple such that:
a) $\tilde{G} = (V, \ E)$ is a simple graph.
b) $\mathcal{A}$ is a nonempty set of parameters.
c) $(\Psi, \mathcal{A})$ is a fuzzy soft set over V.
d) $(\Phi, \mathcal{A})$ is a fuzzy soft set over E.
e) $(\Psi_\alpha, \Phi_\alpha)$ is a fuzzy graph of $\tilde{G}$ for all $\alpha \in \mathcal{A}$ and $\Psi_\alpha, \ \Phi_\alpha \in [0, 1]$. That is $\Phi_\alpha(\nu_1\nu_2) \leq \Psi_\alpha(\nu_1) \wedge \Psi_\alpha(\nu_2)$ for all $\alpha \in \mathcal{A}$ and $\nu_1, \ \nu_2 \in V$. Note that, $\Phi_\alpha(\nu_1\nu_2) = 0$ for all $\nu_1\nu_2 \in V \times V - E$ and for all $\alpha \in \mathcal{A}$.
Then for fuzzy graph $(\Psi_\alpha, \Phi_\alpha) = \mathcal{H}(\alpha)$, the FSGr is a parameterized family of fuzzy graphs of $\tilde{G}$. That is, $\mathcal{G} = (\tilde{G}, \Psi, \Phi, \mathcal{A}) = \{\mathcal{H}(\alpha) : \ \alpha \in \mathcal{A}\}$.

Example 2.5. Consider $\tilde{G} = (V,\ E)$ is a simple graph, where $V = \{\nu_1, \nu_2, \nu_3\}$, $E = \{\nu_1\nu_2, \nu_1\nu_3, \nu_2\nu_3\}$ and let $\mathcal{A} = \{a, b, c\}$ be the set of parameters of the FSGr $\mathcal{G} = \{\mathcal{H}(a), \mathcal{H}(b), \mathcal{H}(c)\}$. Whereas, $(\Psi, \mathcal{A})$ is a fuzzy soft set over V and $(\Phi, \mathcal{A})$ is a fuzzy soft set over E; Ψ and Φ are fuzzy approximate functions over V and E, respectively. Then Figure 1 is FSGr presentation of this example.

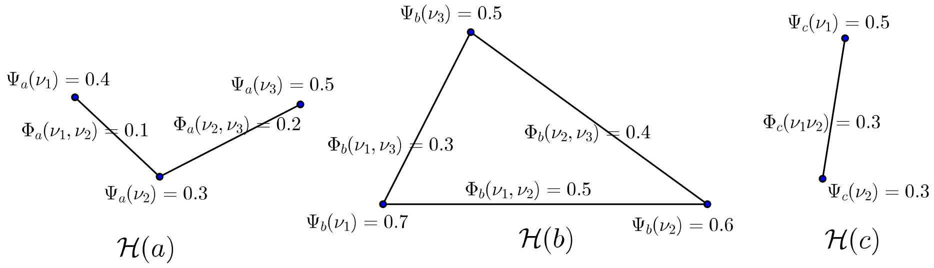

Figure 1: FSGr $\mathcal{G}$

The following are order and size of FSGr, see.[12]

Definition 2.6. For a FSGr $\mathcal{G} = (\tilde{G}, \Psi, \Phi, \mathcal{A})$, the order of $\mathcal{G}$ is:
$O(\mathcal{G}) = \bigoplus\limits_{\nu_i \in V,\ \forall \alpha_\ell \in \mathcal{A}} \Psi_{\alpha_\ell}(\nu_i) = \sum\limits_{\ell} \sum\limits_{i} \eta_{\alpha_\ell}(\nu_i)$

Definition 2.7. For a FSGr $\mathcal{G} = (\tilde{G}, \Psi, \Phi, \mathcal{A})$, the size of $\mathcal{G}$ is:
$S(\mathcal{G}) = \bigoplus\limits_{e_{ij} \in E,\ \forall \alpha_\ell \in \mathcal{A}} \Phi_{\alpha_\ell}(e_{ij}) = \sum\limits_{\ell} \sum\limits_{e_{ij} \in E} \eta_{\alpha_\ell}(e_{ij})$

3 Complex fuzzy soft graph

If we extend a FSGr to be complex, we get a complex fuzzy soft graph (CFSGr).

Definition 3.1. A complex fuzzy soft graph (CFSGr) $\mathcal{G} = (\tilde{G}, \Psi, \Phi, \mathcal{A})$ is a 4-tuple such that:
a) $\tilde{G} = (V,\ E)$ is a simple graph.
b) $\mathcal{A}$ is a nonempty set of parameters.
c) $(\Psi, \mathcal{A})$ is a complex fuzzy soft set over V.
d) $(\Phi, \mathcal{A})$ is a complex fuzzy soft set over E.
e) $(\Psi_\alpha, \Phi_\alpha)$ is a complex fuzzy graph of $\tilde{G}$ for all $\alpha \in \mathcal{A}$ with $\Psi_\alpha :\ V \to \{\zeta \in \mathbb{C} :\ |\zeta| \leq 1\}$, and $\Phi_\alpha :\ E \to \{\zeta \in \mathbb{C} :\ |\zeta| \leq 1\}$. Moreover,

$$\Phi_\alpha(\nu_1\nu_2) \leq \Psi_\alpha(\nu_1) \wedge \Psi_\alpha(\nu_2) \text{ for all } \alpha \in \mathcal{A} \text{ and } \nu_1,\ \nu_2 \in V. \tag{1}$$

Such that, $\Phi_\alpha(\nu_1\nu_2) = 0e^{2\pi i\ 0}$ for all $\nu_1\nu_2 \in V \times V - E$ and for all $\alpha \in \mathcal{A}$. Then for complex fuzzy graph (CFGr) $(\Psi_\alpha, \Phi_\alpha) = \mathcal{H}(\alpha)$, the CFSGr is a parameterized family of complex fuzzy graphs of $\tilde{G}$. That is, $\mathcal{G} = (\tilde{G}, \Psi, \Phi, \mathcal{A}) = \{\mathcal{H}(\alpha) :\ \alpha \in \mathcal{A}\}$.

Note that:
1. e_{ij} is an edge incident to ν_i and ν_j, i.e. $e_{ij} = \nu_i\nu_j$.
2. $\Psi_\alpha(\nu_i) = \zeta_\alpha(\nu_i) = \eta_\alpha(\nu_i)exp(2\pi i\theta_\alpha(\nu_i))$.
3. $\Phi_\alpha(e_{ij}) = \zeta_\alpha(e_{ij}) = \eta_\alpha(e_{ij})exp(2\pi i\theta_\alpha(e_{ij}))$.
4. $\Phi_\alpha(e_{ij}) = \eta_\alpha(e_{ij})exp(2\pi i\theta_\alpha(e_{ij})) \leq \min\{\Psi_\alpha(\nu_i), \Psi_\alpha(\nu_j)\}$
$= \min\{\eta_\alpha(\nu_i)exp(2\pi i\theta_\alpha(\nu_i)), \eta_\alpha(\nu_j)exp(2\pi i\theta_\alpha(\nu_j))\}$
$= \min\{\eta_\alpha(\nu_i), \eta_\alpha(\nu_j)\}exp(2\pi i \min\{\theta_\alpha(\nu_j), \theta_\alpha(\nu_j)\})$.

Definition 3.2. For a CFSGr $\mathcal{G} = (\tilde{G}, \Psi, \Phi, \mathcal{A})$, the order of $\mathcal{G}$ is:
$O(\mathcal{G}) = \bigoplus\limits_{\nu_i \in V,\ \forall \alpha_\ell \in \mathcal{A}} \Psi_{\alpha_\ell}(\nu_i) = \sum\limits_{\ell} \sum\limits_{i} \zeta_{\alpha_\ell}(\nu_i) =$
$(\sum\limits_{\ell} \sum\limits_{i} \eta_{\alpha_\ell}(\nu_i))exp(2\pi i(\sum\limits_{\ell} \sum\limits_{i} \theta_{\alpha_\ell}(\nu_i)))$.

Definition 3.3. For a CFSGr $\mathcal{G} = (\tilde{G}, \Psi, \Phi, \mathcal{A})$, the size of $\mathcal{G}$ is:
$S(\mathcal{G}) = \bigoplus\limits_{e_{ij}\in E,\ \forall\alpha_\ell\in\mathcal{A}} \Phi_{\alpha_\ell}(e_{ij}) = \sum\limits_{\ell}\sum\limits_{e_{ij}\in E} \zeta_{\alpha_\ell}(e_{ij}) =$
$(\sum\limits_{\ell}\sum\limits_{e_{ij}\in E} \eta_{\alpha_\ell}(e_{ij}))exp(2\pi i(\sum\limits_{\ell}\sum\limits_{e_{ij}\in E} \theta_{\alpha_\ell}(e_{ij}))).$

Example 3.4. Consider $\tilde{G} = (V,\ E)$ is a simple graph where $V = \{\nu_1, \nu_2, \nu_3\}$, $E = \{\nu_1\nu_2, \nu_1\nu_3, \nu_2\nu_3\}$ and let $\mathcal{A} = \{a, b, c\}$ be the set of parameters of the CFSGr $\mathcal{G} = \{\mathcal{H}(a), \mathcal{H}(b), \mathcal{H}(c)\}$. Whereas, $(\Psi, \mathcal{A})$ is a CFSS over V and $(\Phi, \mathcal{A})$ is a CFSS over E; Ψ and Φ are complex fuzzy approximate functions over V and E, respectively. Moreover:
1. Figure 2 is presented the CFSGr of this example.
2. $O(\mathcal{G}) = \sum\limits_{i} \Psi_a(\nu_i) + \sum\limits_{i} \Psi_b(\nu_i) + \sum\limits_{i} \Psi_c(\nu_i) = (1.1e^{2\pi i\ 2.0}) + (1.8e^{2\pi i\ 1.7}) + (1.2e^{2\pi i\ 1.1}) = 4.1e^{2\pi i\ 4.8}$.
3. $S(\mathcal{G}) = \sum\limits_{e_{ij}} \Psi_a(\nu_i\nu_j) + \sum\limits_{e_{ij}} \Psi_b(\nu_i\nu_j) + \sum\limits_{e_{ij}} \Psi_c(\nu_i\nu_j) = (10.4e^{2\pi i\ 0.9}) +$
$(1.4e^{2\pi i\ 1.2}) + (0.5e^{2\pi i\ 0.5}) = 2.3e^{2\pi i\ 2.6}$.

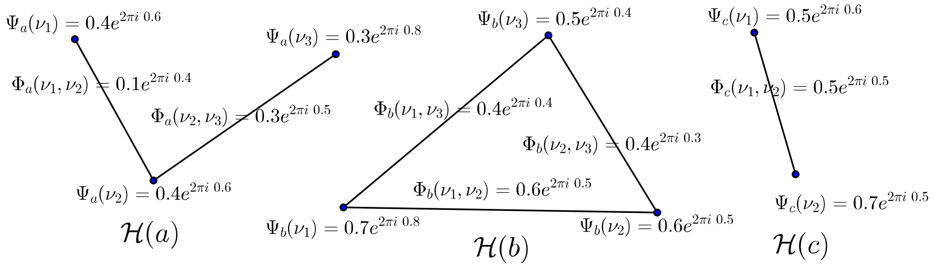

Figure 2: CFSGr $\mathcal{G}$

The complex fuzzy soft subgraph of CFSGr defined below.

Definition 3.5. Let $\mathcal{G}_1 = (\tilde{G_1}, \Psi_1, \Phi_1, \mathcal{A}_1)$ and $\mathcal{G}_2 = (\tilde{G_2}, \Psi_2, \Phi_2, \mathcal{A}_2)$ be two complex fuzzy soft graphs (CFSGrs). Then $\mathcal{G}_2$ is a complex fuzzy soft subgraph of $\mathcal{G}_1$ if $\mathcal{A}_2 \subset \mathcal{A}_1$; $\mathcal{H}_2(\alpha)$ is a partial fuzzy subgraph of $\mathcal{H}_1(\alpha)$ for all $\alpha \in \mathcal{A}_2$.

The CFSGr $\{\mathcal{H}(a), \mathcal{H}(b)\}$, in Example 3.4, is complex fuzzy soft subgraph of $\mathcal{G}$.

4 Some operations over two complex fuzzy soft graphs

Inspiring Akram and Nawaz (2016),[13] we produced some operations over two complex fuzzy soft graphs, as follows:

4.1 Union of two complex fuzzy soft graphs

An improvement of definition of union of two FSGrs, by Akram and Nawaz, is for two CFSGrs.

Definition 4.1. The union graph of two complex fuzzy soft graphs (CFSGrs) $\mathcal{G}_1 = (\tilde{G_1}, \Psi_1, \Phi_1, \mathcal{A}_1)$ and $\mathcal{G}_2 = (\tilde{G_2}, \Psi_2, \Phi_2, \mathcal{A}_2)$, is $\mathcal{G} = \mathcal{G}_1 \bigsqcup \mathcal{G}_2 = (\tilde{G}, \Psi, \Phi, \mathcal{A})$. Whereas, $\tilde{G} = \tilde{G_1} \cup \tilde{G_2}$, $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$, $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. Then:

$$\mathcal{G} = \mathcal{H}(\alpha) = \left\{ \begin{array}{ll} \mathcal{H}(\alpha_1) & : \alpha_1 \in \mathcal{A}_1 \\ \mathcal{H}(\alpha_2) & : \alpha_2 \in \mathcal{A}_2 \\ \mathcal{H}(\alpha) & : \alpha \in \mathcal{A}_1 \cap \mathcal{A}_2 \end{array} \right\},$$

Hence, for $\alpha \in \mathcal{A}_1 \cap \mathcal{A}_2$ the CFSGr $\mathcal{H}(\alpha)$ has:

$$\Psi_\alpha(\nu) = \left\{ \begin{array}{ll} \Psi_{1,\alpha}(\nu) & : \nu \in V_1 \\ \Psi_{2,\alpha}(\nu) & : \nu \in V_2 \\ \max\{\Psi_{1,\alpha}(\nu), \Psi_{2,\alpha}(\nu)\} & : \nu \in V_1 \cap V_2 \end{array} \right\},$$

and

$$\Phi_\alpha(\nu\omega) = \left\{ \begin{array}{ll} \Phi_{1,\alpha}(\nu\omega) & : \nu\omega \in E_1 \\ \Phi_{2,\alpha}(\nu\omega) & : \nu\omega \in E_2 \\ \max\{\Phi_{1,\alpha}(\nu\omega), \Phi_{2,\alpha}(\nu\omega)\} & : \nu\omega \in E_1 \cap E_2 \end{array} \right\}.$$

So that, for $\mathcal{G} = \mathcal{G}_1 \bigsqcup \mathcal{G}_2$, if $\mathcal{A}_1 \cap \mathcal{A}_2 = \emptyset$, then it will CFSGr with all components of $\mathcal{G}_1,\ \mathcal{G}_2$ together.

Example 4.2. The following example present union of two CFSGrs, see Figure 3, whereas their membership values of vertices and edges are shown in the Table 1. Besides, the union of $\mathcal{G}_1$ and $\mathcal{G}_2$ is in the graph of Figure 4.

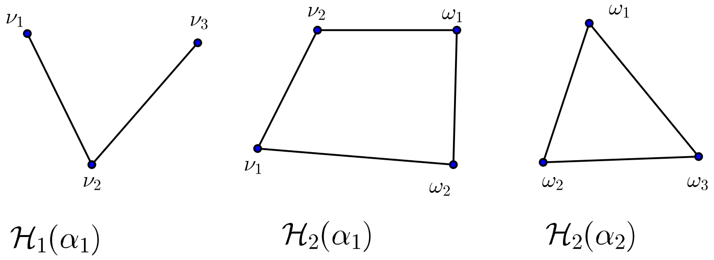

Figure 3: $\mathcal{G}_1$ and $\mathcal{G}_2$

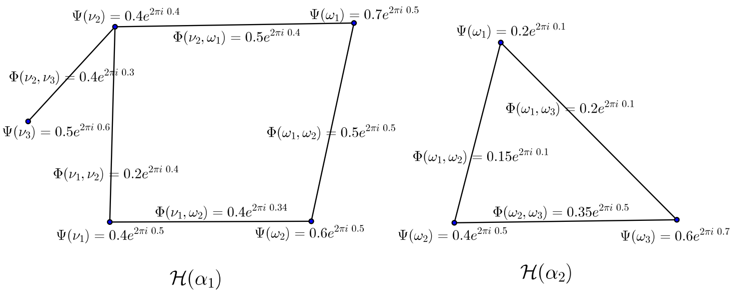

Figure 4: $\mathcal{G} = \mathcal{G}_1 \bigsqcup \mathcal{G}_2$.

It is an easy task to prove the following proposition.

Proposition 4.3. *The union of two CFSGrs is CFSGr.*

4.2 Cartesian product of two complex fuzzy soft graphs

An improvement of definition of cartesian product of two FSGrs, by Akram and Nawaz, is for two CFSGrs.

Definition 4.4. Let $\mathcal{G}_1 = (\tilde{G_1}, \Psi_1, \Phi_1, \mathcal{A}_1)$ and $\mathcal{G}_2 = (\tilde{G_2}, \Psi_2, \Phi_2, \mathcal{A}_2)$ be two complex fuzzy soft graphs (CFSGrs). The cartesian product of $\mathcal{G}_1$ and $\mathcal{G}_2$ is a CFSGr $\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 = (\tilde{G}, \Psi, \Phi, \mathcal{A})$ with $\tilde{G} = \tilde{G_1} \times \tilde{G_2}$, $(\Psi = \Psi_1 \times \Psi_2, \mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2)$ is a CFSS over $V = V_1 \times V_2$ and $(\Phi = \Phi_1 \times \Phi_2, \mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2)$ is a CFSS over $E = \{((\nu_1, \omega_1), (\nu_2, \omega_2)) :\ \nu_1 = \nu_2,\ (\omega_1, \omega_2) \in E_2 \text{ or } \omega_1 = \omega_2,\ (\nu_1, \nu_2) \in E_1\}$, such that:
1. $(\Psi_{1,\alpha_{\ell_1}} \times \Psi_{2,\alpha_{\ell_2}})(\nu, \omega) = \Psi_{(\alpha_{\ell_1}, \alpha_{\ell_2})}(\nu, \omega) = \Psi_{1,\alpha_{\ell_1}}(\nu) \wedge \Psi_{2,\alpha_{\ell_2}}(\omega)$.
2. $(\Phi_{1,\alpha_{\ell_1}} \times \Phi_{2,\alpha_{\ell_2}})((\nu_1, \omega_1), (\nu_2, \omega_2)) = \Phi_{(\alpha_{\ell_1}, \alpha_{\ell_2})}((\nu_1, \omega_1), (\nu_2, \omega_2)) =$

Ψ_1	ν_1	ν_2	ν_3		Φ_1		$\nu_1\nu_2$	$\nu_2\nu_3$
α_1	$0.2e^{2\pi i\ 0.4}$	$0.4e^{2\pi i\ 0.4}$	$0.5e^{2\pi i\ 0.6}$		α_1		$0.2e^{2\pi i\ 0.4}$	$0.4e^{2\pi i\ 0.3}$
Ψ_2	ν_1	ν_2	ω_1	ω_2	ω_3			
α_1	$0.4e^{2\pi i\ 0.5}$	$0.3e^{2\pi i\ 0.3}$	$0.6e^{2\pi i\ 0.5}$	$0.7e^{2\pi i\ 0.5}$	$0e^{2\pi i\ 0}$			
α_2	$0e^{2\pi i\ 0}$	$0e^{2\pi i\ 0}$	$0.2e^{2\pi i\ 0.1}$	$0.4e^{2\pi i\ 0.5}$	$0.6e^{2\pi i\ 0.7}$			
Φ_2	$\nu_1\nu_2$	$\nu_2\omega_1$	$\omega_1\omega_2$	$\omega_2\nu_1$	$\omega_2\omega_3$	$\omega_3\omega_1$		
α_1	$0.1e^{2\pi i\ 0.2}$	$0.5e^{2\pi i\ 0.4}$	$0.5e^{2\pi i\ 0.5}$	$0.4e^{2\pi i\ 0.3}$	$0e^{2\pi i\ 0}$	$0e^{2\pi i\ 0}$		
α_2	$0e^{2\pi i\ 0}$	$0e^{2\pi i\ 0}$	$0.15e^{2\pi i\ 0.1}$	$0e^{2\pi i\ 0}$	$0.35e^{2\pi i\ 0.5}$	$0.2e^{2\pi i\ 0.1}$		

Table 1: $\mathcal{G}_1$ and $\mathcal{G}_2$

$$\begin{cases} \Psi_{1,\alpha_{\ell_1}}(\nu_1) \wedge \Phi_{2,\alpha_{\ell_2}}(\omega_1,\omega_2): & \nu_1=\nu_2,\ (\omega_1,\omega_2)\in E_2 \\ \Phi_{1,\alpha_{\ell_1}}(\nu_1,\nu_2) \wedge \Psi_{2,\alpha_{\ell_2}}(\omega_1): & \omega_1=\omega_2,\ (\nu_1,\nu_2)\in E_1 \end{cases}$$

, where $\mathcal{H}(\alpha_{\ell_1},\alpha_{\ell_2}) = \mathcal{H}_1(\alpha_{\ell_1}) \times \mathcal{H}_2(\alpha_{\ell_2})\ \forall(\alpha_{\ell_1},\alpha_{\ell_2}) \in \mathcal{A}$.

The following example produces cartesian product of two CFSGrs.

Example 4.5. Let $\mathcal{A}_1 = \{a, b\}$ and $\mathcal{A}_2 = \{c, d\}$ be the sets of parameters. Consider two CFSGrs $\mathcal{G}_1 = \{\mathcal{H}_1(a), \mathcal{H}_1(b)\}$ and $\mathcal{G}_2 = \{\mathcal{H}_2(c), \mathcal{H}_2(d)\}$, see Table 2. Note that, $\mathcal{H}_1(a)$ and $\mathcal{H}_2(c)$ are in the Figure 5.
The cartesian product $\mathcal{H}(\alpha_{\ell_1},\alpha_{\ell_2}) = \{\mathcal{H}(a,c), \mathcal{H}(a,d), \mathcal{H}(b,c), \mathcal{H}(b,d)\}$, is CFSGr too, see Theorem 4.6. As an example we produce the subgraph $\mathcal{H}(a,c) = \mathcal{H}_1(a)\times\mathcal{H}_2(c)$, in the Figure 6. Then vertices and edges labelings will follow definition 4.4, for example, $\Psi_{(a,c)}(\nu_1,\omega_2) = \Psi_{1,a}(\nu_1)\wedge\Psi_{2,c}(\omega_2) = \min\{0.4e^{2\pi i\ 0.6}, 0.6e^{2\pi i\ 0.5}\} = 0.4e^{2\pi i\ 0.5}$, and $\Phi_{(a,c)}((\nu_1,\omega_1),(\nu_1,\omega_2)) =$
$\Psi_{1,a}(\nu_1) \wedge \Phi_{2,c}(\omega_1,\omega_2) = \min\{0.4e^{2\pi i\ 0.6}, 0.5e^{2\pi i\ 0.3}\} = 0.4e^{2\pi i\ 0.3}$.

Ψ_1	ν_1	ν_2	ν_3	Φ_1	$\nu_1\nu_2$	$\nu_2\nu_3$	$\nu_1\nu_3$
a	$0.4e^{2\pi i\ 0.6}$	$0.3e^{2\pi i\ 0.5}$	$0.4e^{2\pi i\ 0.3}$	a	$0.1e^{2\pi i\ 0.2}$	$0.3e^{2\pi i\ 0.2}$	$0e^{2\pi i\ 0}$
b	$0.5e^{2\pi i\ 0.4}$	$0.7e^{2\pi i\ 0.6}$	$0e^{2\pi i\ 0}$	b	$0.3e^{2\pi i\ 0.4}$	$0e^{2\pi i\ 0}$	$0e^{2\pi i\ 0}$
Ψ_2	ω_1	ω_2	ω_3	Φ_2	$\omega_1\omega_2$	$\omega_2\omega_3$	$\omega_1\omega_3$
c	$0.7e^{2\pi i\ 0.8}$	$0.6e^{2\pi i\ 0.5}$	$0.5e^{2\pi i\ 0.4}$	c	$0.5e^{2\pi i\ 0.3}$	$0.4e^{2\pi i\ 0.3}$	$0.3e^{2\pi i\ 0.2}$
d	$0.5e^{2\pi i\ 0.4}$	$0.7e^{2\pi i\ 0.6}$	$0.4e^{2\pi i\ 0.7}$	d	$0.3e^{2\pi i\ 0.4}$	$0.4e^{2\pi i\ 0.6}$	$0e^{2\pi i\ 0}$

Table 2: $\mathcal{G}_1$ and $\mathcal{G}_2$

To show that $\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2$ is CFSGr, where $\mathcal{G}_1$ and $\mathcal{G}_2$ are CFSGrs. It suffices to show that the inequality 1 in the Definition 3.1 is correct for cartesian product graph, hence we have the following theorem.

Theorem 4.6. *The cartesian product of two complex fuzzy soft graphs is a complex fuzzy soft graph.*

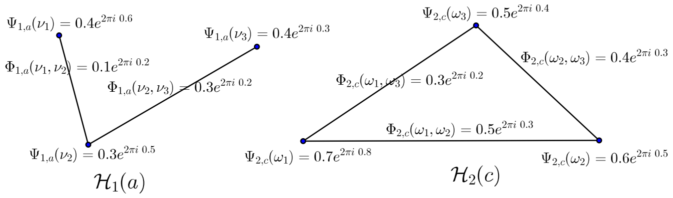

Figure 5: CFGrs $\mathcal{H}_1(a)$ and $\mathcal{H}_2(c)$

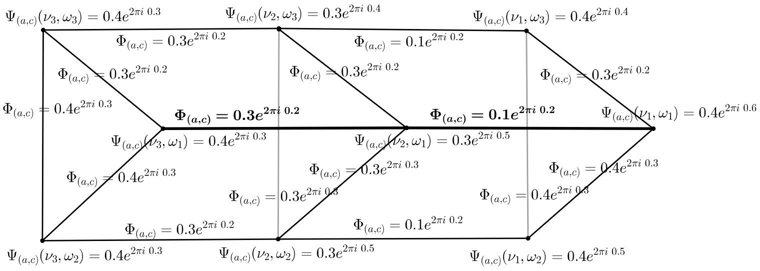

Figure 6: CFGr $\mathcal{H}(a, c) = \mathcal{H}_1(a) \times \mathcal{H}_2(c)$

Proof. Consider the Definition 3.1 of CFSGr and the Definition 4.4 of the cartesian product of two CFSGrs. Then we need only to ensure that the inequality 1 satisfied for cartesian product graph, which leads to show that:
$\eta_{(\alpha_{\ell_1},\alpha_{\ell_2})}((\nu_1, \omega_1), (\nu_2, \omega_2))exp(2\pi i\theta_{(\alpha_{\ell_1},\alpha_{\ell_2})}((\nu_1, \omega_1), (\nu_2, \omega_2))) \leq$
$\min\{\eta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_1, \omega_1)exp(2\pi i\theta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_1, \omega_1)),\ \eta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_2, \omega_2)*$
$exp(2\pi i\theta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_2, \omega_2))\}$.
We have two cases, either $\nu_1 = \nu_2$ and $(\omega_1, \omega_2) \in E_2$ so that the left hand side of previous inequality, by the Definition 4.4, is:
$\eta_{(\alpha_{\ell_1},\alpha_{\ell_2})}((\nu_1, \omega_1), (\nu_1, \omega_2))exp(2\pi i\theta_{(\alpha_{\ell_1},\alpha_{\ell_2})}((\nu_1, \omega_1), (\nu_1, \omega_2))) = \eta_{1,\alpha_{\ell_1}}(\nu_1)$
$*exp(2\pi i\theta_{1,\alpha_{\ell_1}}(\nu_1)) \wedge \eta_{2,\alpha_{\ell_2}}(\omega_1, \omega_2)exp(2\pi i\theta_{2,\alpha_{\ell_2}}(\omega_1, \omega_2)) \leq \eta_{1,\alpha_{\ell_1}}(\nu_1)$
$*exp(2\pi i\theta_{1,\alpha_{\ell_1}}(\nu_1)) \wedge \eta_{2,\alpha_{\ell_2}}(\omega_1)exp(2\pi i\theta_{2,\alpha_{\ell_2}}(\omega_1)) \wedge \eta_{2,\alpha_{\ell_2}}(\omega_2)exp(2\pi i\theta_{2,\alpha_{\ell_2}}(\omega_2))$; since $\mathcal{G}_2$ is CFSGr.

In addition, by the Definition 4.4, the right hand side of the first inequality, is:
$\min\{\eta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_1, \omega_1)exp(2\pi i\theta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_1, \omega_1)),\ \eta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_1, \omega_2)$
$*exp(2\pi i\theta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_1, \omega_2))\} = [\eta_{1,\alpha_{\ell_1}}(\nu_1)exp(2\pi i\theta_{1,\alpha_{\ell_1}}(\nu_1)) \wedge \eta_{2,\alpha_{\ell_2}}(\omega_1)$
$*exp(2\pi i\theta_{2,\alpha_{\ell_2}}(\omega_1))] \wedge [\eta_{1,\alpha_{\ell_1}}(\nu)exp(2\pi i\theta_{1,\alpha_{\ell_1}}(\nu)) \wedge \eta_{2,\alpha_{\ell_2}}(\omega_2)exp(2\pi i\theta_{2,\alpha_{\ell_2}}(\omega_2))]$
$= \eta_{1,\alpha_{\ell_1}}(\nu_1)exp(2\pi i\theta_{1,\alpha_{\ell_1}}(\nu_1)) \wedge \eta_{2,\alpha_{\ell_2}}(\omega_1)exp(2\pi i\theta_{2,\alpha_{\ell_2}}(\omega_1)) \wedge \eta_{2,\alpha_{\ell_2}}(\omega_2)$
$*exp(2\pi i\theta_{2,\alpha_{\ell_2}}(\omega_2))$. Hence, the first inequality satisfied.
Second case, we have, $\omega_1 = \omega_2,\ (\nu_1, \nu_2) \in E_1$ and the result will derived by same strategy, then the result follows. □

4.3 Tensor product of two complex fuzzy soft graphs

An improvement of definition of tensor product of two FSGrs, by Akram and Nawaz, is for two CFSGrs. It is named by cross product in their paper.

Definition 4.7. Let $\mathcal{G}_1 = (\tilde{G_1}, \Psi_1, \Phi_1, \mathcal{A}_1)$ and $\mathcal{G}_2 = (\tilde{G_2}, \Psi_2, \Phi_2, \mathcal{A}_2)$ be two complex fuzzy soft graphs (CFSGrs). The tensor product of $\mathcal{G}_1$ and $\mathcal{G}_2$ is a CFSGr $\mathcal{G} = \mathcal{G}_1 \Box \mathcal{G}_2 = (\tilde{G}, \Psi, \Phi, \mathcal{A})$; $\tilde{G} = \tilde{G_1} \Box \tilde{G_2}$ and $(\Psi = \Psi_1 \Box \Psi_2, \mathcal{A} = \mathcal{A}_1 \Box \mathcal{A}_2)$ is a CFSS over $V = V_1 \Box V_2$, also $(\Phi = \Phi_1 \Box \Phi_2, \mathcal{A} = \mathcal{A}_1 \Box \mathcal{A}_2)$ is a CFSS over $E = \{((\nu_1, \omega_1), (\nu_2, \omega_2)) :\ (\nu_1, \nu_2) \in E_1,\ (\omega_1, \omega_2) \in E_2\}$, such that:
1. $(\Psi_{1,\alpha_{\ell_1}} \Box \Psi_{2,\alpha_{\ell_2}})(\nu, \omega) = \Psi_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu, \omega) = \Psi_{1,\alpha_{\ell_1}}(\nu) \wedge \Psi_{2,\alpha_{\ell_2}}(\omega)$.

2. $(\Phi_{1,\alpha_{\ell_1}}\Box\Phi_{2,\alpha_{\ell_2}})((\nu_1,\omega_1),(\nu_2,\omega_2)) = \Phi_{(\alpha_{\ell_1},\alpha_{\ell_2})}((\nu_1,\omega_1),(\nu_2,\omega_2)) =$
$\Phi_{1,\alpha_{\ell_1}}(\nu_1,\nu_2)\wedge\Phi_{2,\alpha_{\ell_2}}(\omega_1,\omega_2);\ (\nu_1,\nu_2)\in E_1,\ (\omega_1,\omega_2)\in E_2$, where $\mathcal{H}(\alpha_{\ell_1},\alpha_{\ell_2}) = \mathcal{H}_1(\alpha_{\ell_1})\Box\mathcal{H}_2(\alpha_{\ell_2})$, $\forall\ (\alpha_{\ell_1},\alpha_{\ell_2})\in\mathcal{A}$.

Example 4.8. Using Figure 5, we can get the subgraph of tensor cross product; $\mathcal{H}(a,c) = \mathcal{H}_1(a)\Box\mathcal{H}_2(c)$, see Figure 7. The vertices labeling were written in the graph, and for the labeling of edges, we have $\Phi_{(a,c)}((\nu_2,\omega_1),(\nu_1,\omega_3)) = \Phi_{1,a}(\nu_1,\nu_2)\wedge\Phi_{2,c}(\omega_1,\omega_3) = \min\{0.1e^{2\pi i\ 0.2}, 0.3e^{2\pi i\ 0.2}\} = 0.1e^{2\pi i\ 0.2}$, as an example.
Moreover, using Table 2, we can define $\mathcal{G} = \mathcal{G}_1\Box\mathcal{G}_2$, that is CFSGr too, see the following theorem. Hence, we have 4 components and one of them is in the Figure 7.

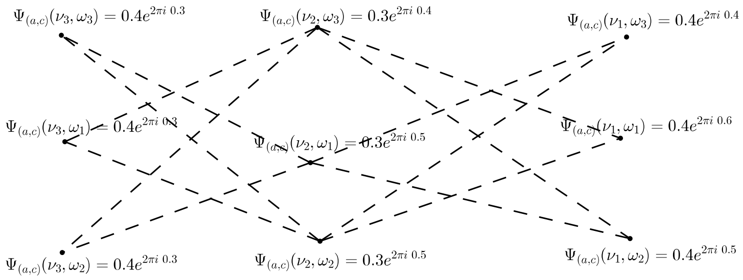

Figure 7: CFGr $\mathcal{H}(a,c) = \mathcal{H}_1(a)\Box\mathcal{H}_2(c)$

The tensor product of two CFSGrs is CFSGr too and that by using the Definition 4.7. Then, it suffices to show that the inequality 1 in the Definition 3.1 is correct for tensor product graph, hence we have the following theorem.

Theorem 4.9. *The tensor product of two complex fuzzy soft graphs is complex fuzzy soft graph.*

Proof. By using Definitions 3.1 and 4.7, we just need to satisfy inequality 1. Hence, consider $(\nu_1,\nu_2)\in E_1$ and $(\omega_1,\omega_2)\in E_2$, so that:
$\Phi(\alpha_{\ell_1},\alpha_{\ell_2})((\nu_1,\omega_1),(\nu_2,\omega_2)) = \eta_{(\alpha_{\ell_1},\alpha_{\ell_2})}((\nu_1,\omega_1),(\nu_2,\omega_2))*$
$exp(2\pi i\theta_{(\alpha_{\ell_1},\alpha_{\ell_2})}((\nu_1,\omega_1),(\nu_2,\omega_2))) = \eta_{1,\alpha_{\ell_1}}(\nu_1,\nu_2)exp(2\pi i\theta_{1,\alpha_{\ell_1}}(\nu_1,\nu_2))$
$\wedge\eta_{2,\alpha_{\ell_2}}(\omega_1,\omega_2)exp(2\pi i\theta_{2,\alpha_{\ell_2}}(\omega_1,\omega_2)) \leq \eta_{1,\alpha_{\ell_1}}(\nu_1)exp(2\pi i\theta_{1,\alpha_{\ell_1}}(\nu_2))\wedge\eta_{1,\alpha_{\ell_1}}(\nu_2)*$
$exp(2\pi i\theta_{1,\alpha_{\ell_1}}(\nu_2))\wedge\eta_{2,\alpha_{\ell_2}}(\omega_1)exp(2\pi i\theta_{2,\alpha_{\ell_2}}(\omega_1))\wedge\eta_{2,\alpha_{\ell_2}}(\omega_2)exp(2\pi i\theta_{2,\alpha_{\ell_2}}(\omega_2)) = \eta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_1,\omega_1)exp(2\pi i\theta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_1,$
$\eta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_2,\omega_2)*$
$exp(2\pi i\theta_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_2,\omega_2)) = (\Psi_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_1,\omega_1))\wedge(\Psi_{(\alpha_{\ell_1},\alpha_{\ell_2})}(\nu_2,\omega_2))$. Whereas, $\mathcal{G}_1$ and $\mathcal{G}_2$ are CFSGrs, hence result follows. $\Box$

4.4 Normal product of two complex fuzzy soft graphs

An improvement of definition of normal product of two FSGrs, by Akram and Nawaz, is for two CFSGrs. It is named by strong product in their paper.

Definition 4.10. Let $\mathcal{G}_1 = (\tilde{G_1},\Psi_1,\Phi_1,\mathcal{A}_1)$ and $\mathcal{G}_2 = (\tilde{G_2},\Psi_2,\Phi_2,\mathcal{A}_2)$ be two complex fuzzy soft graphs (CFSGrs). The normal product of $\mathcal{G}_1$ and $\mathcal{G}_2$ is a CFSGr $\mathcal{G} = \mathcal{G}_1\otimes\mathcal{G}_2 = (\tilde{G},\Psi,\Phi,\mathcal{A})$, with $\tilde{G} = \tilde{G_1}\otimes\tilde{G_2}$, $(\Psi = \Psi_1\otimes\Psi_2, \mathcal{A}_1\otimes\mathcal{A}_2)$ is a CFSS over $V = V_1\otimes V_2$, and $(\Phi = \Phi_1\otimes\Phi_2, \mathcal{A}_1\otimes\mathcal{A}_2)$ is a CFSS over $E = \{((\nu_1,\omega_1),(\nu_2,\omega_2)) :\ \nu_1 = \nu_2,\ (\omega_1,\omega_2)\in E_2$ or $\omega_1 = \omega_2,\ (\nu_1,\nu_2)\in E_1\}\cup\{((\nu_1,\omega_1),(\nu_2,\omega_2)) : (\nu_1,\nu_2)\in E_1,\ (\omega_1,\omega_2)\in E_2\}$, such that:
1. $(\Psi_{1,\alpha_{\ell_1}}\otimes\Psi_{2,\alpha_{\ell_2}})(\nu,\omega) = \Psi_{1,\alpha_{\ell_1}}(\nu)\wedge\Psi_{2,\alpha_{\ell_2}}(\omega)$.
2. $(\Phi_{1,\alpha_{\ell_1}}\otimes\Phi_{2,\alpha_{\ell_2}})((\nu_1,\omega_1),(\nu_2,\omega_2)) =$

$$\begin{cases} \Psi_{1,\alpha_{\ell_1}}(\nu_1)\wedge\Phi_{2,\alpha_{\ell_2}}(\omega_1,\omega_2) : & \nu_1 = \nu_2,\ (\omega_1,\omega_2)\in E_2 \\ \Phi_{1,\alpha_{\ell_1}}(\nu_1,\nu_2)\wedge\Psi_{2,\alpha_{\ell_2}}(\omega_1) : & \omega_1 = \omega_2,\ (\nu_1,\nu_2)\in E_1 \\ \Phi_{1,\alpha_{\ell_1}}(\nu_1,\nu_2)\wedge\Phi_{2,\alpha_{\ell_2}}(\omega_1,\omega_2) : & (\nu_1,\nu_2)\in E_1,\ (\omega_1,\omega_2)\in E_2 \end{cases}$$

, where $\mathcal{H}(\alpha_{\ell_1},\alpha_{\ell_2}) = \mathcal{H}_1(\alpha_{\ell_1})\otimes\mathcal{H}_2(\alpha_{\ell_2})\ \forall(\alpha_{\ell_1},\alpha_{\ell_2})\in\mathcal{A}$.

Using Figure 5 and Table 2, we can construct the subgraph of normal product as shown in the Figure 8.

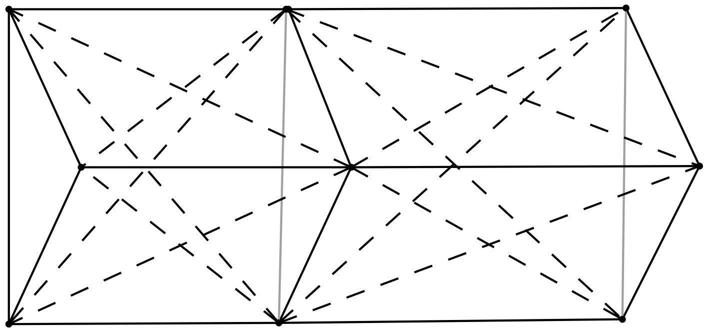

Figure 8: CFGr $\mathcal{H}(a,c)=\mathcal{H}_1(a)\otimes\mathcal{H}_2(c)$

Theorem 4.11. *The normal product of two complex fuzzy soft graphs is complex fuzzy soft graph.*

Proof. From Definition 4.10, one can show that $\mathcal{G}=(\mathcal{G}_1\times\mathcal{G}_2)\bigsqcup(\mathcal{G}_1\Box\mathcal{G}_2)$. Hence, by Theorem 4.6 and Theorem 4.9 result follows. ☐

4.5 Composition of two complex fuzzy soft graphs

An improvement of definition of composition of two FSGrs, by Akram and Nawaz, is for two CFSGrs.

Definition 4.12. Let $\mathcal{G}_1=(\tilde{G_1},\Psi_1,\Phi_1,\mathcal{A}_1)$ and $\mathcal{G}_2=(\tilde{G_2},\Psi_2,\Phi_2,\mathcal{A}_2)$ be two complex fuzzy soft graphs (CFSGrs). The composition of $\mathcal{G}_1$ and $\mathcal{G}_2$ is a CFSGr $\mathcal{G}=\mathcal{G}_1\circ\mathcal{G}_2=(\tilde{G},\Psi,\Phi,\mathcal{A})$, with $\tilde{G}=\tilde{G_1}\circ\tilde{G_2}$, $(\Psi=\Psi_1\circ\Psi_2,\mathcal{A}=\mathcal{A}_1\circ\mathcal{A}_2)$ is a CFSS over $V=V_1\circ V_2$, and $(\Phi=\Phi_1\circ\Phi_2,\mathcal{A}=\mathcal{A}_1\circ\mathcal{A}_2)$ is a CFSS over $E=\{((\nu_1,\omega_1),(\nu_2,\omega_2)):\ \nu_1=\nu_2,\ (\omega_1,\omega_2)\in E_2$ or $\omega_1=\omega_2,\ (\nu_1,\nu_2)\in E_1\}\ \cup\{((\nu_1,\omega_1),(\nu_2,\omega_2)):$ $(\nu_1,\nu_2)\in E_1,\ \omega_1\neq\omega_2\}$, such that:

1. $(\Psi_{1,\alpha_{\ell_1}}\circ\Psi_{2,\alpha_{\ell_2}})(\nu,\omega)=\Psi_{1,\alpha_{\ell_1}}(\nu)\wedge\Psi_{2,\alpha_{\ell_2}}(\omega)$.
2. $(\Phi_{1,\alpha_{\ell_1}}\circ\Phi_{2,\alpha_{\ell_2}})((\nu_1,\omega_1),(\nu_2,\omega_2))=$

$$\begin{cases}\Psi_{1,\alpha_{\ell_1}}(\nu_1)\wedge\Phi_{2,\alpha_{\ell_2}}(\omega_1,\omega_2): & \nu_1=\nu_2,\ (\omega_1,\omega_2)\in E_2\\ \Phi_{1,\alpha_{\ell_1}}(\nu_1,\nu_2)\wedge\Psi_{2,\alpha_{\ell_2}}(\omega_1): & \omega_1=\omega_2,\ (\nu_1,\nu_2)\in E_1\\ \Phi_{1,\alpha_{\ell_1}}(\nu_1,\nu_2)\wedge\Psi_{2,\alpha_{\ell_2}}(\omega_1)\wedge\Psi_{2,\alpha_{\ell_2}}(\omega_2): & (\nu_1,\nu_2)\in E_1,\ \omega_1\neq\omega_2\end{cases}$$

, where $\mathcal{H}(\alpha_{\ell_1},\alpha_{\ell_2})=\mathcal{H}_1(\alpha_{\ell_1})\circ\mathcal{H}_2(\alpha_{\ell_2})\ \forall(\alpha_{\ell_1},\alpha_{\ell_2})\in\mathcal{A}$.

Note that, if $\mathcal{H}_2(\alpha)$ complete graph then the composition of two CFSGr is normal product of these two complex fuzzy soft graphs, hence in Figure 8, we have $\mathcal{H}(a,c)=\mathcal{H}_1(a)\otimes\mathcal{H}_2(c)=\mathcal{H}_1(a)\circ\mathcal{H}_2(c)$.

Consequently, we have the following theorem.

Theorem 4.13. *The composition of two complex fuzzy soft graphs is complex fuzzy soft graph.*

By same structure of proof of Theorem 4.6, we can prove Theorem 4.13.

5 DM-Problem

For many years ago, the typical to purchase and sell products locally, if the goods transferred long distances then they were too expensive and very difficult to get. On the other hand, world logistics has also rapidly changed. The most important concept of logistics is Supply Change Management (SCM), which defined as an integral part of the businesses and is very essential to company success.[14] It is developed from the simple

processing to complex networks nowadays, for more details see.[15]
Also, some authors defined SCM as the systematic strategic coordination of traditional business. The aim or goal of this section is to apply two CFSGrs models on SCM and represent the terms of SCM, in the following DM-problem.

In this problem, we constructed two CFSGr models on SCM as follows:

1. Let nodes represent manufactory, farmer, retail store, and wholesale store. The labeling of nodes has complex values, where the amplitude term is proportion value of volume of goods and the phase term is the mission completion rate.
2. Edges represent brokers, for example transport cars, with complex labeling values such that the amplitude term is proportion value of volume of goods, and the phase term is the mission completion rate.
3. We have four parameters, i.e. goods, of CFSGr are:
$\alpha_1 =$ honey, $\alpha_2 =$ vegetables and fruits, $\alpha_3 =$ dates, and $\alpha_4 =$ olives.
4. $\mathcal{G}_1$ is CFSGr in Jordan, where $V_1 = \{\omega_1 = \text{manufactory}, \omega_2 = \text{farmer}, \omega_3 = \text{retail store}\}$ and $E_1 = \{\omega_1\omega_2, \omega_2\omega_3, \omega_3\omega_1\}$. In contrast, $\mathcal{G}_2$ is CFSGr in UAE, where $V_2 = \{\nu_1 = \text{manufactory}, \nu_2 = \text{farmer}, \nu_3 = \text{retail store}, \nu_4 = \text{wholesale store}\}$ and $E_2 = \{\nu_1\nu_2, \nu_2\nu_3, \nu_3\nu_4, \nu_4\nu_1, \nu_2\nu_4\}$, see the model in the Figure 9. Note that, complex membership of these two models are given in the table 3.
5. In the model 1 the first component is $\mathcal{H}_1(\alpha_1)$. We have here a farmer is deal with manufactory then from manufactory to retail store, where we have the farmer prepare 60% of objective amount with 60% of the mission completion, manufactory prepare 65% of honey products with 60% for the mission completion, and retail store sold 55% of this good with 65% of the mission completion. Moreover, the broker holds 55% from his amount of honey to send to the manufactory with 45% of the mission completion, then another broker holds 50% from his amount of honey product to retail store with 60% of the mission completion. That is for honey as a parameter. In the same manner can be discuss the rest of components; the graph of $\mathcal{H}_1(\alpha_2)$ is to flow the vegetables and fruits from farmer to retail store direct. For graph of $\mathcal{H}_1(\alpha_4)$ is to flow olives from farmer to manufactory direct or from farmer to retail story direct.

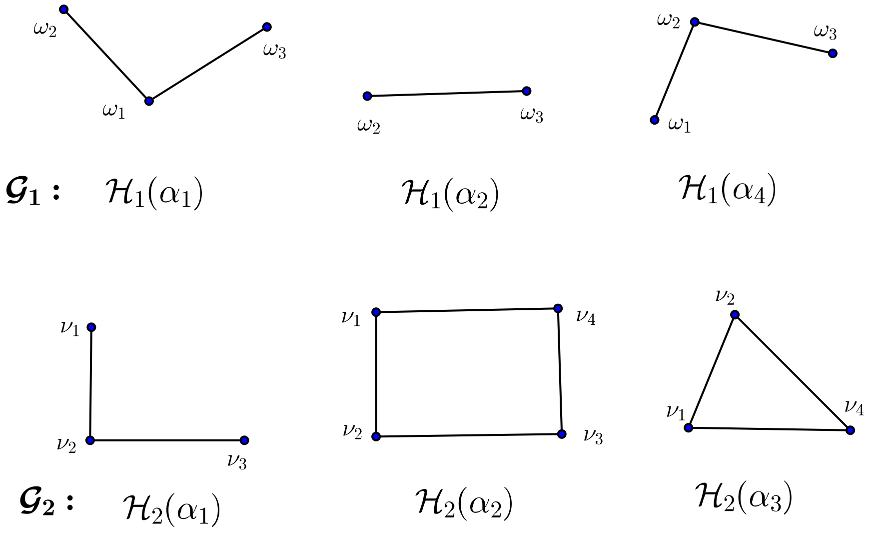

Figure 9: $\mathcal{G}_1$ and $\mathcal{G}_2$

a) Another approach is the order of each component; $O(\mathcal{H}_1(\alpha_1)) = 1.8e^{2\pi i\ 1.85}$, $O(\mathcal{H}_1(\alpha_2)) = 1.35e^{2\pi i\ 1.55}$, and $O(\mathcal{H}_1(\alpha_4)) = 1.1e^{2\pi i\ 1.4}$, see the definition 3.2. The previous order values show that the amplitude(volume) with phase(mission completion rate) in the first good(honey) are better than other goods. In addition, the size of each component; $S(\mathcal{H}_1(\alpha_1)) = 1.05e^{2\pi i\ 1.05}$, $S(\mathcal{H}_1(\alpha_2)) = 0.4e^{2\pi i\ 0.3}$, and $S(\mathcal{H}_1(\alpha_4)) = 0.6e^{2\pi i\ 0.7}$, see the definition 3.3. These values of size of components in Model 1 are showing that the broker with the first good(honey) is effective more than other goods.
b) Another view is the relation between amplitude(volume) with phase(mission completion rate) in the same graph. For example, in the component $\mathcal{H}_1(\alpha_1)$, we have $O(\mathcal{H}_1(\alpha_1)) = 1.8e^{2\pi i\ 1.85}$, which means for more

Ψ_1	ω_1	ω_2	ω_3	Φ_1	$\omega_1\omega_2$	$\omega_2\omega_3$	$\omega_3\omega_1$
α_1	$0.65e^{2\pi i\ 0.6}$	$0.6e^{2\pi i\ 0.6}$	$0.55e^{2\pi i\ 0.65}$	α_1	$0.55e^{2\pi i\ 0.45}$	$0e^{2\pi i\ 0}$	$0.5e^{2\pi i\ 0.6}$
α_2	$0.4e^{2\pi i\ 0.5}$	$0.45e^{2\pi i\ 0.45}$	$0.5e^{2\pi i\ 0.6}$	α_2	$0e^{2\pi i\ 0}$	$0.4e^{2\pi i\ 0.3}$	$0e^{2\pi i\ 0}$
α_4	$0.2e^{2\pi i\ 0.4}$	$0.4e^{2\pi i\ 0.4}$	$0.5e^{2\pi i\ 0.6}$	α_4	$0.2e^{2\pi i\ 0.4}$	$0.4e^{2\pi i\ 0.3}$	$0e^{2\pi i\ 0}$

Ψ_2	ν_1	ν_2	ν_3	ν_4
α_1	$0.4e^{2\pi i\ 0.5}$	$0.3e^{2\pi i\ 0.3}$	$0.6e^{2\pi i\ 0.5}$	$0e^{2\pi i\ 0}$
α_2	$0.6e^{2\pi i\ 0.7}$	$0.65e^{2\pi i\ 0.6}$	$0.45e^{2\pi i\ 0.5}$	$0.5e^{2\pi i\ 0.6}$
α_3	$0.5e^{2\pi i\ 0.55}$	$0.45e^{2\pi i\ 0.5}$	$0e^{2\pi i\ 0}$	$0.6e^{2\pi i\ 0.4}$

Φ_2	$\nu_1\nu_2$	$\nu_2\nu_3$	$\nu_3\nu_4$	$\nu_4\nu_1$	$\nu_1\nu_3$	$\nu_2\nu_4$
α_1	$0.3e^{2\pi i\ 0.2}$	$0.25e^{2\pi i\ 0.3}$	$0e^{2\pi i\ 0}$	$0e^{2\pi i\ 0}$	$0e^{2\pi i\ 0}$	$0e^{2\pi i\ 0}$
α_2	$0.55e^{2\pi i\ 0.6}$	$0.35e^{2\pi i\ 0.4}$	$0.4e^{2\pi i\ 0.5}$	$0.5e^{2\pi i\ 0.6}$	$0e^{2\pi i\ 0}$	$0e^{2\pi i\ 0}$
α_3	$0.35e^{2\pi i\ 0.5}$	$0e^{2\pi i\ 0}$	$0e^{2\pi i\ 0}$	$0.45e^{2\pi i\ 0.3}$	$0e^{2\pi i\ 0}$	$0.4e^{2\pi i\ 0.35}$

Table 3: $\mathcal{G}_1$ and $\mathcal{G}_2$

value of volume, the result more value of mission completion rate will be. Also, we have $S(\mathcal{H}_1(\alpha_1)) = 1.05e^{2\pi i\ 1.05}$, which leads that if the broker flows more volume, his mission completion rate will be enhance. Hence, amplitude(volume) will be affect on phase(mission completion rate).

6. More general, to find the order and the size for CFSGrs $\mathcal{G}_1$ and $\mathcal{G}_2$:

i) We have $O(\mathcal{G}_1) = 4.25e^{2\pi i\ 4.8}$ and $O(\mathcal{G}_2) = 5.05e^{2\pi i\ 4.55}$, by the Definition 3.2. The interpretation of these values says that orders are not comparable. Whereas, we have the volume in model 2 is better than model 1 because $5.05 > 4.25$. But the mission completion rate in model 1 better than model 2 because $4.8 > 4.55$.

ii) We have $S(\mathcal{G}_1) = 2.05e^{2\pi i\ 2.05}$ and $S(\mathcal{G}_2) = 3.55e^{2\pi i\ 3.75}$, by the Definition 3.3. Here we get that model 2 is better than model 1. Note that both parameters the volume of goods and the mission completion rate in the model 2 are better than in the model 1.

Moreover, in the model $\mathcal{G}_1$ we have positive affectiveness between amplitude(volume) and phase(mission completion rate) which is true for vertices and for edges. Similarly for the model $\mathcal{G}_2$ we have the same result.

There are many elements in CFSGr can be discussed in the subgraphs(components) of the given model or consider a CFSGr model as a complete unit. The two important coefficients that affect results are volume of goods and mission completion rate. Hence model 2 can be discussed as model 1 in the point above with number 5. Another direction, we can make a comparison between two components in the two models with same good for example honey, see $\mathcal{H}_1(\alpha_1)$ vs. $\mathcal{H}_2(\alpha_1)$. In conclusion, we have presented ideas and discussion points. Perhaps the topic will be directed at specialists in this field to use CFSGr to improve supply chain management (SCM).

6 Discussion

In conclusion, this paper gives the first formalization of complex fuzzy soft graphs (CFSGrs), where we applied the principles of CFSSs to graph theory. We introduced the concept of CFSGr and provided comprehensive definitions and notations for CFSGr, including the union, cartesian product, tensor product, normal product and composition of two CFSGrs. To ensure clarity and understanding, we presented numerous examples that effectively illustrate the concept. One of the contributions is to upgrade in the fuzzy graph field, like as investigate lexicographic product of two CFSGrs, complement of CFSGr, joint of two CFSGrs, and regular CFSGr. Another contribution is to employ CFSGr in applications through sciences.

References

[1] D.A. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications, 37 (1999), 19–31.

[2] M.I. Ali, F. Feng, X.Y. Liu,W.K. Min and M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications, 57 (2009), 1547–1553.

[3] P.K. Maji, A.R. Roy and R. Biswas, Fuzzy soft sets, Journal of fuzzy Mathematics, 9(3) (2001), 589–602.

[4] Naim. Cagman, S. Enginoglu and F. Citak, Fuzzy Soft Set Theory and Its Applications, Iranian Journal of Fuzzy Systems, 8(3) (2011), 137-147.

[5] D. Ramot, R. Milo, M. Friedman and A. Kandel, Complex fuzzy sets, IEEE Trans. Fuzzy Syst., Vol.10 (2002), pp. 171-186.

[6] P. Thirunavukarasu, R. Suresh and V. Ashokkumar, Theory of complex fuzzy soft set and its applications, International Journal for Innovative Research in Science & Technology, 3(10) (2017), 13-18.

[7] A. Kauffman, Introduction a la Theorie des Sous-emsembles Flous, Masson et Cie, Vol.1, 1973.

[8] A. Rosenfeld, Fuzzy graphs, in: L.A. Zadeh, K.S. Fu and M. Shimura (Eds.), Fuzzy Sets and Their Applications, Academic Press, New York, 1975, pp. 77–95.

[9] P. Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognition Letter, 6 (1987), 297–302.

[10] J.N. Mordeson and C.S. Peng, Operations on fuzzy graphs, Information Sciences, 79 (1994), 159–170.

[11] M.S. Sunitha and A. Vijayakumar, Complement of a fuzzy graph, Indian J Pure Appl Mat,h 33 (2002), 1451–1464.

[12] M. Akram and S. Nawaz, On fuzzy soft graphs. Italian J Pure Appl Math 34 (2015), 497–514.

[13] M. Akram and S. Nawaz, Fuzzy soft graphs with applications. Journal of Intelligent & Fuzzy Systems, 30(6) (2016), 3619-3632.

[14] Zdenka Vidrova, Supply chain management in the aspect of globalization, Globalization and its Socio-Economic Consequences 2019, Vol.74, 2020.

[15] L. Horvath, Collaboration: The key to value creation in supply chain management. Supply Chain Management: An International Journal, 6 (2001), 205-207.