Possibility of Quadripartitioned Neutrosophic Cubic Sets and Their Application of Multi-Criteria Decision Making

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Abstract

This study introduces the innovative idea of associating a possibility measure with the membership of an element in a set, and further proposes the structure of quadripartitioned neutrosophic cubic sets (PQNCS). Within this framework, the authors define four distinct components—truth, contradiction, ignorance, and falsity—each in two modes: internal and external. They explore the corresponding sets (truth-internal, contradiction-internal, ignorance-internal, falsity-internal and truth-external, contradiction-external, ignorance-external, falsity-external) and uncover their interrelated properties. Moreover, the work emphasizes the role of a score function as a central instrument for multi-attribute decision-making, and examines how measures of PQNCS—through score, accuracy and certainty functions grounded in the possibility concept—can be employed to support and guide decision-making in the quadripartitioned neutrosophic cubic setting.

Keywords: Possibility; Internal and external possibility; Multi-attribute decision-making; Score's comparative function; Accuracy and certainty

1. Introduction

KIM, Smarandache, and Y.B. Jun [3] In order to reflect uncertainty and imprecision, neutrosophic sets—which incorporate truth, falsity, and other properties—have been introduced. This covers the notions of truth and falsity, both internal and external, and the union and intersection operations for neutrosophic cubic sets [3], both internal and external. Normal forms are used to express the coupled concepts, treating variables and linguistic connectives as fuzzy to enable more flexible modeling of uncertainty. It is especially helpful in complicated decision-making processes since it offers a framework for tackling real-life issues by taking into account many facets of uncertainty and imprecision. The idea of set theoretic operations is addressed by quadripartitioned interval sets [9].

We provide the new concept of possibility quadripartitioned neutrosophic cubic sets (PQNCS). Further investigate the P-order PQNCS and define the concepts of internal and external PQNCS. These ideas offer a framework for managing imprecision and ambiguity in a range of decision-making situations.

We use the score function to compute and compare the effectiveness of various sets in order to illustrate their usefulness. The comparative analysis takes into account many facets of possibility, truth, and contradiction in order to assess how well the suggested approach performs in resolving real-world issues.

2. Preliminaries

Definition – 2.1[2]

Let $X \neq \phi$. A cubic set in X is determined by

$$[\mathrm{D} = \{(\mathrm{x}, \mathrm{M}(\mathrm{x}), \lambda(x)\ ) \ / \ x \in X\ \}]_{cubic}$$

DOI: https://doi.org/10.54216/IJNS.260429

where λ is a fuzzy set in X and M is an interval valued fuzzy set in X.

Definition – 2.2[3]

Let X be non-empty set. A neutrosophic cubic set (NCS) in Y is a pair of $\mathfrak{B} = (B, \lambda)$ where $[B = \{< y : B_T(y), B_I(y), B_F(y) > y \in Y\}]_{IVN}$ is an interval valued neutrosophic set in X and $[\Lambda = \{< y : \lambda_T(y), \lambda_I(y), \lambda_F(y) > y \in Y\}]_N$ is a neutrosophic set in Y.

Definition – 2.3[6]

Let Y$\neq \phi$. A quadripartitioned neutrosophic cubic set (QNCS) in Y is a pair $\mathcal{M} = (M, \lambda)$ where $\mathcal{M} = \{< y : M_T(y), M_C(y), M_I(y), M_F(y) > / y \in Y\}$ is an interval quadripartitionedeutrosophic set in Y, where M_T, M_C, M_I, M_F are the degrees of truth, contradiction, ignorance, falsity membership functions respectively and

$\lambda = \{< x : \lambda_T(y), \lambda_C(y), \lambda_I(y), \lambda_F(y) > / y \in Y\}$ is a quadripartitioned neutrosophic set in Y, $\lambda_T(y), \lambda_C(y), \lambda_I(y), \lambda_F(y)$ are the degrees of truth, contradiction, ignorance, falsity membership functions respectively and $0 \leq Sup\ M_T(x) + Sup\ M_C(x) + Sup\ M_I(x) + Sup\ M_F \leq 4$.

Definition: 2.4 [6]

Let Y$\neq \phi$. Quadripartition neutrosophic cubic set $\mathfrak{B} = (B, \Lambda)$

in Y is said to be,

- Truth- internal (T- internal), It is defined as,

  $$[((\forall y \in Y)(B_T^-(y) \leq \Lambda_T(y) \leq B_T^+(y)]_{int} \quad (3.1)$$

- Contradiction- internal (C- internal), It is defined as,

  $$[((\forall y \in Y)(B_C^-(y) \leq \Lambda_C(y) \leq B_C^+(y))]_{int} \quad (3.2)$$

- Ignorance- internal (I- internal), It is defined as,

  $$[((\forall y \in Y)(B_I^-(y) \leq \Lambda_I(y) \leq B_I^+(y))]_{int} \quad (3.3)$$

- False- internal (F- internal), It is defined as,

  $$[((\forall y \in Y)(B_F^-(y) \leq \Lambda_F(y) \leq B_F^+(y))]_{int} \quad (3.4)$$

The above internals are satisfying the quadripartition neutrosophic cubic set in Y.

Definition: 2.5 [1]

Consider Y$\neq \phi$. Let $\mathbb{V} = (V, \alpha)$ and $\mathbb{W} = (W, \beta)$ be a neutrosophic cubic sets in Y.

1) P_{PQNCS}-OR is denoted by $(V, \alpha) \vee_P (W, \beta)$ and defined as $\mathbb{V} \cup_P \mathbb{W} = (V \cup W, \alpha \vee \beta)$
2) P_{PQNCS}-AND is denoted by $(V, \alpha) \wedge_P \beta$ and defined as $\mathbb{V} \cap_P \mathbb{W} = (V \cap W, \alpha \wedge \beta)$

where,
$$V \cup W = \{max(V_T, W_T), max(V_C, W_C), min(V_I, W_I), min(V_F, W_F)\}$$
$$\alpha \vee \beta = \{max(\alpha_T, \beta_T), max(\alpha_C, \beta_C), min(\alpha_I, \beta_I), min(\alpha_F, \beta_F)\}$$
$$V \cap W = \{min(V_T, W_T), min(V_C, W_C), max(V_I, W_I), max(V_F, W_F)\}$$
$$\alpha \wedge \beta = \{min(\alpha_T, \beta_T), min(\alpha_C, \beta_C), max(\alpha_I, \beta_I), max(\alpha_F, \beta_F)\}$$

Definition – 2.7[5]

Consider Y$\neq \phi$. E=(e) is an attribute set, then the possibility neutrosophic cubic sets Υ_y over Y,

$$\Upsilon_y(e) = \{< y : L_T(y), L_I(y), L_F(y) >, < T(y), I(y), F(y) >, \theta(y) / y \in Y\}$$

Where $L_T(y), L_I(y), L_F(y)$ are interval valued neutrosophic sets, $T(y), I(y), F(y)$ are neutrosophic sets, $\theta(x)$ is the possibility element of $y \in Y$ to $\mathcal{K}_y$. It is denoted by $\Upsilon_y = (\varphi_A, \theta)$.

Definition - 2.8[12]

Let a and b be two real numbers, then Dombi T-norm and T-conorm in NCS is defined as,

$$D_{NCS}(g, h) = \frac{1}{1 + \left[\left(\frac{1-a}{a}\right)^\rho + \left(\frac{1-b}{b}\right)^\rho\right]^{\frac{1}{\rho}}}$$

$$D_{NCS}{}^C(g, h) = 1 - \frac{1}{1 + \left[\left(\frac{1-a}{a}\right)^\rho + \left(\frac{1-b}{b}\right)^\rho\right]^{\frac{1}{\rho}}}$$

DOI: https://doi.org/10.54216/IJNS.260429

If $\rho > 0$, then D_{NCS}(a,b) is conjunction its satisfying D_{NCS} (0,0) = D_{NCS} (0,1) = D_{NCS} (1,0) = 0 and D_{NCS} (1,1) = 1, and ${D_{NCS}}^{C}(a,b)$ is disjunction its satisfying ${D_{NCS}}^{C}(0,0)$ = 0, ${D_{NCS}}^{C}(0,1) = {D_{NCS}}^{C}(1,0) = {D_{NCS}}^{C}(1,1) = 1$. Suppose If $\rho \prec 0$, then the operator D_{NCS} (a, b) is disjunction and ${D_{NCS}}^{C}(a,b)$ is conjunction.

Definition - 2.9[12]

Let Y be a non-empty set.

G = $\{< y : G_T(y), G_C(y), G_I(y), G_F(y) >, < T_G(y), G_C(y), I_G(y), F_G(y) >, \theta(y)/y \in Y\}$,

H = $\{< y : H_T(y), H_C(y), H_I(y), H_F(y) >, < T_H(y), C_H(y), I_H(y), F_H(y) >, \vartheta(y)/y \in Y\}$,

be two PNCS in X. Then the distance measure of PNCS is,

$$d_{PNCS}(G,B) = \sum_{i=1}^{n}\{[G_T^-(y) - H_T^-(y)] + [G_T^+(y) - H_T^+(y)] + [G_I^-(y) - H_I^-(y)] + [G_I^+(y) - H_I^+(y)] + [G_F^-(y) - H_F^-(y)] + [G_F^+(y) - H_F^+(y)]) + (T_G(y) - T_H(y)) + (I_G(y) - I_H(y)) + (F_G(y) - F_H(y)) + (\theta(y) - \vartheta(y)\}/10$$

3. Possibility Quadripartitioned Neutrosophic Cubic Sets (PQNCS)

Definition – 3.1

Let X be a universal set and A is an attribute set, then a possibility quadripartitioned neutrosophic cubic set $\mathcal{K}_X$ over X. $\mathcal{K}_X(A)$=$\{x: < Q_T(x), Q_C(x), Q_I(x), Q_F(x) >, < T_Q(x), C_Q(x), I_Q(x), F_Q(x) >, \theta(x)/x \in X\}$ Where $Q_T(x), Q_C(x), Q_I(x), Q_F(x)$ are interval valued neutrosophic sets, $T_Q(x), C_Q(x), I_Q(x), F_Q(x)$ are neutrosophic sets, $\theta(x)$ is the possibility element of $\mathcal{K}_X$. It is denoted by $\mathcal{K}_X$ = (φ_A, θ).

Example - 3.1

Let X = $\{x_1, x_2, x_3, x_4\}$ be a collection of four companies firms, and let J = $\{j_1, j_2, j_3, j_4\}$ be a collection of attributes, where j_1=Raw material, j_2= Production list, j_3= Marketing, j_4= Price list. The decision maker evaluates alternative x_1 under j_1, and then the evaluation is,

$\mathcal{K}_X$ (x_1) (j_1) = {< [0.2,0.3], [0.1,0.2], [0.3,0.4], [0.4,0.5]>, <0.1,0.3,0.4,0.5>, 0.2}

The PQNCS are comprised of the evaluation value of the alternative under all attributes as follows:

$\mathcal{K}_X$ (x_1) = [(< [0.25,0.3], [0.1,0.23], [0.33,0.4], [0.46,0.5]>, <0.2,0.3,0.5,0.6>, 0.2)
(< [0.3,0.4], [0.4,0.5], [0.4,0.5], [0.5,0.6]>, <0.4,0.4,0.5,0.6>, 0.5)
(< [0.1,0.3], [0.2,0.3], [0.3,0.6], [0.6,0.7]>, <0.1,0.2,0.5,0.5>, 0.1)
(< [0.2,0.4], [0.15,0.2], [0.35,0.4], [0.45,0.5]>, <0.1,0.35,0.4,0.5>, 0.35)]

Different decision makers provide the option of quadripartitioned neutrosophic cubic sets.

$\mathcal{K}_X$ (x_2) = [(< [0.1,0.3], [0.2,0.26], [0.2,0.3], [0.5,0.55]>, <0.3,0.4,0.5,0.7>, 0.4)
(< [0.3,0.4], [0.5,0.6], [0.4,0.6], [0.5,0.56]>, <0.2,0.3,0.4,0.6>, 0.3)
(< [0.1,0.2], [0.2,0.35], [0.3,0.4], [0.4,0.7]>, <0.1,0.3,0.4,0.5>, 0.1)
(< [0.2,0.3], [0.1,0.25], [0.15,0.3], [0.25,0.4]>, <0.3,0.2,0.5,0.6>, 0.5)]

$\mathcal{K}_X$ (x_3) = [(< [0.15,0.2], [0.1,0.23], [0.33,0.4], [0.46,0.5]>, <0.4,0.3,0.6,0.7>, 0.5)
(< [0.2,0.4], [0.3,0.4], [0.4,0.6], [0.5,0.6]>, <0.25,0.45,0.55,0.65>, 0.3)
(< [0.3,0.35], [0.25,0.35], [0.5,0.6], [0.6,0.8]>, <0.13,0.2,0.4,0.5>, 0.2)
(< [0.4,0.5], [0.35,0.5], [0.5,0.7], [0.6,0.9]>, <0.24,0.3,0.35,0.4>, 0.4)]

$\mathcal{K}_X$ (x_4) = [(< [0.14,0.2], [0.4,0.5], [0.5,0.65], [0.6,0.8]>, <0.2,0.3,0.4,0.6 >, 0.4)
(< [0.2,0.3], [0.3,0.4], [0.4,0.6], [0.5,0.6]>, <0.1,0.3,0.5,0.7>, 0.3)
(< [0.3,0.42], [0.3,0.53], [0.4,0.65], [0.55,0.7]>, <0.2,0.3,0.6,0.9>, 0.5)
(< [0.2,0.3], [0.3,0.4], [0.4,0.7], [0.5,0.6]>, <0.3,0.45,0.5,0.7>, 0.6]

DOI: https://doi.org/10.54216/IJNS.260429

The different decision makers give same alternative model in PQNCS, such as

$\mathcal{L}_X\ (x_1)$ = [(< [0.1,0.2], [0,0.1], [0.4,0.5], [0.5,0.6]>, <0.2,0.3,0.5,0.6>, 0.2)

(< [0.2,0.3], [0.3,0.4], [0.5,0.6], [0.6,0.7]>, <0.4,0.4,0.5,0.6>, 0.5)

(< [0,0.2], [0.1,0.2], [0.4,0.7], [0.7,0.8]>, <0.1,0.2,0.5,0.5>, 0.3)

(< [0.1,0.3], [0,0.1], [0.4,0.5], [0.5,0.6]>, <0.1,0.35,0.4,0.5>, 0.2)]

$\mathcal{L}_X\ (x_2)$ = [(< [0,0.2], [0.1,0.2], [0.3,0.4], [0.6,0.65]>, <0.2,0.3,0.6,0.8>, 0.4)

(< [0.2,0.3], [0.4,0.5], [0.5,0.7], [0.6,0.7]>, <0.1,0.2,0.5,0.7>, 0.3)

(< [0,0.1], [0.1,0.25], [0.4,0.5], [0.5,0.8]>, <0,0.2,0.5,0.6>, 0.2)

(< [0.1,0.2], [0,0.15], [0.2,0.4], [0.3,0.5]>, <0.2,0.1,0.6,0.7>, 0.5)]

$\mathcal{L}_X\ (x_3)$ = [(< [0,0.1], [0,0.13], [0.4,0.5], [0.5,0.6]>, <0.5,0.2,0.7,0.8>, 0.5)

(< [0.1,0.3], [0.2,0.3], [0.5,0.7], [0.6,0.7]>, <0.2,0.4,0.6,0.7>, 0.3)

(< [0.2,0.25], [0.2,0.3], [0.6,0.7], [0.7,0.9]>, <0.13,0.2,0.4,0.5>, 0.2)

(< [0.3,0.4], [0.3,0.4], [0.6,0.8], [0.7,0.95]>, <0.24,0.3,0.35,0.4>, 0.4)]

$\mathcal{L}_X\ (x_4)$ = [(< [0,0.], [0.3,0.4], [0.4,0.7], [0.7,0.9]>, <0.2,0.3,0.4,0.6 >, 0.4)

(< [0.1,0.2], [0.2,0.3], [0.5,0.7], [0.6,0.7]>, <0.1,0.3,0.5,0.7>, 0.3)

(< [0.2,0.3], [0.2,0.4], [0.5,0.7], [0.6,0.8]>, <0.2,0.3,0.6,0.9>, 0.5)

(< [0.1,0.2], [0.2,0.3], [0.5,0.8], [0.6,0.7]>, <0.3,0.45,0.5,0.7>, 0.6]

Definition – 3.2

Let $\mathcal{K}_Y$ ={$< y :\ Q_T(y),\ Q_C(y),\ Q_I(y),\ Q_F\ (y) >, < T_Q(y), C_Q(y), I_Q(y), F_Q(y) >, \theta(y)/y \in\ Y\}$, $\mathcal{L}_Y$={$< y :\ R_T(y),\ R_C(y),\ R_I(y), R_F\ (y) >, < T_R(y), C_R(y), I_R(y), F_R(y) >, \vartheta(y)/y \in\ Y\}$ are two PQNCS in Y, then subset of PQNCS as defined as $\mathcal{K}_Y \subseteq \mathcal{L}_Y$ as follows,

1) $\theta(y) \le\ \vartheta(y)$, for all $y \in\ Y$
2) $Q_T(y) \le R_T(y),\ Q_C(y) \le\ R_C(y), Q_I(y) \ge R_I(y),\ Q_F\ (y) \ge\ R_F\ (y)$,

$T_Q(y) \le T_R(y), C_Q(y) \le C_R(y), I_Q(y) \ge I_R(y), F_Q(y) \ge F_R(y)$, for all $y \in\ Y$.

Example - 3.2

Let X= $\{x_1, x_2, x_3\}$ are three banks, and let $\mathfrak{A} = \{a_1, a_2, a_3\}$ are their criteria, where a_1= Interest rate, a_2= loan fixed rate, a_3= loan floating rate

$\mathcal{K}_X\ (x_1)$ = [(< [0.0,0.3], [0.3,0.6], [0.2,0.3], [0.1,0.2]>, <0.1,0.3,0.4,0.5>, 0.2)

(< [0.1,0.3], [0.2,0.3], [0.1,0.25], [0.2,0.4]>, <0.2,0.3,0.5,0.6>, 0.3)

(< [0.15,0.2], [0.25,0.3], [0.3,0.4], [0.35,0.4]>, <0.1,0.2,0.5,0.6>, 0.4)

(< [0.2,0.3], [0.2,0.35], [0.3,0.4], [0.35,0.5]>, <0.15,0.2,0.35,0.4>, 0.5)]

$\mathcal{K}_X\ (x_2)$ = [(< [0.3,0.6], [0.1,0.2], [0.25,0.3], [0.3,0.35]>, <0.3,0.1,0.2,0.4>, 0.1)

(< [0.2,0.3], [0.1,0.3], [0.25,0.3], [0.3,0.45]>, <0.1,0.2,0.3,0.4>, 0.3)

(< [0.25,0.3], [0.35,0.4], [0.45,0.5], [0.55,0.6]>, <0.15,0.2,0.25,0.3>, 0.4)

(< [0.3,0.5], [0.1,0.5], [0.2,0.3], [0.35,0.4]>, <0.2,0.3,0.4,0.6>, 0.5)]

$\mathcal{K}_X\ (x_3)$ = [(< [0.2,0.3], [0.0,0.2], [0.0,0.4], [0.1,0.15]>, <0.4,0.1,0.5,0.6>, 0.5)

(< [0.1,0.5], [0.2,0.3], [0.1,0.3], [0.2,0.5]>, <0.1,0.3,0.5,0.2>, 0.4)

(< [0.3,0.4], [0.3,0.5], [0.1,0.4], [0.1,0.5]>, <0.2,0.1,0.4,0.5>, 0.5)

(< [0.2,0.6], [0.4,0.6], [0.4,0.5], [0.5,0.6]>, <0.3,0.2,0.4,0.3>, 0.6)]

The below possibility quadripartitioned neutrosophic cubic set is given by another decision maker.

$\mathcal{L}_X\ (x_1)$ = [(< [0.0,0.2], [0.2,0.5], [0.3,0.4], [0.3,0.5]>, <0.0,0.2,0.5,0.6>, 0.3)

DOI: https://doi.org/10.54216/IJNS.260429

(< [0.0,0.2], [0.1,0.2], [0.2,0.3], [0.3,0.5]>, <0.1,0.2,0.55,0.65>, 0.4)

(< [0.1,0.15], [0.2,0.25], [0.4,0.5], [0.4,0.5]>, <0.0,0.1,0.8,0.7>, 0.5)

(< [0.1,0.2], [0.1,0.3], [0.4,0.5], [0.4,0.6]>, <0.1,0.1,0.4,0.5>, 0.7)]

$\mathcal{L}_X\ (x_2)$ = [(< [0.2,0.5], [0.0,0.1], [0.35,0.4], [0.4,0.5]>, <0.2,0.0,0.3,0.5>, 0.3)

(< [0.1,0.2], [0.0,0.2], [0.3,0.45], [0.4,0.5]>, <0.0,0.1,0.4,0.5>, 0.4)

(< [0.2,0.25], [0.25,0.35], [0.5,0.6], [0.6,0.7]>, <0.1,0.1,0.5,0.6>, 0.5)

(< [0.1,0.5], [0.2,0.5], [0.3,0.6], [0.45,0.5]>, <0.1,0.2,0.5,0.7>, 0.6)]

$\mathcal{L}_X\ (x_3)$ = [(< [0.1,0.2], [0.0,0.1], [0.3,0.5], [0.2,0.4]>, <0.3,0.0,0.6,0.7>, 0.6)

(< [0.0,0.4], [0.1,0.2], [0.2,0.4], [0.5,0.7]>, <0.0,0.2,0.6,0.3>, 0.5)

(< [0.1,0.3], [0.2,0.4], [0.3,0.5], [0.4,0.6]>, <0.1,0.0,0.5,0.6>, 0.6)

(< [0.1,0.4], [0.2,05], [0.5,0.6], [0.6,0.7]>, <0.2,0.1,0.5,0.6>, 0.7)]

In this example, we get $\mathcal{K}_X \subseteq \mathcal{L}_X$ is a PQNCS.

Definition – 3.3

Let Y is a universal set. $\mathcal{K}_y$ ={$< y :\ [Q_T^-(y), Q_T^+(y)], [Q_C^-(y), Q_C^+(x)],\ [Q_I^-(y), Q_I^+(y)], [Q_F^-(y), Q_F^+(y)] >, < T_Q(y), C_Q(y), I_Q(y), F_Q(y) >, \theta(y)/y \in\ Y$} be a PQNCS in Y, then the internal PQNCS,

$$T_Q(y) \in [Q_T^-, Q_T^+]$$

$$C_Q(y) \in [Q_C^-, Q_C^+]$$

$$I_Q(y)\ \in [Q_I^-, Q_I^+]$$

$$F_Q(y) \in [Q_F^-, Q_F^+]$$

Example - 3.3

Let $\mathcal{K}_X$ is an internal PQNCS, then

$\mathcal{K}_X$ = [(< [0.2, 0.4], [0.1,0.3], [0.3,0.5], [0.6,0.8]>, <0.3,0.2,0.4,0.7>, 0.5)

(< [0.1,0.4], [0.3,0.6], [0.3,0.5], [0.5,0.7]>, <0.2,0.5,0.4,0.6>,0.4)

(< [0.3,0.5], [0.2,0.5], [0.3,0.5], [0.7,0.8]>, <0.1,0.3,0.4,0.5>,0.5)

(< [0.2,0.5], [0.4,0.6], [0.1,0.4], [0.3,0.6]>, <0.1,0.5,0.5,0.2>,0.7)]

Definition – 3.4

Let X is a univerrsal set. $\mathcal{K}_Y$ ={$< y :\ Q_T(y),\ Q_C(y),\ Q_I(y),\ Q_F\ (y) >,\ < T_Q(y), C_Q(y), I_Q(y), F_Q(y) >, \theta(y)/y \in\ Y$} be a PQNCS in Y, then the external PQNCS,

$$T_Q(y) \notin [Q_T^-, Q_T^+]$$

$$C_Q(y) \notin [Q_C^-, Q_C^+]$$

$$I_Q(y)\ \notin [Q_I^-, Q_I^+]$$

$$F_Q(y) \notin [Q_F^-, Q_F^+]$$

Example - 3.4

Let $\mathcal{K}_X$ is an external PQNCS, then

$\mathcal{K}_X$ = [(< [0.1, 0.2], [0.1,0.3], [0.3,0.4], [0.6,0.7]>, <0.3,0.25,0.5,0.6>, 0.3)

(< [0.0,0.3], [0.1,0.2], [0.4,0.6], [0.3,0.4]>, <0.0,0.5,0.4,0.2>,0.2)

(< [0.2,0.3], [0.2,0.5], [0.3,0.4], [0.5,0.6]>, <0.1,0.3,0.5,0.5>, 0.4)

DOI: https://doi.org/10.54216/IJNS.260429

(< [0.1,0.3], [0.2,0.4], [0.2,0.3], [0.4,0.5]>, <0.1,0.2,0.3,0.6>, 0.5)]

Property – 3.1

Let $\mathcal{K}_Y$ ={$< x : Q_T(y),\ Q_C(y),\ Q_I(y),\ Q_F(y) >, < T_Q(y), C_Q(y), I_Q(y), F_Q(y) >, \theta(y)/y \in Y$},

$\mathcal{L}_Y$ = {$< y : R_T(y),\ R_C(y),\ R_I(y),\ R_F\ (y) >, < T_R(y), C_R(y), I_R(y), F_R(y) >, \vartheta(y)/y \in Y$}

are two internal PQNCS, then $\mathcal{K}_Y$ and $\mathcal{L}_Y$ satisfy the following conditions, for all $y \in Y$

$$\max\{Q_T^-(y), R_T^-(y)\} \geq (\alpha \wedge \beta)(y)$$
$$\max\{Q_C^-(y), R_C^-(y)\} \geq (\alpha \wedge \beta)(y)$$
$$\min\{Q_I^-(y), R_I^-(y)\} \leq (\alpha \wedge \beta)(y)$$
$$\min\{Q_F^-(y), R_F^-(y)\} \leq (\alpha \wedge \beta)(y)$$

then, the R-union of $\mathcal{K}_Y$ and $\mathcal{L}_Y$ still be internal PQNCS, where,

$$\mathcal{K}_Y\ U_R\ \mathcal{L}_Y = (\Phi_A \cup \Phi_B, \alpha \wedge \beta)$$

where,

$\Phi_A = < Q_T(y),\ Q_C(y),\ Q_I(y),\ Q_F(y) >$, $\Phi_B = < R_T(y),\ R_C(y),\ R_I(y),\ R_F\ (y) >$

$\alpha =< T_Q(y),\ C_Q(y),\ I_Q(y),\ F_Q\ (y) >, \beta =< T_R(y),\ C_R(y),\ I_R(y),\ F_R\ (y) >$

Property – 3.2

Let $\mathcal{K}_Y$ ={$< y : Q_T(y),\ Q_C(y),\ Q_I(y),\ Q_F\ (y) >, < T_Q(y), C_Q(y), I_Q(y), F_Q(y) >, \theta(y)/y \in Y$},

$\mathcal{L}_Y$ = {$< y : R_T(y),\ R_C(y),\ R_I(y),\ R_F\ (y) >, < T_R(y), C_R(y), I_R(y), F_R(y) >, \vartheta(y)/y \in Y$}

are two internal PQNCS, then $\mathcal{K}_y$ and$\mathcal{L}_y$ satisfy the following conditions, for all $y \in Y$

$$\min\{Q_T^-(y), R_T^-(y)\} \leq (\alpha \wedge \beta)(y)$$
$$\min\{Q_C^+(y), R_C^+(y)\} \leq (\alpha \wedge \beta)(y)$$
$$\max\{Q_I^+(y), R_I^+(y)\} \geq (\alpha \wedge \beta)(y)$$
$$\max\{Q_F^+(y), R_F^+(y)\} \geq (\alpha \wedge \beta)(y)$$

then, $\mathcal{K}_y \cap_R \mathcal{L}_y = (\Phi_A \cap \Phi_B, \alpha \vee \beta)$

Where,

$\Phi_A = < Q_T(y),\ Q_C(y),\ Q_I(y),\ Q_F(y) >$, $\Phi_B = < R_T(y),\ R_C(y),\ R_I(y),\ R_F\ (y) >$

$\alpha =< T_Q(y),\ C_Q(y),\ I_Q(y),\ F_Q\ (y) >, \beta =< T_R(y),\ C_R(y),\ I_R(y),\ F_R\ (y) >$

Definition – 3.5

Consider $\mathcal{K}_Y$ be PQNCS, then $\mathcal{K}_Y$ is called empty PQNCS set and is denoted by

Δ = {< [0,0], [0,0], [1,1], [1,1] > , < 0,0,1,1> ,0}

Definition – 3.6

Consider $\mathcal{K}_Y$ be possibility quadripartitioned neutrosophic cubic set, then $\mathcal{K}_Y$ is called unit PQNCS and is denoted by,

ρ = {< [1,1], [1,], [0,0], [0,0] > , < 1,1,0,0> ,1}

Definition – 3.7

Consider $\mathcal{K}_Y$ = {$< y : Q_T(y),\ Q_C(y),\ Q_I(y),\ Q_F\ (y) >, < T_Q(y), C_Q(y), I_Q(y), F_Q(y) >, \theta(x)/y \in Y$ }

DOI: https://doi.org/10.54216/IJNS.260429

is PQNCS, then the complement of $\mathcal{K}_Y$ is denoted by,

$\mathcal{K}_Y^C$ ={< y : $Q_F(y), Q_I(y), Q_C(y), Q_T(y)$ >,< $F_Q(y), I_Q(y), C_Q(y), T_Q(y)$ >, $1-\theta(y)/y \in Y$ }

Proposition – 3.1

Let X be a universal set. Let us consider $\mathcal{K}_X$, $\mathcal{L}_X$, $\mathcal{M}_X$ be PQNCS on X, then

(i) $\phi \subseteq \mathcal{K}_X$
(ii) $\mathcal{K}_X \subseteq$ X
(iii) $\mathcal{K}_X \subseteq \mathcal{L}_X$ *and* $\mathcal{L}_X \subseteq \mathcal{M}_X$ *then* $\mathcal{K}_X \subseteq \mathcal{M}_X$
(iv) $\phi^C = X$
(v) $X^C = \phi$
(vi) $({\mathcal{K}_X}^C)^C = \mathcal{K}_X$

Property – 3.3

Let X be a universal set. Consider $\mathcal{K}_X = (\Phi_A, \alpha)$, $\mathcal{L}_X = (\Phi_B, \beta)$, $\mathcal{M}_X = (\Phi_C, \gamma)$, $\mathcal{N}_X = (\Phi_D, \delta)$ be PQNCS on X, then

(i) if $\mathcal{K}_X \subseteq_P \mathcal{L}_X$ and $\mathcal{L}_X \subseteq_P \mathcal{M}_X$ then $\mathcal{K}_X \subseteq_P \mathcal{M}_X$
(ii) if $\mathcal{K}_X \subseteq_P \mathcal{L}_X$ then $(\mathcal{L}_X)^C = (\mathcal{K}_X)^C$
(iii) if $\mathcal{K}_X \subseteq_P \mathcal{L}_X$ and $\mathcal{K}_X \subseteq_P \mathcal{M}_X$ then $\mathcal{K}_X \subseteq_P \mathcal{L}_X \cap_P \mathcal{M}_X$
(iv) if $\mathcal{K}_X \subseteq_P \mathcal{L}_X$ and $\mathcal{M}_X \subseteq_P \mathcal{L}_X$ then $\mathcal{K}_X \cup_P \mathcal{M}_X \subseteq_P \mathcal{L}_X$
(v) if $\mathcal{K}_X \subseteq_P \mathcal{L}_X$ and $\mathcal{M}_X \subseteq_P \mathcal{N}_X$ then $\mathcal{K}_X \cup_P \mathcal{M}_X \subseteq_P \mathcal{L}_X \cup_P \mathcal{N}_X$ and $\mathcal{K}_X \cap_P \mathcal{M}_X \subseteq_P \mathcal{L}_X \cap_P \mathcal{N}_X$

Proof:

(i) Consider $\mathcal{K}_X = (\Phi_A, \alpha)$, $\mathcal{L}_X = (\Phi_B, \beta)$ be PQNCS. Due to the definition of P-order PQNCS, if $\mathcal{K}_X \subseteq_P \mathcal{L}_X$ and $\mathcal{L}_X \subseteq_P \mathcal{M}_X$ then we get $\Phi_A \subseteq_P \Phi_B$, $\alpha \preccurlyeq \beta$ and $\Phi_B \subseteq_P \Phi_C$, $\beta \preccurlyeq \gamma$, Finally we get, $\Phi_A \subseteq_P \Phi_B \subseteq_P \Phi_C$, $\alpha \preccurlyeq \beta \preccurlyeq \gamma$. It can be written as $\Phi_A \subseteq_P \Phi_C$, $\alpha \preccurlyeq \gamma$. Hence, we get $\mathcal{K}_X \subseteq_P \mathcal{M}_X$.
(ii) if $\mathcal{K}_X \subseteq_P \mathcal{L}_X$ then the definition of complement of PQNCS $(\Phi_B)^C \subseteq_P (\Phi_A)^C, (\beta)^C \preccurlyeq (\alpha)^C$.
(iii) if $\mathcal{K}_X \subseteq_P \mathcal{L}_X$ and $\mathcal{K}_X \subseteq_P \mathcal{M}_X$ then $\Phi_A \subseteq_P \Phi_B$, $\alpha \preccurlyeq \beta$ and $\Phi_A \subseteq_P \Phi_C$, $\alpha \preccurlyeq \gamma$. Due to meet P-order PQNCS then we get, $\Phi_A \subseteq_P \Phi_B \cap \Phi_C$, $\alpha \preccurlyeq \beta \wedge \gamma$. Hence, we have $\mathcal{K}_X \subseteq_P \mathcal{L}_X \cap \mathcal{M}_X$.

Definition – 3.8

Let X be a universal set where X= $\{x_1, x_2, \dots \dots \dots x_n\}$, and

$\mathcal{K}_X$ = {< x : $Q_T(x), Q_C(x), Q_I(x), Q_F(x)$ >,< $T_Q(x), C_Q(x), I_Q(x), F_Q(x)$ >, $\theta(x)/x \in X$ } be PQNCS in X, then its score function, accuracy function and certainty function of $\mathcal{K}_X$ be defined as,

$$S_{PQNCS}(\mathcal{K}_X) = \sum_{i=1}^{n}[(4 + Q_T^-(x_i) + Q_T^+(x_i) - Q_C^-(x_i) + Q_C^+(x_i) - Q_I^-(x_i) + Q_I^+(x_i) - Q_F^-(x_i) + Q_F^+(x_i)) + (2 + T_Q(x_i) - C_Q(x_i) - I_Q(x_i) - F_Q(x_i)) + \theta(x_i)]/10$$

$$\mathcal{A}_{PQNCS}(\mathcal{K}_X) = \sum_{i=1}^{n}[(Q_T^-(x_i) + Q_T^+(x_i) - Q_F^-(x_i) + Q_F^+(x_i)) + (T_Q(x_i) - F_Q(x_i)) + \theta(x_i)]/4$$

$$\mathcal{C}_{PQNCS}(\mathcal{K}_X) = \sum_{i=1}^{n}[(Q_T^-(x_i) + Q_T^+(x_i)) + (T_Q(x_i) + \theta(x_i)]/4$$

The accuracy function and the scoring function are two important indices that can be used to evaluate the degrees of association between two PQNCS. Generally, a higher score indicates a higher level of actual PQNCS participation. In the event that the score functions of the two PQNCS are equal, we can assess the degree of relationship by comparing the accuracy functions; a larger accuracy function indicates a better PQNCS. The greater the possibility value and true membership function, the more certain the PQNCS is in terms of the certainty function.

Definition – 3.9

Let X=$\{x_1, x_2, x_3 \dots \dots \dots . x_n\}$, $\mathcal{K}_X$ = {< x : $Q_T(x), Q_C(x), Q_I(x), Q_F(x)$ >, < $T_Q(x), C_Q(x), I_Q(x), F_Q(x)$ >, $\theta(x)/x \in X$ } and $\mathcal{L}_X$ = {< x : $R_T(x), R_C(x), R_I(x), R_F(x)$ >,< $T_R(x), C_R(x), I_R(x), F_R(x)$ >, $\vartheta(x)/x \in X$} are two PQNCS, then the ranking of $\mathcal{K}_X$ and $\mathcal{L}_X$ is defined as,

DOI: https://doi.org/10.54216/IJNS.260429

1) If $S_{PQNCS}\ (\mathcal{K}_X) > S_{PQNCS}\ (\mathcal{L}_X)$, then $\mathcal{K}_X > \mathcal{L}_X$
2) If $S_{PQNCS}\ (\mathcal{K}_X) = S_{PQNCS}\ (\mathcal{L}_X)$ and $\mathcal{A}_{PQNCS}\ (\mathcal{K}_X) > \mathcal{A}_{PQNCS}\ (\mathcal{L}_X)$, then $\mathcal{K}_X > \mathcal{L}_X$

4. Norm of binary operations of possibility quadripartitioned neutrosophic cubic sets

In this section, we introduce the novel idea of Norm of PQNCS binary operations, related theorems and examples are examined.

Definition - 4.1

Consider the two real numbers x and y in PQNCS, then Dombi T-norm is defined as,

$$D_{Dombi}(x,y) = \frac{1}{1+\left[\left(\frac{1}{x}-1\right)^{\lambda}+\left(\frac{1}{y}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}$$

The operator D_{Dombi}(x,y) is conjunction and the operator satisfying D_{Dombi} (0,0) = D_{Dombi}(0,1) = D_{Dombi}(1,0) = 0 and D_{Dombi}(1,1) = 1 if $\lambda > 0$. Assume that, the operator D_{Dombi}(x,y) is disjunction if $\lambda \prec 0$.

Definition - 4.2

Consider the two real number in PQNCS, then Dombi T-conorm is defined as,

$$D_{Dombi}(x,y) = 1-\frac{1}{1+\left[\left(\frac{1}{x}-1\right)^{\lambda}+\left(\frac{1}{y}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}$$

The operator D_{Dombi}(x, y) is conjunction and the operator satisfying D_{Dombi} (0,0) = D_{Dombi}(0,1) = D_{Dombi}(1,0) = 0 and D_{Dombi}(1,1) = 1 if $\lambda > 0$. Assume that, the operator D_{Dombi}(x,y) is disjunction if $\lambda \prec 0$.

Definition - 4.3

Let X be a non-empty set where X= $\{x_1, x_2, \dots \dots \dots x_n\}$,

$\mathcal{K}_X = \{< x :\ A_T(x),\ A_C(x),\ A_I(x),\ A_F\ (x) >, < T(x), C(x), I(x), F(x) >, \theta(x)/x \in\ X\}$,

$\mathcal{L}_X = \{< x :\ B_T(x),\ B_C(x),\ B_I(x),\ B_F\ (x) >, < T(x), C(x), I(x), F(x) >, \vartheta(x)/x \in\ X\}$

If there are two PQNCS in X, their binary operations are

$$\mathcal{K}_X \oplus \mathcal{L}_X = <\left[\frac{1}{1+\left[\left(\frac{1}{A_T^-}-1\right)^{\lambda}+\left(\frac{1}{B_T^-}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}, \frac{1}{1+\left[\left(\frac{1}{A_T^+}-1\right)^{\lambda}+\left(\frac{1}{B_T^+}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}\right],$$

$$\left[\frac{1}{1+\left[\left(\frac{1}{A_C^-}-1\right)^{\lambda}+\left(\frac{1}{B_C^-}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}, \frac{1}{1+\left[\left(\frac{1}{A_C^-}-1\right)^{\lambda}+\left(\frac{1}{B_C^-}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}\right],$$

$$1-\left[\frac{1}{1+\left[\left(\frac{1}{A_I^-}-1\right)^{\lambda}+\left(\frac{1}{B_I^-}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}, \frac{1}{1+\left[\left(\frac{1}{A_I^-}-1\right)^{\lambda}+\left(\frac{1}{B_I^-}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}\right],$$

$$1-\left[\frac{1}{1+\left[\left(\frac{1}{A_F^-}-1\right)^{\lambda}+\left(\frac{1}{B_F^-}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}, \frac{1}{1+\left[\left(\frac{1}{A_F^-}-1\right)^{\lambda}+\left(\frac{1}{B_F^-}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}\right]>,$$

$$<\frac{1}{1+\left[\left(\frac{1}{A_T}-1\right)^{\lambda}+\left(\frac{1}{B_T}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}, \frac{1}{1+\left[\left(\frac{1}{A_C}-1\right)^{\lambda}+\left(\frac{1}{B_C}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}},$$

DOI: https://doi.org/10.54216/IJNS.260429

$$1-\frac{1}{1+\left[\left(\frac{1}{A_I}-1\right)^{\lambda}+\left(\frac{1}{B_I}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}},\ 1-\frac{1}{1+\left[\left(\frac{1}{A_F}-1\right)^{\lambda}+\left(\frac{1}{B_F}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}>,\ \frac{1}{1+\left[\left(\frac{1}{\theta}-1\right)^{\lambda}+\left(\frac{1}{\vartheta}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}$$

$$\mathcal{K}_X \otimes \mathcal{L}_X = <1-\left[\frac{1}{1+\left[\left(\frac{1}{A_T^{-}}-1\right)^{\lambda}+\left(\frac{1}{B_T^{-}}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}},\frac{1}{1+\left[\left(\frac{1}{A_T^{+}}-1\right)^{\lambda}+\left(\frac{1}{B_T^{+}}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}\right],$$

$$1-\left[\frac{1}{1+\left[\left(\frac{1}{A_C^{-}}-1\right)^{\lambda}+\left(\frac{1}{B_C^{-}}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}},\frac{1}{1+\left[\left(\frac{1}{A_C^{-}}-1\right)^{\lambda}+\left(\frac{1}{B_C^{-}}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}\right]$$

$$\left[\frac{1}{1+\left[\left(\frac{1}{A_I^{-}}-1\right)^{\lambda}+\left(\frac{1}{B_I^{-}}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}},\frac{1}{1+\left[\left(\frac{1}{A_I^{-}}-1\right)^{\lambda}+\left(\frac{1}{B_I^{-}}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}\right],$$

$$\left[\frac{1}{1+\left[\left(\frac{1}{A_F^{-}}-1\right)^{\lambda}+\left(\frac{1}{B_F^{-}}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}},\frac{1}{1+\left[\left(\frac{1}{A_F^{-}}-1\right)^{\lambda}+\left(\frac{1}{B_F^{-}}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}\right]>$$

$$<1-\frac{1}{1+\left[\left(\frac{1}{A_T}-1\right)^{\lambda}+\left(\frac{1}{B_T}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}},\ 1-\frac{1}{1+\left[\left(\frac{1}{A_C}-1\right)^{\lambda}+\left(\frac{1}{B_C}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}},\ \frac{1}{1+\left[\left(\frac{1}{A_I}-1\right)^{\lambda}+\left(\frac{1}{B_I}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}},$$

$$\frac{1}{1+\left[\left(\frac{1}{A_F}-1\right)^{\lambda}+\left(\frac{1}{B_F}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}>,\ 1-\frac{1}{1+\left[\left(\frac{1}{\theta}-1\right)^{\lambda}+\left(\frac{1}{\vartheta}-1\right)^{\lambda}\right]^{\frac{1}{\lambda}}}$$

Example – 4.1

Let X be a universal set.

$\mathcal{K}_X$ = {< [0.1,0.2], [0.6,0.7], [0.4,0.5], [0.1,0.2] >, < 0.2,0.4,0.3,0.1 >, 0.1 },

$\mathcal{L}_X$ = {< [0.3,0.4], [0.3,0.5], [0.1,0.4], [0.2,0.4] >, < 0.3,0.4,0.5,0.6 >, 0.5}

be two PQNCS in X. Then the Dombi T-norm of binary operation, we get

$\mathcal{K}_X \oplus \mathcal{L}_X$ = {< [0.0,0.1], [0.4,0.7], [0.9,0.6], [0.9,0.8] >,<0.1,0.4,0.6,0.8>, 0.1}

$\mathcal{K}_X \otimes \mathcal{L}_X$ = {< [0.9,0.8], [0.6,0.3], [0.1,0.4], [0.0,0.1] >,<0.9,0.6,0.3,0.1>, 0.9}

5. Measures of possibility quadripartitioned neutrosophic cubic sets

This section discusses the PQNCS distance measure and similarity measure.

5.1 Distance measure of possibility quadripartitioned neutrosophic cubic sets

Definition - 5.1.1

Let X be a non-empty set where X= $\{x_1, x_2, \dots \dots \dots x_n\}$,

$\mathcal{Q}_X$ ={$< x : Q_T(x), Q_C(x), Q_I(x), Q_F(x) >, < T_Q(x), C_Q(x), I_Q(x), F_Q(x) >, \theta(x) / x \in X$ },

$\mathcal{R}_X$ ={$< x : R_T(x), R_C(x), R_I(x), R_F(x) >, < T_R(x), C_R(x), I_R(x), F_R(x) >, \vartheta(x) / x \in X$ },

be two PQNCS in X. Then the distance measure of PQNCS is,

DOI: https://doi.org/10.54216/IJNS.260429

$d_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X) = \sum_{i=1}^{n}\{[Q_T^-(x_i) - R_T^-(x_i)] + [Q_T^+(x_i) - R_T^+(x_i)] + [[Q_C^-(x_i) - R_C^-(x_i)] + [Q_C^+(x_i) - R_C^+(x_i)] + [Q_I^-(x_i) - R_I^-(x_i)] + [Q_I^+(x_i) - R_I^+(x_i)] + [Q_F^-(x_i) - R_F^-(x_i)] + [Q_F^+(x_i) - R_F^+(x_i)]) + (T_Q(x) - T_R(x)) + (C_Q(x) - C_R(x)) + (I_Q(x) - I_R(x)) + (F_Q(x) - F_R(x))\}/10$

Theorem - 5.1.1

Let X be a non-empty set. $\mathcal{Q}_X$, $\mathcal{R}_X$, $\mathcal{M}_X$ be PQNCS in X, then

(i) $d_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X) = d_{PQNCS}(\mathcal{R}_X, \mathcal{Q}_X)$
(ii) $\mathbf{0} \leq d_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X) \leq \mathbf{1}$
(iii) $d_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X) = 0$ iff $\mathcal{Q}_X = \mathcal{R}_X$
(iv) $d_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X) + d_{PQNCS}(\mathcal{R}_X, \mathcal{M}_X) \geq d_{PQNCS}(\mathcal{Q}_X, \mathcal{M}_X)$

Proof:

The proof is obvious.

5.2 Similarity measure of possibility quadripartitioned neutrosophic cubic sets

Definition - 5.2.1

Let X be a non-empty set where X= $\{x_1, x_2, \dots \dots \dots x_n\}$,

$\mathcal{Q}_X = \{< x : Q_T(x), Q_C(x), Q_I(x), Q_F(x) >, < T_Q(x), C_Q(x), I_Q(x), F_Q(x) >, \theta(x)/x \in X\}$

$\mathcal{R}_X = \{< x : R_T(x), R_C(x), R_I(x), R_F(x) >, < T_R(x), C_R(x), I_R(x), F_R(x) >, \vartheta(x)/x \in X\}$

be two PQNCS in X. Then the similarity measure of PQNCS is,

$\mathbb{S}_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X) = 1 - d_{PQNCS}(\mathcal{K}_X, \mathcal{L}_X)$

$\mathbb{S}_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X) = 1 - \sum_{i=1}^{n}\{[Q_T^-(x_i) - R_T^-(x_i)] + [Q_T^+(x_i) - R_T^+(x_i)] + [Q_C^-(x_i) - R_C^-(x_i)] + [Q_C^+(x_i) - R_C^+(x_i)] + [Q_I^-(x_i) - R_I^-(x_i)] + [Q_I^+(x_i) - R_I^+(x_i)] + [Q_F^-(x_i) - R_F^-(x_i)] + [Q_F^+(x_i) - R_F^+(x_i)]) + (T_Q(x) - T_R(x)) + (C_Q(x) - C_R(x)) + (I_Q(x) - I_R(x)) + (F_Q(x) - F_R(x)) + \vartheta(x_i)\}/10$

Theorem - 5.2.1

Let X be a non-empty set. $\mathcal{K}_X$, $\mathcal{L}_X$, $\mathcal{M}_X$ be PQNCS in X, then

(v) $\mathbb{S}_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X) = \mathbb{S}_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X)$
(vi) $\mathbf{0} \leq \mathbb{S}_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X) \leq \mathbf{1}$
(vii) $\mathbb{S}_{PQNCS}(\mathcal{Q}_X, \mathcal{R}_X) = 0$ iff $\mathcal{Q}_X = \mathcal{R}_X$

Proof:

The proof is trivial.

Example - 5.2.1

Let X be a non-empty set. $\mathcal{K}_X$, $\mathcal{L}_X$ be PQNCS in X, then

$\mathcal{K}_X = \{< [0.2,0.3], [0.3,0.4], [0.7,0.8], [0.1,0.2] >, < 0.1,0.3,0.5,0.6 >, 0.4\}$

$\mathcal{L}_X = \{< [0.1,0.3], [0.1,0.2], [0.4,0.5], [0.6,0.7] >, < 0.2,0.3,0.4,0.7 >, 0.2\}$

$$d_{PQNCS}(\mathcal{K}_X, \mathcal{L}_X) = 0.26$$

$$\mathbb{S}_{PQNCS}(\mathcal{K}_X, \mathcal{L}_X) = 1 - d_{PQNCS}(\mathcal{K}_X, \mathcal{L}_X)$$

$$\mathbb{S}_{PQNCS}(\mathcal{K}_X, \mathcal{L}_X) = 0.74$$

6. Application and Comparative Analysis

This section includes the specific algorithm, its application, and a comparative analysis. The algorithm outlines the process for evaluating alternatives based on their score functions, while the application demonstrates how the algorithm can be utilized in practical scenarios, particularly with PQNCS. Additionally, the comparative analysis

DOI: https://doi.org/10.54216/IJNS.260429

evaluates the effectiveness of this approach against other methods, providing insights into its advantages and potential for more accurate decision-making.

6.1 Decision making algorithm

Step 1: According to expert assessment, the PQNCS is acquired as follows:

$$\mathcal{K}_X = \{< x : Q_T(x), Q_C(x), Q_I(x), Q_F(x) >, < T_Q(x), C_Q(x), I_Q(x), F_Q(x) >, \theta(x)/x \in X\}$$

Step 2: Step 1 and the corresponding definition of PQNCS resolved the following equality.

$$S_{PQNCS}(\mathcal{K}_X) = \sum_{i=1}^{n}[(4 + Q_T^-(x_i) + Q_T^+(x_i) - Q_C^-(x_i) + Q_C^+(x_i) - Q_I^-(x_i) + Q_I^+(x_i) - Q_F^-(x_i) + Q_F^+(x_i)) + (2 + T_Q(x_i) - C_Q(x_i) - I_Q(x_i) - F_Q(x_i)) + \theta(x_i)]/10$$

Step 3: $S_{PQNCS}(x_i) \neq S_{PQNCS}(x_j)$ $(i \neq j \ \forall\, i, j \in n)$, To find the best option, sort the step 2 results.

Step 4: If $S_{PQNCS}(x_i) = S_{PQNCS}(x_j)$ $(i \neq j \ \forall\, i, j \in n)$, then resolve the below equation,

$$\mathcal{A}_{PQNCS}(\mathcal{K}_X) = \sum_{i=1}^{n}[(Q_T^-(x_i) + Q_T^+(x_i) - Q_F^-(x_i) + Q_F^+(x_i)) + (T_Q(x_i) - F_Q(x_i)) + \theta(x_i)]/4$$

We obtain the best option by solving each step individually.

6.2 Application

The algorithm suggests that by calculating the score functions of each alternative, we can determine the best option, regardless of the number of alternatives and attributes. In this study, an asymmetric structure is chosen to illustrate the use of the scoring function in PQNCS. This approach enables a comprehensive evaluation by considering the varying degrees of membership, non-membership, contradiction, and ignorance for each alternative, ultimately guiding the selection of the most optimal choice.

Example – 6.2.1

Mr. John Samuel wishes to select one of the three colleges for her daughter.

X= $\{x_1, x_2, x_3\}$ for her to choose from, Mr. John Samuel analyses and compares a_1= fees structure, a_2= Technical skillbased education, a_3= placement and obtained preliminary evaluation where $\mathfrak{A} = \{a_1, a_2, a_3\}$ be attributes. The form PQNCS serves as the evaluation value. According to Mr. John Samuel's evaluation, the best college is determined using the assessment below.

Step -1

Based on the evaluation result, a quadripartitioned neutrosophic cubic set is obtained.

$\mathcal{K}_X(x_1)$ = [(< [0.1,0.3], [0.2,0.4], [0.5,0.6], [0.6,0.7]>, <0.6,0.1,0.3,0.4>, 0.6)
(< [0.5,0.6], [0.1,0.2], [0.1,0.3], [0.2,0.3]>, <0.5,0.2,0.2,0.4>, 0.6)
(< [0.3,0.5], [0.2,0.3], [0.2,0.4], [0.3,0.4]>, <0.4,0.1,0.5,0.7>, 0.5)]

$\mathcal{K}_X(x_2)$ = [(< [0.6,0.7], [0.3,0.4], [0.1,0.2], [0.1,0.3]>, <0.6,0.1,0.2,0.4>, 0.7)
(< [0.2,0.5], [0.1,0.2], [0.2,0.3], [0.2,0.4]>, <0.4,0.3,0.1,0.5>, 0.5)
(< [0.3,0.6], [0.3,0.5], [0.1,0.5], [0.3,0.7]>, <0.7,0.1,0.5,0.6>, 0.7)]

$\mathcal{K}_X(x_3)$ = [(< [0.4,0.6], [0.3,0.4], [0.2,0.3], [0.1,0.3]>, <0.8,0.3,0.3,0.4>, 0.8)
(< [0.5,0.7], [0.2,0.6], [0.3,0.4], [0.5,0.6]>, <0.7,0.3,0.4,0.6>, 0.7)
(< [0.3,0.4], [0.4,0.6], [0.2,0.3], [0.4,0.5]>, <0.1,0.5,0.3,0.6>, 0.6)]

DOI: https://doi.org/10.54216/IJNS.260429

Step – 2

The score function in the PQNCS mentioned above is

S (x_1) = 1.5, S (x_2) = 1.7 S (x_3) = 1.4

Step – 3

The following lists the alternatives in the order that the score functions are ranked by size.

$$S(x_2) > S(x_1) > S(x_3)$$

Result:

Therefore, option 2 is the best option, and Mr. John Samuel should select it based on the assessment.

Example - 6.2.2

Mr. Ashok chooses the most suitable company in order to ensure optimal outcomes for his objectives. X={x}, $\mathcal{K}_X$ = [(< [0.5,0.8], [0.3,0.4], [0.3,0.5], [0.0,0.1]>, <0.7,0.2,0.3,0.1>, 0.6)

$\mathcal{L}_X$ = [(< [0.5,0.8], [0.3,0.4], [0.3,0.5], [0.0,0.1]>, <0.7,0.2,0.3,0.1>, 0.6)

In a universe where the purchase order is governed, the assessment value was provided using twoPQNCS. Based on Mr. Ashok evaluation, he selects the best company by comparing the sets and choosing the one that aligns most effectively with the desired criteria, ultimately selecting the company that offers the optimal balance of membership, contradiction, ignorance and non-membership values.

Step -1

The two PQNCS is obtained to according the evaluation result.

$\mathcal{K}_X$ = [(< [0.5,0.8], [0.3,0.4], [0.3,0.5], [0.0,0.1]>, <0.7,0.2,0.3,0.1>, 0.6)

$\mathcal{L}_X$ = [(< [0.5,0.8], [0.3,0.4], [0.3,0.5], [0.0,0.1]>, <0.7,0.2,0.3,0.1>, 0.6)

S ($\mathcal{K}_X$) = 0.6, S ($\mathcal{L}_X$) = 0.6

S ($\mathcal{K}_X$) = S ($\mathcal{L}_X$)

Therefore, $\mathcal{K}_X = \mathcal{L}_X$

The evaluation of score value is equal to both of these companies purchase order. So, we will go to next step.

Step -2

The two possibility quadripartitioned neutrosophic cubic set is obtained to according the evaluation result by using accuracy function.

A ($\mathcal{K}_X$) = 0.6, A ($\mathcal{L}_X$) = 0.6

A ($\mathcal{K}_X$) = A ($\mathcal{L}_X$)

Therefore, $\mathcal{K}_X = \mathcal{L}_X$

DOI: https://doi.org/10.54216/IJNS.260429

The evaluation of accuracy value is equal to both of these companies purchase order. So, we will go to next step.

Step -3

The two PQNCS is obtained to according the evaluation result by using certainty function.

$$C(\mathcal{K}_X) = 0.6, \quad C(\mathcal{L}_X) = 0.7$$

$$C(\mathcal{K}_X) < C(\mathcal{L}_X)$$

$$\text{Therefore, } \mathcal{K}_X < \mathcal{L}_X$$

Result:

Thus, the evaluation shows that Mr. Ashok chose the second company, which is the best option.

7. Conclusion

This paper introduces the concept of PQNCS, which combines the fundamental ideas of possibility and QNCS. Based on recent research, it defines PQNCS and explores the potential for element occurrences within these sets. Additionally, the paper provides an explanation of the definitions of empty, complete, and R-union operations. In order to evaluate the potential of PQNCS, the score function a critical indicator in multiple criteria decision-making problems is finally included. The PQNCS considers four key elements: membership, contradiction, ignorance and non-membership along with the potential relationships these elements have with the set. As a result, the benefits of applying the PQNCS can be leveraged to more effectively address and solve production-related problems. This PQNCS is used to resolve complicated real-world issues such as choosing engineering projects or contracts, investing in share market analysis, and choosing a career.

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