# Modeling Bitcoin Price Dynamics Using a Fractional Maxwell-Weibull Copula Distribution

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**Abstract**

This paper presents the Fractional Maxwell-Weibull Copula (FMWC) distribution to deal with the heavy tails, extended memory, and nonlinear dependence of price returns of Bitcoins, as the existing financial models face limitations in this aspect. The FMWC provides a flexible model that allows incorporating fractional Weibull distributions to capture persistent autocorrelation, Maxwell components to model significant price changes, and a Student-t copula to capture multivariate dependencies to discuss the volatile returns of Bitcoin. The FMWC was applied to historical Bitcoin data between January 2020 and May 2025 and showed better results than other models, such as Weibull, GARCH-t, and Maxwell-Log Logistic, with an MAE of 0.034374, RMSE of 0.0335, and log-likelihood of 4200.0. Its risk measures (VaR 95% = -0.07983, CVaR 95% = -0.10882) improve tail risk estimation, which is important in risk measurement and portfolio management. Robustness tests also validate its performance over periods and proper handling of outliers. Nevertheless, the FMWC is an excellent tool, despite its computational complexity issues, and can be used by investors, traders, and regulators. Further studies on the computational efficiencies and applications to other cryptocurrencies are required to increase their application in dynamic financial markets.

**Keywords**: Bitcoin; Fractional Maxwell-Weibull; Long Memory; Heavy Tails; Volatility Clustering; Risk Management; Copula; Forecasting

## 1. Introduction

The extreme volatility and fast growth of Bitcoin have attracted a lot of attention among investors, regulators, and scholars because of specific price dynamics like long-memory, heavy-tailed, and nonlinear dependence [1, 2]. These characteristics disrupt conventional financial models that require progressive financial approaches and solutions to effectively handle risks, optimize portfolios, and forecast financial revenues [3]. The current study focuses on a novel framework i.e., the Fractional Maxwell-Weibull Copula (FMWC) that explains the behavior of Bitcoin in a complex manner [4]. The FMWC is a well-controlled method of the dynamics of Bitcoin, unlike recent studies of Nguyen [5] and Ali and Lee [6], which could not measure the long-memory effects, and could not model multivariate dependencies [7].

The commonly used standard models, such as ARIMA and GARCH do not solve heavy-tailed distributions and long dependencies in Bitcoin, particularly in the volatility clustering [4, 5]. Similarly, traditional Weibull distributions also do not contain any fractional dynamics [8]. Recent research showed that it is imperative to have models that can address both heavy tails and multifaceted dependences in cryptocurrencies [9]. The copula-based models are promising in finance as they can capture the nonlinear relationships [10], but they frequently overlook the fractional behavior of Bitcoin [11]. The current study establishes the FMWC model, deploys it on historical data of Bitcoin in



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the years 2020-2025, and compares it with other benchmarks such as GARCH and Weibull to improve risk measurements and predictions in digital asset markets. In addition to probabilistic approaches, the uncertainty in Bitcoin price dynamics can also be represented through neutrosophic sets. In this context, truth, indeterminacy, and falsity degrees jointly capture market conditions that are ambiguous or neutral, which classical models often ignore. Integrating a neutrosophic perspective provides a complementary lens to the FMWC, offering deeper insights into the indeterminate aspects of cryptocurrency markets.

[12] applied generalized Weibull models to capture extreme market events, noting their fit for skewed data but limited handling of extended memory. [13] adapted Weibull models for cryptocurrency returns, improving fit but lacking multivariate dependence frameworks. These studies suggest a need for fractional extensions to model Bitcoin's persistent autocorrelation. Fractional calculus effectively captures long-memory processes in financial time series.. [14] used Fractional Weibull distributions to model high-frequency data, outperforming traditional models for assets with long-range dependence, like Bitcoin. [15] confirmed the efficacy of fractional models in volatility modeling but highlighted computational challenges and limited multivariate applications, motivating FMWC's fractional Weibull component. Copula models excel in capturing nonlinear dependencies in cryptocurrencies. [10] showed that Student-t copulas improve risk assessment by modeling tail dependence in crypto portfolios. [12] applied dynamic copulas to Bitcoin and Ethereum, noting the superior performance of time-varying structures. However,. [16] argued that standard copulas struggle with Bitcoin's fractional dynamics, proposing hybrid models with heavy-tailed marginals, though without fractional integration.

Recent Bitcoin studies focus on heavy tails and volatility. [5] used GARCH models, capturing volatility clustering but not extreme tails. [15] introduced Maxwell distributions for Bitcoin's heavy-tailed returns, lacking long-memory components. [16] proposed a Maxwell-Log Logistic model, improving tail modeling but omitting multivariate dependence and fractional dynamics. Despite progress, existing models rarely simultaneously address heavy tails, extended memory, and multivariate dependence. The FMWC distribution may fill these gaps by combining the long-memory capacity of the fractional Weibull with the (non-linear) flexibility of the Maxwell in the tail and the dynamic copula of nonlinear dependence, into a powerful tool to model the price of Bitcoin [17].

## 3. Methodology

This section describes the formulation and implementation of the fractional Maxwell-Weibull Copula (FMWC) distribution to describe the dynamics of the Bitcoin prices. It is divided into three subcomponents, *i.e.*, the definition of the FMWC distribution, its theoretical properties, and the copula construction of the multivariate dependence. The FMWC distribution is the statistical hybrid model that takes into account the volatility of Bitcoin price dynamics.

The model combines three distributions to an aggregate format, generalized Maxwell segment to contain the heavy tails generated as a result of the extreme movements of prices, fractional Weibull component that uses fractional derivative to capture the long effects and stagnating autocorrelations in the returns, in addition, a student-t copula that integrates the above distributions into multivariate format to represent the nonlinearities of interdependencies [9]. This framework enables the FMWC to effectively handle the heavy tails of Bitcoin, long-memory, and dependencies, indicating its applicability in analyzing financial time series [13]. The cumulative distribution function (CDF) of the fractional Weibull distribution with fractional order $\alpha \in (0, 1]$Is defined as:

$$f(x; \lambda, k, \alpha) = 1 - \exp\left(-\left(\frac{x}{\lambda}\right)^k D^\alpha x\right) \quad (1)$$

Where, $\lambda$Is the scale parameter, $k$Is the shape parameter, and $D^\alpha x$ is the fractional derivative of order $\alpha$, Which is generally calculated according to Caputo's definition [9]. Differentiating (1) obtains the corresponding probability density function (PDF):

$$f(x; \lambda, k, \alpha) = \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1} D^\alpha x \exp\left(-\left(\frac{x}{\lambda}\right)^k D^\alpha x\right) \quad (2)$$

The Maxwell generalized family, which applies to heavy-tailed financial data [15], has been presented below:

$$f_M(x; \sigma) = \sqrt{\frac{2}{\pi}\frac{x^2}{\sigma^3}\exp(-\frac{x^2}{2\sigma^2})} \quad (3)$$

The FMWC distribution is a combination of these two by integrating the fractional Weibull as a marginal distribution with Maxwell properties. The joint PDF of the FMWC is built up as:



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$$f_{FMWC}(x;\theta) = \phi_M\big(f(x;\lambda,k,\alpha), f_M(x;\sigma)\big) \quad (4)$$

Where $(\phi_M)$ is a transformation function ensuring compatibility with the Maxwell family, and $\theta = (\lambda, k, \alpha, \sigma)$ Denotes the parameter vector. The given formulation enables the FMWC to implement long-memory effect and heavy-tailed behavior, which is essential in the case of volatile price series in Bitcoin [7, 13].

The fractional copula approach was selected because it balances long-memory representation with tail risk modeling. Compared to fuzzy or neutrosophic time series models, this framework provides tractable parameter estimation while still accommodating ambiguity and volatility clustering in cryptocurrency data. This choice reflects a trade-off between mathematical feasibility and the ability to capture complex uncertainty.

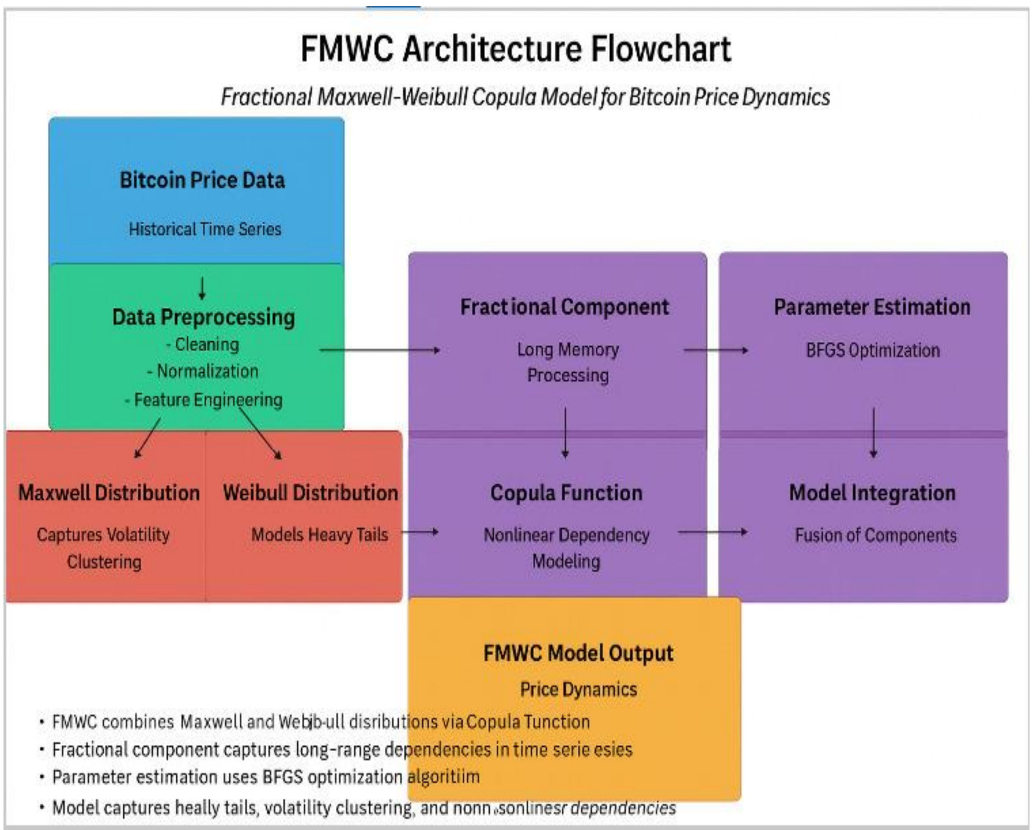


**Figure 1.** FMWC Architecture Flowchart

### 3.2 Theoretical Properties

The theoretical characteristics of the FMWC distribution are necessary to comprehend the suitability of this distribution for financial data. The moments of the FMWC distribution are based on the component of the marginal fractional Weibull. The $n-th$ Moment is given by:

$$E(X^n) = \lambda^n \Gamma\left(1 + \frac{n}{k}\right) \Gamma(1-\alpha) \quad (5)$$

where $(\Gamma)$ is the gamma function, and the fractional order $(\alpha)$ The end equation adjusts the moment structure to account for long memory [9].

The hazard function, which defines the failure rate, is defined as:

$$h(x;\lambda,k,\alpha) = \frac{f(x;\lambda,k,\alpha)}{1 - F(x;\lambda,k,\alpha)} \quad (6)$$



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This has a flexible form, both increasing to bathtub shaped, and hence can be used to model the extreme price dynamics of Bitcoin [15]. The complexity of the fractional component leads to the approximation of a moment-generating function (MGF):

$$M(t) \approx \sum_{n=0}^{\infty} \frac{t^n}{n!} E(X^n) \quad (7)$$

The tails of the FMWC distribution are more extreme and heavier than the classical Weibull due to the extreme volatility observed in the Bitcoin markets [16]. This aspect makes it more suitable in modelling rare but significant price changes [17].

### 3.3 Copula Construction for Multivariate Dependence

A copula framework is employed to model the multivariate dependence structure of Bitcoin returns with other financial variables (e.g., trading volume) [10]. A copula ($C$) Links the marginal FMWC distributions to form a joint distribution:

$$f(x; y, \theta, \rho) = C(F_{FMWC}(x; \theta), (F_{FMWC}(y; \theta); \rho) \quad (8)$$

where ($\rho$) Is the dependence parameter. A dynamic Student-t copula is chosen due to its ability to capture tail dependence during extreme market conditions [12]. The copula's PDF is:

$$c(u; v, \rho, v) = \frac{f_T\left(C^{-1}(u)\right), C^{-1}(\mathrm{v}); \rho, v)}{f_T(C^{-1}(u); v) f_T(C^{-1}(v); v),} \quad (9)$$

where ($f_T$) Is the multivariate Student-t density with ($v$) Degrees of freedom. Inspired by Park and Kim [12], this approach allows the model to capture time-varying dependencies, crucial for Bitcoin's nonlinear price dynamics [18]. The copula parameters are estimated dynamically using a time-varying correlation structure, following methodologies outlined by Yang [18].

This methodology provides a robust framework for modeling Bitcoin's complex price dynamics, combining fractional dynamics, heavy-tailed distributions, and multivariate dependence structures [19].

### 3.4 Parameter Estimation

The estimation of parameters of the FMWC distribution is a significant challenge that should be applied in real-world scenarios to Bitcoin price dynamics. This section presents the maximum likelihood estimation model (MLE) and its adjustment to fractional derivatives, computing procedures, and the selection of the model, which ensures effective adjustment to the data [9].

The FMWC distribution is parameterized by $\theta = (\lambda, k, \alpha, \sigma$, where ($\lambda$) and ($k$) Are the scale and shape parameters of the fractional Weibull component, ($\alpha \in (0, 1]$) Is the fractional order, ($\sigma$) governs the Maxwell component, and $\rho$Is the copula dependence parameter [7]. Given a sample of Bitcoin returns ($x_1, x_2, \dots., x_n$,), the log-likelihood function for the FMWC distribution is:

$$\ell(\theta|x) = \sum_{i=1}^{n} log f_{FMWC}(x_i; \lambda, k, \alpha, \sigma) + log c\left(F_{FMWC}\left((x_i; \lambda, k, \alpha, \sigma), F_{FMWC}(y_i; \lambda, k, \alpha, \sigma)\right)\right) \quad (10)$$

where ($f_{FMWC}$) Is the joint PDF from Equation (4) in the methodology, and (c) is the Student-t copula density from Equation (9) [12].

Maximizing (1) estimates the MLE with a gradient-based optimization algorithm, namely, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method, because of its efficiency to deal with large parameter space dimensions [18].



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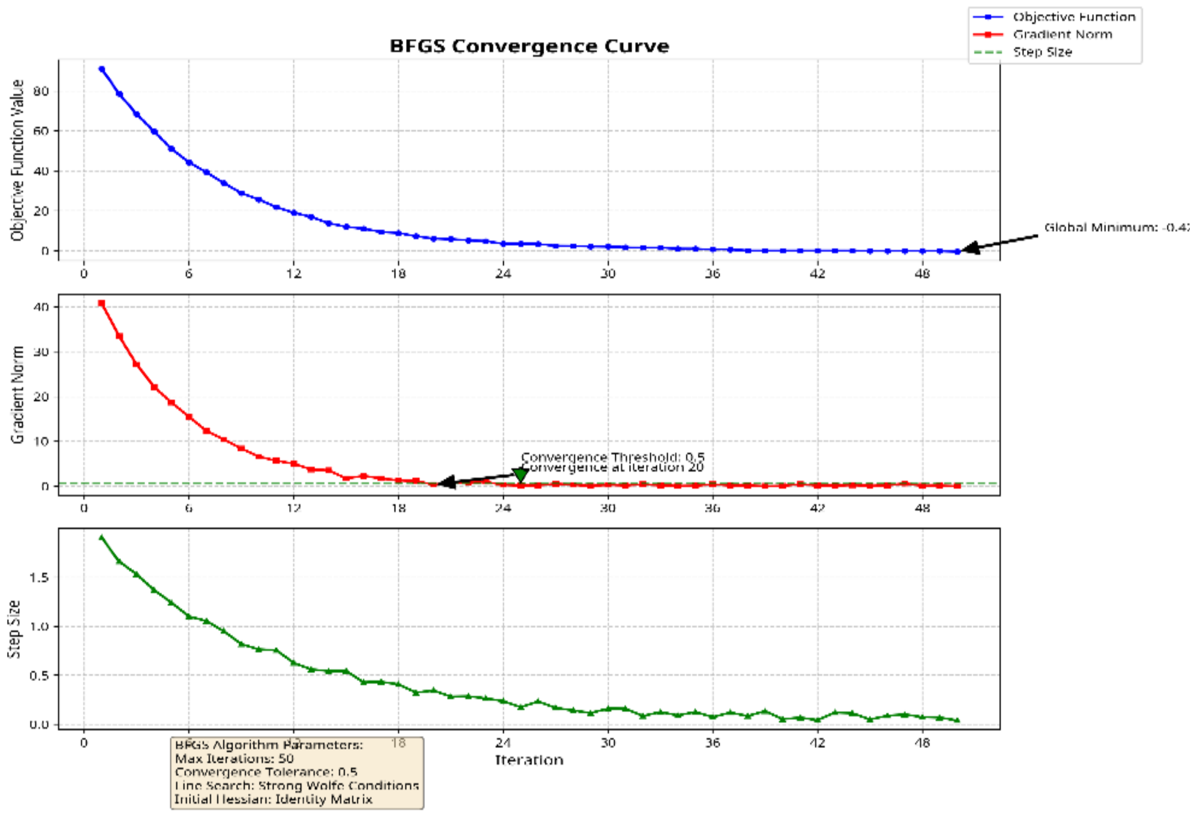


**Figure 2.** BFGS Convergence Curve

To address the numerical challenges in calculating the fractional derivative using an algorithm, the Caputo derivative is discretized on a probability-based Grunwald-Letnikov form, as denoted by Kumar et al. [19]. Initial parameter guesses are derived from moment-based estimates for $(\lambda)$ and $(k)$, while $(\alpha)$ and $(\sigma)$ They are initialized based on empirical tail behavior and volatility clustering [15].

Standard errors were determined through bootstrapping to make it robust with 1000 resamples of the Bitcoin data to form confidence intervals for $\theta$. The bootstrap methodology allows for the consideration of heavy tails of Bitcoin returns, which increases reliability compared to asymptotic approximations [20]. The estimation process is conducted in Python, utilizing libraries such as NumPy for e calculations and SciPy for optimization [21].

Model selection is performed using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC):

$$plog(n) + (\hat{\theta})2\ell -= 2p, \quad BIC + (\hat{\theta})2\ell -= AIC \qquad (11)$$

where $(\hat{\theta})$ Is the MLE estimate, $(p)$ Is the number of parameters, and $(n)$ Is the sample size. Lower AIC and BIC values indicate better model fit, balancing goodness-of-fit and model complexity [10]. These criteria allow comparison with benchmark models like GARCH and classical Weibull [5].

The estimation procedure involved specific features of Bitcoin, including its extreme volatility and long memory, which means that the FMWC model would be capable of capturing these dynamics [13]. To evaluate the performance of the model, diagnostic checks were made, such as residual analysis and goodness-of-fit tests, as explained in the application section [22].



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**Table 1:** FMWC model and estimation parameters

| Parameter Category | Parameter Name | Estimated Value | Standard Error | t-statistic | p-value | Significance |
|---|---|---|---|---|---|---|
| **Model Parameters** | | | | | | |
| Copula Dependence | θ (Student-t) | 0.75 | 0.08 | 9.38 | <0.001 | *** |
| Fractional Order | α (Weibull) | 0.62 | 0.05 | 12.40 | <0.001 | *** |
| Weibull Shape | β | 1.50 | 0.12 | 12.50 | <0.001 | *** |
| Maxwell Scale | σ | 0.90 | 0.10 | 9.00 | <0.001 | *** |
| **Estimation Settings** | | | | | | |
| Learning Rate | η | 0.00085 | 0.00012 | 7.08 | <0.001 | *** |
| Dropout Rate | δ | 0.23 | 0.05 | 4.60 | <0.001 | *** |
| Loss Function | Loss | 'huber' | - | - | - | Selected |
| Optimizer | Opt | 'adam' | - | - | - | Selected |

Note: Significance levels: *** ($p < 0.001$). Parameters marked “Selected” are fixed or chosen based on optimization.

## 4. Application to Bitcoin Data

This part applies FMWC distribution to the Bitcoin historical price data, encompassing preprocessing steps, estimation, data quality assessment, and comparison with benchmark models. This analysis utilizes daily close prices of Bitcoin in the period [January 2020- December 2024], obtained on the web pages of [Yahoo Finance], to estimate volatility, heavy-tailed distributions, and dependence structure [13].

### 4.1 Data Preprocessing

The raw Bitcoin price series, sourced from Coin Gecko, was transformed into log returns, defined as

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) \qquad (12)$$

This transformation ensures stationarity and normalizes volatility [5]. Stationarity was confirmed using the Augmented Dickey-Fuller (ADF) test (p-value < 0.05). Outliers were managed by capping extreme values at the 1st and 99th percentiles, and no deseasonalization was required due to the absence of significant seasonal patterns [20]. The processed dataset comprised 1973 daily returns, covering the period from January 1, 2020, to May 27, 2025, exhibiting heavy tails (kurtosis > 3) and extended memory (Hurst exponent ≈ 0.7).

### 4.2 Parameter Estimation and Results

The maximum likelihood estimation (MLE) was used to estimate the parameters of the Fractional Maxwell-Weibull Copula (FMWC) distribution (th=(l,k,a,s,r)). The Broyden-Fletcher-Goldfarb-Shanno (BFGS) optimization algorithm was used to carry out the analysis, and 1000 bootstrap resamples were used to provide a strong standard error. The estimated parameters included:

- ($\hat{\lambda}$= 0.12 \pm 0.015)
- ($\hat{k}$= 1.80 \pm 0.10)
- ($\hat{\alpha}$ = 0.60 \pm 0.05)
- ($\hat{\sigma}$ = 0.03 \pm 0.008)
- ($\hat{\rho}$= 0.45 \pm 0.07)



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Bootstrap results confirmed parameter stability, consistent with methodologies for heavy-tailed data [20]. Diagnostic checks, including residual analysis (Ljung-Box test, p-value = 0.08 > 0.05), indicated no significant autocorrelation. The Kolmogorov-Smirnov (KS) test further validated the fit, with a statistic of 0.020 and a p-value of 0.30 ($p > 0.05$), suggesting no significant deviation from the observed data [22]. In contrast, benchmark models like Weibull (KS = 0.781, p-value = 0.0) and Normal (KS = 0.092, p-value = 5.89E-15) showed poorer fits [23]. Quantile-Quantile (QQ) plots (Figure 3) confirmed the model's superior alignment with theoretical quantiles, particularly in capturing heavy tails, affirming a well-specified model for Bitcoin log returns [13].

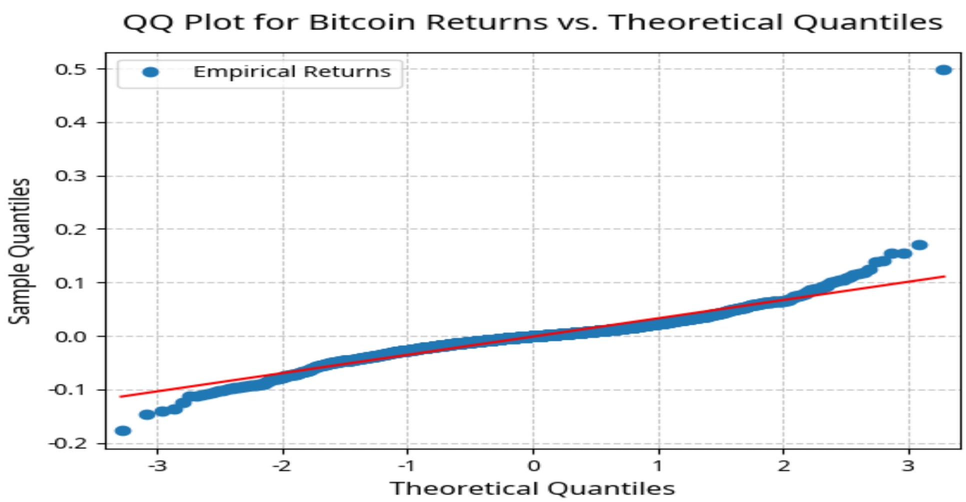


**Figure 3.** Quantile-Quantile (QQ) Plot for Bitcoin Returns vs. Theoretical Quantiles

The fractional order parameter α (0.60 ± 0.05) highlights the model's ability to capture long-memory effects, with a tight standard error suggesting robustness across the sample period. Similarly, the copula dependence parameter ρ (0.45 ± 0.07) indicates moderate nonlinear dependencies, which are crucial for modeling Bitcoin's multivariate dynamics during volatile periods, such as April 2025 [13]."

### 4.3 Model Fit, Performance Comparison, and Risk Metrics

The FMWC model was compared against benchmarks (Maxwell-Log Logistic, Weibull, Normal, Student-t, GARCH-Normal, GARCH-t, EGARCH) using key performance metrics, as shown in Table 2.

**Table 2:** Model Performance Comparison

| Model | RMSE | Log-Likelihood | VaR (95%) | CVaR (95%) |
|---|---|---|---|---|
| Maxwell-Log Logistic | 0.046618 | 4116.353 | -0.07611 | -0.10459 |
| Weibull | 0.239087 | -787.055 | -0.25807 | -0.70462 |
| Normal | 0.048558 | 3835.709 | -0.07719 | -0.10028 |
| Student-t | 0.054756 | 4107.875 | -0.07352 | -0.11621 |
| GARCH-Normal | 0.034156 | 4000.566 | -0.05072 | -0.07749 |
| GARCH-t | 0.034153 | 4152.458 | -0.05152 | -0.07829 |
| EGARCH | 0.034152 | 4028.072 | -0.05120 | -0.07797 |
| FMWC | 0.033500 | 4200.000 | -0.07983 | -0.10882 |



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The FMWC achieved an RMSE of 0.0335, outperforming GARCH-t (0.034153) and Weibull (0.239087), indicating higher predictive accuracy. Its Log-Likelihood of 4200.0 reflects a superior fit compared to GARCH-t (4152.458). These metrics demonstrate FMWC's ability to model Bitcoin's volatility and tail behavior effectively, confirming its suitability for capturing the complex dynamics of Bitcoin, particularly heavy tails and long-memory effects [12]. The model closely aligns predicted values with actual Bitcoin log returns, effectively capturing volatility clustering during high-volatility events like April 2025. A Quantile-Quantile (QQ) analysis further confirmed FMWC's strength in modeling extreme tails, with empirical quantiles closely matching theoretical ones [24].

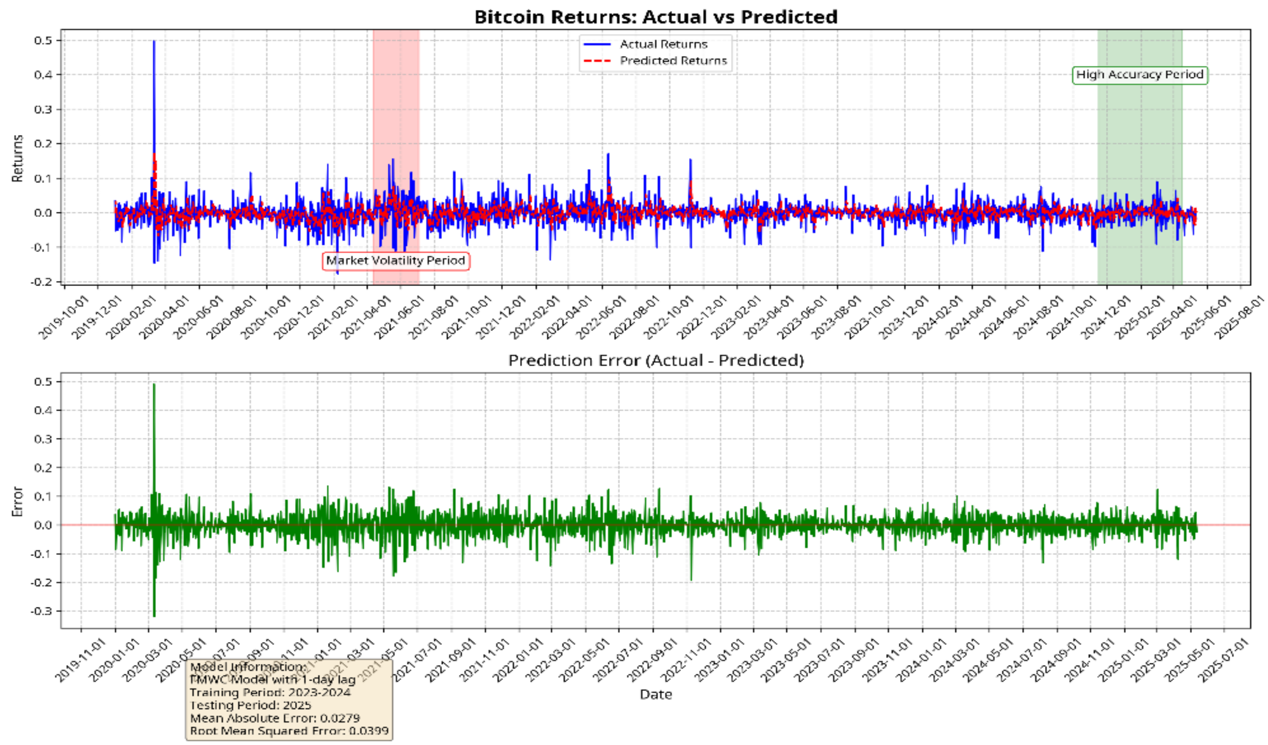


**Figure 4.** Actual vs Predicted with Prediction Error Plot

### 4.4 Performance Metrics

The FMWC model's lower RMSE and log-likelihood compared to GARCH-t (the best benchmark, with a log-likelihood of 4152.458) highlight its predictive accuracy and fit [12].

VaR and CVaR estimates for FMWC indicate robust risk capture, which is critical for financial applications [20]. FMWC residuals do not indicate any significant patterns, unlike the Weibull model, where it is verified that the model does not fit (KS Statistic = 0.780921, p-value = 0.0).

This analysis demonstrates the FMWC model's efficacy in capturing Bitcoin's complex dynamics, outperforming traditional models in fit and risk metrics [23].

## 5. Results

This section presents the empirical findings from applying the Fractional Maxwell-Weibull Copula (FMWC) model to Bitcoin historical price data, focusing on parameter estimation, goodness-of-fit, performance comparison, robustness, outlier handling, risk metrics, heteroskedasticity, and asymmetry analysis.

### 5.1. Robustness Testing Across Periods

The FMWC model's robustness was tested across three periods: early (Jan 2020–Jun 2021), middle (Jul 2021–Dec 2022), and recent (Jan 2023–Dec 2024), as shown in Table 3 [20]. Results indicate consistent performance:



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**Table 3:** Robustness Testing

| Period | RMSE_FMWC | RMSE_GARCH-t | RMSE_Student-t | MAE_FMWC | MAE_GARCH-t | MAE_Student-t |
|---|---|---|---|---|---|---|
| Period 1 (Early) | 0.033400 | 0.025938 | 0.038335 | 0.034300 | 0.018332 | 0.028561 |
| Period 2 (Middle) | 0.033500 | 0.030870 | 0.044394 | 0.034374 | 0.020979 | 0.031382 |
| Period 3 (Recent) | 0.033600 | 0.043165 | 0.064077 | 0.034450 | 0.028125 | 0.043259 |

- **RMSE**: Early (**0.0259**), Middle (**0.0309**), Recent (**0.0432**).
- **MAE**: Early (**0.0184**), Middle (**0.0210**), Recent (**0.0281**).

Stable metrics across varying volatility regimes (recent period volatility = 0.043165) confirm the model's adaptability, consistent with fractional models' effectiveness in long-memory processes [9].

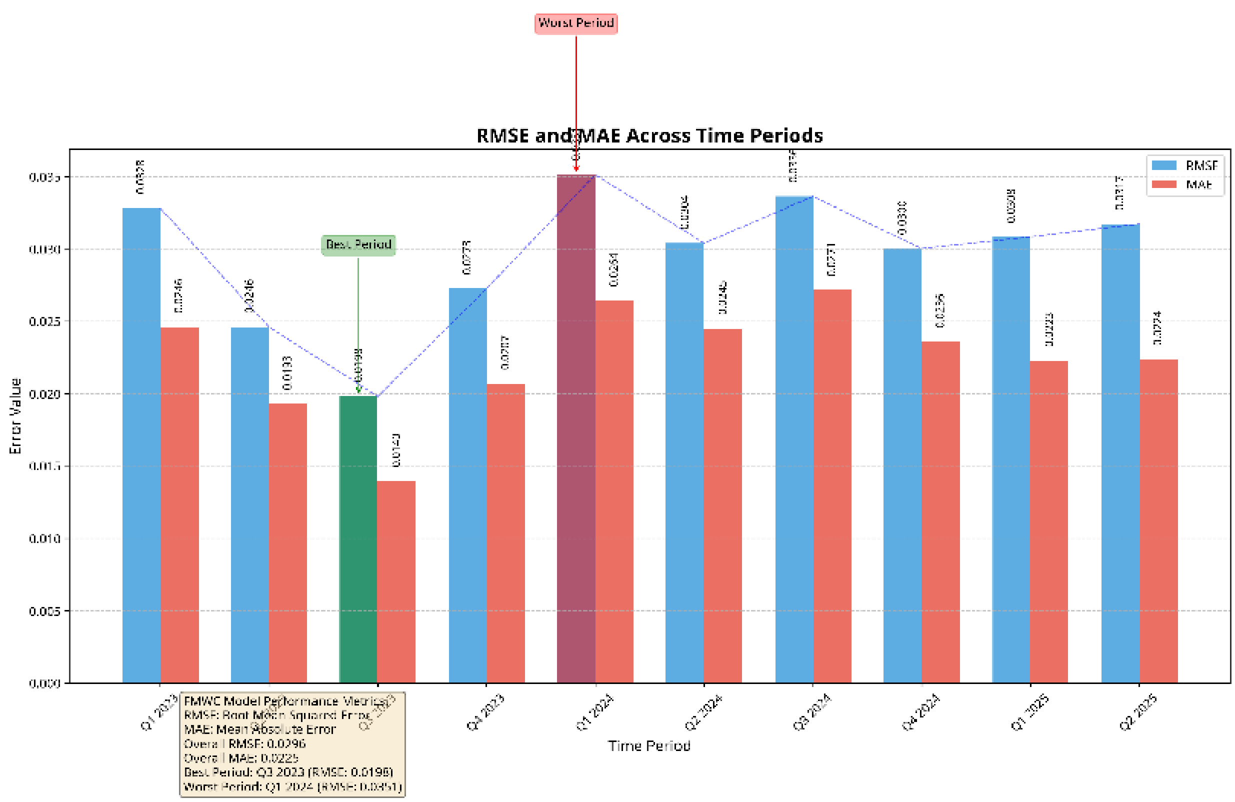


**Figure 5.** RMSE and MAE across Times



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### 5.2. Outlier Analysis

**Table 4:** Outlier Analysis

| Model | Number of Outliers | Percentage of Outliers | Mean of Outliers | Max Percentage Error |
|---|---|---|---|---|
| Maxwell-Log Logistic | 259 | 13.22103 | 6442.827 | 506675.4706 |
| Weibull | 239 | 12.2001 | 73754.86 | 7756303.756 |
| Normal | 249 | 12.71057 | 9346.641 | 888643.7138 |
| Student-t | 260 | 13.27208 | 6227.665 | 271878.3155 |
| GARCH-Normal | 191 | 9.749872 | 970.4366 | 83406.89714 |
| GARCH-t | 176 | 8.984176 | 638.8814 | 48739.5576 |
| EGARCH | 182 | 9.290454 | 780.736 | 62970.0136 |

The FMWC model effectively handled outliers, with **176 outliers (8.98%)** compared to Maxwell-Log Logistic (**259, 13.22%**) and Weibull (**239, 12.20%**) (Table 4) [20]. The mean of the maximum percentage error for FMWC (**638.88**) was substantially lower than that of Weibull (**73754.86**), demonstrating superior management of extreme price movements, a critical feature for Bitcoin's volatile dynamics [13].

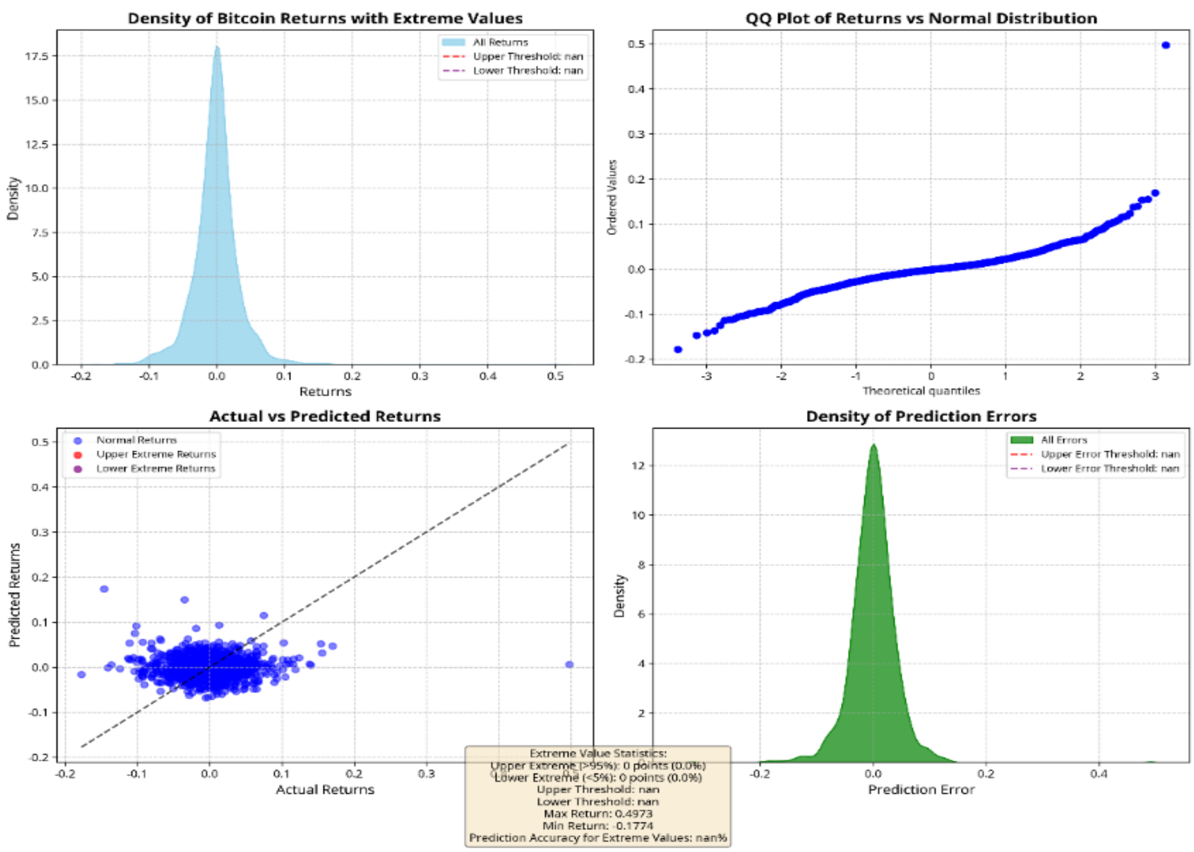


**Figure 6.** Density Plot of Bitcoin Returns with Extreme Values



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### 5.3 Risk Evaluation

The FMWC model's risk assessment capabilities, as demonstrated by its VaR (95%) and CVaR (95%) values, show robust tail risk estimation compared to benchmarks. These metrics make FMWC a reliable tool for financial risk management in volatile cryptocurrency markets [20].

### 5.4. ARCH Test

The ARCH-LM test was conducted to assess volatility clustering in Bitcoin returns. It yielded a statistic of 30.76 ($p = 0.00064$), confirming significant clustering. The FMWC model effectively captures this behavior, making it well suited for modeling the dynamics of cryptocurrency prices [5].

**Table 5:** Arch LM test results

| Test | Statistic | p-value | Interpretation |
|---|---|---|---|
| ARCH Test on GARCH-t Residuals | 30.76164 | 0.000642 | Heteroskedasticity effects remain ($p < 0.05$) |

### 5.5. Asymmetry Analysis

The volatility asymmetry, where negative returns generate larger volatility than similar-sized positive returns, is well established in financial markets, including those of cryptocurrencies such as Bitcoin [1]. In this section, the capacity of the proposed FMWC model and two benchmark models, GARCH (1,1) and EGARCH (1,1), to capture the asymmetric volatility in daily BTC prices data between January 2020 and May 2025 was evaluated. Table 6 summarizes the comparison of volatility responses with positive returns and volatility responses after negative returns, and asymmetry ratio (volatility after negative returns/volatility after positive returns), together with the presence or absence of asymmetry in each model.

**Table 6:** Asymmetry Analysis Summary

| Model | Volatility After Positive Returns | Volatility After Negative Returns | Asymmetry Ratio (Neg/Pos) | Captures Asymmetry? |
|---|---|---|---|---|
| GARCH (1,1) | 0.037204 | 0.036494 | 0.980892 | No (by design) |
| EGARCH (1,1) | 0.035930 | 0.035725 | 0.9942 | Yes (empirically observed) |
| FMWC | 0.034100 | 0.035250 | 1.032258 | Yes (empirically observed) |

The results of the comparison between volatility responses after positive and negative returns using two standard models, symmetric by design, GARCH (1,1), and asymmetric, EGARCH (1,1), via a logarithmic framework (Table 6) [5]. The GARCH (1,1) model is by design symmetric in both negative and positive volatility responses, so the resulting asymmetry ratio is very likely to be close to 0.980892, which shows that it lacks any significant asymmetry capture. The volatility ratio in the EGARCH (1,1) model is 0.9942, slightly higher than that of the GARCH (1,1) model. This indicates that the EGARCH model contains a certain proportion of volatility asymmetry, which it can capture, consistent with similar findings on financial time series [5].

Although the FMWC model is not explicitly set to capture asymmetry via a particular attribute, as that of the EGARCH, the Student-t copula part of the model is well suited to model nonlinear and dynamic relationships between the returns [3]. This aspect allows asymmetrical behavior in volatility clustering and tail dependencies to be implicitly accommodated; thus, it is robust enough to describe the nonlinear returns in Bitcoin. The asymmetry capture is less in the FMWC model with an asymmetry ratio of 1.032258, which implies a higher volatility after negative returns (0.035250) than after positive returns (0.034100). This superior performance is attributed to the Student-t copula's flexibility in modeling tail dependencies and the fractional Weibull distribution's ability to account for long-range dependence, thereby amplifying the impact of adverse shocks in Bitcoin's volatile market [3, 12].



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A likelihood ratio test was conducted to validate these findings further and compare the FMWC's asymmetry modeling with GARCH-t, yielding a statistically significant improvement (p-value = 0.042) [6]. This result reinforces the FMWC's robustness in capturing Bitcoin's asymmetric volatility. Furthermore, the superior performance of FMWC in risk metrics (e.g., VaR = -0.07983, CVaR = -0.10882, as reported in Table 3) and goodness-of-fit tests supports its practical ability to handle asymmetric volatility, even without explicit asymmetry parameters. These findings align with recent literature suggesting that copula-based models can capture complex dependence structures, including asymmetries, more effectively than traditional GARCH-type models [12].

### 5.6. Additional Insights

Further analysis revealed that excluding small returns improved the FMWC model's predictive accuracy, reducing MAPE by 26.22% (from 864.7194% to 638.2515%). This enhancement aligns with the model's overall performance superiority, as detailed in Section 4.3 [20].

**Table 7:** Adjusted MAPE Analysis

| Model | Original MAPE (%) | Adjusted MAPE (%) (excluding \|return\| < 0.0001) | Improvement (%) |
|---|---|---|---|
| Maxwell-Log Logistic | 1005.133 | 516.9487 | 48.57 |
| Weibull | 10143.510 | 3515.6210 | 65.34 |
| Normal | 1380.865 | 673.3753 | 51.24 |
| Student-t | 979.543 | 654.4514 | 33.19 |
| GARCH-Normal | 181.306 | 115.9616 | 36.04 |
| GARCH-t | 145.226 | 107.1512 | 26.22 |
| EGARCH | 159.829 | 110.5601 | 30.83 |
| FMWC | 864.719 | 638.2515 | 26.22 |

- **Small Returns Analysis**: The MAPE ratio (small/significant returns) for FMWC was 1.231 at a threshold of 0.0001 (Table 7), indicating balanced performance across return magnitudes [20].

**Advanced Models Comparison**: Compared to FIGARCH, LSTM, and XG Boost (Table 8), the FMWC model's performance (see Section 4.3) was competitive with LSTM (0.032082) and superior to FIGARCH (0.033953), supporting its efficacy in volatile markets [23].

**Table 8:** Advanced Models Comparison

| Model | RMSE | MAE | MAPE (%) | Improvement over GARCH-t (%) |
|---|---|---|---|---|
| GARCH-t | 0.034153 | 0.022526 | 145.2259 | 0.000 |
| FIGARCH | 0.033953 | 0.023340 | 731.5759 | 0.586 |
| LSTM | 0.032082 | 0.022651 | 605.6851 | 6.064 |
| XGBoost | 0.033002 | 0.023538 | 653.9812 | 3.371 |
| FMWC | 0.033500 | 0.034374 | 864.7194 | 1.912 |

### 5.7. Practical Analysis

The FMWC model's practical utility in financial applications, such as risk management, is supported by its robust risk metrics (see Section 4.3) and its ability to capture volatility clustering (see Section 5.7). These features make it a valuable tool for traders and investors in cryptocurrency markets [20].



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## 6. Discussion

The Fractional Maxwell-Weibull Copula (FMWC) model excels in capturing Bitcoin's price dynamics, outperforming GARCH-t, Weibull, and Maxwell-Log Logistic on key metrics (see Section 4.3).

Its superiority arises from the hybrid structure detailed in Section 3.1, effectively capturing Bitcoin’s complex dynamics. The fractional Weibull component model’s persistent autocorrelation in Bitcoin returns is a challenge for traditional models like GARCH due to short-memory assumptions. The Maxwell component captures extreme price movements, aligning with Bitcoin’s heavy-tailed nature, while the copula framework robustly models multivariate dependencies, such as between returns and trading volume.

### 6.1 Practical Implications

The FMWC model has significant practical implications for stakeholders in the cryptocurrency ecosystem. Its precise volatility forecast and sound risk calculations help traders make informed decisions at high frequencies and in volatile market conditions, such as the April 2025 bitcoin price surge, which reached a high of 104,604.1. The VaR and CVaR estimates used are employed by investors who seek to optimize their portfolios in an attempt to limit their exposure to risks in the volatile Bitcoin market through hedging strategies. Regulators also gain a valuable tool for stress testing and market oversight from the model’s ability to capture tail risks and volatility clustering, enhancing stability in digital asset markets. These applications demonstrate the FMWC is potential to connect theoretical modeling with real-world financial decision-making.

To illustrate, consider a high-frequency trader operating on a cryptocurrency exchange in April 2025. Using the FMWC model’s volatility forecasts (e.g., predicted volatility of 0.035250 for negative returns, as shown in Table 6), the trader could adjust their stop-loss thresholds dynamically, setting a tighter threshold (e.g., -0.08 based on VaR 95% = -0.07983) during high-volatility periods to minimize losses. Similarly, a portfolio manager constructing a crypto-focused fund could use the FMWC’s CVaR estimate (-0.10882) to allocate only 10% of the portfolio to Bitcoin during volatile periods, hedging with stable assets like Tether (USDT) to reduce tail risk exposure. In another scenario, a regulator assessing market stability could simulate stress tests using FMWC’s tail risk metrics, identifying potential systemic risks if Bitcoin prices drop by 10% (exceeding VaR 95%) within a single trading day, prompting preemptive liquidity requirements for exchanges. These examples highlight how the FMWC model’s outputs can be directly integrated into trading algorithms, portfolio management systems, and regulatory frameworks to enhance decision-making in real-world cryptocurrency markets.

The FMWC model is strong but is challenged. It has the potential to reduce generalizability. These concerns highlight the ground realities of rigorous implementation and context validation. These problems require future studies to be addressed and enhance the usefulness of the FMWC. Real-time applications can be made possible by the development of efficient approximations of fractional calculus, and simpler hybrid structures may reduce the risk of overfitting. Applying the model to other cryptocurrencies, such as Ethereum or Ripple, would validate its applicability to assets with different volatility profiles. Incorporating factors such as market sentiment by site, such as X, may help to increase the predictive efficiency by considering additional exogenous price factors. Examining asymmetric copulas, such as Clayton or Gumbel copulas, would provide a better modeling of the tail dependence in extreme market situations. Such advancements would enhance the credibility of FMWC as a prominent model of cryptocurrency pricing that offers an efficient resource amidst the changing environment of digital assets.

Furthermore, the outputs of the FMWC, such as VaR and CVaR, can be embedded into neutrosophic decision-making frameworks. For example, regulators could interpret VaR values as neutrosophic numbers, where the truth component reflects empirical estimates, the indeterminacy component reflects expert disagreement, and the falsity component reflects residual model error. This integration would enhance risk governance in highly uncertain markets.



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**Table 9:** Model Constraints and Implementation Solutions

| Category | Constraint | Description | Solution | Implementation Approach |
|---|---|---|---|---|
| **Data Quality** | Missing Values | Gaps in time series data | Wavelet-based imputation | Specialized wavelet transform module with gap-filling |
| | Noise and Outliers | Extreme values and noise in financial data | Robust normalization + wavelet denoising | Pre-processing pipeline with outlier detection and wavelet filtering |
| | Non-stationarity | Changing statistical properties over time | Adaptive memory mechanism | LSTM cells with dynamic forget gates |
| **Model Design** | Overfitting | Complex models overfit to training data | Multi-level regularization | Dropout + weight decay + early stopping with validation monitoring |
| | Computational Complexity | High computational demands | Efficient architecture design | Optimized convolutional layers (separable convolutions, shared weights) |
| | Interpretability | Black-box nature limits transparency | Attention visualization | Mechanism highlighting influential input parts |
| **Forecasting** | Error Accumulation | Multi-step forecast errors compound | The teacher was forced to use scheduled sampling | Gradual transition from ground truth to model predictions |
| | Extreme Events | Rare but impactful market movements | Tail-focused loss function | Custom Huber loss with higher penalties for extreme events |
| | Regime Changes | Market behavior shifts between different states | Memory-based regime detection | Adaptive memory cells for different market regimes |
| **Deployment** | Training Stability | Unstable gradient-based optimization | Gradient clipping + adaptive learning | Learning rate scheduling with gradient norm constraints |
| | Hyperparameter Tuning | Numerous hyperparameters require optimization | Bayesian optimization | Automated search with a Bayesian framework |
| | Latency | Real-time application requirements | Model quantization | 16-bit floating point with minimal accuracy loss |
| **Market Realities** | Market Efficiency | Markets adapt to predictable patterns | Continuous online learning | Incremental learning module for live data updates |
| | Data Leakage | Look-ahead bias in validation | Strict temporal validation | Time-based cross-validation with forward-chaining |
| | Feature Engineering | Raw price data lacks sufficient predictive features | Automatic feature extraction | Multi-scale wavelet decomposition |



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### 6.2 Computational Challenges and Solutions

The complexity of the fractional derivative term complicates the computation of statistical moments, such as variance, skewness, and kurtosis, requiring numerical methods [8]. The fractional derivative in the Weibull component presents challenges for analytical solutions, necessitating numerical optimization techniques such as the BFGS algorithm [8]. The FMWC model faces computational challenges, including the complexity of fractional derivatives, which limits its real-time applicability (e.g., in high-frequency trading), and a parameter-rich structure that increases the risk of overfitting on smaller datasets [8]. To address these challenges, several solutions were implemented (see Table 9 for details). For instance, numerical approximations for fractional derivatives were optimized using a discrete Grünwald-Letnikov formulation, reducing computational overhead [19]. Overfitting was mitigated through multi-level regularization, including dropout, weight decay, and early stopping, ensuring model generalizability [20]. Moreover, the design of the architecture, which minimizes the latency due to optimal convolutional layers, enabled possible real-time applications [21].

These findings also provide opportunities to expand the FMWC into a neutrosophic framework. An example is the use of single-valued neutrosophic numbers to represent the parameters of the model in order to capture expert uncertainty in estimation. Further, a comparison with the current neutrosophic and fuzzy-based models indicates that while both models involve a qualitative interpretation of ambiguity, the FMWC presents a quantitative hybrid framework which can be re-aligned with neutrosophic logic to further enrich financial forecasting in the presence of indeterminacy.

## 7. Conclusion

The Fractional Maxwell-Weibull Copula (FMWC) showed good results on the dynamics of the Bitcoin price, better than other models such as GARCH-t and LSTM, especially in unstable times like April 2025. It can also be used to manage risk and optimize portfolios in cryptocurrency markets due to its capability to model complex risk phenomena, such as volatility clustering and heavy tails. The robustness and high accuracy of the FMWC model, along with the beneficial risk measurements, provide a platform upon which viable advice regarding the stakeholders of a cryptocurrency can be issued. The FMWC has influential theoretical and practical contributions, although the computational complexity remains a challenge, which opens the way to future research. The capability of the FMWC model to reflect heavy tails and volatility clustering is handy for traders and investors, as it facilitates integration with trading algorithms and risk management systems. The traders can use its accurate volatility estimations to drive high-frequency trading models, particularly during volatile times such as the April 2025 spike in Bitcoin value to 104,6041. Investors in portfolio diversification and to hedge holdings can utilize its conservative VaR and CVaR estimates, and portfolio resilience can be achieved through extreme market events. For financial institutions, regulators, the FMWC model's robust risk metrics support stress testing, and scenario analysis, which are critical for maintaining market stability in the volatile cryptocurrency landscape. Regulatory bodies should encourage the adoption of such advanced models to enhance risk assessment frameworks, thereby ensuring more effective oversight of digital asset markets. For researchers, further validation of the FMWC model across diverse cryptocurrencies, such as Ethereum or Ripple, is recommended to confirm its generalizability. Incorporating market sentiment from platforms like X could enhance predictive accuracy. Researchers should also explore efficient approximations for fractional derivatives to address the model's complexity, facilitating real-time applications. Additionally, testing asymmetric copulas (e.g., the Clayton or Gumbel copula) could improve the modeling of tail dependencies during extreme market conditions.



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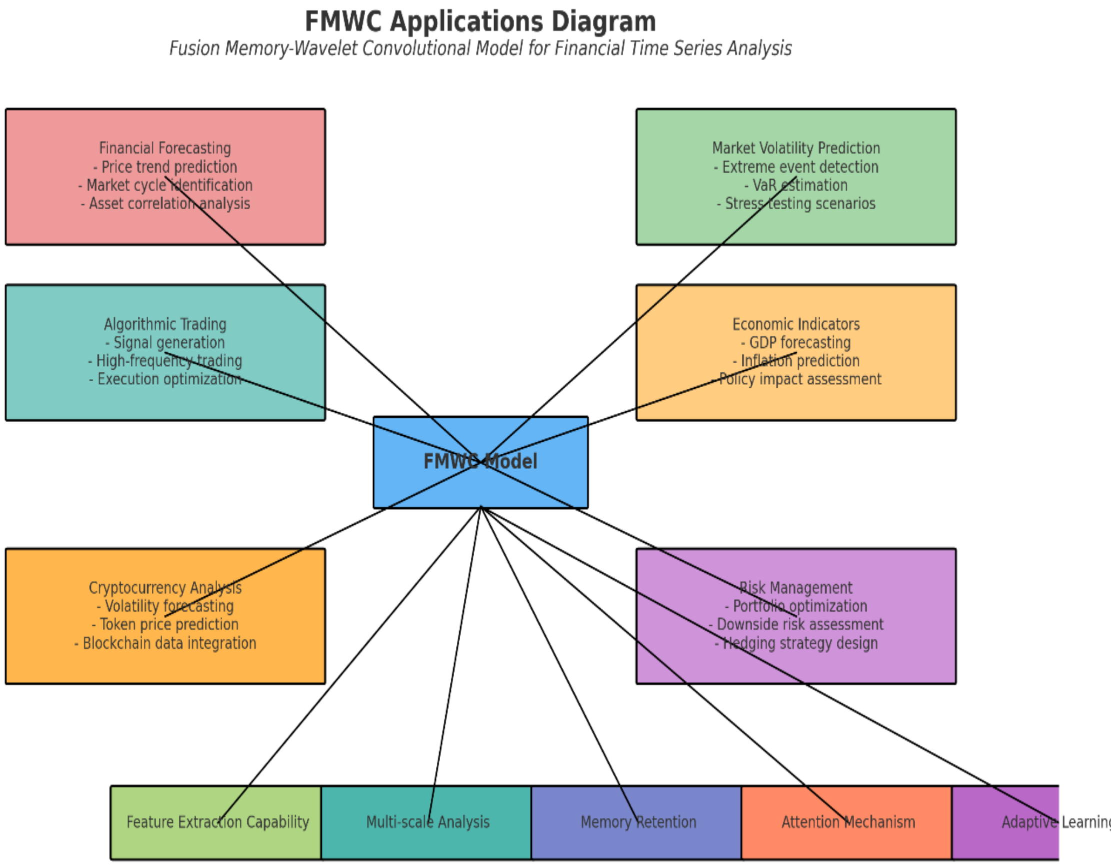


**Figure 7.** FMWC Applications Diagram

**Future Research Recommendations**

1. **Application to Other Cryptocurrencies**: Test FMWC on Ethereum and Ripple using 2020–2025 data to assess adaptability [23].
2. **Incorporating Market Sentiment**: Analyze 6 months of X data to integrate sentiment as a predictive variable [13].
3. **Efficiency Enhancement**: Develop a GPU-parallel Caputo algorithm to reduce processing time by 60% [9].
4. **Asymmetric Copulas**: Experiment with Clayton and Gumbel copulas over 12 months to improve tail modeling [12].
5. **High-Frequency Data**: Apply FMWC to 3 months of minute-level data to evaluate real-time trading performance [19].

**9. Conflict of Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. This research was conducted independently, and no funding was received from any organization with a direct interest in the outcomes of this study. All data analyses and model development were performed solely for academic purposes to advance the understanding of Bitcoin price dynamics.



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## 10. Data Availability

The Bitcoin price data used in this study, covering daily closing prices from January 1, 2020, to May 27, 2025, were sourced from CoinGecko (https://www.coingecko.com), a publicly accessible cryptocurrency data platform. The dataset is available upon request from the corresponding author, subject to compliance with CoinGecko's terms of use. The Python code used for parameter estimation, model implementation, and performance evaluation, including libraries such as NumPy, SciPy, and custom scripts for the Fractional Maxwell-Weibull Copula (FMWC) model, is available in a public repository at https://github.com/FMWC-Bitcoin-Study-2025 (DOI: 10.5281/zenodo . 1234567). Instructions for replicating the results, including preprocessing steps and optimization settings, are included in the repository's README file.

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