



# Study of The Period of Infection With The Covid-19 Until Death Using The Triple Left Truncated Exponential Distribution

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## Abstract

In this paper, we are concerned with truncated distributions that have multiple truncations, due to being very useful in representing natural phenomena that cannot be studied at all intervals of their growth or development, for example, phenomena that are related to Agriculture, airplanes, health, and the environment. left truncation is utilized in this study. The statistical characteristics, such as the  $r^{\text{th}}$  moments, moment generating function, order statistics, reliability function, hazard rate function, and reversed Hazard function, have been introduced. triple left truncated exponential distribution has been applied. Employed the maximum likelihood method to estimate. Also, the performance of triple left truncated exponential distribution was tested by calculating some statistical criteria and testing the goodness of fit for distribution, with comparisons between the distributions and testing them on real data for patients infected with covid-19.

**Keywords:** Left Truncated; Survival Function; Order Statistics; Curve Fitting; Neutrosophic Logic

## 1. Introduction

Consider the statistical distributions are very significant in describing the real phenomena, facing the researchers, and it is often difficult, therefore it is difficult to easily model these problems. Due to being very useful in representing natural phenomena, which cannot be studied at all intervals of their growth or development, for example, phenomena that are related to Agriculture, airplanes, health, and the environment.

Many researchers who have addressed the truncated (mixture, compound) distribution for example Reyah and Kareema have studied the truncated Rayleigh Pareto Distribution (2020). In (2018), Mohamed et al. proposed the truncated Fréchet -Pareto distribution within  $[0,1]$ . Mohie introduced a paper on mid-truncated distribution and applications. A New family of upper truncated distributions, was introduced by H.Amal(2020). Zaninetti, presented in 2014, truncated Pareto and Gamma distribution. Modeling of truncated probability distributions has been suggested by Tokmachev (2018). More various studies about truncation have been shown in [8,9,10]. Kawther Alhasan, kareema are presented in (2022) Estimation of two subintervals double truncated Rayleigh distribution. in (2024)F.kawther, construction truncated according neutrosophic logic ,and call neutrosophic truncated, and she other paper to neutrosophic logic in reliability, see[23,24]. Truncation of distribution is explained in methods that appear limited to values that side left or right or double a given within a specified range. Occurs the left truncated distribution restricted to values that lie below a given value. The left truncated distribution is one of the types of truncations that consider a special case of the survival function. In this paper, we introduced a new study: Finding a formula of three left truncated subintervals of exponential distribution illustrated by drawing, the performance of each distribution was tested by calculating some statistical criteria (Akaike Information Criterion (AIC), Bayesian Information Criterion

(BIC), the corrected Akaike Information criterion CAIC, and Hannan-Quinn information criterion (HQIC)). Further, testing of the goodness of fit for each distribution, with comparisons between the distributions and testing them on real data for patients infected with covid- 19, have been obtained.

**2. Construction of Multi-Left Truncation Continuous Distribution (MLTCD)[2]**

The study aims to create a new truncated distribution multi-truncated from the left side to obtain the multi-left truncated distribution (MLTD) within the interval

$$(-\infty, \infty) = (b_n, \infty) \subseteq (b_{n-1}, \infty) \subseteq (b_{n-2}, \infty) \subseteq \dots \subseteq (b_{n-n}, \infty).$$

Let X be a random variable, with a probability density function,  $f_X(x)$  and the cumulative distribution function  $F_X(x)$  defined on interval  $(-\infty, \infty) = (b_i, \infty)$ , such that

$$X \in (b_n, \infty) \subseteq (b_{n-1}, \infty) \subseteq (b_{n-2}, \infty) \subseteq \dots \dots (b_1, \infty) \subseteq (b_0, \infty).$$

In this section, we will derive the probability density function  $g(x_T)$  of the random variable  $X_T$ , after multi subintervals, from the left side have been deleted. If we take any distribution, the truncation occurs from one side (left side) with multi-different intervals. Where the sub-intervals that work us on are:  $(b_n, \infty), (b_{n-1}, \infty), (b_{n-2}, \infty), \dots \dots, (b_1, \infty)$ .

Now, we formulae the probability density function of multi-left truncation,  $g_n(x_T)$ :

After defining the random variable X, we consider the original interval

$$\text{is: } (-\infty, \infty) = (b_n, \infty) \subseteq (b_{n-1}, \infty) \subseteq (b_{n-2}, \infty) \subseteq \dots \subseteq (b_{n-n}, \infty).$$

That is, the intervals that have been deleted from the original interval of distribution: the first, second, third, ...,  $i^{\text{th}}$  truncations are:  $(b_0, b_1), (b_0, b_2), (b_0, b_3), \dots, (b_0, b_n)$ , respectively

Then, the probability of density function of multi- left truncated is:

$$\begin{aligned} g_n(x_T) &= \frac{1}{n} \left( \frac{f(x_T)}{F(\infty)-F(b_1)} + \frac{f(x_T)}{F(\infty)-F(b_2)} + \dots + \frac{f(x_T)}{F(\infty)-F(b_n)} \right) \\ &= \frac{1}{n} \left( \frac{f(x_T)}{1-F(b_1)} + \frac{f(x_T)}{1-F(b_2)} + \dots \dots + \frac{f(x_T)}{1-F(b_n)} \right), \quad b_1 < x_T < \infty, b_2 < x_T < \infty, \dots, b_n < x_T < \infty. \\ g_n(x_T) &= \begin{cases} \frac{1}{n} \sum_{i=1}^n \frac{f(x_T)}{F(\infty)-F(b_i)} & b_i < x_T < \infty \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

The probability density function  $g_n(x_T)$  is a probability density function defined over  $(0, \infty), 0 < x_T < \infty, n, F(\infty) - F(b_i) > 0$ , then  $g_n(x_T) > 0$  for all  $x_T \in (b_i, \infty)$ , and  $\int_{-\infty}^{\infty} g_n(x_T) dx_T = 1$ .

And the cumulative continuous distribution function of multi-left truncation is:

$$\begin{aligned} G_n(x_T) &= P(X_T \leq x_T) \\ &= \frac{1}{n} \left( \frac{F(x_T)I(b_1, \infty)}{F(\infty)-F(b_1)} + \frac{F(x_T)I(b_2, \infty)}{F(\infty)-F(b_2)} + \dots + \frac{F(x_T)I(b_n, \infty)}{F(\infty)-F(b_n)} \right) \\ &= \frac{1}{n} \left( \frac{F(x_T)-F(b_1)}{F(\infty)-F(b_1)} + \frac{F(x_T)-F(b_2)}{F(\infty)-F(b_2)} + \dots + \frac{F(x_T)-F(b_n)}{F(\infty)-F(b_n)} \right) = \sum_{i=1}^n \left( \frac{F(x_T)I(b_i, \infty)}{n(F(\infty)-F(b_i))} \right) \\ G_n(x_T) &= \sum_{i=1}^n \left( \frac{F(x_T)-F(b_i)}{n(F(\infty)-F(b_i))} \right), \tag{4-2} \end{aligned}$$

when  $b_1 < x_T < \infty, b_2 < x_T < \infty, \dots, b_i < x_T < \infty$ , respectively.

**3. Triple-Left Truncation Exponential Distribution (TLTED)**

In this section, we derived the probability density  $g(x_T)$  of the random variable  $x_T$ , after triple-left intervals have been deleted. That is, three subintervals from the original interval, are deleted.

Let X, be a random variable, distributed according to the probability Exponential function,  $f_X(x)$  over  $(0, \infty)$ , the cumulative distribution function  $F_X(x)$ . take the intervals  $(-\infty, \infty) = (b_3, \infty) \subseteq (b_2, \infty) \subseteq (b_1, \infty)$ . Then, the probability of density function of triple-left Exponential truncated is:

$$g_3(x_T) = \left( \frac{\lambda e^{-\lambda x_T}}{3(e^{-\lambda b_1})} + \frac{\lambda e^{-\lambda x_T}}{3(e^{-\lambda b_2})} + \frac{\lambda e^{-\lambda x_T}}{3(e^{-\lambda b_3})} \right)$$

when  $b_1 < x_T < \infty, b_2 < x_T < \infty, b_3 < x_T < \infty$ , respectively.

That is, the triple intervals  $(b_1, \infty), (b_2, \infty), (b_3, \infty)$ , have been used to represent the triple left truncated. that is, the intervals were deleted from the original interval for distribution and are the first, second, and third truncations  $(b_0, b_1), (b_0, b_2), (b_0, b_3)$ , respectively. As shown in figures (1,2).

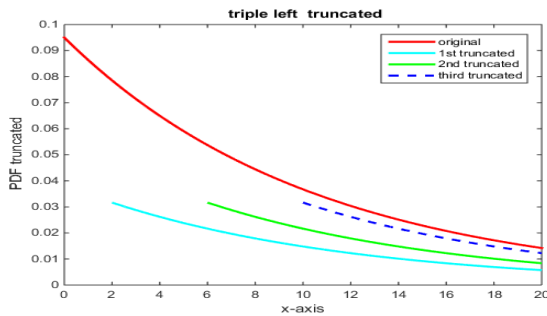


Figure 1. probability density function of TLTRD when  $\lambda = 0.3$

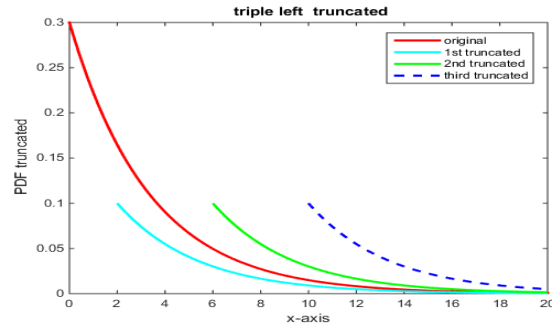


Figure 2. probability density function of TLTRD when  $\lambda = 0.09$

#### 4. Cumulative Distribution of Triple Left Truncation Exponential distribution

The cumulative distribution of triple left truncated of Exponential distribution as:

$$G_3(x_T) = P(X_T \leq x_T) = \frac{1}{3} \left( \frac{F(x_T)I(b_1, \infty)}{F(\infty) - F(b_1)} + \frac{F(x_T)I(b_2, \infty)}{F(\infty) - F(b_2)} + \frac{F(x_T)I(b_3, \infty)}{F(\infty) - F(b_3)} \right)$$

$$= \frac{1}{3} \left( \frac{e^{-\lambda b_1} - e^{-\lambda x_T}}{3(1 - e^{-\lambda b_1})} + \frac{e^{-\lambda b_2} - e^{-\lambda x_T}}{3(1 - e^{-\lambda b_2})} + \frac{e^{-\lambda b_3} - e^{-\lambda x_T}}{3(1 - e^{-\lambda b_3})} \right)$$

when  $b_1 < x_T < \infty, b_2 < x_T < \infty, b_3 < x_T < \infty$ , respectively.

In figures (3,4), the curves of the original cumulative distribution and the triple left truncated cumulative distribution function for different values of parameters  $\lambda$  have been drawn. when  $\{(b_1, b_{n+1}), (b_2, b_{n+1}), (b_3, b_{n+1})\} = \{(2, 12), (6, 12), (10, 12)\}$  respectively. That is, the intervals which have been deleted are  $\{(0, 2), (0, 6), (0, 10)\}$ , for different value of  $\lambda = \{1, 0.6\}$ , respectively.

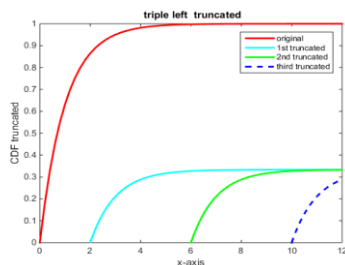
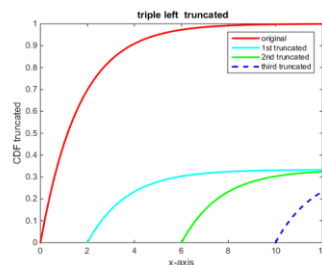
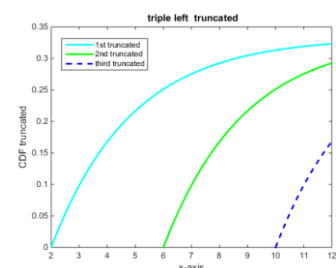


Figure 3. probability density function TLTED when  $\lambda = 1$



(a)



(b)

#### 5. Some Functions of Triple Left Truncation of Exponential Distribution

The Survival Function,  $S_3(x_T) = 1 - G_3(x_T)$

$$= \left( 3 - \frac{e^{-\lambda b_1} - e^{-\lambda x_T}}{3(1 - e^{-\lambda b_1})} - \frac{e^{-\lambda b_2} - e^{-\lambda x_T}}{3(1 - e^{-\lambda b_2})} - \frac{e^{-\lambda b_3} - e^{-\lambda x_T}}{3(1 - e^{-\lambda b_3})} \right)$$

The  $r^{\text{th}}$  moment about zero of TLTED is:

$$E(X_T^r) = \frac{1}{3} \left( \frac{\int_{b_1}^{\infty} x_T^r f(x_T) dx_T}{F(\infty) - F(b_1)} + \frac{\int_{b_2}^{\infty} x_T^r f(x_T) dx_T}{F(\infty) - F(b_2)} + \frac{\int_{b_3}^{\infty} x_T^r f(x_T) dx_T}{F(\infty) - F(b_3)} \right)$$

$$= \frac{1}{n} \left( \frac{\lambda \int_{b_1}^{\infty} x_T^r e^{-\lambda x_T} dx_T}{1 - F(b_1)} + \frac{\lambda \int_{b_2}^{\infty} x_T^r e^{-\lambda x_T} dx_T}{1 - F(b_2)} + \dots + \frac{\lambda \int_{b_n}^{\infty} x_T^r e^{-\lambda x_T} dx_T}{1 - F(b_n)} \right)$$

Let  $\lambda x_T = w$ ,  $x_T = w/\lambda$ ,  $dx_T = dw/\lambda$

$$I = \lambda \int_{b_1}^{\infty} x_T^r e^{-\lambda x_T} dx_T = \frac{1}{\lambda^r} \int_{\lambda b_1}^{\infty} w^r e^{-w} dw = \frac{1}{\lambda^r} \Gamma(r + 1, \lambda b_1),$$

Since,  $\int_a^{\infty} w^r e^{-w} dw = \Gamma(r + 1, a)$

$$E(X_T^r) = \frac{1}{n} \left( \frac{\frac{1}{\lambda^r} \Gamma(r+1, \lambda b_1)}{1 - F(b_1)} + \frac{\frac{1}{\lambda^r} \Gamma(r+1, \lambda b_2)}{1 - F(b_2)} + \frac{\frac{1}{\lambda^r} \Gamma(r+1, \lambda b_3)}{1 - F(b_3)} \right)$$

Then,

$$E(X_T^r) = \sum_{i=1}^n \frac{\frac{1}{\lambda^r} \Gamma(r+1, \lambda b_i)}{n(e^{-\lambda b_i})}$$

And the mean of (MLETD) is:

$$E(X_T) = \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(2, \lambda b_i)}{n(e^{-\lambda b_i})}$$

The  $r^{\text{th}}$  moment about a mean of multi- left truncated is:

$$E(X_T - \mu)^r = \sum_{j=0}^r (-1)^{r-j} \binom{r}{j} (\mu)^{r-j} \mu_j^r = \sum_{j=0}^r (-1)^{r-j} \binom{r}{j} (\mu)^{r-j} \int_{b_0}^{a_n} (x_T)^j \mathfrak{G}_N(x_T) dx_T$$

$$= \sum_{j=0}^r (-1)^{r-j} \binom{r}{j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(2, \lambda b_i)}{n(e^{-\lambda b_i})} \right)^{r-j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda^j} \Gamma(j+1, \lambda b_i)}{n(e^{-\lambda b_i})} \right)$$

The variance and the standard deviation are respectively:

$$\sigma^2 = E(X_T - \mu)^2 = \sum_{j=0}^2 (-1)^{2-j} \binom{2}{j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(2, \lambda b_i)}{n(e^{-\lambda b_i})} \right)^{2-j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda^j} \Gamma(j+1, \lambda b_i)}{n(e^{-\lambda b_i})} \right)$$

$$\sigma = \sqrt{\sum_{j=0}^2 (-1)^{2-j} \binom{2}{j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(2, \lambda b_i)}{n(e^{-\lambda b_i})} \right)^{2-j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda^j} \Gamma(j+1, \lambda b_i)}{n(e^{-\lambda b_i})} \right)}$$

Therefore the coefficients of variation, kurtosis, and skewness are respectively:

$$CV = \frac{\left[ \sum_{j=0}^2 (-1)^{2-j} \binom{2}{j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(2, \lambda b_i)}{n(e^{-\lambda b_i})} \right)^{2-j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda^j} \Gamma(j+1, \lambda b_i)}{n(e^{-\lambda b_i})} \right) \right]^{1/2}}{\sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(2, \lambda b_i)}{n(e^{-\lambda b_i})}}$$

$$CK = \frac{\sum_{j=0}^4 (-1)^{4-j} \binom{4}{j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(2, \lambda b_i)}{n(e^{-\lambda b_i})} \right)^{4-j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda^j} \Gamma(j+1, \lambda b_i)}{n(e^{-\lambda b_i})} \right)}{\left[ \sum_{j=0}^2 (-1)^{2-j} \binom{2}{j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(2, \lambda b_i)}{n(e^{-\lambda b_i})} \right)^{2-j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda^j} \Gamma(j+1, \lambda b_i)}{n(e^{-\lambda b_i})} \right) \right]^2}$$

$$CS = \frac{\sum_{j=0}^3 (-1)^{r-j} \binom{3}{j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(2, \lambda b_i)}{n(e^{-\lambda b_i})} \right)^{r-j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(j+1, \lambda b_i)}{n(e^{-\lambda b_i})} \right)}{\left[ \sum_{j=0}^2 (-1)^{2-j} \binom{2}{j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(2, \lambda b_i)}{n(e^{-\lambda b_i})} \right)^{r-j} \left( \sum_{i=1}^n \frac{\frac{1}{\lambda} \Gamma(j+1, \lambda b_i)}{n(e^{-\lambda b_i})} \right) \right]^{3/2}}$$

**6. Real Data Analysis of Study (TLTED)**

In this section, we illustrated the applicability of the TLTED model by considering real data of the infected with Covid-19 in the city of Babylon-Iraq. Data was collected for the period of their infection with the virus until their death for the two sets. Comparison of the triple left truncated Exponential distribution with Maxwell distribution, Chi-Square distribution, and Rayleigh with two parameters (RATO), has been discussed according to many criteria, Akaike Information Criterion (AIC), Corrected Akaike Information Criterion (CAIC), and Bayesian Information Criterion (BIC). Also, we test four types of curve fitting quantities to identify the best-fitted curve among the tested models they are: the sum of Squared due to Error, R-square, Adjusted R-square, and root mean square error (RMSE).

**Data in set(1):** (1/10/2020-30/12/2020).

The data set are:  $t=[11,22,16,9,17,6,12,7,29,11,7,10,15,8,1,21,8,1,17,1,10,21,16,12,1,2,7,4,11,5,5,8,4,15,1,23,16,8,10,10,11,27,9,18,1,6,3,26,4,1,3,11,2,12,3,32,12,1,5,10,14,3,66,51,20,8,1,15,18,22,16,19,8,21,9,12,8,3,9,17,7,12,44,11,23,12,10,8,11,27,10,4,5,6,11,28,22,6,7,31,6,16,2,12,21,19,2,19,8,18,39]$ , with size 112.

**Data in set(2):** (1/7/2020-31/8/2020).

The data set are:

$t=[5,13,8,5,4,3,4,2,16,7,6,22,24,14,6,4,9,1,1,6,5,2,12,4,2,1,1,7,13,3,1,12,7,14,3,1,2,10,1,17,8,11,10,1,1,1,1,1,2,6,10,1,13,1,1,1,17,15,6,3,5,19,3,1,1,12,6,3,23,6,14,11,12,8,9,3,1,6,7,1,20,2,3,2,9,8,5,12,6,7,6,1,3,5,1,9,11,1,25,6,5,15,1,2,13,19,3,1,3,7,8,5,5,11,3,10,4,13,4,7,4,14,3,9,2,9,9,2,4,17,2,9,6,9,2,3,8,11,1,9,21,12,13,5,6,1,1,6,1,1,13,22,7,4,3,7,16,6,5,4,11,6,24,6,16,4,1,16,6,1,5,12,4,15,3,1,25,1,10,11,2,22,4,9,2,5,1,5,1,2,16,1,6,2,4,21,4,17,12,5,21,7,8,6,7,15,5,7,9,57,6,1,1,3,7,10,2,1,13,3,7,6]$ , with size 221.

**Table 1:** The values of statistics  $\ell^{\wedge}$ , AIC, BIC, CAIC, HQIC for data in set (1)

Model	$\ell^{\wedge}$	AIC	BIC	CAIC	HQIC
TLTED	-165.132	332.264	334.98249	332.30036	334.98249
MAWE	-1,592.892	3,187.784	3,190.50249	3,185.82036	3,188.88694
CH-SQ	-783.545	1,567.09	1,571.80849	1,567.12636	1,570.19294
RA-TO	-1.368.956	2,749.912	2,747.34898	2,738.2209	2,750.32375

**Table 2:** Estimations parameters of TLTED and others for data in set(1)

Model	parameter	Parameter estimates
TLTED( $\lambda$ )	$\lambda^{\wedge}$	0.622
CH-SQ	$\alpha^{\wedge}$	9.00
MAWE ( $\theta$ )	$\theta^{\wedge}$	1.939
RA-TO ( $\alpha, \delta$ )	$\alpha^{\wedge}, \delta^{\wedge}$	0.09, 8.937

**Table 3:** The values of statistics  $\ell^{\wedge}$ , AIC, BIC, CAIC, HQIC for TLTED and others for data in set (2)

Model	$\ell^{\wedge}$	AIC	BIC	CAIC	HQIC
TLTED	-633.995	1,269.99	1,278.78632	1,268.00826	1,269.67606
MAWE	-935.238	1,872.476	1,875.87416	1,870.49926	1,872.81620
CH-SQ	-1,125.799	2,253.598	2,256.99616	2,251.61626	2,253.28406
RA-TO	-1,600,936	3,205,872	3,212,6683	3,208,24412	3,205,24412

In this table, the numerical values of statistics for all models are given and show the model TLTED is a better fit of data of set (2) than TLTRD and other models.

**Table 4:**Estimations parameters for TLTED of data in set (2)

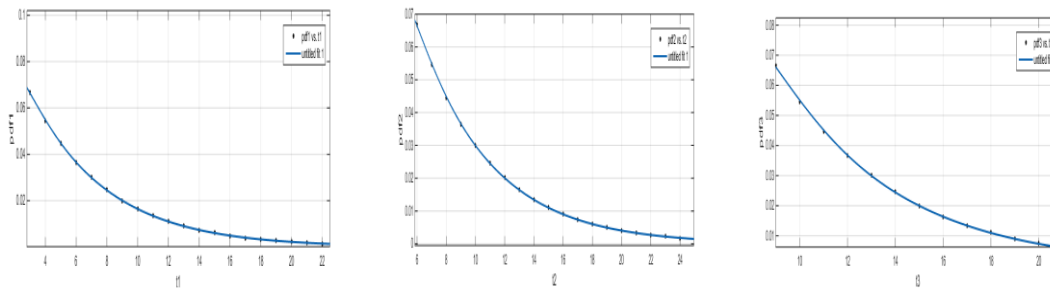
Model	parameter	Parameter estimates
TLTRD ( $\sigma^2$ )	$\sigma^2^{\wedge}$	4.674
TLTED( $\lambda$ )	$\lambda^{\wedge}$	0.2
CH-SQ( $\alpha$ )	$\alpha^{\wedge}$	1.903
MAWE ( $\theta$ )	$\theta^{\wedge}$	9.0
RA-TO ( $\alpha, \delta$ )	$\alpha^{\wedge}, \delta^{\wedge}$	0.2, 7.47

By using statistical information criteria, we can note that the (TLTED) distribution which has the least criteria in set (1) is better than set (2). Furthermore, the (TLTED) is better than other models, this is shown in tables (3)and (1). The maximal likelihood estimation of all model parameters for the data for set (1) and set(2) are shown in tables (2), and (4), respectively.

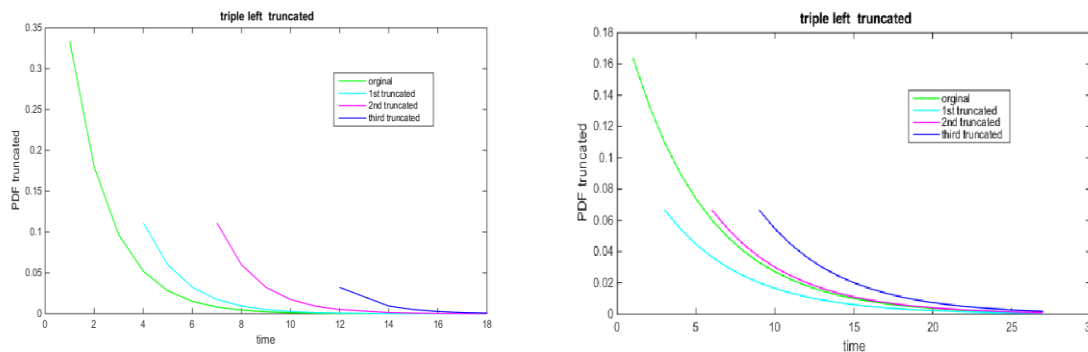
**Table 5:** Statistics to the goodness of fit (TLTED)

Data	SSE	R-squared	Adj-R squared	RMSE	
TLTED	set 1	1.6284933e-5	0.9997	0.999633333	0.00010122
	set 2	6.034e-6	0.9999	0.999833333	0.0003727

For TLTED, the values of (SSE, RMSE) of data in set(1) are the smallest compared to the data in set(2) and it fitted the data better in set(2), and this result is compatible with the results of Tables (3,1). And SSE with RMSE values closer to 0 indicates a fit that is more useful for prediction, this is shown in sets (1) and set(2).



**Figure 4.** Curving fitting of the model (TLTED) for data in set (1)



**Figure 6.** PDF of TLTED of data sets (1), and (2) with the estimated parameter, respectively.

## 7. Conclusion

The left truncated distribution is one of the types of truncations, which consider a special case of the survival function. In this paper, triple-left truncated exponential distribution has been derived. Statistical properties, such as survival, reliability, hazard function, and  $r^{\text{th}}$  moment, moment generating function are discussed. Employed the maximum likelihood method to estimate. Also, the performance of triple left truncated exponential distribution was tested by calculating some statistical criteria and testing the goodness of fit for distribution, with comparisons between the distributions and testing them on real data for patients infected with covid-19.

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