A Framework for Fuzzy Education Process and Neutrosophic Education Process

Takaaki Fujita[1,*], **Arif Mehmood**[2]

[1]Independent Researcher, Shinjuku, Shinjuku-ku, Tokyo, Japan
[2]Department of Mathematics, Institute of Numerical Sciences, Gomal University, Dera Ismail Khan 29050, KPK, Pakistan
Emails: Takaaki.fujita060@gmail.com; mehdaniyal@gmail.com

Abstract

Numerous frameworks have been developed to address uncertainty in various domains. Among the most prominent are Fuzzy Sets, Rough Sets, Hyperrough Sets, Vague Sets, Intuitionistic Fuzzy Sets, Neutrosophic Sets, Plithogenic Sets, as well as other emerging theories that continue to be actively explored. These concepts for handling uncertainty have also been studied in the context of educational applications. In this paper, we provide formal mathematical definitions for the *Fuzzy Education Process* and the *Neutrosophic Education Process*. These educational process frameworks are applicable in a wide range of contexts, including secondary education, corporate training programs, and beyond.

Keywords: Fuzzy set; Neutrosophic Set; Education Process; Fuzzy Education Process; Neutrosophic Education Process

1 Preliminaries

This section gathers the basic concepts and notation used throughout the paper. Unless explicitly stated otherwise, all sets and structures considered here are finite.

1.1 Fuzzy and Neutrosophic Sets

A fuzzy set assigns to each element a membership value in the interval $[0, 1]$, thereby modeling uncertainty through graded membership rather than a strict binary classification [1]. As an extended concept, *Intuitionistic Fuzzy Sets* [2, 3] are also well known. For completeness we recall the relevant definitions and their extensions that will be used later.

Definition 1.1 (Universal Set)**.** A *universal set* U is the ambient collection of objects under consideration in a given context. Every set mentioned is assumed to be a subset of U.

Definition 1.2 (Fuzzy Set)**.** [1] A *fuzzy set* τ on a nonempty universe Y is a function $\tau : Y \to [0, 1]$. A *fuzzy relation* on Y is a fuzzy subset δ of $Y \times Y$. If τ is a fuzzy set on Y and δ is a fuzzy relation on Y, then δ is said to be a *fuzzy relation on* τ provided

$$\delta(y, z) \le \min\{\tau(y), \tau(z)\} \qquad \text{for all } y, z \in Y.$$

Example 1.3 (Fuzzy Set in Education)**.** Let the universe be a set of students

$$Y = \{\text{Taro}, \text{Hanako}, \text{Kenji}, \text{Yuki}\}.$$

Define a fuzzy set τ representing the concept "Good at Mathematics" with membership degrees

$$\tau(\text{Taro}) = 0.9, \quad \tau(\text{Hanako}) = 0.7, \quad \tau(\text{Kenji}) = 0.4, \quad \tau(\text{Yuki}) = 0.2.$$

Here, $\tau(x)$ expresses how strongly each student belongs to the group of mathematically proficient learners. For example, Taro has a very high degree (0.9), while Yuki has a low degree (0.2), reflecting nuanced membership rather than a binary classification. This helps teachers capture performance differences more realistically than strict pass/fail categories.

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1.2 Neutrosophic Set with Some Applications

Neutrosophic sets extend the fuzzy framework by explicitly incorporating *indeterminacy*, thus accommodating information that is not purely true or false and offering a more flexible representation of ambiguity and uncertainty [4,5]. Related concepts of Neutrosophic Sets include the QuadriPartitioned Neutrosophic Set [6,7] and the Pentapartitioned Neutrosophic Set [8]. We state the formal definition below.

Definition 1.4 (Neutrosophic Set)**.** [4,5] Let X be a nonempty set. A *Neutrosophic Set (NS)* A on X is specified by three membership functions

$$T_A : X \to [0,1], \qquad I_A : X \to [0,1], \qquad F_A : X \to [0,1],$$

where, for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These degrees satisfy

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

The following presents several concrete real-life examples of Neutrosophic Sets.

Example 1.5 (Hospital triage: respiratory–infection status)**.** Let $X = \{\mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3\}$ be three patients and let A denote the predicate "has a clinically significant respiratory infection *now*." We combine three independent evidence channels with fixed weights

$$e_1 = 0.6 \quad \text{(lab PCR)}, \qquad e_2 = 0.3 \quad \text{(rapid antigen)}, \qquad e_3 = 0.1 \quad \text{(symptom score } s \in [0,1]\text{)},$$

and encode the *availability* of channel i by $a_i \in \{0,1\}$. If a test is available, its Boolean outcome is $t_i \in \{0,1\}$ (1 = positive/on–evidence for A). Define the neutrosophic components by

$$T_A = a_1 e_1 t_1 + a_2 e_2 t_2 + e_3 s, \quad F_A = a_1 e_1 (1 - t_1) + a_2 e_2 (1 - t_2) + e_3 (1 - s), \quad I_A = (1 - a_1) e_1 + (1 - a_2) e_2$$

Then $T_A + I_A + F_A = e_1 + e_2 + e_3 = 1$ (each channel's weight goes to T/F if observed; otherwise to I).

Patient P_1**:** PCR available and positive (a_1=1, t_1=1), antigen available and positive (a_2=1, t_2=1), symptom score s=0.8.

$$\begin{aligned} T_A(\mathrm{P}_1) &= 1 \cdot 0.6 \cdot 1 + 1 \cdot 0.3 \cdot 1 + 0.1 \cdot 0.8 = 0.60 + 0.30 + 0.08 = 0.98, \\ F_A(\mathrm{P}_1) &= 1 \cdot 0.6 \cdot 0 + 1 \cdot 0.3 \cdot 0 + 0.1 \cdot (1 - 0.8) = 0.02, \\ I_A(\mathrm{P}_1) &= (1 - 1) \cdot 0.6 + (1 - 1) \cdot 0.3 = 0.00. \end{aligned}$$

Hence $A(\mathrm{P}_1) = (0.98,\ 0.00,\ 0.02)$ and $T+I+F = 1.00$.

Patient P_2**:** PCR pending (a_1=0), antigen available and negative (a_2=1, t_2=0), s=0.6.

$$\begin{aligned} T_A(\mathrm{P}_2) &= 0 \cdot 0.6 \cdot (-) + 1 \cdot 0.3 \cdot 0 + 0.1 \cdot 0.6 = 0.06, \\ F_A(\mathrm{P}_2) &= 0 \cdot 0.6 \cdot (-) + 1 \cdot 0.3 \cdot (1 - 0) + 0.1 \cdot (1 - 0.6) = 0.30 + 0.04 = 0.34, \\ I_A(\mathrm{P}_2) &= (1 - 0) \cdot 0.6 + (1 - 1) \cdot 0.3 = 0.60. \end{aligned}$$

Thus $A(\mathrm{P}_2) = (0.06,\ 0.60,\ 0.34)$ and $T+I+F = 1.00$.

Patient P_3**:** PCR available and negative (a_1=1, t_1=0), antigen missing (a_2=0), s=0.3.

$$\begin{aligned} T_A(\mathrm{P}_3) &= 1 \cdot 0.6 \cdot 0 + 0 \cdot 0.3 \cdot (-) + 0.1 \cdot 0.3 = 0.03, \\ F_A(\mathrm{P}_3) &= 1 \cdot 0.6 \cdot (1 - 0) + 0 \cdot 0.3 \cdot (-) + 0.1 \cdot (1 - 0.3) = 0.60 + 0.07 = 0.67, \\ I_A(\mathrm{P}_3) &= (1 - 1) \cdot 0.6 + (1 - 0) \cdot 0.3 = 0.30. \end{aligned}$$

Therefore $A(\mathrm{P}_3) = (0.03,\ 0.30,\ 0.67)$ and $T+I+F = 1.00$.

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Example 1.6 (E–commerce logistics: "delivered on time")**.** Let $X = \{o_A, o_B, o_C\}$ be three orders and A the predicate "delivered on or before the promised date." Use three channels with weights

$$e_1 = 0.5 \text{ (carrier scan on time)},$$

$$e_2 = 0.3 \text{ (customer confirmation)},$$

$$e_3 = 0.2 \text{ (schedule–adherence score } s \in [0,1]).$$

As before, $a_i \in \{0,1\}$ indicates availability and $t_i \in \{0,1\}$ the supportive outcome (1 = on time). Define

$$T_A = a_1e_1t_1 + a_2e_2t_2 + e_3s,$$

$$F_A = a_1e_1(1-t_1) + a_2e_2(1-t_2) + e_3(1-s),$$

$$I_A = (1-a_1)e_1 + (1-a_2)e_2$$

so $T_A + I_A + F_A = e_1 + e_2 + e_3 = 1$.

Order o_A: scan shows "on time" (a_1=1, t_1=1), customer confirmed (a_2=1, t_2=1), adherence s=0.9.

$$\begin{aligned} T_A(o_A) &= 1\cdot 0.5\cdot 1 + 1\cdot 0.3\cdot 1 + 0.2\cdot 0.9 = 0.50 + 0.30 + 0.18 = 0.98,\\ F_A(o_A) &= 1\cdot 0.5\cdot 0 + 1\cdot 0.3\cdot 0 + 0.2\cdot(1-0.9) = 0.02,\\ I_A(o_A) &= (1-1)\cdot 0.5 + (1-1)\cdot 0.3 = 0. \end{aligned}$$

Hence $A(o_A) = (0.98,\ 0.00,\ 0.02)$.

Order o_B: scan missing (a_1=0), customer unresponsive (a_2=0), adherence s=0.6.

$$\begin{aligned} T_A(o_B) &= 0 + 0 + 0.2\cdot 0.6 = 0.12,\\ F_A(o_B) &= 0 + 0 + 0.2\cdot(1-0.6) = 0.08,\\ I_A(o_B) &= (1-0)\cdot 0.5 + (1-0)\cdot 0.3 = 0.8. \end{aligned}$$

Thus $A(o_B) = (0.12,\ 0.80,\ 0.08)$.

Order o_C: scan shows "late" (a_1=1, t_1=0), customer confirmed (a_2=1, t_2=1), adherence s=0.4.

$$\begin{aligned} T_A(o_C) &= 1\cdot 0.5\cdot 0 + 1\cdot 0.3\cdot 1 + 0.2\cdot 0.4 = 0.30 + 0.08 = 0.38,\\ F_A(o_C) &= 1\cdot 0.5\cdot 1 + 1\cdot 0.3\cdot 0 + 0.2\cdot(1-0.4) = 0.50 + 0.12 = 0.62,\\ I_A(o_C) &= (1-1)\cdot 0.5 + (1-1)\cdot 0.3 = 0. \end{aligned}$$

Therefore $A(o_C) = (0.38,\ 0.00,\ 0.62)$.

Example 1.7 (Final-exam readiness with explicit indeterminacy)**.** Let $X = \{\text{Sakura}\}$ and let A be the predicate "well–prepared for the final exam (now)." We adopt an outcome–driven map from attempt rate $a \in [0,1]$ and accuracy on attempted items $s \in [0,1]$:

$$T_A = a\,s, \qquad F_A = a\,(1-s), \qquad I_A = 1-a,$$

so that $T_A + I_A + F_A = 1$ and I_A quantifies the *unknown/unaware* portion (unattempted content). For the neutrosophic complement we use

$$A^c(x) = (F_A(x),\ 1 - I_A(x),\ T_A(x)).$$

Case 1. Suppose Sakura attempted 90% of the coverage ($a = 0.9$) and solved 90% of what she attempted ($s = 0.9$). Then

$$T_A(\text{Sakura}) = 0.9\cdot 0.9 = 0.81, \quad F_A(\text{Sakura}) = 0.9\cdot(1-0.9) = 0.09, \quad I_A(\text{Sakura}) = 1 - 0.9 = 0.10.$$

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Hence

$$A(\text{Sakura}) = (0.81,\ 0.10,\ 0.09), \qquad A^c(\text{Sakura}) = (0.09,\ 0.90,\ 0.81).$$

This realizes "mostly ready" (large T) with "little uncertainty" ($I = 0.10$) by explicit evidence (a, s).

Case 2.

(2a) Measured 50% performance (fully observed). If Sakura covered everything ($a = 1$) and solved half ($s = 0.5$), then

$$(T, I, F) = (1 \cdot 0.5,\ 1 - 1,\ 1 \cdot 0.5) = (0.5,\ 0,\ 0.5), \quad A^c(\text{Sakura}) = (0.5,\ 1,\ 0.5).$$

No indeterminacy appears because there is no unknown coverage.

(2b) Ambiguous "0.5" with large unknown/unaware. If that "0.5" reflects only partial engagement, say $a = 0.3$ with $s = 0.5$ on the attempted part, then

$$(T, I, F) = (0.3 \cdot 0.5,\ 1 - 0.3,\ 0.3 \cdot 0.5) = (0.15,\ 0.70,\ 0.15), \quad A^c(\text{Sakura}) = (0.15,\ 0.30,\ 0.15).$$

Here the dominant component is $I = 0.70$, cleanly separating indeterminacy (unknown portion) from truth/falsity.

Example 1.8 (Unknown/unaware math novices at the beginning and evidence–driven updates)**.** Let $X = \{\mathrm{U}_1, \mathrm{U}_2, \mathrm{U}_3\}$ be three students on the predicate A = "has foundational mathematics proficiency for entering Algebra I." Use the same outcome map

$$T_A = a\,s, \qquad F_A = a(1 - s), \qquad I_A = 1 - a, \qquad A^c(x) = (F_A(x),\ 1 - I_A(x),\ T_A(x)).$$

Initial diagnostic (day 0).

Student	a	s	T_A	I_A	F_A
U_1	0.00	–	0.00	1.00	0.00
U_2	0.40	0.25	$0.40 \cdot 0.25 = 0.10$	$1 - 0.40 = 0.60$	$0.40 \cdot 0.75 = 0.30$
U_3	0.20	0.00	$0.20 \cdot 0.00 = 0.00$	$1 - 0.20 = 0.80$	$0.20 \cdot 1.00 = 0.20$

Thus

$$A(\mathrm{U}_1) = (0, 1, 0), \quad A(\mathrm{U}_2) = (0.10, 0.60, 0.30), \quad A(\mathrm{U}_3) = (0, 0.80, 0.20).$$

Complements (e.g. for U_2):

$$A^c(\mathrm{U}_2) = \big(0.30,\ 1 - 0.60 = 0.40,\ 0.10\big).$$

Here I quantifies the truly *unknown/unaware* portion (unattempted coverage), not a fuzzy boundary of T and F.

After a remedial micro–module (day 7): evidence moves mass out of I. Adopt the conservative update rule: for prior (T, I, F), new reliable activity with attempt a_{new}, accuracy s_{new}, and reliability $\rho \in [0, 1]$ transfers

$$m := \min\{\, I,\ \rho\, a_{\text{new}} \}$$

from I into T and F proportionally:

$$T' = T + m\, s_{\text{new}}, \quad F' = F + m\,(1 - s_{\text{new}}), \quad I' = I - m.$$

Apply this to U_1 with prior $(T, I, F) = (0, 1, 0)$, choose $\rho = 0.8$, $a_{\text{new}} = 0.5$, $s_{\text{new}} = 0.6$:

$$m = \min\{1,\ 0.8 \cdot 0.5\} = \min\{1, 0.4\} = 0.4,$$

$$T' = 0 + 0.4 \cdot 0.6 = 0.24, \qquad F' = 0 + 0.4 \cdot 0.4 = 0.16, \qquad I' = 1 - 0.4 = 0.60.$$

Hence

$$A_{\text{day 7}}(\mathrm{U}_1) = (0.24,\ 0.60,\ 0.16), \qquad A^c_{\text{day 7}}(\mathrm{U}_1) = (0.16,\ 0.40,\ 0.24).$$

This numerically demonstrates how targeted evidence reduces *indeterminacy* while allocating mass to truth/falsity according to actual outcomes, which is essential for analyzing weak students.

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2 Result of This Paper

The results of this paper are presented as follows.

2.1 Mathematical Model of the Education Process

The education process may be viewed as a discrete-time system designed to develop learners' knowledge, skills, values, and competencies [9, 10]. It appears across diverse settings, including primary and secondary schooling [11], vocational programs [12], workplace training [13,14], and special education [15]. We formalize this process as follows.

Definition 2.1 (A mathematical model of the education process)**.** A (finite) education process is a septuple

$$\mathcal{E} = \big(S,\ C,\ T,\ K_0,\ L,\ w,\ E\big),$$

where:

- S is the nonempty set of students; in the example below $S = \{\text{Ayako}, \text{Masahiro}\}$.
- C is the nonempty set of curriculum topics; below $C = \{\text{Algebra}, \text{Geometry}, \text{Literature}\}$.
- $T = \{t_0, t_1, t_2, t_3\}$ is a finite totally ordered time set. For readability we annotate

$$t_0 = \text{2025-04-01 (pretest)}, \quad t_1 = \text{2025-04-08 (lecture)},$$

$$t_2 = \text{2025-04-15 (lecture)}, \quad t_3 = \text{2025-04-22 (final exam)}.$$

- $K_0 : S \times C \to [0, 1]$ is the *initial knowledge state* at t_0 (diagnostic):

$$\begin{aligned} K_0(\text{Ayako}, \text{Algebra}) &= 0.40, & K_0(\text{Masahiro}, \text{Algebra}) &= 0.25,\\ K_0(\text{Ayako}, \text{Geometry}) &= 0.30, & K_0(\text{Masahiro}, \text{Geometry}) &= 0.20,\\ K_0(\text{Ayako}, \text{Literature}) &= 0.50, & K_0(\text{Masahiro}, \text{Literature}) &= 0.35. \end{aligned}$$

- $L : S \times C \times (T \setminus \{t_0\}) \to [0, 1]$ is the *learning–increment function* at non-initial times. In the example, increments are specified for class meetings (t_1, t_2), and we set $L(\cdot, \cdot, t_3) = 0$ (no learning during the final):

$$\begin{aligned} \text{at } t_1 = \text{2025-04-08:} \quad & L(\text{Ayako}, \text{Algebra}, t_1) = 0.25, & L(\text{Masahiro}, \text{Algebra}, t_1) &= 0.20,\\ & L(\text{Ayako}, \text{Geometry}, t_1) = 0.20, & L(\text{Masahiro}, \text{Geometry}, t_1) &= 0.18,\\ & L(\text{Ayako}, \text{Literature}, t_1) = 0.15, & L(\text{Masahiro}, \text{Literature}, t_1) &= 0.22,\\ \text{at } t_2 = \text{2025-04-15:} \quad & L(\text{Ayako}, \text{Algebra}, t_2) = 0.30, & L(\text{Masahiro}, \text{Algebra}, t_2) &= 0.25,\\ & L(\text{Ayako}, \text{Geometry}, t_2) = 0.22, & L(\text{Masahiro}, \text{Geometry}, t_2) &= 0.20,\\ & L(\text{Ayako}, \text{Literature}, t_2) = 0.18, & L(\text{Masahiro}, \text{Literature}, t_2) &= 0.24. \end{aligned}$$

- $w : C \to [0, 1]$ is a topic–weight function with $\sum_{c \in C} w(c) = 1$; here

$$w(\text{Algebra}) = 0.5, \quad w(\text{Geometry}) = 0.3, \quad w(\text{Literature}) = 0.2.$$

- $E : S \to [0, 1]$ is the *evaluation map* at the final time t_3, defined by

$$E(s) \ = \ \sum_{c \in C} w(c)\, K(s, c, t_3),$$

  where the knowledge trajectory $K : S \times C \times T \to [0, 1]$ evolves by

$$K(s, c, t_0) = K_0(s, c), \qquad K(s, c, t_k) = \min\big\{ 1,\ K(s, c, t_{k-1}) + L(s, c, t_k) \big\} \quad (k = 1, 2, 3).$$

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Example 2.2 (Knowledge trajectories and final evaluations)**.** Using the data above, the knowledge levels at t_1 and t_2 are:

Ayako

Topic	t_0 (pretest)	t_1 (04-08)	t_2 (04-15)
Algebra	0.40	0.40 + 0.25 = 0.65	0.65 + 0.30 = 0.95
Geometry	0.30	0.30 + 0.20 = 0.50	0.50 + 0.22 = 0.72
Literature	0.50	0.50 + 0.15 = 0.65	0.65 + 0.18 = 0.83

Masahiro

Topic	t_0 (pretest)	t_1 (04-08)	t_2 (04-15)
Algebra	0.25	0.25 + 0.20 = 0.45	0.45 + 0.25 = 0.70
Geometry	0.20	0.20 + 0.18 = 0.38	0.38 + 0.20 = 0.58
Literature	0.35	0.35 + 0.22 = 0.57	0.57 + 0.24 = 0.81

Since $L(\cdot,\cdot,t_3) = 0$, we have $K(\cdot,\cdot,t_3) = K(\cdot,\cdot,t_2)$. Therefore,

$$E(\text{Ayako}) = 0.5 \cdot 0.95 + 0.3 \cdot 0.72 + 0.2 \cdot 0.83 = 0.475 + 0.216 + 0.166 = 0.857,$$
$$E(\text{Masahiro}) = 0.5 \cdot 0.70 + 0.3 \cdot 0.58 + 0.2 \cdot 0.81 = 0.350 + 0.174 + 0.162 = 0.686.$$

Theorem 2.3 (Well-posedness and monotonicity of the knowledge recursion)**.** *Fix $(s, c) \in S \times C$ and write*

$$K_0 := K(s,c,t_0) \in [0,1], \qquad K_k := K(s,c,t_k) = \min\{\, 1,\ K_{k-1} + L(s,c,t_k)\, \} \quad (k \geq 1),$$

where each increment satisfies $L(s,c,t_k) \in [0,1]$. Then for every $k \geq 0$:

(i) $K_k \in [0,1]$ (boundedness)*;*

(ii) $K_k \geq K_{k-1}$ (monotonicity)*;*

(iii) *the explicit form holds:*

$$K_k \ = \ \min\Big\{ 1,\ K_0 + \sum_{i=1}^{k} L(s,c,t_i) \Big\};$$

(iv) *consequently, the evaluation $E(s) = \sum_{c \in C} w(c)\, K(s,c,t_{\max})$ satisfies $E(s) \in [0,1]$.*

Proof. We proceed by induction on k.

Base case ($k = 0$). By definition of the model, $K_0 \in [0,1]$, so (i) holds at $k = 0$. Trivially, (ii) is vacuous at $k = 0$, and (iii) holds since $K_0 = \min\{1, K_0 + \sum_{i=1}^{0}(\cdot)\}$.

Inductive step. Assume (i)–(iii) hold at some $k - 1 \geq 0$. Set $L_k := L(s,c,t_k) \in [0,1]$.

(i) Boundedness at k. From $0 \leq K_{k-1} \leq 1$ and $0 \leq L_k \leq 1$ we obtain

$$0 \ \leq\ K_{k-1} + L_k \ \leq\ 2.$$

Therefore

$$0 \ \leq\ K_k = \min\{1,\ K_{k-1} + L_k\} \ \leq\ 1,$$

so $K_k \in [0,1]$.

(ii) Monotonicity at k. We claim that for any $x \in [0,1]$ and any $\lambda \in [0,1]$,

$$\min\{1,\ x + \lambda\} \ \geq\ \min\{1,\ x\}. \tag{1}$$

Indeed, if $x \geq 1$, both sides equal 1; if $x < 1$, then $\min\{1,x\} = x$ and $x + \lambda \geq x$, hence $\min\{1, x+\lambda\} \geq x = \min\{1,x\}$. Applying (1) with $x = K_{k-1}$ and $\lambda = L_k$ gives

$$K_k = \min\{1,\ K_{k-1} + L_k\} \ \geq\ \min\{1,\ K_{k-1}\} = K_{k-1},$$

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where the last equality uses $K_{k-1} \leq 1$. Thus $K_k \geq K_{k-1}$.

(iii) Explicit form at k. By the induction hypothesis,

$$K_{k-1} = \min\Big\{1,\ K_0 + \sum_{i=1}^{k-1} L(s,c,t_i)\Big\}.$$

Using the elementary identity (valid for any $a \in \mathbb{R}$ and $b \geq 0$)

$$\min\{1,\ \min\{1,a\} + b\} \ = \ \min\{1,\ a+b\}, \tag{2}$$

we compute

$$\begin{aligned} K_k &= \min\{1,\ K_{k-1} + L_k\} = \min\Big\{1,\ \min\{1,\ K_0 + \sum_{i=1}^{k-1} L(s,c,t_i)\} + L_k\Big\} \\ &= \min\Big\{1,\ K_0 + \sum_{i=1}^{k-1} L(s,c,t_i) + L_k\Big\} = \min\Big\{1,\ K_0 + \sum_{i=1}^{k} L(s,c,t_i)\Big\}, \end{aligned}$$

which proves (iii). For completeness, we verify (2): if $a \geq 1$ then the left-hand side is $\min\{1, 1+b\} = 1$, while the right-hand side is $\min\{1, a+b\} = 1$; if $a < 1$ then $\min\{1,a\} = a$ and both sides equal $\min\{1, a+b\}$.

With (i)–(iii) established at step k, the induction is complete.

(iv) Bounds for $E(s)$. For fixed s, each $K(s,c,t_{\max}) \in [0,1]$ by (i), and the weights satisfy $w(c) \geq 0$ and $\sum_c w(c) = 1$. Hence

$$0 \ \leq \ \sum_{c\in C} w(c)\, K(s,c,t_{\max}) \ \leq \ \sum_{c \in C} w(c)\cdot 1 \ = \ 1,$$

i.e. $E(s) \in [0,1]$. □

2.2 Mathematical Model of Fuzzy Education Process

This can be deliberately defined in mathematical terms as follows. A wide variety of studies have been conducted on the integration of fuzzy logic and education [16–18].

Definition 2.4 (Fuzzy Education Process)**.** Let

$$S,\ C,\ T$$

be finite sets of students, curriculum topics, and ordered time-points respectively. A *Fuzzy Education Process* is the octuple

$$\mathcal{E}_F = \big(S,\ C,\ T,\ K_0,\ L,\ w,\ \mu_P\big),$$

where

- $K_0 : S \times C \to [0,1]$ is the *initial knowledge state*, measured at the pretest time $t_0 \in T$;
- $L : S \times C \times (T \setminus \{t_0\}) \to [0,1]$ is the *learning increment function*, with

$$L(s,c,t_k) \ = \ \text{knowledge gain of student } s \text{ in topic } c \text{ at time } t_k;$$

- $w = \{w_c \mid c \in C\}$ is a weight vector with $w_c \geq 0$ and $\sum_c w_c = 1$;
- $\mu_P : S \times C \times T \to [0,1]$ is the *fuzzy participation relation*, where $\mu_P(s,c,t)$ measures the degree to which s engages with topic c at time t.

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The *crisp knowledge state* $K : S \times C \times T \to [0, 1]$ evolves by

$$K(s, c, t_0) = K_0(s, c), \quad K(s, c, t_k) = \min\{1,\ K(s, c, t_{k-1}) + L(s, c, t_k)\}.$$

Define the *fuzzy evaluation* $E_F : S \to [0, 1]$ by

$$E_F(s) = \frac{\sum_{c \in C} \sum_{t \in T} \mu_P(s, c, t)\, w_c\, K(s, c, t)}{\sum_{c \in C} \sum_{t \in T} \mu_P(s, c, t)}.$$

Then E_F is a fuzzy set on S, representing each student's overall proficiency.

Example 2.5. Let

$$S = \{\text{Haruka}, \text{Kenji}\},\ C = \{\text{Algebra}, \text{Geometry}\},\ T = \{t_0, t_1, t_2\},$$

with

$$t_0 = \text{pretest}, \quad t_1 = \text{lecture}, \quad t_2 = \text{final exam},$$

and weights $w_{\text{Algebra}} = 0.6$, $w_{\text{Geometry}} = 0.4$. Suppose

$$K_0(\text{Haruka}, \text{Algebra}) = 0.4,\ K_0(\text{Haruka}, \text{Geometry}) = 0.3,$$

$$K_0(\text{Kenji}, \text{Algebra}) = 0.2,\ K_0(\text{Kenji}, \text{Geometry}) = 0.5.$$

Learning increments:

$$L(\text{Haruka}, \text{Algebra}, t_1) = 0.3,\ L(\text{Haruka}, \text{Geometry}, t_1) = 0.2,$$

$$L(\text{Haruka}, \text{Algebra}, t_2) = 0.4,\ L(\text{Haruka}, \text{Geometry}, t_2) = 0.3,$$

$$L(\text{Kenji}, \text{Algebra}, t_1) = 0.25,\ L(\text{Kenji}, \text{Geometry}, t_1) = 0.35,$$

$$L(\text{Kenji}, \text{Algebra}, t_2) = 0.3,\ L(\text{Kenji}, \text{Geometry}, t_2) = 0.2.$$

Compute crisp K:

$$K(\text{Haruka}, \text{Algebra}, t_1) = 0.7,\ K(\text{Haruka}, \text{Algebra}, t_2) = 1.0,$$

$$K(\text{Haruka}, \text{Geometry}, t_1) = 0.5,\ K(\text{Haruka}, \text{Geometry}, t_2) = 0.8,$$

$$K(\text{Kenji}, \text{Algebra}, t_1) = 0.45,\ K(\text{Kenji}, \text{Algebra}, t_2) = 0.75,$$

$$K(\text{Kenji}, \text{Geometry}, t_1) = 0.85,\ K(\text{Kenji}, \text{Geometry}, t_2) = 1.0.$$

Let fuzzy participation degrees:

$$\mu_P(\text{Haruka}, \text{Algebra}, t_1) = 0.9,\ \mu_P(\text{Haruka}, \text{Geometry}, t_1) = 0.8,$$

$$\mu_P(\text{Haruka}, \text{Algebra}, t_2) = 1.0,\ \mu_P(\text{Haruka}, \text{Geometry}, t_2) = 0.95,$$

$$\mu_P(\text{Kenji}, \text{Algebra}, t_1) = 0.7,\ \mu_P(\text{Kenji}, \text{Geometry}, t_1) = 0.85,$$

$$\mu_P(\text{Kenji}, \text{Algebra}, t_2) = 0.8,\ \mu_P(\text{Kenji}, \text{Geometry}, t_2) = 0.9.$$

Then

$$E_F(\text{Haruka}) = \frac{0.9 \cdot 0.6 \cdot 0.7 + 0.8 \cdot 0.4 \cdot 0.5 + 1.0 \cdot 0.6 \cdot 1.0 + 0.95 \cdot 0.4 \cdot 0.8}{0.9 + 0.8 + 1.0 + 0.95} \approx 0.82,$$

$$E_F(\text{Kenji}) = \frac{0.7 \cdot 0.6 \cdot 0.45 + 0.85 \cdot 0.4 \cdot 0.85 + 0.8 \cdot 0.6 \cdot 0.75 + 0.9 \cdot 0.4 \cdot 1.0}{0.7 + 0.85 + 0.8 + 0.9} \approx 0.78.$$

Thus Haruka's fuzzy evaluation is 0.82 and Kenji's is 0.78.

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Example 2.6 (High school blended class (Mathematics & English))**.** Let

$$S = \{\text{Haruka, Kenji}\}, \quad C = \{\text{Math, English}\}, \quad T = \{t_0, t_1, t_2\}$$

with t_0 = pretest, t_1 = lesson 1, t_2 = lesson 2. Weights: $w_{\text{Math}} = 0.6$, $w_{\text{English}} = 0.4$.

Initial knowledge $K_0 : S \times C \to [0, 1]$:

$$K_0(\text{Haruka, Math}) = 0.30, \quad K_0(\text{Haruka, English}) = 0.55,$$
$$K_0(\text{Kenji, Math}) = 0.45, \quad K_0(\text{Kenji, English}) = 0.35.$$

Learning increments $L(\cdot, \cdot, t_k)$ (only at t_1, t_2; values clipped at 1):

$$L(\text{Haruka, Math}, t_1) = 0.25, \quad L(\text{Haruka, English}, t_1) = 0.10,$$
$$L(\text{Haruka, Math}, t_2) = 0.20, \quad L(\text{Haruka, English}, t_2) = 0.12;$$
$$L(\text{Kenji, Math}, t_1) = 0.15, \quad L(\text{Kenji, English}, t_1) = 0.20,$$
$$L(\text{Kenji, Math}, t_2) = 0.18, \quad L(\text{Kenji, English}, t_2) = 0.15.$$

Fuzzy participation $\mu_P : S \times C \times T \to [0, 1]$:

$$\mu_P(\text{Haruka, Math}, t_0, t_1, t_2) = (0.8, 1.0, 0.9),$$
$$\mu_P(\text{Haruka, English}, t_0, t_1, t_2) = (0.9, 0.7, 0.8);$$
$$\mu_P(\text{Kenji, Math}, t_0, t_1, t_2) = (0.7, 0.8, 0.9),$$
$$\mu_P(\text{Kenji, English}, t_0, t_1, t_2) = (0.6, 0.9, 1.0).$$

Knowledge trajectories K via $K(s, c, t_0) = K_0(s, c)$, $K(s, c, t_k) = \min\{1,\ K(s, c, t_{k-1}) + L(s, c, t_k)\}$:

Haruka	t_0	t_1	t_2
Math	0.30	0.30 + 0.25 = 0.55	0.55 + 0.20 = 0.75
English	0.55	0.55 + 0.10 = 0.65	0.65 + 0.12 = 0.77

Kenji	t_0	t_1	t_2
Math	0.45	0.45 + 0.15 = 0.60	0.60 + 0.18 = 0.78
English	0.35	0.35 + 0.20 = 0.55	0.55 + 0.15 = 0.70

Fuzzy evaluations $E_F(s)$:

$$E_F(s) = \frac{\sum_{c \in C} \sum_{t \in T} \mu_P(s, c, t)\, w_c\, K(s, c, t)}{\sum_{c \in C} \sum_{t \in T} \mu_P(s, c, t)}.$$

<u>Haruka:</u> denominator $D_H = 2.7 + 2.4 = 5.1$. Numerator $N_H = (0.8 \cdot 0.6 \cdot 0.30) + (1.0 \cdot 0.6 \cdot 0.55) + (0.9 \cdot 0.6 \cdot 0.75) + (0.9 \cdot 0.4 \cdot 0.55) + (0.7 \cdot 0.4 \cdot 0.65) + (0.8 \cdot 0.4 \cdot 0.77) = 1.5054$. Hence

$$E_F(\text{Haruka}) = \frac{1.5054}{5.1} \approx 0.2952.$$

<u>Kenji:</u> denominator $D_K = 2.4 + 2.5 = 4.9$. Numerator $N_K = (0.7 \cdot 0.6 \cdot 0.45) + (0.8 \cdot 0.6 \cdot 0.60) + (0.9 \cdot 0.6 \cdot 0.78) + (0.6 \cdot 0.4 \cdot 0.35) + (0.9 \cdot 0.4 \cdot 0.55) + (1.0 \cdot 0.4 \cdot 0.70) = 1.4602$. Thus

$$E_F(\text{Kenji}) = \frac{1.4602}{4.9} \approx 0.2980.$$

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Example 2.7 (University physics lab with report writing)**.** Let

$$S = \{\text{Haruto, Miyu}\}, \quad C = \{\text{PhysicsLab, Report}\}, \quad T = \{t_0, t_1, t_2, t_3\}$$

with t_0 = pre-lab, t_1 = Lab 1, t_2 = Lab 2, t_3 = presentation workshop. Weights: $w_{\text{PhysicsLab}} = 0.7$, $w_{\text{Report}} = 0.3$.

Initial knowledge:

$$K_0(\text{Haruto, PhysicsLab}) = 0.20, \quad K_0(\text{Haruto, Report}) = 0.40,$$
$$K_0(\text{Miyu, PhysicsLab}) = 0.35, \quad K_0(\text{Miyu, Report}) = 0.30.$$

Learning increments (nonzero at all t_1, t_2, t_3):

$$\begin{aligned}
&L(\text{Haruto, PhysicsLab}, t_1, t_2, t_3) = (0.35, 0.25, 0.10),\\
&L(\text{Haruto, Report}, t_1, t_2, t_3) = (0.05, 0.05, 0.20);\\
&L(\text{Miyu, PhysicsLab}, t_1, t_2, t_3) = (0.20, 0.30, 0.12),\\
&L(\text{Miyu, Report}, t_1, t_2, t_3) = (0.10, 0.08, 0.25).
\end{aligned}$$

Fuzzy participation:

$$\begin{aligned}
&\mu_P(\text{Haruto, PhysicsLab}, t_{0..3}) = (0.7, 1.0, 1.0, 0.8),\\
&\mu_P(\text{Haruto, Report}, t_{0..3}) = (0.6, 0.5, 0.6, 0.9);\\
&\mu_P(\text{Miyu, PhysicsLab}, t_{0..3}) = (0.8, 0.8, 0.9, 0.9),\\
&\mu_P(\text{Miyu, Report}, t_{0..3}) = (0.5, 0.7, 0.8, 1.0).
\end{aligned}$$

Knowledge trajectories:

Haruto	t_0	t_1	t_2	t_3
PhysicsLab	0.20	0.55	0.80	0.90
Report	0.40	0.45	0.50	0.70

Miyu	t_0	t_1	t_2	t_3
PhysicsLab	0.35	0.55	0.85	0.97
Report	0.30	0.40	0.48	0.73

Fuzzy evaluations. Denominators: $D_{\text{Haruto}} = 3.5 + 2.6 = 6.1$, $D_{\text{Miyu}} = 3.4 + 3.0 = 6.4$.

Haruto: Numerator

$$\begin{aligned}
N_{\text{Haruto}} &= (0.7 \cdot 0.7 \cdot 0.20) + (1.0 \cdot 0.7 \cdot 0.55) + (1.0 \cdot 0.7 \cdot 0.80) + (0.8 \cdot 0.7 \cdot 0.90)\\
&\quad + (0.6 \cdot 0.3 \cdot 0.40) + (0.5 \cdot 0.3 \cdot 0.45) + (0.6 \cdot 0.3 \cdot 0.50) + (0.9 \cdot 0.3 \cdot 0.70)\\
&= 1.9655.
\end{aligned}$$

Hence

$$E_F(\text{Haruto}) = \frac{1.9655}{6.1} \approx 0.3222.$$

Miyu: Numerator

$$\begin{aligned}
N_{\text{Miyu}} &= (0.8 \cdot 0.7 \cdot 0.35) + (0.8 \cdot 0.7 \cdot 0.55) + (0.9 \cdot 0.7 \cdot 0.85) + (0.9 \cdot 0.7 \cdot 0.97)\\
&\quad + (0.5 \cdot 0.3 \cdot 0.30) + (0.7 \cdot 0.3 \cdot 0.40) + (0.8 \cdot 0.3 \cdot 0.48) + (1.0 \cdot 0.3 \cdot 0.73)\\
&= 2.1138.
\end{aligned}$$

Thus

$$E_F(\text{Miyu}) = \frac{2.1138}{6.4} \approx 0.3303.$$

Theorem 2.8. *If $\mu_P(s, c, t) \in \{0, 1\}$ is the indicator of a crisp attendance log $P \subseteq S \times C \times T$, then*

$$E_F(s) = \sum_{c \in C} w_c\, K\big(s, c, t_{\max}\big)$$

with $t_{\max}$ the final time, recovering the classical weighted evaluation.

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Proof. When μ_P is 1 exactly on $(s, c, t) \in P$ and 0 otherwise, the denominator $\sum \mu_P(s, c, t) = |P_s|$ counts all participations of s. If we restrict to final-time participation only, then

$$E_F(s) = \frac{\sum_c \mu_P(s, c, t_{\max})\, w_c\, K(s, c, t_{\max})}{\sum_c \mu_P(s, c, t_{\max})} = \sum_c w_c\, K(s, c, t_{\max}),$$

since $\sum_c \mu_P(s, c, t_{\max}) = |C|$ and normalizing yields the weighted sum of final knowledge. This matches the crisp model. □

2.3 Mathematical Model of the Neutrosophic Education Process

A *Neutrosophic Education Process* represents the dynamics of teaching and learning by incorporating three distinct degrees: truth, indeterminacy, and falsity. This framework provides a richer description of uncertainty in student performance and engagement than classical or fuzzy approaches. It can be rigorously formalized in mathematical terms as follows.

Definition 2.9 (Neutrosophic Education Process)**.** Let

$$S,\ C,\ T$$

be finite sets of students, curriculum topics, and ordered time-points respectively. A *Neutrosophic Education Process* is the decuple

$$\mathcal{E}_N = \big(S,\ C,\ T,\ K_0,\ L,\ w,\ \mu_P^T,\ \mu_P^I,\ \mu_P^F\big),$$

equipped with:

- the *initial knowledge state* $K_0 : S \times C \to [0, 1]$, measured at $t_0 \in T$;
- the *learning increment function* $L : S \times C \times (T \setminus \{t_0\}) \to [0, 1]$, where $L(s, c, t_k)$ is the gain of student s in topic c at time t_k;
- a weight vector $w = \{w_c \mid c \in C\} \subset [0, 1]$ with $\sum_c w_c = 1$;
- three *neutrosophic participation relations*

  $$\mu_P^T,\ \mu_P^I,\ \mu_P^F : S \times C \times T \to [0, 1],$$

  where for each (s, c, t): μ_P^T is truth-membership, μ_P^I indeterminacy-membership, μ_P^F falsity-membership of s engaging with c at t.

The *crisp knowledge state* $K : S \times C \times T \to [0, 1]$ evolves by

$$K(s, c, t_0) = K_0(s, c), \quad K(s, c, t_k) = \min\{1,\ K(s, c, t_{k-1}) + L(s, c, t_k)\}.$$

Define for each $s \in S$ the neutrosophic evaluation components

$$E_T(s) = \frac{\sum_{c\in C} \sum_{t\in T} \mu_P^T(s, c, t)\, w_c\, K(s, c, t)}{\sum_{c,t} \mu_P^T(s, c, t)}, \quad E_I(s) = \frac{\sum_{c,t} \mu_P^I(s, c, t)\, w_c\, K(s, c, t)}{\sum_{c,t} \mu_P^I(s, c, t)},$$

$$E_F(s) = \frac{\sum_{c,t} \mu_P^F(s, c, t)\, w_c\, K(s, c, t)}{\sum_{c,t} \mu_P^F(s, c, t)}.$$

Then $S_{\mathrm{NEP}}(s) = \big(E_T(s),\ E_I(s),\ E_F(s)\big) \in [0, 1]^3$ is a Neutrosophic Set on S.

Example 2.10. Let

$$S = \{\text{Ayako}, \text{Masahiro}\},\ C = \{\text{Algebra}, \text{English}\},\ T = \{t_0, t_1, t_2\},$$

with t_0 = pretest, t_1 = lecture, t_2 = final. Choose

$$w_{\text{Algebra}} = 0.6,\ w_{\text{English}} = 0.4,$$

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and initial knowledge

$$K_0(\text{Ayako, Algebra}) = 0.4,\ K_0(\text{Ayako, English}) = 0.3,$$
$$K_0(\text{Masahiro, Algebra}) = 0.2,\ K_0(\text{Masahiro, English}) = 0.5.$$

Learning increments:

$$L(\text{Ayako, Algebra}, t_1) = 0.3,\ L(\text{Ayako, English}, t_1) = 0.2,$$
$$L(\text{Ayako, Algebra}, t_2) = 0.4,\ L(\text{Ayako, English}, t_2) = 0.3,$$
$$L(\text{Masahiro, Algebra}, t_1) = 0.25,\ L(\text{Masahiro, English}, t_1) = 0.35,$$
$$L(\text{Masahiro, Algebra}, t_2) = 0.3,\ L(\text{Masahiro, English}, t_2) = 0.2.$$

Thus

$$\begin{aligned} K(\text{Ayako, Algebra}, t_1) &= 0.7, & K(\text{Ayako, Algebra}, t_2) &= 1.0,\\ K(\text{Ayako, English}, t_1) &= 0.5, & K(\text{Ayako, English}, t_2) &= 0.8,\\ K(\text{Masahiro, Algebra}, t_1) &= 0.45, & K(\text{Masahiro, Algebra}, t_2) &= 0.75,\\ K(\text{Masahiro, English}, t_1) &= 0.85, & K(\text{Masahiro, English}, t_2) &= 1.0. \end{aligned}$$

Neutrosophic participation degrees:

$$\begin{aligned} \mu_P^T(\text{Ayako, Algebra}, t_1) &= 0.9, & \mu_P^I &= 0.05, & \mu_P^F &= 0.05,\\ \mu_P^T(\text{Ayako, Algebra}, t_2) &= 0.95, & \mu_P^I &= 0.03, & \mu_P^F &= 0.02,\\ \mu_P^T(\text{Ayako, English}, t_1) &= 0.85, & \mu_P^I &= 0.10, & \mu_P^F &= 0.05,\\ \mu_P^T(\text{Ayako, English}, t_2) &= 0.90, & \mu_P^I &= 0.05, & \mu_P^F &= 0.05,\\ \mu_P^T(\text{Masahiro, Algebra}, t_1) &= 0.80, & \mu_P^I &= 0.10, & \mu_P^F &= 0.10,\\ \mu_P^T(\text{Masahiro, Algebra}, t_2) &= 0.85, & \mu_P^I &= 0.10, & \mu_P^F &= 0.05,\\ \mu_P^T(\text{Masahiro, English}, t_1) &= 0.70, & \mu_P^I &= 0.20, & \mu_P^F &= 0.10,\\ \mu_P^T(\text{Masahiro, English}, t_2) &= 0.75, & \mu_P^I &= 0.15, & \mu_P^F &= 0.10. \end{aligned}$$

Computing yields approximately

$$S_{\text{NEP}}(\text{Ayako}) \approx (0.39,\ 0.33,\ 0.35), \quad S_{\text{NEP}}(\text{Masahiro}) \approx (0.37,\ 0.36,\ 0.35).$$

Example 2.11 (University Course Sequence)**.** Consider two students $S = \{\text{David, Emma}\}$ enrolled in two core courses $C = \{\text{Calculus\_I, Linear\_Algebra}\}$ over three key time-points $T = \{t_0 = \text{2025-02-01 (start)},\ t_1 = \text{2025-03-15 (midterm)},\ t_2 = \text{2025-05-01 (final)}\}$.

Weights for course importance are

$$w_{\text{Calculus\_I}} = 0.6, \quad w_{\text{Linear\_Algebra}} = 0.4.$$

Initial knowledge levels at t_0 (diagnostic test):

$$K_0(\text{David, Calculus\_I}) = 0.30, \quad K_0(\text{David, Linear\_Algebra}) = 0.40,$$
$$K_0(\text{Emma, Calculus\_I}) = 0.50, \quad K_0(\text{Emma, Linear\_Algebra}) = 0.20.$$

Learning increments:

$$\begin{aligned} L(\text{David, Calculus\_I}, t_1) &= 0.25, & L(\text{David, Linear\_Algebra}, t_1) &= 0.20,\\ L(\text{David, Calculus\_I}, t_2) &= 0.35, & L(\text{David, Linear\_Algebra}, t_2) &= 0.30,\\ L(\text{Emma, Calculus\_I}, t_1) &= 0.30, & L(\text{Emma, Linear\_Algebra}, t_1) &= 0.25,\\ L(\text{Emma, Calculus\_I}, t_2) &= 0.40, & L(\text{Emma, Linear\_Algebra}, t_2) &= 0.35. \end{aligned}$$

These yield crisp knowledge:

$$\begin{aligned} K(\text{David, Calculus\_I}, t_1) &= 0.55, & K(\text{David, Calculus\_I}, t_2) &= 0.90,\\ K(\text{David, Linear\_Algebra}, t_1) &= 0.60, & K(\text{David, Linear\_Algebra}, t_2) &= 0.90,\\ K(\text{Emma, Calculus\_I}, t_1) &= 0.80, & K(\text{Emma, Calculus\_I}, t_2) &= 1.00,\\ K(\text{Emma, Linear\_Algebra}, t_1) &= 0.45, & K(\text{Emma, Linear\_Algebra}, t_2) &= 0.80. \end{aligned}$$

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Neutrosophic participation degrees $\mu_P^T, \mu_P^I, \mu_P^F$ on (s, c, t):

Interaction	μ_P^T	μ_P^I	μ_P^F
(David, Calculus_I, t_1)	0.95	0.03	0.02
(David, Linear_Algebra, t_1)	0.90	0.05	0.05
(David, Calculus_I, t_2)	1.00	0.00	0.00
(David, Linear_Algebra, t_2)	0.98	0.01	0.01
(Emma, Calculus_I, t_1)	0.88	0.07	0.05
(Emma, Linear_Algebra, t_1)	0.85	0.10	0.05
(Emma, Calculus_I, t_2)	0.92	0.05	0.03
(Emma, Linear_Algebra, t_2)	0.90	0.07	0.03

Compute neutrosophic evaluations, e.g. for David:

$$E_T(\text{David}) = \frac{0.95 \cdot 0.6 \cdot 0.55 + 0.90 \cdot 0.4 \cdot 0.60 + 1.00 \cdot 0.6 \cdot 0.90 + 0.98 \cdot 0.4 \cdot 0.90}{0.95 + 0.90 + 1.00 + 0.98} \approx 0.85,$$

$$E_I(\text{David}) = \frac{0.03 \cdot 0.6 \cdot 0.55 + 0.05 \cdot 0.4 \cdot 0.60 + 0.00 \cdot 0.6 \cdot 0.90 + 0.01 \cdot 0.4 \cdot 0.90}{0.03 + 0.05 + 0.00 + 0.01} \approx 0.51,$$

$$E_F(\text{David}) = \frac{0.02 \cdot 0.6 \cdot 0.55 + 0.05 \cdot 0.4 \cdot 0.60 + 0.00 \cdot 0.6 \cdot 0.90 + 0.01 \cdot 0.4 \cdot 0.90}{0.02 + 0.05 + 0.00 + 0.01} \approx 0.42.$$

Similarly for Emma one obtains $S_{\text{NEP}}(\text{Emma}) \approx (0.84,\ 0.52,\ 0.38)$.

Example 2.12 (High school biology class (single topic)). Let

$$S = \{\text{Haruka, Kenji}\}, \qquad C = \{\text{Biology}\}, \qquad T = \{t_0, t_1, t_2\},$$

with t_0=pretest, t_1=lesson, t_2=quiz, and weight $w_{\text{Biology}} = 1$. Initial knowledge and learning increments:

$$K_0(\text{Haruka}, \text{Biology}) = 0.40, \quad L(\text{Haruka}, \text{Biology}, t_1) = 0.30, \ L(\cdot, t_2) = 0.20,$$

$$K_0(\text{Kenji}, \text{Biology}) = 0.35, \quad L(\text{Kenji}, \text{Biology}, t_1) = 0.25, \ L(\cdot, t_2) = 0.30.$$

Thus (with $K(s, c, t_k) = \min\{1,\ K(s, c, t_{k-1}) + L(s, c, t_k)\}$)

	t_0	t_1	t_2
$K(\text{Haruka}, \text{Biology}, \cdot)$	0.40	0.70	0.90
$K(\text{Kenji}, \text{Biology}, \cdot)$	0.35	0.60	0.90

Neutrosophic participation degrees $(\mu_P^T, \mu_P^I, \mu_P^F)$:

$$(\text{Haruka}, \text{Biology}): \ \mu^T = (0.9, 1.0, 0.8), \ \mu^I = (0.1, 0.0, 0.2), \ \mu^F = (0.0, 0.0, 0.1),$$

$$(\text{Kenji}, \text{Biology}): \ \mu^T = (0.8, 0.6, 0.7), \ \mu^I = (0.1, 0.3, 0.2), \ \mu^F = (0.1, 0.2, 0.1).$$

Neutrosophic evaluations $E_X(s) = \dfrac{\sum_t \mu_P^X(s, \text{Biology}, t)\, K(s, \text{Biology}, t)}{\sum_t \mu_P^X(s, \text{Biology}, t)}$ for $X \in \{T, I, F\}$:

$$E_T(\text{Haruka}) = \frac{0.9 \cdot 0.40 + 1.0 \cdot 0.70 + 0.8 \cdot 0.90}{0.9 + 1.0 + 0.8} = \frac{1.78}{2.7} \approx 0.659,$$

$$E_I(\text{Haruka}) = \frac{0.1 \cdot 0.40 + 0.0 \cdot 0.70 + 0.2 \cdot 0.90}{0.1 + 0.0 + 0.2} = \frac{0.22}{0.3} \approx 0.733,$$

$$E_F(\text{Haruka}) = \frac{0.0 \cdot 0.40 + 0.0 \cdot 0.70 + 0.1 \cdot 0.90}{0.0 + 0.0 + 0.1} = \frac{0.09}{0.1} = 0.900;$$

$$E_T(\text{Kenji}) = \frac{0.8 \cdot 0.35 + 0.6 \cdot 0.60 + 0.7 \cdot 0.90}{0.8 + 0.6 + 0.7} = \frac{1.27}{2.1} \approx 0.605,$$

$$E_I(\text{Kenji}) = \frac{0.1 \cdot 0.35 + 0.3 \cdot 0.60 + 0.2 \cdot 0.90}{0.1 + 0.3 + 0.2} = \frac{0.395}{0.6} \approx 0.658,$$

$$E_F(\text{Kenji}) = \frac{0.1 \cdot 0.35 + 0.2 \cdot 0.60 + 0.1 \cdot 0.90}{0.1 + 0.2 + 0.1} = \frac{0.245}{0.4} = 0.613.$$

These triplets summarize readiness (truth), ambiguity (indeterminacy), and shortfall (falsity) under a single-topic class.

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Example 2.13 (Corporate training (Compliance & Data Security)). Let

$$S = \{\text{Miyu}, \text{ Daichi}\}, \quad C = \{\text{Compliance}, \text{ Security}\}, \quad T = \{t_0, t_1, t_2\},$$

with t_0=pretest, t_1=workshop, t_2=assessment, and weights $w_{\text{Compliance}} = 0.4$, $w_{\text{Security}} = 0.6$.

Initial knowledge and learning increments:

$$K_0(\text{Miyu}, \text{Compliance}) = 0.50, \quad L(\text{Miyu}, \text{Compliance}, t_1, t_2) = (0.20, 0.10);$$
$$K_0(\text{Miyu}, \text{Security}) = 0.30, \quad L(\text{Miyu}, \text{Security}, t_1, t_2) = (0.25, 0.30);$$
$$K_0(\text{Daichi}, \text{Compliance}) = 0.40, \quad L(\text{Daichi}, \text{Compliance}, t_1, t_2) = (0.25, 0.15);$$
$$K_0(\text{Daichi}, \text{Security}) = 0.35, \quad L(\text{Daichi}, \text{Security}, t_1, t_2) = (0.20, 0.25).$$

Knowledge trajectories:

Miyu	t_0	t_1	t_2
Compliance	0.50	0.70	0.80
Security	0.30	0.55	0.85

Daichi	t_0	t_1	t_2
Compliance	0.40	0.65	0.80
Security	0.35	0.55	0.80

Neutrosophic participation:

$$(\text{Miyu}, \text{Compliance}):\ \mu^T = (0.9, 0.8, 0.9),\ \mu^I = (0.1, 0.2, 0.1),\ \mu^F = (0.0, 0.0, 0.1),$$
$$(\text{Miyu}, \text{Security}):\ \mu^T = (0.7, 0.9, 1.0),\ \mu^I = (0.3, 0.2, 0.1),\ \mu^F = (0.1, 0.0, 0.0);$$
$$(\text{Daichi}, \text{Compliance}):\ \mu^T = (0.8, 0.7, 0.8),\ \mu^I = (0.2, 0.2, 0.1),\ \mu^F = (0.0, 0.1, 0.1),$$
$$(\text{Daichi}, \text{Security}):\ \mu^T = (0.6, 0.8, 0.9),\ \mu^I = (0.3, 0.1, 0.1),\ \mu^F = (0.1, 0.1, 0.0).$$

For $X \in \{T, I, F\}$,

$$E_X(s) = \frac{\sum_{c\in C} \sum_{t\in T} \mu_P^X(s, c, t)\, w_c\, K(s, c, t)}{\sum_{c\in C} \sum_{t\in T} \mu_P^X(s, c, t)}.$$

Miyu. Denominators: $D_T = 2.6 + 2.6 = 5.2$, $D_I = 0.4 + 0.6 = 1.0$, $D_F = 0.1 + 0.1 = 0.2$. Numerators:

$$N_T = \underbrace{0.9 \cdot 0.4 \cdot 0.50 + 0.8 \cdot 0.4 \cdot 0.70 + 0.9 \cdot 0.4 \cdot 0.80}_{\text{Compliance}=0.692} + \underbrace{0.7 \cdot 0.6 \cdot 0.30 + 0.9 \cdot 0.6 \cdot 0.55 + 1.0 \cdot 0.6 \cdot 0.85}_{\text{Security}=0.933} = 1.625,$$

$$N_I = \underbrace{0.1 \cdot 0.4 \cdot 0.50 + 0.2 \cdot 0.4 \cdot 0.70 + 0.1 \cdot 0.4 \cdot 0.80}_{0.108} + \underbrace{0.3 \cdot 0.6 \cdot 0.30 + 0.2 \cdot 0.6 \cdot 0.55 + 0.1 \cdot 0.6 \cdot 0.85}_{0.171} = 0.279,$$

$$N_F = \underbrace{0.0 \cdot 0.4 \cdot 0.50 + 0.0 \cdot 0.4 \cdot 0.70 + 0.1 \cdot 0.4 \cdot 0.80}_{0.032} + \underbrace{0.1 \cdot 0.6 \cdot 0.30 + 0.0 \cdot 0.6 \cdot 0.55 + 0.0 \cdot 0.6 \cdot 0.85}_{0.018} = 0.050.$$

Hence

$$E_T(\text{Miyu}) = \frac{1.625}{5.2} = 0.313, \quad E_I(\text{Miyu}) = \frac{0.279}{1.0} = 0.279, \quad E_F(\text{Miyu}) = \frac{0.050}{0.2} = 0.250.$$

Daichi. Denominators: $D_T = 2.3 + 2.3 = 4.6$, $D_I = 0.5 + 0.5 = 1.0$, $D_F = 0.2 + 0.2 = 0.4$. Numerators:

$$N_T = \underbrace{0.8 \cdot 0.4 \cdot 0.40 + 0.7 \cdot 0.4 \cdot 0.65 + 0.8 \cdot 0.4 \cdot 0.80}_{0.566} + \underbrace{0.6 \cdot 0.6 \cdot 0.35 + 0.8 \cdot 0.6 \cdot 0.55 + 0.9 \cdot 0.6 \cdot 0.80}_{0.822} = 1.388,$$

$$N_I = \underbrace{0.2 \cdot 0.4 \cdot 0.40 + 0.2 \cdot 0.4 \cdot 0.65 + 0.1 \cdot 0.4 \cdot 0.80}_{0.116} + \underbrace{0.3 \cdot 0.6 \cdot 0.35 + 0.1 \cdot 0.6 \cdot 0.55 + 0.1 \cdot 0.6 \cdot 0.80}_{0.144} = 0.260,$$

$$N_F = \underbrace{0.0 \cdot 0.4 \cdot 0.40 + 0.1 \cdot 0.4 \cdot 0.65 + 0.1 \cdot 0.4 \cdot 0.80}_{0.058} + \underbrace{0.1 \cdot 0.6 \cdot 0.35 + 0.1 \cdot 0.6 \cdot 0.55 + 0.0 \cdot 0.6 \cdot 0.80}_{0.054} = 0.112.$$

Therefore

$$E_T(\text{Daichi}) = \frac{1.388}{4.6} \approx 0.302, \quad E_I(\text{Daichi}) = \frac{0.260}{1.0} = 0.260, \quad E_F(\text{Daichi}) = \frac{0.112}{0.4} = 0.280.$$

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Theorem 2.14. *If $\mu_P^I \equiv 0$ and $\mu_P^F = 1 - \mu_P^T$, then the Neutrosophic Education Process reduces to the Fuzzy Education Process with $\mu_P^T = \mu_P$. In particular,*

$$E_T(s) = E_F(s), \quad E_I(s) = 0, \quad E_F(s) = 1 - E_F(s),$$

recovering the classical fuzzy evaluation.

Proof. Under $\mu_P^I(s,c,t) = 0$ and $\mu_P^F(s,c,t) = 1 - \mu_P^T(s,c,t)$, the denominator $\sum \mu_P^T$ coincides with $\sum \mu_P$ of the fuzzy model, and the numerator for E_T matches the fuzzy weighted sum. Meanwhile $\sum \mu_P^I = 0$ forces $E_I = 0$, and $\sum \mu_P^F = \sum(1 - \mu_P^T)$ gives $E_F = 1 - E_T$. Hence the specialization holds. □

3 Additional Results: Scope-Fitting operator for Neutrosophic Sets in Education

As additional results, we introduce the Scope-Fitting operator for Neutrosophic Sets. A Scope-Fitting operator adjusts the truth, indeterminacy, and falsity degrees of Neutrosophic Sets relative to contextual application domains (cf. [19]). This mechanism also allows the addition or removal of new elements in the considered universe set. We further examine how the Scope-Fitting operator for Neutrosophic Sets can be effectively applied in the field of education.

Definition 3.1 (Scope-Fitting operator for Neutrosophic Sets (SFNS))**.** (cf. [19]) Let $A = (Y, T, I, F)$ be a neutrosophic set on $Y \subseteq \Omega$ with $T, I, F : Y \to [0,1]$. Fix $\alpha, \beta \in [0,1]$, kernel K, prior π, attenuation parameters $\lambda_T, \lambda_I, \lambda_F \in [0,1]$, extension gains $\gamma_T, \gamma_I, \gamma_F \in [0,1]$, and baseline priors $(\tau_0, \iota_0, \varphi_0) \in [0,1]^3$ for newly added elements. Use the fitted universe $\widehat{Y}$. For retained $y \in Y \setminus R_\beta$ (write $w_{\text{out}}(y) := 1 - \pi(y)$, $w_{\text{in}}(y) := \pi(y)$):

$$\widehat{T}(y) = \big(1 - \lambda_T\, w_{\text{out}}(y)\big)\, T(y), \tag{3}$$

$$\widehat{I}(y) = (1 - \lambda_I\, w_{\text{in}}(y))\, I(y) \; + \; \lambda_I\, w_{\text{out}}(y)\, \iota_0, \tag{4}$$

$$\widehat{F}(y) = (1 - \lambda_F\, w_{\text{in}}(y))\, F(y) \; + \; \lambda_F\, w_{\text{out}}(y)\, \varphi_0. \tag{5}$$

For newly added $x \in E_\alpha$:

$$\widehat{T}(x) = \gamma_T \sup_{y \in Y}\big(K(x,y)\, T(y)\big) \; + \; (1 - \gamma_T)\, \tau_0, \tag{6}$$

$$\widehat{I}(x) = \gamma_I \sup_{y \in Y}\big(K(x,y)\, I(y)\big) \; + \; (1 - \gamma_I)\, \iota_0, \tag{7}$$

$$\widehat{F}(x) = \gamma_F \sup_{y \in Y}\big(K(x,y)\, F(y)\big) \; + \; (1 - \gamma_F)\, \varphi_0. \tag{8}$$

Then $\widehat{T}, \widehat{I}, \widehat{F} : \widehat{Y} \to [0,1]$, and we denote

$$\mathrm{SFitN}^{\pi,K}_{\alpha,\beta;\Lambda,\Gamma}(Y, T, I, F) := (\widehat{Y}, \widehat{T}, \widehat{I}, \widehat{F}),$$

with $\Lambda = (\lambda_T, \lambda_I, \lambda_F)$ and $\Gamma = (\gamma_T, \gamma_I, \gamma_F)$.

Example 3.2 (Scope shift: Algebra readiness $\to$ Calculus I readiness with a transfer student)**.** Let $Y = \{\text{Aiko}, \text{Ben}\}$ be students evaluated for *Algebra readiness*. Define the neutrosophic set $A = (Y, T, I, F)$ by

$$(T, I, F)(\text{Aiko}) = (0.70,\ 0.20,\ 0.10), \qquad (T, I, F)(\text{Ben}) = (0.40,\ 0.40,\ 0.20).$$

We scope-fit these to *Calculus I readiness* using Definition 3.1. Choose parameters for retained elements:

$$\lambda_T = 0.5,\ \lambda_I = 0.6,\ \lambda_F = 0.4, \quad \iota_0 = 0.5,\ \varphi_0 = 0.2,$$

and priors $\pi(\text{Aiko}) = 0.9$, $\pi(\text{Ben}) = 0.4$ (higher prior = more in-scope evidence). Let $w_{\text{in}} = \pi$, $w_{\text{out}} = 1 - \pi$. For a new transfer student $x = \text{Sakura} \in E_\alpha$, take

$$\gamma_T = \gamma_I = \gamma_F = 0.7, \quad \tau_0 = 0.3,\ \iota_0 = 0.5,\ \varphi_0 = 0.2,$$

and a similarity kernel K with $K(x, \text{Aiko}) = 0.8$, $K(x, \text{Ben}) = 0.5$.

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Retained: Aiko ($\pi = 0.9$: w_{in}=0.9, w_{out}=0.1).

$$\begin{aligned}
\widehat{T}(\text{Aiko}) &= (1-\lambda_T w_{\text{out}})\,T = (1-0.5\cdot 0.1)\cdot 0.70 = 0.95\cdot 0.70 = 0.665,\\
\widehat{I}(\text{Aiko}) &= (1-\lambda_I w_{\text{in}})\,I + \lambda_I w_{\text{out}}\iota_0 = (1-0.6\cdot 0.9)\cdot 0.20 + 0.6\cdot 0.1\cdot 0.5\\
&= 0.46\cdot 0.20 + 0.03 = 0.092 + 0.03 = 0.122,\\
\widehat{F}(\text{Aiko}) &= (1-\lambda_F w_{\text{in}})\,F + \lambda_F w_{\text{out}}\varphi_0 = (1-0.4\cdot 0.9)\cdot 0.10 + 0.4\cdot 0.1\cdot 0.2\\
&= 0.64\cdot 0.10 + 0.008 = 0.064 + 0.008 = 0.072.
\end{aligned}$$

Retained: Ben ($\pi = 0.4$: w_{in}=0.4, w_{out}=0.6).

$$\begin{aligned}
\widehat{T}(\text{Ben}) &= (1-0.5\cdot 0.6)\cdot 0.40 = 0.7\cdot 0.40 = 0.28,\\
\widehat{I}(\text{Ben}) &= (1-0.6\cdot 0.4)\cdot 0.40 + 0.6\cdot 0.6\cdot 0.5 = 0.76\cdot 0.40 + 0.18 = 0.304 + 0.18 = 0.484,\\
\widehat{F}(\text{Ben}) &= (1-0.4\cdot 0.4)\cdot 0.20 + 0.4\cdot 0.6\cdot 0.2 = 0.84\cdot 0.20 + 0.048 = 0.168 + 0.048 = 0.216.
\end{aligned}$$

Extension: new x = Sakura with kernel K.

$$\begin{aligned}
\sup_{y\in Y} K(x,y)T(y) &= \max\{0.8\cdot 0.70,\ 0.5\cdot 0.40\} = \max\{0.56, 0.20\} = 0.56,\\
\sup_{y\in Y} K(x,y)I(y) &= \max\{0.8\cdot 0.20,\ 0.5\cdot 0.40\} = \max\{0.16, 0.20\} = 0.20,\\
\sup_{y\in Y} K(x,y)F(y) &= \max\{0.8\cdot 0.10,\ 0.5\cdot 0.20\} = \max\{0.08, 0.10\} = 0.10.
\end{aligned}$$

Hence, by (6)–(8),

$$\begin{aligned}
\widehat{T}(x) &= 0.7\cdot 0.56 + 0.3\cdot 0.3 = 0.392 + 0.09 = 0.482,\\
\widehat{I}(x) &= 0.7\cdot 0.20 + 0.3\cdot 0.5 = 0.14 + 0.15 = 0.29,\\
\widehat{F}(x) &= 0.7\cdot 0.10 + 0.3\cdot 0.2 = 0.07 + 0.06 = 0.13.
\end{aligned}$$

High in-scope prior preserves Aiko's truth and reduces her indeterminacy; Ben's lower in-scope prior raises $\widehat{I}$ (unknown/unaware under the Calculus I scope). The transfer student's values are synthesized from similar peers and scope baselines via K and $(\tau_0, \iota_0, \varphi_0)$.

Example 3.3 (Scope narrowing: General English $\to$ Academic Writing (no new students))**.** Let $Y = \{\text{Haruka}, \text{Kenji}\}$ be evaluated for *General English proficiency* with

$$(T, I, F)(\text{Haruka}) = (0.65,\ 0.25,\ 0.10), \qquad (T, I, F)(\text{Kenji}) = (0.45,\ 0.35,\ 0.20).$$

We scope-fit to *Academic Writing*. Choose

$$\lambda_T = 0.4,\ \lambda_I = 0.6,\ \lambda_F = 0.4, \quad \iota_0 = 0.6,\ \varphi_0 = 0.3,$$

and priors reflecting how much prior evidence is writing-relevant: $\pi(\text{Haruka}) = 0.5$ (balanced), $\pi(\text{Kenji}) = 0.2$ (mostly speaking/listening). Thus $w_{\text{in}} = \pi$, $w_{\text{out}} = 1-\pi$, and $E_\alpha = \varnothing$.

Haruka (w_{in}=0.5, w_{out}=0.5).

$$\begin{aligned}
\widehat{T} &= (1-0.4\cdot 0.5)\cdot 0.65 = 0.8\cdot 0.65 = 0.52,\\
\widehat{I} &= (1-0.6\cdot 0.5)\cdot 0.25 + 0.6\cdot 0.5\cdot 0.6 = 0.7\cdot 0.25 + 0.18 = 0.175 + 0.18 = 0.355,\\
\widehat{F} &= (1-0.4\cdot 0.5)\cdot 0.10 + 0.4\cdot 0.5\cdot 0.3 = 0.8\cdot 0.10 + 0.06 = 0.08 + 0.06 = 0.14.
\end{aligned}$$

Kenji (w_{in}=0.2, w_{out}=0.8).

$$\begin{aligned}
\widehat{T} &= (1-0.4\cdot 0.8)\cdot 0.45 = 0.68\cdot 0.45 = 0.306,\\
\widehat{I} &= (1-0.6\cdot 0.2)\cdot 0.35 + 0.6\cdot 0.8\cdot 0.6 = 0.88\cdot 0.35 + 0.288 = 0.308 + 0.288 = 0.596,\\
\widehat{F} &= (1-0.4\cdot 0.2)\cdot 0.20 + 0.4\cdot 0.8\cdot 0.3 = 0.92\cdot 0.20 + 0.096 = 0.184 + 0.096 = 0.28.
\end{aligned}$$

Narrowing to Academic Writing attenuates truth for out-of-scope evidence and explicitly reassigns a portion to indeterminacy via ι_0. Kenji's low writing-relevant prior yields a large $\widehat{I}$, highlighting unknown/unaware areas for targeted instruction.

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4 Conclusion

In this paper, we provided formal mathematical definitions for the *Fuzzy Education Process* and the *Neutrosophic Education Process*. For future research, we anticipate progress in machine learning studies utilizing datasets, as well as quantitative investigations based on computational experiments. Moreover, we expect further developments in extended frameworks employing advanced models such as HyperFuzzy Sets [20,21], Hesitant Fuzzy Sets [22], Graphs [23], HyperGraphs [24,25], SuperHyperGraphs [26,27], and Plithogenic Sets [28,29].

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Use of Generative AI and AI-Assisted Tools

I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

Author's Contributions

Conceptualization, Takaaki Fujita; Investigation, Takaaki Fujita; Methodology, Takaaki Fujita; Writing – original draft, Takaaki Fujita; Writing – review & editing, All authors.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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