Decision-Making Approach by Using Choice Value and Weighted Choice Value of Interval-Valued Fuzzy Sets

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Abstract

This paper tackles the difficulty of accurately modeling uncertainty in complicated DM settings, where conventional FS models frequently fail. The IVFS theory, a broadening FS theory, is a potent tool that can offer the potential to approach uncertain data in vague environment in order to get over these restrictions. This paper presents an application of IVFS in a DM challenges, where on CV and WCV of an IVFS are used to select a qualified applicant for the HR manager position. Additionally, sensitivity analysis has demonstrated the stability of the final decision.

Keywords: FS; IVFS; CVIVFS; WCVIVFS

1. Introduction

DM in real-world environments is involved with uncertainty, imprecision, and vagueness that traditional crisp mathematical models struggle to handle effectively. To address these challenges, Zadeh [19] put forth the idea of FS, which is an effective means, enabling the modeling of uncertainty through degrees of membership rather than binary classifications. Bellman and Zadeh [3] initially shown the value of FS theory in DM in 1970. Numerous scholars have since been using FS theory to address DM issues. Examples of these include the hypothesis of probability, the idea of FS's [19], the concept of IFS's [2], the proposition of ambiguous sets [5], the idea of rough sets [10], and others. Coroiu [4], who also showed how fuzzy logic might appear in the DM approach, discussed the benefit of the fuzzy approach over other paradigms. In order to help decision-makers achieve the best consistency of fuzzy judgments, Wei [17] demonstrated a capability project opting framework that merges an effective fuzzy weighted mean with a best aggregation approach. This representation creates a single synergistic index study fuzzy simulated rating that takes efficiency and hazard into account.

Among the various extensions of FS ‘s, the IVFS introduced by Zadeh [20] which offers risk a more flexible and expressive framework by assigning a range (interval) of membership values to elements, instead of a single exact value. This added expressiveness makes IVFS particularly suitable for MCDM and other complex DM problems, where subjective judgments, incomplete information, and linguistic assessments are common. Gorzałczany [6] laid the groundwork for the use of IVFS‘s in DM by offering an approach to inference in approximation reasoning. In order to gather a lot more data for the multiple attribute group DM, He et al. [7] looked into the ideal characteristics of the new IVHF aggregation operations. Singh and Haung [14], who demonstrate that the most important decision guidelines are incorporated in the interval-valued gray fuzzy area, presented a four-way interval-valued decision space. In order to make urgent and stochastic multivariable decisions, Peng et al. [11] suggested using the merged weights for the criteria in IVFSS. Ma et al. [9] proposed a novel, effective approach for DM. It is more flexible and

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helps with the broadening of IVFSS and the combination of multiple IVFSS's. A new process of DM based on IVFSS was initiated by Qin et al. [13] using a contrast table with symmetry between the entities. Their suggested algorithm bases its decisions on the number of higher-level criteria points sooner than score points, which is a narrative way to make decisions. Petry and Yoger [12] presented an example to show how to aggregate IVFS has and provide information measures to guide the aggregation. In order to efficiently update the three-way regions in a dynamic environment, Yang et al. [18] looked into the active preservation of the variable exactness fuzzy locality three-way regions against the backdrop of an IVF decision approach. Vo et al., [16] suggested a novel way for using IVFS's to rank the characteristics of a particular DM approach.

Using CVFS & WCVFS, Aggarwal [1] demonstrated the use of FS's in a DM strategy. While CV and WCV in FS 's are useful tools for decision-making under some uncertainty, but it falls short when the uncertainty is high or vague, lack the flexibility to express ambiguity or hesitation and Can produce unstable results in sensitive or complex decision environments. As IVFS is a subset of the closed interval [0,1], it would be more suitable in this situation for decision-makers to voice their thoughts. In this paper, the role and effectiveness of IVFS 's in a DM approach with the aid of CV & WCV of IVFS's is explored and discussed how this model improves decision quality in uncertain and imprecise environments.

This work's next portions are arranged as follows: The relevant preliminaries, including definitions of FS, IVFS, CVFS, and WCVFS, are covered in Section 2. The CVIVFS and WCVIVFS are proposed in Section 3 to address DM approach, demonstrating the effectiveness of the recommended approach. The IVFS is contrasted with current FS models in Section 4.

2. Preliminaries

This section offers some basic concepts that are important for this work.

Definition: 2.1

The component of $I^{\mho}$ is an FS in $\mho$, which is a mapping from $\mho$ into I= [0, 1].

Definition: 2.2

IVFS in $\mho$, [I] are closed subintervals of [0, 1], according to $\mathcal{A}_{IV} : \mho \to [I]$. IVFS ($\mho$) is used to indicate all IVFS 's on $\mho$. $\mathcal{A}_{IV}(v) = [\mathcal{A}^{LF}(v), \mathcal{A}^{UF}(v)]$ called membership for element v to $\mathcal{A}_{IV}$, where $\mathcal{A}^{LF} : \mho \to$ I and $\mathcal{A}^{UF} : \mho \to$ I called lower FS and upper FS in $\mho$, for each $\mathcal{A}_{IV} \in$ IVFS($\mho$) and $v \in \mho$. Another way to express $\mathcal{A}_{IV}$ is as $\mathcal{A}_{IV} = [\mathcal{A}^{LF}, \mathcal{A}^{UF}]$.

Definition: 2.3

The CVFS $(C_F)_i$ of fuzzy membership value $((\mathcal{A}^F)_i)(v_j)$ defined as $(C_F)_i = \sum_j((\mathcal{A}^F)_i)(v_j)$

Definition: 2.4

The WCVFS $((C_F)_i)_w$ of Fuzzy membership value $((\mathcal{A}^F)_i)v_j$ is defined as $((C_{IF})_i)_w = \sum_j w_j$ x $((\mathcal{A}^F)_i)(v_j)$, where w_j are respective weights for j = 1, 2, . . , n.

3. DM Approach using CVIVFS's and WCVIVFS's

In this Section, CVIVFS and WCVIVFS are defined and applied in DM approach in order to pick an appropriate applicant for the HR manager position. In addition, we applied sensitivity analysis to strengthen the result.

Definition: 3.1

The CVIVFS $(C_{IV})_i$ of IVF membership value $((\mathcal{A}_{IV})_i)(v_j)$ defined as $(C_{IV})_i = [\sum_j((\mathcal{A}^{LF})_i)(v_j), \sum_j((\mathcal{A}^{UF})_i)(v_j)]$,

Definition: 3.2

The WCVIVFS $((C_{IV})_i)_w$ of IVF membership value $((\mathcal{A}_{IV})_i)v_j$ is defined as $((C_{IV})_i)_w = [\sum_j w_j$ x $((\mathcal{A}^{LF})_i)(v_j), \sum_j w_j$ x $((\mathcal{A}^{UF})_i)(v_j)]$, where w_j are espective weights for j = 1, 2, . . . , n.

Algorithm: 3.3

Step 1: Construct a linguistic term and IVF membership value table.

Step 2: Construct the IVFS's with IVF membership value.

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Step 3: Calculate the CVIVF membership value of IVFS as $(C_{IV})_i$

Step 4: Find κ, for which $(C_{IV})_\kappa$ = max $(C_{IV})_i$, (i.e.,) choose an entity with the highest IVF membership value.

(In case of tie i.e., when more than one object with same highest IVF membership value, WCVIVF membership of IVFS as $((C_{IV})_i)_w$ is chosen).

Step 5: Construct the weighted value table for each parameter.

Step 6: Calculate the WCVIVFS's according to the weighted choice value of each parameter.

Step 7: Find κ, $((C_{IV})_\kappa)_w$ = max $((C_{IV})_i)_w$ (i.e.,) Choose an entity with the highest IVF membership value.

Step 8: Sensitivity analysis used to strengthen the result.

Example: 3.

Suppose a company wants to hire an HR Manager. Ten candidates have applied for the HR Manager posts. Let U = $\{(\mathrm{NM})_1, (\mathrm{NM})_2, (\mathrm{NM})_3, (\mathrm{NM})_4, (\mathrm{NM})_5, (\mathrm{NM})_6, (\mathrm{NM})_7, (\mathrm{NM})_8, (\mathrm{NM})_9, (\mathrm{NM})_{10}\}$ be the set of applicants and set of criteria for the position of HR Manager be $\mho$ = {$\mho_1$("experience"), $\mho_2$("leadership "), $\mho_3$("patience "), $\mho_4$ ("communication skills"), $\mho_5$ ("sympathetic attitude"), $\mho_6$ ("quick decision"), $\mho_7$("integrity"), $\mho_8$ ("computer knowledge")} considered by the committee of experts who are conducting the interview. They give their opinion in terms of linguistic & transformed into IVF membership value displayed in Table 1.

Step 1

Table 1: IVF membership value

Linguistic terms	**IVF membership value**
"Excellent"	[0.91, 1]
"Very Good"	[.71, .90]
"Good"	[.51, .70]
"Neutral"	[.31, .5]
"Poor'	[.2, .3]
'Very Poor"	[.06, .19]
Extremely Poor	[0, 0.05]

Step 2:

For example a committee of experts have given the statement about the candidate A_1 as follows: Good of the expectation on parameter experience ($\mho_1$) is satisfied, Very Good of the expectation on parameter leadership ($\mho_2$) is satisfied, Very Poor of the expectation on parameter patience ($\mho_3$) is satisfied, Neutral of the expectation on parameter good communication skills ($\mho_4$) is satisfied, Neutral of the expectation on parameter sympathetic attitude ($\mho_5$) is satisfied, Poor of the expectation on parameter quick decision ($\mho_6$) is satisfied, Very Good of the expectation on parameter integrity ($\mho_7$) is satisfied and Excellent of the expectation on parameter computer knowledge ($\mho_8$) is satisfied. Thus, based on committee of expert's opinion we construct IVFS as follows:

$(\mathcal{A}_{IV})_1$ = {(($\mho_1$), [.51, .7]), (($\mho_2$), [.71, .9]), (($\mho_3$), [.71, .9]}, (($\mho_4$), [.91, 1]), (($\mho_5$), [.31, .5]), (($\mho_6$), [.2, .3]), (($\mho_7$), [.71, .9]), (($\mho_8$), [.31, .5])}

In a similar way for the other candidates the following IVFS's is constructed.

$(\mathcal{A}_{IV})_2$ = {(($\mho_1$), [.31, .5]), (($\mho_2$), [.2, .3]), (($\mho_3$), [.51, .7]}, (($\mho_4$), [.31, .5]), (($\mho_5$), [.31, .5]), (($\mho_6$), [.06, .19]), (($\mho_7$), [.31, .5]), (($\mho_8$), [.31, .5])}

$(\mathcal{A}_{IV})_3$ = {(($\mho_1$), [.2, .3]), (($\mho_2$), [.51, .7]), (($\mho_3$), [.2, .3]}, (($\mho_4$), [.2, .3]), (($\mho_5$),

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[.51, .7]), (($\mathcal{v}_6$), [.06, .19]), (($\mathcal{v}_7$), [.2, 0.3]), (($\mathcal{v}_8$), [.2, .3])}

$(\mathcal{A}_{IV})_4$ = {(($\mathcal{v}_1$), [.2, .3]), (($\mathcal{v}_2$), [.51, .7]), (($\mathcal{v}_3$), [.31, .5]}, (($\mathcal{v}_4$), [.2, .3]), (($\mathcal{v}_5$), [.2, .3]), (($\mathcal{v}_6$), [.2, .3]), (($\mathcal{v}_7$), [.2, .3]), (($\mathcal{v}_8$), [.31, .5])}

$(\mathcal{A}_{IV})_5$ = {(($\mathcal{v}_1$), [.31, .5]), (($\mathcal{v}_2$), [.2, .3]), (($\mathcal{v}_3$), [.2, .3]}, (($\mathcal{v}_4$), [.06, .19]), (($\mathcal{v}_5$), [.31, .5]), (($\mathcal{v}_6$), [.2, .3]), (($\mathcal{v}_7$), [.06, .19]), (($\mathcal{v}_8$), [.2, 0.3])}

$(\mathcal{A}_{IV})_6$ = {(($\mathcal{v}_1$), [.06, .19]), (($\mathcal{v}_2$), [.2, .3]), (($\mathcal{v}_3$), [.31, .5]}, (($\mathcal{v}_4$), [.06, .19]), (($\mathcal{v}_5$), [.06, .19]), (($\mathcal{v}_6$), [.2, .3]), (($\mathcal{v}_7$), [.06, .19]), (($\mathcal{v}_8$), [.06, .19])}

$(\mathcal{A}_{IV})_7$ = {(($\mathcal{v}_1$), [.31, .5]), (($\mathcal{v}_2$), [.06, .19]), (($\mathcal{v}_3$), [.2, .3]}, (($\mathcal{v}_4$), [.31, .5]), (($\mathcal{v}_5$), [.2, .3]), (($\mathcal{v}_6$), [.2, .3]), (($\mathcal{v}_7$), [.06, .19]), (($\mathcal{v}_8$), [.2, .3])}

$(\mathcal{A}_{IV})_8$ = {(($\mathcal{v}_1$), [.51, .7]), (($\mathcal{v}_2$), [.51, .7]), (($\mathcal{v}_3$), [.31, .5]}, (($\mathcal{v}_4$), [.31, .5]), (($\mathcal{v}_5$), [.31, .5]), (($\mathcal{v}_6$), [.2, .3]), (($\mathcal{v}_7$), [.31, .5]), (($\mathcal{v}_8$), [.71, .9])}

$(\mathcal{A}_{IV})_9$= {(($\mathcal{v}_1$), [.2, .3]), (($\mathcal{v}_2$), [.31, .5]), (($\mathcal{v}_3$), [.31, .5]}, (($\mathcal{v}_4$), [.06, .19]), (($\mathcal{v}_5$), [.06, .19]), (($\mathcal{v}_6$), [.06, .19]), (($\mathcal{v}_7$), [.31, .5]), (($\mathcal{v}_8$), [0, .05])}

$(\mathcal{A}_{IV})_{10}$= {(($\mathcal{v}_1$), [.71, .9]), (($\mathcal{v}_2$), [.71, .9]), (($\mathcal{v}_3$), [.71, .9]}, (($\mathcal{v}_4$), [.31, .5]), (($\mathcal{v}_5$), [.31, .5]), (($\mathcal{v}_6$), [.2, .3]), (($\mathcal{v}_7$), [.51, .7]), (($\mathcal{v}_8$), [.91, 1])}

The above IVFS's in terms of IVF membership value displayed in Table 2.

Table 2: IVFS of each applicant

U	$(\mathcal{v}_1)$	$(\mathcal{v}_2)$	$(\mathcal{v}_3)$	$(\mathcal{v}_4)$	$(\mathcal{v}_5)$	$(\mathcal{v}_6)$	$(\mathcal{v}_7)$	$(\mathcal{v}_8)$
$(\mathcal{A}_{IV})_1$	[.51, .7]	[.71, 0.9]	[0.71, 0.9]	[0.91, 1.0]	[0.31, 0.5]	[0.2, 0.3]	[0.71, 0.9]	[0.31, 0.5]
$(\mathcal{A}_{IV})_2$	[0.31, 0.5]	[0.2, 0.3]	[0.51, 0.7]	[0.31, 0.5]	[0.31, 0.5]	[0.06, 0.19]	[0.31, 0.5]	[0.31, 0.5]
$(\mathcal{A}_{IV})_3$	[0.2, 0.3]	[0.51, 0.7]	[0.2, 0.3]	[0.2, 0.3]	[0.51, 0.7]	[0.06, 0.19]	[0.2, 0.3]	[0.2, 0.3]
$(\mathcal{A}_{IV})_4$	[0.2, 0.3]	[0.51, 0.7]	[0.31, 0.5]	[0.2, 0.3]	[0.2, 0.3]	[0.2, 0.3]	[0.2, 0.3]	[0.31, 0.5]
$(\mathcal{A}_{IV})_5$	[0.31, 0.5]	[0.2, 0.3]	[0.2, 0.3]	[0.06, 0.19]	[0.31, 0.5]	[0.2, 0.3]	[0.06, 0.19]	[0.2, 0.3]
$(\mathcal{A}_{IV})_6$	[0.06, 0.19]	[0.2, 0.3]	[0.31, 0.5]	[0.06, 0.19]	[0.06, 0.19]	[0.2, 0.3]	[0.06, 0.19]	[0.06, 0.19]
$(\mathcal{A}_{IV})_7$	[0.31, 0.5]	[0.06, 0.19]	[0.2, 0.3]	[0.31, 0.5]	[0.2, 0.3]	[0.2, 0.3]	[0.06, 0.19]	[0.2, 0.3]
$(\mathcal{A}_{IV})_8$	[0.51, 0.7]	[0.51, 0.7]	[0.31, 0.5]	[0.31, 0.5]	[0.31, 0.5]	[0.2, 0.3]	[0.31, 0.5]	[0.71, 0.9]
$(\mathcal{A}_{IV})_9$	[0.2, 0.3]	[0.31, 0.5]	[0.31, 0.5]	[0.06, 0.19]	[0.06, 0.19]	[0.06, 0.19]	[0.31, 0.5]	[0, 0.05]
$(\mathcal{A}_{IV})_{10}$	[0.71, 0.9]	[0.71, 0.9]	[0.71, 0.9]	[0.31, 0.5]	[0.31, 0.5]	[0.2, 0.3]	[0.51, 0.7]	[0.91, 1]

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Step 3

Table 3: CVIVFS of each candidate

U	$(\mathcal{v}_1)$	$(\mathcal{v}_2)$	$(\mathcal{v}_3)$	$(\mathcal{v}_4)$	$(\mathcal{v}_5)$	$(\mathcal{v}_6)$	$(\mathcal{v}_7)$	$(\mathcal{v}_8)$	$(C_{IV})_i$
$(\mathcal{A}_{IV})_1$	[.51, .7]	[.71, 0.9]	[0.71, 0.9]	[0.91, 1.0]	[0.31, 0.5]	[0.2, 0.3]	[0.71, 0.9]	[0.31, 0.5]	[4.37, 5.7]
$(\mathcal{A}_{IV})_2$	[0.31, 0.5]	[0.2, 0.3]	[0.51, 0.7]	[0.31, 0.5]	[0.31, 0.5]	[.06, .19]	[0.31, 0.5]	[0.31, 0.5]	[2.32, 3.69]
$(\mathcal{A}_{IV})_3$	[0.2, 0.3]	[0.51, 0.7]	[0.2, 0.3]	[0.2, 0.3]	[0.51, 0.7]	[.06, .19]	[0.2, 0.3]	[0.2, 0.3]	[2.08, 3.09]
$(\mathcal{A}_{IV})_4$	[0.2, 0.3]	[0.51, 0.7]	[0.31, 0.5]	[0.2, 0.3]	[0.2, 0.3]	[0.2, 0.3]	[0.2, 0.3]	[0.31, 0.5]	[2.13, 3.2]
$(\mathcal{A}_{IV})_5$	[0.31, 0.5]	[0.2, 0.3]	[0.2, 0.3]	[.06, 0.19]	[0.31, 0.5]	[0.2, 0.3]	[.06, 0.19]	[0.2, 0.3]	[1.54, 2.58]
$(\mathcal{A}_{IV})_6$	[0.06, 0.19]	[0.2, 0.3]	[0.31, 0.5]	[.06, 0.19]	[.06, 0.19]	[0.2, 0.3]	[.06, 0.19]	[.06, 0.19]	[1.01, 2.05]
$(\mathcal{A}_{IV})_7$	[0.31, 0.5]	.06, 0.19]	[0.2, 0.3]	[0.31, 0.5]	[0.2, 0.3]	[0.2, 0.3]	[.06, 0.19]	[0.2, 0.3]	[1.54, 2.58]
$(\mathcal{A}_{IV})_8$	[0.51, 0.7]	[0.51, 0.7]	[0.31, 0.5]	[0.31, 0.5]	[0.31, 0.5]	[0.2, 0.3]	[0.31, 0.5]	[0.71, 0.9]	[3.17, 4.6]
$(\mathcal{A}_{IV})_9$	[0.2, 0.3]	[0.31, 0.5]	[0.31, 0.5]	[.06, 0.19]	[.06, 0.19]	[.06, .19]	[0.31, 0.5]	[0.0, 0.05]	[1.31, 2.42]
$(\mathcal{A}_{IV})_{10}$	[0.71, 0.9]	[0.71, 0.9]	[0.71, 0.9]	[0.31, 0.5]	[0.31, 0.5]	[0.2, 0.3]	[0.51, 0.7]	[0.91, 1]	[4.37, 5.7]

Step 4:

From table 3, $(C_{IV})_i$ is highest to the applicants $(\mathrm{NM})_1$ & $(\mathrm{NM})_{10}$. Hence $(\mathrm{NM})_1$& $(\mathrm{NM})_{10}$ is appropriate for the HR Manager position. There is a tie i.e., the candidates $(\mathrm{NM})_1$ and $(\mathrm{NM})_{10}$ having same highest IVF membership value.

Step 5:

The experts have the following weights to assign for each of the parameters.

Table 4: Weights decided to assign for each of the parameters

Parameters	$(\mathcal{v}_1)$	$(\mathcal{v}_2)$	$(\mathcal{v}_3)$	$(\mathcal{v}_4)$	$(\mathcal{v}_5)$	$(\mathcal{v}_6)$	$(\mathcal{v}_7)$	$(\mathcal{v}_8)$
Weighted Value	.10	.11	.13	.15	.10	.15	.13	.13

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Step 6:

Table 5: WCVIVFS of each candidate

U	$\mathcal{v}_1$ ($w_1 = .1$)	$\mathcal{v}_2$ ($w_2 = .11$)	$\mathcal{v}_3$ $w_3 = .13$	$\mathcal{v}_4$ ($w_4 = .15$)	$\mathcal{v}_5$ ($w_5 = .10$)	$\mathcal{v}_6$ ($w_6 = .15$)	$\mathcal{v}_7$ ($w_7 = .13$)	$\mathcal{v}_8$ ($w_8 = .13$)	$((C_{IV})_i)_w$
$(\mathcal{A}_{IV})_1$	[0.051, 0.070]	[0.078, 0.099]	[0.092, 0.117]	[0.137, 0.150]	[0.062, 0.100]	[0.030, 0.045]	[0.092, 0.117]	[0.040, 0.065]	[0.582, 0.763]
$(\mathcal{A}_{IV})_2$	[0.031, 0.050]	[0.022, 0.033]	[0.066, 0.091]	[0.046, 0.075]	[0.028, 0.042]	[0.009, 0.0028]	[0.040, 0.065]	[0.040, 0.065]	[0.282, 0.449]
$(\mathcal{A}_{IV})_3$	[0.020, 0.030]	[0.056, 0.077]	[0.026, 0.039]	[0.030, 0.045]	[0.051, 0.070]	[0.009, 0.028]	[0.026, 0.039]	[0.026, 0.039]	[0.244, 0.367]
$(\mathcal{A}_{IV})_4$	[0.022, 0.030]	[0.056, 0.077]	[0.040, 0.065]	[0.030, 0.045]	[0.024, 0.036]	[0.030, 0.045]	[0.026, 0.039]	[0.040 0.065]	[0.268, 0.402]
$(\mathcal{A}_{IV})_5$	[0.031, 0.050]	[0.022, 0.033]	[0.026, 0.039]	[0.009, 0.028]	[0.051, 0.070]	[0.030, 0.045]	[0.007, 0.002]	[0.026, 0.039]	[0.201, 0.368]
$(\mathcal{A}_{IV})_6$	[0.006, 0.019]	[0.022, 0.033]	[0.040, 0.065]	[0.009, 0.028]	[0.006, 0.019]	[0.030, 0.045]	[0.007, 0.024]	[0.007, 0.024]	[0.127, 0.357]
$(\mathcal{A}_{IV})_7$	[0.031, 0.050]	[0.006, 0.021]	[0.026, 0.039]	[0.046, 0.075]	[0.040, 0.065]	[0.030, 0.042]	[0.007, 0.024]	[0.007 0.024]	[0.193, 0.343
$(\mathcal{A}_{IV})_8$	[0.051, 0.070]	[0.056, 0.071]	[0.040, 0.065]	[0.046, 0.075]	[0.051, 0.070	[0.030, 0.045]	[0.040, 0.065]	[0.092, 0.117]	[0.406, 0.578
$(\mathcal{A}_{IV})_9$	[0.020, 0.030]	[0.034, 0.055]	[0.040, 0.065]	[0.009, 0.028]	[0.006, 0.019]	[0.009, 0.028]	[0.040, 0.065]	[0.000 0.006]	[0.158, 0.296]
$(\mathcal{A}_{IV})_{10}$	[0.071, 0.090]	[0.078, 0.099]	[0.092, 0.117]	[0.046, 0.075]	[0.051, 0.070]	[0.030, 0.045]	[0.066, 0.091]	[0.118, 0.130]	[0.552, 0.717

Step: 7 From the above data, $(NM)_1$ is the suitable candidate as the WCVIVFS $(\mathcal{A}_{IV})_1$ is more compared to other WCVIVFS's.

Step 8: In comparison to other requirements, we considered that Communication skill $(\mathcal{v}_4)$, Integrity $(\mathcal{v}_7)$ and Patience $(\mathcal{v}_3)$ are the most important for the HR manager position. The following cases have been presented to show how sensitivity analysis of the changes in criteria weights leads to the final decision.

Case (i):

Change in criteria weight considering the relative importance of communication skills $(\mathcal{v}_4)$ as high compared to other criteria.

Table 6: Change in criteria weight, Case (i)

criteria's	$(\mathcal{v}_1)$	$(\mathcal{v}_2)$	$(\mathcal{v}_3)$	$(\mathcal{v}_4)$	$(\mathcal{v}_5)$	$(\mathcal{v}_6)$	$(\mathcal{v}_7)$	$(\mathcal{v}_8)$
Weighted Value	.10	.11	.12	.20	.10	.14	.12	.11

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Table 7: WCVIVFS of each candidate

Observation:

U	$(\mathcal{v}_1)$ $(w_1 = .1)$	$(\mathcal{v}_2)$ $(w_2 = .11)$	$(\mathcal{v}_3)$ $(w_3 = .12)$	$(\mathcal{v}_4)$ $(w_4 = .2)$	$(\mathcal{v}_5)$ $(w_5 = .1)$	$(\mathcal{v}_6)$ $(w_6 = .14)$	$(\mathcal{v}_7)$ $(w_7 = .12)$	$(\mathcal{v}_8)$ $(w_8 = .11)$	$((C_{IV})_i)_w$
$(\mathcal{A}_{IV})_1$	[0.051, 0.070]	[0.078, 0.099]	[0.085, 0.108]	[0.182, 0.200]	[0.031, 0.050]	[0.028, 0.042]	[0.085, 0.108]	[0.034, 0.055]	[0.574, 0.727]
$(\mathcal{A}_{IV})_2$	[0.031, 0.050]	[0.022, 0.033]	[0.061, 0.084]	[0.035, 0.050]	[0.028, 0.042]	[0.008, 0.0266]	[0.037, 0.060]	[0.034, 0.055]	[0.253, 0.401]
$(\mathcal{A}_{IV})_3$	[0.020, 0.030]	[0.056, 0.077]	[0.024, 0.036]	[0.040, 0.060]	[0.051, 0.070]	[0.008, 0.026]	[0.024, 0.036]	[0.022, 0.033]	[0.246, 0.369]
$(\mathcal{A}_{IV})_4$	[0.022, 0.030]	[0.056, 0.077]	[0.037, 0.060]	[0.040, 0.060]	[0.024, 0.036]	[0.028, 0.042]	[0.024, 0.036]	[0.034 0.05]	[0.263, 0.397]
$(\mathcal{A}_{IV})_5$	[0.031, 0.050]	[0.022, 0.033]	[0.024, 0.036]	[0.012, 0.038]	[0.051, 0.070]	[0.028, 0.042]	[0.007, 0.002]	[0.022, 0.033]	[0.195, 0.310]
$(\mathcal{A}_{IV})_6$	[0.006, 0.019]	[0.022, 0.033]	[0.037, 0.060]	[0.012, 0.038]	[0.006, 0.019]	[0.028, 0.042]	[0.024, 0.036]	[0.006, 0.021]	[0.140, 0.263]
$(\mathcal{A}_{IV})_7$	[0.031, 0.050]	[0.006, 0.021]	[0.024, 0.036]	[0.031, 0.050]	[0.040, 0.060]	[0.028, 0.042]	[0.024, 0.036]	[0.034 0.055]	[0.219, 0.350
$(\mathcal{A}_{IV})_8$	[0.051, 0.070]	[0.056, 0.071]	[0.037, 0.060]	[0.031, 0.050]	[0.051, 0.070	[0.028, 0.042]	[0.037, 0.06]	[0.078, 0.099]	[0.369, 0.522
$(\mathcal{A}_{IV})_9$	[0.020, 0.030]	[0.034, 0.055]	[0.037, 0.060]	[0.012, 0.038]	[0.006, 0.019]	[0.008, 0.020]	[0037, 0.060]	[0.000 0.005]	[0.155, 0.291]
$(\mathcal{A}_{IV})_{10}$	[0.071, 0.090]	[0.078, 0.099]	[0.085, 0.108]	[0.031, 0.050]	[0.051, 0.070]	[0.028, 0.042]	[0.085, 0.108]	[0.099, 0.110]	[0.528, 0.677]

The WCVIVFS $(\mathcal{A}_{IV})_1$ is more compared to other WCVIVFS's. In addition, the candidate $(\mathrm{NM})_1$ has highest communication skill $(\mathcal{v}_4)$ with membership value as [0.182, 0.200]. Therefore, with more emphasis on communication skill, the candidate $(\mathrm{NM})_1$ is suitable for the post.

Case (ii):

Change in criteria weight considering the relative importance of integrity $(\mathcal{v}_7)$ as high compared to other criteria.

Table 8: Change in criteria weight, Case (ii)

criteria's	$(\mathcal{v}_1)$	$(\mathcal{v}_2)$	$(\mathcal{v}_3)$	$(\mathcal{v}_4)$	$(\mathcal{v}_5)$	$(\mathcal{v}_6)$	$(\mathcal{v}_7)$	$(\mathcal{v}_8)$
Weighted Value	.11	.13	.12	.13	.14	.12	.15	.10

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Table 9: WCVIVFS of each candidate

Observation:

U	$(\mathcal{v}_1)$ $(w_1 = .11)$	$(\mathcal{v}_2)$ $(w_2 = .13)$	$(\mathcal{v}_3)$ $(w_3 = .12)$	$(\mathcal{v}_4)$ $(w_4 = .13)$	$(\mathcal{v}_5)$ $(w_5 = .14)$	$(\mathcal{v}_6)$ $(w_6 = .12)$	$(\mathcal{v}_7)$ $(w_7 = .15)$	$(\mathcal{v}_8)$ $(w_8 = .10)$	$((C_{IV})_i)_w$
$(\mathcal{A}_{IV})_1$	[0.056, 0.077]	[0.092, 0.117]	[0.085, 0.108]	[0.118, 0.130]	[0.043, 0.070]	[0.076, 0.099]	[0.106, 0.135]	[0.031, 0.050]	[0.607, 0.786]
$(\mathcal{A}_{IV})_2$	[0.034, 0.055]	[0.026, 0.039]	[0.061, 0.084]	[0.040, 0.065]	[0.043, 0.070]	[0.007, 0.022]	[0.046, 0.075]	[0.031, 0.050]	[0.243, 0.460]
$(\mathcal{A}_{IV})_3$	[0.022, 0.033]	[0.066, 0.091]	[0.024, 0.036]	[0.026, 0.039]	[0.071, 0.098]	[0.007, 0.022]	[0.030, 0.045]	[0.020, 0.030]	[0.266, 0.394]
$(\mathcal{A}_{IV})_4$	[0.022, 0.033]	[0.066, 0.091]	[0.037, 0.060]	[0.026, 0.039]	[0.028, 0.042]	[0.026, 0.039]	[0.030, 0.045]	[0.031 0.050]	[0.266, 0.399]
$(\mathcal{A}_{IV})_5$	[0.034, 0.055]	[0.026, 0.033]	[0.024, 0.036]	[0.008, 0.025]	[0.043, 0.070]	[0.024, 0.036]	[0.009, 0.028]	[0.020, 0.030]	[0.188, 0.313]
$(\mathcal{A}_{IV})_6$	[0.006, 0.021]	[0.026, 0.039]	[0.037, 0.060]	[0.008, 0.025]	[0.008, 0.076]	[0.024, 0.036]	[0.009, 0.028]	[0.006, 0.019]	[0.124, 0.304]
$(\mathcal{A}_{IV})_7$	[0.034, 0.055]	[0.008, 0.076]	[0.024, 0.036]	[0.040, 0.065]	[0.028, 0.042]	[0.024, 0.036]	[0.009, 0.028]	[0.020 0.030]	[0.187, 0.368
$(\mathcal{A}_{IV})_8$	[0.051, 0.070]	[0.066, 0.091]	[0.037, 0.060]	[0.040, 0.065]	[0.043, 0.070	[0.006, 0.021]	[0.046, 0.075]	[0.071, 0.090]	[0.360, 0.542
$(\mathcal{A}_{IV})_9$	[0.056, 0.077]	[0.040, 0.065]	[0.037, 0.060]	[0.008, 0.025]	[0.008, 0.076]	[0.007, 0.023]	[0.046, 0.075]	[0.000 0.005]	[0.202, 0.406]
$(\mathcal{A}_{IV})_{10}$	[0.078, 0.099]	[0.092, 0.117]	[0.085, 0.118]	[0.040, 0.065]	[0.043, 0.070]	[0.024, 0.036]	[0.076, 0.105]	[0.091, 0.100]	[0.529, 0.710]

The WCVIVFS $(\mathcal{A}_{IV})_1$ is more compared to other WCVIVFS's. In addition, the candidate $(\mathrm{NM})_1$ has highest integrity $(\mathcal{v}_7)$ with membership value as [0.106, 0.135]. Therefore, with more emphasis on integrity, the candidate $(\mathrm{NM})_1$ is suitable for the post.

Case (iii):

Change in criteria weight considering the relative importance of patience $(\mathcal{v}_3)$ as high compared to other criteria.

Table 10: Change in criteria weight, Case (iii)

criteria's	$(\mathcal{v}_1)$	$(\mathcal{v}_2)$	$(\mathcal{v}_3)$	$(\mathcal{v}_4)$	$(\mathcal{v}_5)$	$(\mathcal{v}_6)$	$(\mathcal{v}_7)$	$(\mathcal{v}_8)$
Weighted Value	.12	.12	.15	.14	.11	.14	.12	.10

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Table 11: WCVIVFS of each candidate

Observation:

U	$(\mathcal{v}_1)$ $(w_1 = .12)$	$(\mathcal{v}_2)$ $(w_2 = .12)$	$(\mathcal{v}_3)$ $(w_3 = .15)$	$(\mathcal{v}_4)$ $(w_4 = .14)$	$(\mathcal{v}_5)$ $(w_5 = .11)$	$(\mathcal{v}_6)$ $(w_6 = .14)$	$(\mathcal{v}_7)$ $(w_7 = .12)$	$(\mathcal{v}_8)$ $(w_8 = .10)$	$((C_{IV})_i)_w$
$(\mathcal{A}_{IV})_1$	[0.061, 0.084]	[0.085, 0.108]	[0.107, 0.135]	[0.127, 0.140]	[0.034, 0.055]	[0.028, 0.042]	[0.085, 0.108]	[0.031, 0.050]	[0.558, 0.722]
$(\mathcal{A}_{IV})_2$	[0.037, 0.060]	[0.024, 0.036]	[0.078, 0.105]	[0.043, 0.065]	[0.034, 0.055]	[0.008, 0.026]	[0.037, 0.060]	[0.031, 0.050]	[0.292, 0.450]
$(\mathcal{A}_{IV})_3$	[0.024, 0.036]	[0.061, 0.084]	[0.030, 0.045]	[0.028, 0.042]	[0.056, 0.077]	[0.008, 0.026]	[0.024, 0.036]	[0.020, 0.030]	[0.251, 0.376]
$(\mathcal{A}_{IV})_4$	[0.024, 0.036]	[0.061, 0.084]	[0.047, 0.075]	[0.028, 0.042]	[0.022, 0.033]	[0.024, 0.036]	[0.028, 0.042]	[0.031 0.050]	[0.265, 0.351]
$(\mathcal{A}_{IV})_5$	[0.037, 0.060]	[0.024, 0.036]	[0.030, 0.045]	[0.008, 0.026]	[0.034, 0.055]	[0.028, 0.042]	[0.007, 0.023]	[0.020, 0.030]	[0.188, 0.317]
$(\mathcal{A}_{IV})_6$	[0.007, 0.023]	[0.024, 0.036]	[0.047, 0.075]	[0.008, 0.026]	[0.066, 0.020]	[0.028, 0.042]	[0.007, 0.023]	[0.006, 0.019]	[0.193, 0.264]
$(\mathcal{A}_{IV})_7$	[0.037, 0.060]	[0.007, 0.023]	[0.030, 0.045]	[0.043, 0.065]	[0.034, 0.055]	[0.007, 0.023]	[0.026, 0.039]	[0.020 0.030]	[0.186, 0.327
$(\mathcal{A}_{IV})_8$	[0.061, 0.084]	[0.085, 0.108]	[0.047, 0.075]	[0.043, 0.065]	[0.034, 0.055]	[0.008, 0.026]	[0.085, 0.108]	[0.071, 0.090]	[0.434, 0.608
$(\mathcal{A}_{IV})_9$	[0.024, 0.036]	[0.037, 0.060]	[0.047, 0.075]	[0.008, 0.026]	[0.066, 0.020]	[0.008, 0.026]	[0.037, 0.060]	[0.000 0.005]	[0.227, 0.308]
$(\mathcal{A}_{IV})_{10}$	[0.085, 0.108]	[0.085, 0.108]	[0.107, 0.135]	[0.043, 0.065]	[0.034, 0.055]	[0.028, 0.042]	[0.061, 0.084]	[0.091, 0.100]	[0.534, 0.697]

The candidate $(\mathrm{NM})_1$ and $(\mathrm{NM})_{10}$ have highest patience $(\mathcal{v}_3)$ with membership value as [0.107, 0.135]. So, both the candidates are suitable for the post, but the WCVIVFS $(\mathcal{A}_{IV})_1$ is more compared to $(\mathcal{A}_{IV})_{10}$ and to other WCVIVFS's. Therefore, with more emphasis on patience, the candidate $(\mathrm{NM})_1$ is suitable for the post.

Result:

Table 12: Result

Sensitivity Analysis	**Parameter**	**Highest weighted value fix to the parameter**	**Candidates with High Membership value**	
Case (i)	$(\mathcal{v}_4)$	0.2	$(\mathrm{NM})_1$	[0.182, 0.200]
Case (ii)	$(\mathcal{v}_7)$	0.15	$(\mathrm{NM})_1$	[0.106, 0.135]
Case (iii)	$(\mathcal{v}_3)$	0.15	$(\mathrm{NM})_1$ & $(\mathrm{NM})_{10}$	[0.107, 0.135]

From the above table, the candidate $(\mathrm{NM})_1$ is suitable for the HR manager post

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4. Comparison of IVFS and FS

IVFS provide an enhancement over classical FS's in decision-making problems by incorporating uncertainty more effectively. Below is a comparison of IVFS's over traditional FS's:

Table 13: A comparison of IVFS's over traditional FS's

Feature	FS	IVFS
Membership	Single value $\mathcal{A}^F(v) \in [0,1]$	Interval value $[(\mathcal{A}^{LF})(v), (\mathcal{A}^{UF})(v)] \subseteq [0,1]$
CV	$\sum_j((\mathcal{A}^F)_i)(v_j)$,	$[\sum_j((\mathcal{A}^{LF})_i)(v_j), \sum_j((\mathcal{A}^{UF})_i)(v_j)]$
WCV	$\sum_j w_j$ x $((\mathcal{A}^F)_i(v_j))$	$[\sum_j w_j$ x $((\mathcal{A}^{LF})_i(v_j)), \sum_j w_j$ x $((\mathcal{A}^{UF})_i(v_j))]$
Information content	Precise, less flexible	Less precise, more flexible
Computation complexity	Simpler	Slightly more complex due to interval arithmetic

5. Conclusion

Here, ten candidates were evaluated for a role of HR manager using CVIVFS's & WCVIVFS's. The same technique can be applied for the large set also. It has been shown that IVFS 's provides a more sophisticated framework than traditional FS's which can accurately catch the difficulties of real-world DM by adopting the concept of layered uncertainty, leading to more adaptive, robust, and realistic decision representations.

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Notations and Abbreviations:

This manuscript uses the following abbreviations and notations:

Abbreviations:

FS	Fuzzy Set
IVFS	Interval-Valued Fuzzy Set
CV	Choice Value
WCV	Weighted Choice Value
CVFS	Choice Value of Fuzzy Set
WCVFS	Weighted Choice Value of Fuzzy Set
CVIVFS	Choice Value of Interval-Valued Fuzzy Set
WCVIVFS	Weighted Choice Value of Interval-Valued Fuzzy Set
DM	Decision-Making
MCDM	Multi-Criteria Decision-Making
IVFSS	Interval-Valued Fuzzy Soft Set
IVHF	Interval-Valued Hesitant Fuzzy
IFS	Intuitionistic Fuzzy Set"

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Notations:

$\mho$	Crisp Set
I	Closed Unit Interval
$I^{\mho}$	Family of FS 's on $\mho$
$\mathcal{A}^F$	FS
$\mathcal{A}_{IV}$	IVFS
[I]	Set of all Closed Sub Intervals of [0, 1]
$\mathcal{A}^{LF}$	Lower FS
$\mathcal{A}^{UF}$	Upper FS
IVFS ($\mho$)	Collection of IVFS 's
$(C_F)_i$	CVFS
$((C_F)_i)_w$	WCVFS
$(C_{IV})_i$	CVIVFS
$((C_{IV})_i)_w$	WCVIVFS
$(\mathcal{A}_{IV})_1, \ldots . (\mathcal{A}_{IV})_{10}$	IVFS of candidates $(\mathrm{NM})_1, \ldots \ldots, (\mathrm{NM})_{10}$

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