Optimizing Crop Selection for Small Scale Farmers Using Neutrosophic Hypersoft Set Theory and Cubic Spherical Neutrosophic Sets

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Abstract

This study addresses the inherent challenges of uncertainty, vagueness, and imprecision in real-world decision-making, particularly focusing on the problem small-scale farmer's face in optimally selecting short-term crops across diverse planting seasons. The central challenge is the absence of a systematic framework to evaluate multiple, often conflicting, criteria such as initial investment, expected yield, market demand, water and soil requirements, specific fertilizer needs, and pest susceptibility. To overcome this, a robust Multi-Criteria Decision-Making (MCDM) framework is introduced, integrating Cubic Spherical Neutrosophic Sets (CSNS) with Neutrosophic Hyper Soft Sets (NHSS). The research proposes the cubic spherical neutrosophic Bonferroni mean operator as a novel geometric representation for aggregating neutrosophic sets, which enables a more refined modeling of uncertainty and indeterminacy in complex environments. Cubic Spherical Neutrosophic Sets embed neutrosophic information within a spherical structure using interval-based (Truth, Indeterminacy, Falsity) triplets and a radius, offering robust aggregation and ranking capabilities. Neutrosophic hypersoft sets further enhance logical expressiveness by associating each multi-parameter tuple with a neutrosophic triplet, effectively managing complex multi-attribute decision-making tasks with deep interdependencies. The applicability and effectiveness of this approach are demonstrated through a practical case study involving the selection of the most suitable crop for different climatic zones (Pattams) in Tamil Nadu, considering agricultural, environmental, and economic factors. Expert linguistic assessments are converted into neutrosophic values and aligned with seasonal cropping patterns. A subsequent sensitivity analysis confirms the robustness of the model, revealing a perfect correlation between the outcomes of different decision-making methods and thereby validating the consistency and reliability of the proposed approach. This context-aware, data-driven tool aims to enhance decision-making, improve resource utilization, reduce risks, and promote agricultural sustainability and improved farmer livelihoods.

Keywords: Soft sets; Hypersoft sets, Neutrosophic Soft sets; Neutrosophic Hypersoft sets

1. Introduction

Uncertainty, vagueness, and imprecision are inherent in real-world decision-making processes, motivating the development of mathematical frameworks to manage such complexity. Among these, soft set theory, introduced by Molodtsov [16], provides a parameterized approach wherein each parameter maps to a subset of a universal set,

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enabling flexible modelling without the need for membership functions or equivalence relations. To address multidimensional parameter structures, the hypersoft set, proposed by Abbas, Murtaza, and Smarandache [1], generalizes soft sets by allowing the approximate function to act on tuples of attribute values, significantly enriching the decision-making process in hierarchical domains. Parallelly, Neutrosophic set theory, introduced by Smarandache [25], expanded uncertainty modelling by introducing three independent degrees truth (T), indeterminacy (I), and falsity (F) each ranging over [0,1]. This approach accommodates contradiction and incomplete information more comprehensively than fuzzy or intuitionistic sets. Combining this with soft set theory leads to the neutrosophic soft set introduced by Maji, which assigns neutrosophic values to each parameter-object pair, capturing multi-valued uncertainty across various decision attributes. Advancing further, the neutrosophic hypersoft set integrates the hypersoft structure with neutrosophic logic, wherein each multi-parameter tuple is associated with a neutrosophic triplet. This model is particularly effective in complex multi-attribute decision-making (MADM) tasks involving deep interdependencies and indeterminate information.

Recently, cubic spherical neutrosophic sets introduced by Gomathi et al. [9] offer a geometric extension by embedding neutrosophic information within a spherical structure while incorporating cubic interval representations. Each element is represented by an interval-based (T, I, F) triplet along with a radius that constrains its position within a unit sphere. Cubic spherical neutrosophic sets has shown strong potential in MCDM scenarios, offering robust aggregation, ranking, and similarity measures. Together, these models provide a rich, evolving toolkit for intelligent systems that must operate under layered and uncertain information.

1.1 Literature Review

The Bonferroni Mean [6] (BM) operator is a powerful aggregation technique originally designed to handle interrelationships among input data. Over time, it has been widely generalized and integrated into various forms of neutrosophic sets, enhancing its ability to model uncertainty, indeterminacy, and inconsistency in MCDM problems. Its evolution and adaptability have made it an essential tool in solving real-world decision scenarios where traditional methods struggle. Recent research has demonstrated the BM operator's versatility by embedding it in different neutrosophic frameworks, including pythagorean, single-valued, Fermatean, pentapartitioned, and cubic fuzzy neutrosophic sets. These enriched environments enable decision-makers to capture complex and vague information, offering more nuanced assessments. For example, in pythagorean neutrosophic sets, the BM operator has been effectively applied to assess hotel green human resource management performance [2], detect financial fraud using intelligent systems [14], and classify product review sentiments through machine learning models [5]. Similarly, [3] utilized this operator in the supplier selection of halal products, while [17] and [22] employed it within digital social innovation ecosystems to evaluate entrepreneurial barriers. In cubic fuzzy neutrosophic environments, the BM operator has addressed uncertainty in investment decisions [12], healthcare evaluations [7], and group decision-making under conflicting data [4]. These variants offer more robust information representation through combined interval and membership value handling. Furthermore, the operator has found strong footing in single-valued neutrosophic applications, including logistics provider selection [8 &11] and software strategy analysis involving metaverse and AI adoption [17]. Also, Fermatean and pentapartitioned neutrosophic extensions were introduced in [20] and [18], tackling cable selection and multi-attribute aggregation, respectively. Collectively, these studies highlight how the BM operator and its generalized forms serve as vital decision-support mechanisms, deeply interwoven with various neutrosophic paradigms. Their contributions to MCDM across finance, supply chain, HRM, AI, healthcare, and digital innovation domains underscore their real-world value and continued research relevance. The applications of neutrosophic hypersoft sets in MCDM were introduced in [10 & 24], offering a robust approach to handle imprecise, indeterminate, and inconsistent information in complex decision environments. Smarandache further enriched the theoretical landscape by introducing the superhypersoft set in [26], which integrates multi-parameter uncertainty modeling with advanced soft set theory. Expanding on these foundational works, additional practical applications in both MADM and MCDM are demonstrated in [20], highlighting the adaptability and effectiveness of neutrosophic frameworks in real-world decision-making scenarios involving multiple conflicting criteria.

Table 1 presents a comprehensive overview of the applications of the BM operator across different types of neutrosophic sets, highlighting their roles in various real-world multi-criteria decision-making scenarios.

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Table 1: Applications of the Bonferroni Mean Operator across Different Neutrosophic Set Frameworks

Author's	**Sets**	**Application area**
Khan, M. et al. [13]	Neutrosophic Cubic Set	Investment Portfolio Scenarios, Indicating Its Utility in Financial Decision Making
Azam, M. & Jadoon, A. M. [3]	Neutrosophic Cubic Fuzzy Set	A Real-Life Case Example Involving Decision Making Under Uncertainty—Where the Proposed Operator Handles Counterintuitive Scenarios to Demonstrate Its Reliability and Validity
Darvesh, A. et al. [7]	Neutrosophic Cubic Fuzzy Set	They Validated Their Approach Through a Numerical Case in Healthcare Management, Showing Robust Decision-Making Capabilities Under Uncertainty
Kanchana, A. et al. [11]	Neutrosophic Set (triangular and trapezoidal)	A Supplier Selection Problem Using a Real-World Example Where Manufacturing Firms Evaluate Potential Suppliers Based on Multiple Attributes, Effectively Aggregating Experts' Neutrosophic Assessments
Kara, K. et al. [12]	Single-valued Neutrosophic Set	Selecting Finished Vehicle Logistics Service Providers. They Applied it to Rank Logistics Firms Based on a Set of Performance Criteria, Showcasing the Model's Robustness in a Practical Logistics-Provider Selection Scenario
Önden, A. et al. [18]	Single-Valued Neutrosophic Set	The Model to Evaluate Different Approaches for Adopting the Metaverse and ChatGPT-Like Conversational AI In Software Development—Analyzing Multiple Criteria to Identify the Best Strategic Pathway
Rana Muhammad Zulqarnain. et al. [19]	Neutrosophic Hypersoft Set	A Real-World Example, Demonstrating Its Applicability and Effectiveness in Aggregating Complex Neutrosophic Attribute Information
Revathy, A. et al. [21]	Fermatean Neutrosophic Set	Selection of Best Ethernet Cable Within the Existing Cables available in Market.
Ahmed, A., et al. [2]	Pythagorean Neutrosophic Set	The Performance of Hotel GHRM Practices and Optimized Parameters Via the Quasi-Oppositional Teaching-Learning-Based Optimization (QTLBO) Algorithm, Demonstrating High Accuracy and Confidence in Performance Evaluation
Bin Mohammad Kamari, M. S. et al. [4]	Pythagorean Neutrosophic Set	A Practical Case Study on Supplier Selection for Halal Products, Demonstrating Its Effectiveness in Aggregating Uncertainty-Laden Criteria in Real Supplier Evaluations
Badawood, D. [5]	Pythagorean Neutrosophic Set	A Real-World Product-Review Dataset, Demonstrating That The DAFSC-PNBM Technique Outperformed Traditional Aggregation and Classification Methods for Sentiment Detection

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Matyakubov, U. et. al. [15]	Pythagorean Neutrosophic Set	The Detection of Fraudulent Vs. Legitimate Financial Transactions in a Benchmark Dataset. The Experimental Results Showed That The IDSSFFD-PNBM Model Outperformed Existing Approaches in Accuracy and Detection Performance
Noorraha, A. R. [17]	Pythagorean Neutrosophic Set	The PNBM–DEMATEL Method to Analyze Barrier Factors Hindering Young Entrepreneurs within Digital Social Innovation (DSI) Ecosystems
Rodzi, Z., et al. [23]	Pythagorean Neutrosophic Set	The Digital Social Innovation Ecosystem, With A Targeted Emphasis on How Barrier Factors Affect Young Entrepreneurs Within That Environment

1.2 Motivation

The primary motivation for this research stems from the inherent challenges of uncertainty, vagueness, and imprecision in real-world decision-making processes. This is particularly critical for small-scale farmers who face the complex task of optimally selecting short-term crops across diverse planting seasons. This decision-making process is complicated by the need to evaluate multiple, often conflicting, criteria simultaneously. These criteria include initial investment, expected yield, market demand, water availability and requirements, soil characteristics, specific fertilizer needs, and susceptibility to pest impact. Without a structured and comprehensive framework, farmers' crop choices can lead to inefficient resource utilization, reduced profitability, heightened risks, and environmental unsustainability, thereby impeding the progress of smallholder agriculture. The ultimate goal is to provide a practical and data-driven tool to enhance decision-making, improve resource utilization, reduce risks, and promote agricultural sustainability and improved farmer livelihoods.

1.3 Research Gap

While existing mathematical frameworks like Soft Set Theory, hypersoft sets, and neutrosophic set theory have been developed to manage complexity and model uncertainty, there was a notable gap in a systematic and comprehensive framework capable of optimally integrating these advanced concepts for MCDM problems with deep interdependencies and indeterminate information. Specifically, the research identifies the following gaps that this study aims to address:

1. Refined Modeling of Uncertainty: Although neutrosophic set theory introduced independent degrees of truth, indeterminacy, and falsity to more comprehensively accommodate contradiction and incomplete information than fuzzy or intuitionistic sets, there was a need for a more refined geometric representation. The recently introduced cubic spherical neutrosophic sets offer a powerful geometric extension by embedding neutrosophic information within a spherical structure, providing robust aggregation, ranking, and similarity measures in MCDM scenarios.

2. Advanced Aggregation Operator for CSNS: The BM operator is a powerful aggregation technique for handling interrelationships among input data and has been integrated into various neutrosophic sets. However, there was a gap in the development of a novel cubic spherical neutrosophic Bonferroni mean operator specifically designed for the aggregation of neutrosophic sets within the CSNS framework, enabling a more refined modeling of uncertainty and indeterminacy in complex environments.

3. Enhanced Logical Expressiveness for Complex Scenarios: While NHSS integrate the hypersoft structure with neutrosophic logic to manage complex MADM tasks involving deep interdependencies and indeterminate information, the existing frameworks lacked the specific integration with CSNS and a specialized aggregation operator to comprehensively analyze alternatives across diverse decision scenarios and multiple criteria and sub-criteria.

In summary, the research gap lies in the absence of an integrated, robust MCDM framework that leverages the unique strengths of both CSNS(for geometric representation and handling interval-based uncertainty) and NHSS (for logical expressiveness and multi-parameter dependencies), combined with a novel cubic spherical neutrosophic Bonferroni mean operator to provide a systematic and comprehensive solution for complex real-world decision problems, such as optimal crop selection for small-scale farmers

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1.4 Contribution of NHSS

The present study contributes to the advancement of decision-making under uncertainty by extending neutrosophic modeling to NHSS, which generalize hypersoft sets by embedding multi-level, interdependent, and higher-order parameters into the decision framework. Unlike conventional neutrosophic or hypersoft sets, NHSS allow a super-parameterized representation in which each decision factor can be decomposed into nested sub-parameters, each characterized by neutrosophic triplets. This provides a granular and expressive structure capable of capturing both the independence and interdependence of attributes in real-world problems. The integration of the cubic spherical neutrosophic BM operator within the NHSS framework further enhances its aggregation capacity, enabling a refined balance between conflicting criteria. Thus, NHSS serve as a novel and comprehensive mathematical tool that expands the logical and computational expressiveness of existing neutrosophic decision models.

1.5 Importance of NHSS

The importance of NHSS lies in their ability to systematically capture and process uncertainty, vagueness, and interdependencies in complex multi-criteria decision-making scenarios. In the agricultural context of small-scale farmers, where multiple factors such as soil fertility, water availability, fertilizer compatibility, crop yield, and market demand are interrelated rather than independent, NHSS provide a robust framework for constructing context-aware, data-driven, and sustainable decisions. By accommodating multi-dimensional uncertainty across layered parameters, NHSS significantly improve the robustness, reliability, and interpretability of decision outcomes compared with traditional MCDM methods. Beyond agriculture, their versatility ensures broad applicability in domains such as healthcare diagnostics, energy resource allocation, and supply chain management, thereby establishing NHSS as an essential extension of neutrosophic set theory with wide-ranging implications for science and engineering.

1.6 Preliminaries

Definition 1.1. [9] Let Ů be a fixed universe and δ⊆ Ů a subset of interest. A *Cubic Spherical Neutrosophic Set* (CSNS), denoted by $\boldsymbol{\Theta_u}$ is defined as:

$$\Theta_u = \{< \text{ù}, Ŧ(\text{ù}), ₦(\text{ù}), ₣(\text{ù}); ŕ > : \text{ù} \in Ů\}$$

where:

- Ŧ(ù), ₦(ù), ₣(ù) : Ů → [0,1] represent the truth-membership, indeterminacy-membership, and falsity-membership functions, respectively,
- The condition $0 \leq$ Ŧ(ù) + ₦(ù) + ₣(ù) ≤ 3 holds for all ù ∈ Ů,
- ŕ ∈[0,1] denotes the radius of the sphere centred at the point (Ŧ(ù), ₦(ù), ₣(ù)), encapsulating the neutrosophic nature of the element ù.

This spherical representation characterizes the degree of membership, indeterminacy, and non-membership of each element ù ∈ Ů in a three-dimensional neutrosophic space.

Now, let {< ù$_{i,}$: Ŧ$_{i,j}$, ₦$_{i,j}$, ₣$_{i,j}$ >/$i = 1,2, \ldots j = 1,2,,,,, k_i$} be a collection of neutrosophic evaluations assigned to each ù$_{i,}$ ∈ Ů. The center of the sphere for ù$_{i,}$ is computed as the average of all assigned neutrosophic values:

$$< Ŧ(\text{ù}_i), ₦(\text{ù}_i), ₣(\text{ù}_i) > = < \frac{1}{k_i}\Sigma_{j=1}^{k_i} Ŧ_{i,j}, \frac{1}{k_i}\Sigma_{j=1}^{k_i} ₦_{i,j}, \frac{1}{k_i}\Sigma_{j=1}^{k_i} ₣_{i,j} >$$

The corresponding radius ŕ$_i$ is then calculated as:

$$ŕ_i = \min\{\max_{1\leq j\leq k_i} \sqrt{\left(Ŧ(\text{ù}_i) - Ŧ_{i,j}\right)^2 + (₦(\text{ù}_i) - ₦_{i,j})^2 + (₣(\text{ù}_i) - ₣_{i,j})^2} \ ,1 \}$$

Definition 1.2. [10] Let Θ_{u_i} =< Ŧ(u_i), ₦(u_i), ₣(u_i), ŕ(u_i) > (i=1, 2, . . . , n) be CSNSs over the universal set X, and let α > 0. We define the following operations:

1. $\Theta_u \oplus \Theta_v$ = <Ŧ(u) + Ŧ(v) - Ŧ(u) . Ŧ(v), ₦(u) . ₦(v), ₣(u) . ₣(v); ŕ(u) + ŕ(v) - ŕ(u) . ŕ(v)>.
2. $\oplus_{ă=1}^{n}$ =< $1 - \prod_{ă=1}^{n}(1 -$ Ŧ($u_ă$)), $\prod_{ă=1}^{n}$ ₦($u_ă$), $\prod_{ă=1}^{n}$ ₣($u_ă$), $1 - \prod_{ă=1}^{n}(1 -$ ŕ($u_ă$)) >
3. $\Theta_u \otimes \Theta_v$ =< Ŧ(u). Ŧ(v), ₦(u) + ₦(v) - ₦(u) . ₦(v), ₣(u) + ₣(v)-$\mathcal{F}_₣$(u) . ₣(v); ŕ(u). ŕ(v) >.
4. $\otimes_{ă=1}^{n}$ =< $\prod_{ă=1}^{n}$ Ŧ($u_ă$), $1 - \prod_{ă=1}^{n}(1 -$ ₦($u_ă$)), $1 - \prod_{ă=1}^{n}(1 -$ ₣($u_ă$)), $\prod_{ă=1}^{n}$ ŕ($u_ă$) >

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5. $\alpha\Theta_u =< 1-(1-Ŧ(u))^{\alpha}, (₦(u))^{\alpha}, (₣(u))^{\alpha}, 1-(1-ŕ(u))^{\alpha} >$
6. $(\Theta_u)^{\alpha} =< (Ŧ(u))^{\alpha}, 1-(1-₦(u))^{\alpha}, 1-(1-₣(u))^{\alpha}, (ŕ(u))^{\alpha} >$

Definition 1.3 [1] Let X be a fixed universe, Þ(X) be the power set of X, Let $ȕ_1, ȕ_2, \dots\dots, ȕ_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the set $Ű_1, \dots, Ű_n$ with $Ű_i \cap Ű_j = \emptyset$ for $i \neq j$. A *Neutrosophic HyperSoft Set* over X as the pair $(š, Ű_1 \times \dots \times Ű_n)$ where $š : Ű_1 \times \dots \times Ű_n \rightarrow Þ(X)$ and $(š, (Ű_1 \times \dots \times Ű_n)) = \{\alpha, \langle x, Ŧ_{š(\alpha)}(x), ₦_{š(\alpha)}(x), ₣_{š(\alpha)}(x) \rangle : x \in X, \alpha \in (Ű_1 \times \dots \times Ű_n) \subseteq Ű_1 \dots Ű_n\}$.

2. Cubic Spherical Neutrosophic Bonferroni mean operator

Definition 2.1. Let $\Theta_{u_ă} =< Ŧ(u_ă), ₦(u_ă), ₣(u_ă), ŕ(u_ă) >$ (ă =1, . . . , n) is a collection of CSNSs. For any $\mathbb{u}$, $\mathbb{v}$, if

$$CSNBM^{\mathbb{u},\mathbb{v}}(\mathbb{x}_1, \dots, \mathbb{x}_{\mathbb{m}}) = \left(\frac{1}{\mathbb{m}(\mathbb{m}+1)} \oplus_{\substack{\mathbb{a},\mathbb{b}=1 \\ \mathbb{a}\neq\mathbb{b}}}^{\mathbb{m}} (\mathbb{x}_{\mathbb{a}}^{\mathbb{u}} \otimes \mathbb{x}_{\mathbb{b}}^{\mathbb{v}})\right)^{\frac{1}{\mathbb{u}+\mathbb{v}}}$$

Theorem 2.2. For $\mathbb{u}$, $\mathbb{v} > 0$ and $\Theta_{u_i} =< Ŧ(u_i), ₦(u_i), ₣(u_i), ŕ(u_i) >$ (i=1, 2, …, n) the aggregation Cubic Spherical Neutrosophic Bonferroni Mean (CSNBM) is also a aggregation cubic spherical neutrosophic number and is of the form

$$CSNBM^{\mathbb{u},\mathbb{v}}(\mathbb{x}_1, \mathbb{x}_2, \dots, \mathbb{x}_n) = \begin{cases} \left(1 - \prod_{\substack{\mathbb{a},\mathbb{b}=1 \\ \mathbb{a}\neq\mathbb{b}}}^{\mathbb{m}} (1 - Ŧ_{\mathbb{x}_\mathbb{a}}^{\mathbb{u}} Ŧ_{\mathbb{x}_\mathbb{b}}^{\mathbb{v}})^{\frac{1}{\mathbb{m}(\mathbb{m}+1)}}\right)^{\frac{1}{\mathbb{u}+\mathbb{v}}}, \\ 1 - \left(1 - \prod_{\substack{\mathbb{a},\mathbb{b}=1 \\ \mathbb{a}\neq\mathbb{b}}}^{\mathbb{m}} \left(1 - (1 - ₦_{\mathbb{x}_\mathbb{a}})^{\mathbb{u}} (1 - ₦_{\mathbb{x}_\mathbb{b}})^{\mathbb{v}}\right)^{\frac{1}{\mathbb{m}(\mathbb{m}+1)}}\right)^{\frac{1}{\mathbb{u}+\mathbb{v}}}, \\ 1 - \left(1 - \prod_{\substack{\mathbb{a},\mathbb{b}=1 \\ \mathbb{a}\neq\mathbb{b}}}^{\mathbb{m}} \left(1 - (1 - ₣_{\mathbb{x}_\mathbb{a}})^{\mathbb{u}} (1 - ₣_{\mathbb{x}_\mathbb{b}})^{\mathbb{v}}\right)^{\frac{1}{\mathbb{m}(\mathbb{m}+1)}}\right)^{\frac{1}{\mathbb{u}+\mathbb{v}}}, \\ \left(1 - \prod_{\substack{\mathbb{a},\mathbb{b}=1 \\ \mathbb{a}\neq\mathbb{b}}}^{\mathbb{m}} (1 - ŕ_{\mathbb{x}_\mathbb{a}}^{\mathbb{u}} ŕ_{\mathbb{x}_\mathbb{b}}^{\mathbb{v}})^{\frac{1}{\mathbb{m}(\mathbb{m}+1)}}\right)^{\frac{1}{\mathbb{u}+\mathbb{v}}} \end{cases}$$

Proof:

Let $\mathbb{x}_\mathbb{a} = (Ŧ_\mathbb{a}, ₦_\mathbb{a}, ₣_\mathbb{a}, ŕ_\mathbb{a})$ and $\mathbb{x}_\mathbb{b} = (Ŧ_\mathbb{b}, ₦_\mathbb{b}, ₣_\mathbb{b}, ŕ_\mathbb{b})$ be any two CSNSs.

By Definition 1.2,

$\mathbb{x}_\mathbb{a}^{\mathbb{u}} = \left(Ŧ_{\mathbb{x}_\mathbb{a}}^{\mathbb{u}}, 1 - (1 - ₦_{\mathbb{x}_\mathbb{a}})^{\mathbb{u}}, 1 - (1 - ₣_{\mathbb{x}_\mathbb{a}})^{\mathbb{u}}, ŕ_{\mathbb{x}_\mathbb{a}}^{\mathbb{u}}\right)$, and $\mathbb{x}_\mathbb{b}^{\mathbb{v}} = \left(Ŧ_{\mathbb{x}_\mathbb{b}}^{\mathbb{v}}, 1 - (1 - ₦_{\mathbb{x}_\mathbb{b}})^{\mathbb{v}}, 1 - (1 - ₣_{\mathbb{x}_\mathbb{b}})^{\mathbb{v}}, ŕ_{\mathbb{x}_\mathbb{b}}^{\mathbb{v}}\right)$

Then $\mathbb{x}_\mathbb{a}^{\mathbb{u}} \otimes \mathbb{x}_\mathbb{b}^{\mathbb{v}} = \left(Ŧ_{\mathbb{x}_\mathbb{a}}^{\mathbb{u}} Ŧ_{\mathbb{x}_\mathbb{b}}^{\mathbb{v}}, 1 - (1 - ₦_{\mathbb{x}_\mathbb{a}})^{\mathbb{u}} (1 - ₦_{\mathbb{x}_\mathbb{b}})^{\mathbb{v}}, 1 - (1 - ₣_{\mathbb{x}_\mathbb{a}})^{\mathbb{u}} (1 - ₣_{\mathbb{x}_\mathbb{b}})^{\mathbb{v}}, ŕ_{\mathbb{x}_\mathbb{a}}^{\mathbb{u}} ŕ_{\mathbb{x}_\mathbb{b}}^{\mathbb{v}}\right)$

First to prove,

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$$\oplus_{\substack{a,b,=1\\a\neq b}}^{m}(x_a^u \otimes x_b^v) = \begin{cases} 1-\prod\limits_{\substack{a,b,=1\\a\neq b}}^{m}\left(1-Ŧ_{x_a}^{u}Ŧ_{x_b}^{v}\right), \\ \prod\limits_{\substack{a,b,=1\\a\neq b}}^{m}\left(\left(1-₦_{x_a}\right)^{u}\left(1-₦_{x_b}\right)^{v}\right) \\ \prod\limits_{\substack{a,b,=1\\a\neq b}}^{m}\left(\left(1-Ғ_{x_a}\right)^{u}\left(1-Ғ_{x_b}\right)^{v}\right), \\ 1-\prod\limits_{\substack{a,b,=1\\a\neq b}}^{m}\left(1-ŕ_{x_a}^{u}ŕ_{x_b}^{v}\right) \end{cases}$$

Using the mathematical induction on n, let $m = 2$,

$$\oplus_{\substack{a,b,=1\\a\neq b}}^{m}(x_a^u \otimes x_b^v) = (x_1^u \otimes x_2^v)(x_2^u \otimes x_1^v) = \begin{cases} 1-\left(1-Ŧ_{x_1}^{u}Ŧ_{x_2}^{v}\right)\left(1-Ŧ_{x_2}^{u}Ŧ_{x_1}^{v}\right) \\ \left(\left(1-₦_{x_1}\right)^{u}\left(1-₦_{x_2}\right)^{v}\right)\times\left(\left(1-₦_{x_2}\right)^{u}\left(1-₦_{x_1}\right)^{v}\right) \\ \left(\left(1-Ғ_{x_1}\right)^{u}\left(1-Ғ_{x_2}\right)^{v}\right)\times\left(\left(1-Ғ_{x_2}\right)^{u}\left(1-Ғ_{x_1}\right)^{v}\right) \\ 1-\left(1-ŕ_{x_1}^{u}ŕ_{x_2}^{v}\right)\left(1-ŕ_{x_2}^{u}ŕ_{x_1}^{v}\right) \end{cases}$$

$$= \begin{cases} 1-\prod\limits_{\substack{a,b,=1\\a\neq b}}^{2}\left(1-Ŧ_{x_a}^{u}Ŧ_{x_b}^{v}\right) \\ \prod\limits_{\substack{a,b,=1\\a\neq b}}^{2}\left(1-\left(1-₦_{x_a}\right)^{u}\left(1-₦_{x_b}\right)^{v}\right) \\ \prod\limits_{\substack{a,b,=1\\a\neq b}}^{2}\left(1-\left(1-Ғ_{x_a}\right)^{u}\left(1-Ғ_{x_b}\right)^{v}\right) \\ 1-\prod\limits_{\substack{a,b,=1\\a\neq b}}^{2}\left(1-ŕ_{x_a}^{u}ŕ_{x_b}^{v}\right) \end{cases}$$

Assume that, this result is true for $m = k$,

$$\oplus_{\substack{a,b,=1\\a\neq b}}^{k}(x_a^u \otimes x_b^v) = \begin{cases} 1-\prod\limits_{\substack{a,b,=1\\a\neq b}}^{k}\left(1-Ŧ_{x_a}^{u}Ŧ_{x_b}^{v}\right) \\ \prod\limits_{\substack{a,b,=1\\a\neq b}}^{k}\left(1-\left(1-₦_{x_a}\right)^{u}\left(1-₦_{x_b}\right)^{v}\right) \\ \prod\limits_{\substack{a,b,=1\\a\neq b}}^{k}\left(1-\left(1-Ғ_{x_a}\right)^{u}\left(1-Ғ_{x_b}\right)^{v}\right) \\ 1-\prod\limits_{\substack{a,b,=1\\a\neq b}}^{k}\left(1-ŕ_{x_a}^{u}ŕ_{x_b}^{v}\right) \end{cases}$$

If $m = k + 1$,

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$$\oplus_{\substack{a,b,=1\\ a\neq b}}^{k+1}(x_a^u \otimes x_b^v) = \oplus_{\substack{a,b,=1\\ a\neq b}}^{k}(x_a^u \otimes x_b^v) \oplus \left(\oplus_{\substack{a,b,=1\\ a\neq b}}^{k+1}(x_k^u \otimes x_{k+1}^v)\right) = \begin{cases} 1-\prod_{\substack{a,b,=1\\ a\neq b}}^{k+1}\left(1-T_{x_a}^u T_{x_b}^v\right) \\ \prod_{\substack{a,b,=1\\ a\neq b}}^{k+1}\left(1-(1-N_{x_a})^u(1-N_{x_b})^v\right) \\ \prod_{\substack{a,b,=1\\ a\neq b}}^{k+1}\left(1-(1-F_{x_a})^u(1-F_{x_b})^v\right) \\ 1-\prod_{\substack{a,b,=1\\ a\neq b}}^{k+1}\left(1-r_{x_a}^u r_{x_b}^v\right) \end{cases}$$

The above equation holds, $m=k+1$,

$$\oplus_{\substack{a,b,=1\\ a\neq b}}^{m}(x_a^u \otimes x_b^v) = \begin{cases} 1-\prod_{\substack{a,b,=1\\ a\neq b}}^{m}\left(1-T_{x_a}^u T_{x_b}^v\right) \\ \prod_{\substack{a,b,=1\\ a\neq b}}^{m}\left(1-(1-N_{x_a})^u(1-N_{x_b})^v\right) \\ \prod_{\substack{a,b,=1\\ a\neq b}}^{m}\left(1-(1-F_{x_a})^u(1-F_{x_b})^v\right) \\ 1-\prod_{\substack{a,b,=1\\ a\neq b}}^{m}\left(1-r_{x_a}^u r_{x_b}^v\right) \end{cases}$$

$$\frac{1}{m(m+1)}\oplus_{\substack{a,b,=1\\ a\neq b}}^{m}(x_a^u \otimes x_b^v) = \begin{cases} \left(1-\prod_{\substack{a,b,=1\\ a\neq b}}^{m}\left(1-T_{x_a}^u T_{x_b}^v\right)\right)^{\frac{1}{m(m+1)}} \\ \left(\prod_{\substack{a,b,=1\\ a\neq b}}^{m}\left(1-(1-N_{x_a})^u(1-N_{x_b})^v\right)\right)^{\frac{1}{m(m+1)}} \\ \left(\prod_{\substack{a,b,=1\\ a\neq b}}^{m}\left(1-(1-F_{x_a})^u(1-F_{x_b})^v\right)\right)^{\frac{1}{m(m+1)}} \\ \left(1-\prod_{\substack{a,b,=1\\ a\neq b}}^{m}\left(1-r_{x_a}^u r_{x_b}^v\right)\right)^{\frac{1}{m(m+1)}} \end{cases}$$

It is true for , $m=k+1$. Therefore,

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$$CSNBM^{u,v}(x_1, x_2, \dots, x_n) = \begin{cases} \left(1 - \prod_{\substack{a,b=1 \\ a \neq b}}^{m} (1 - T_{x_a}^{u} T_{x_b}^{v})^{\frac{1}{m(m+1)}}\right)^{\frac{1}{u+v}} \\ 1 - \left(1 - \prod_{\substack{a,b=1 \\ a \neq b}}^{m} (1 - (1 - N_{x_a})^{u} (1 - N_{x_b})^{v})^{\frac{1}{m(m+1)}}\right)^{\frac{1}{u+v}} \\ 1 - \left(1 - \prod_{\substack{a,b=1 \\ a \neq b}}^{m} (1 - (1 - F_{x_a})^{u} (1 - F_{x_b})^{v})^{\frac{1}{m(m+1)}}\right)^{\frac{1}{u+v}} \\ \left(1 - \prod_{\substack{a,b=1 \\ a \neq b}}^{m} (1 - r_{x_a}^{u} r_{x_b}^{v})^{\frac{1}{m(m+1)}}\right)^{\frac{1}{u+v}} \end{cases}$$

3. Optimizing Crop Selection for Small Scale Farmers Using Neutrosophic Hypersoft Set Theory

The central challenge for small-scale farmers is the absence of a systematic and comprehensive framework to optimally select the most suitable short-term crops across diverse planting seasons. This decision is complicated by the need to simultaneously evaluate multiple, often conflicting, criteria such as initial investment, expected yield, market demand, water availability and requirement, soil characteristics, specific fertilizer needs, and the susceptibility to pest impact. Without a structured approach, crop choices can lead to inefficient resource utilization, reduced profitability, heightened risks, and environmental unsustainability, thereby impeding the advancement of smallholder agriculture.

This case study aims to develop and demonstrate a robust Multi-Criteria Decision Making (MCDM) methodology to assist small-scale farmers in systematically identifying the most suitable short-term crop alternatives for various planting seasons. By quantitatively integrating diverse, often conflicting, criteria related to input costs, output benefits, environmental factors, and risk considerations, this study seeks to provide a practical and data-driven tool to enhance decision-making in crop selection, thereby promoting agricultural sustainability and improving farmer livelihoods.

Tamil Nadu's agriculture is highly dependent on its distinct cropping seasons, locally known as "Pattams." These seasons are primarily influenced by the monsoons, temperature, and traditional farming practices. Understanding these pattams is crucial for optimal crop selection and management.

Here is a breakdown of the four main agricultural seasons in Tamil Nadu:

1. Jan - Mar (Navarai / Thaladi)

- Months: December - January (start) to March (harvest). Navarai specifically refers to the short-duration rice crop, typically sown in December-January and harvested in March. Thaladi refers to the second crop of paddy in a double-cropped system, usually following Kuruvai and extending into this period.
- Climatic Conditions: This period generally falls within the tail end of the Northeast Monsoon (which peaks Oct-Dec) and transitions into the hotter pre-summer months. Rainfall becomes less reliable, making irrigation crucial for most crops. Temperatures start to rise towards March.
- Significance: It is often considered a 'winter' or 'cool weather' crop season, though temperatures in Tamil Nadu are mild rather than cold. Irrigation facilities are essential for successful cultivation.
- Typical Crops:
    - Paddy (Rice): Short-duration varieties, particularly under irrigation.
    - Vegetables: Small Onion, Brinjal, Tomato, Bhendi, Gourds, Chilli. These crops thrive with controlled irrigation.
    - Flowers: Winter Jasmine.
    - Pulses: Blackgram, Bush Lablab.
    - Oilseeds: Sesame (Gingelly).

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2. April - May (Chithiraipattam / Sornavari)

- Months: April - May (sowing) to August - September (harvest). Chithiraipattam is also known as Sornavari.
- Climatic Conditions: This is the hottest part of the year in Tamil Nadu, characterized by high temperatures and generally dry conditions, preceding the Southwest Monsoon. Rainfall is scarce and highly localized (pre-monsoon showers).
- Significance: Cultivation during this period is highly dependent on assured irrigation. Crops that are drought-tolerant or have short durations and require less water are often preferred if irrigation is limited.
- Typical Crops:
  - Millets: Ragi, Pearl Millet, Sorghum (drought-tolerant varieties, often irrigated).
  - Vegetables: Bhendi, Gourds (require intensive irrigation).
  - Oilseeds: Groundnut (requires significant irrigation).
  - Pulses: Cowpea, Bush Lablab (short-duration varieties).
  - Flowers: Winter Jasmine.

3. June - July (Aadi / Kuruvai Pattam)

- Months: June - July (sowing) to September - October (harvest). Aadi Pattam, often synonymous with Kuruvai, is crucial as it marks the beginning of the Southwest Monsoon in many parts of India, though Tamil Nadu primarily benefits from the Northeast Monsoon. However, some rains are received, especially in western districts.
- Climatic Conditions: This period brings some relief from the summer heat, with the onset of the Southwest Monsoon. Rainfall can be variable, with some areas receiving moderate showers.
- Significance: This is a vital paddy season, especially in delta regions where irrigation from reservoirs (like Mettur Dam) becomes available. The Tamil proverb "Aadi pattam thedi vidhai" (Seek out the seeds for it is Aadi season) highlights its importance for sowing.
- Typical Crops:
  - Paddy (Rice): Short to medium-duration varieties (Kuruvai paddy).
  - Millets: Ragi, Pearl Millet, Kudiraivali, Panivaragu (often rainfed).
  - Vegetables: Tomato, Brinjal, Chilli, Gourds (benefiting from monsoon moisture).
  - Oilseeds: Groundnut (can be rainfed or irrigated).
  - Pulses: Cowpea, Bush Lablab (often rainfed).
  - Flowers: Winter Jasmine.

4. Sep - Oct (Purattaasi / Samba Pattam)

- Months: September - October (sowing) to January - February (harvest). Samba Pattam is the longest duration paddy crop, primarily reliant on the Northeast Monsoon. Purattaasi refers to sowing activities within this broader Samba season.
- Climatic Conditions: This period is dominated by the Northeast Monsoon, which brings the bulk of rainfall to Tamil Nadu. Temperatures are moderate, making it highly favorable for many crops.
- Significance: This is the most important cropping season for paddy in Tamil Nadu, particularly in the Cauvery Delta. It is also suitable for a wide range of other crops due to favorable moisture and temperature conditions.
- Typical Crops:
  - Paddy (Rice): Long-duration varieties (Samba paddy).
  - Millets: Ragi, Pearl Millet, Kudiraivali, Panivaragu (well-suited for rainfed conditions).
  - Vegetables: Small Onion, Brinjal, Gourds (thrive with good monsoon moisture).
  - Oilseeds: Groundnut, Sunflower.
  - Pulses: Cowpea, Blackgram, Greengram (benefit significantly from monsoon rains).
  - Flowers: Winter Jasmine.

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These "Pattams" dictate the agricultural calendar and play a crucial role in shaping crop choices, resource management, and overall agricultural productivity in Tamil Nadu.

Figure 1. Four types of "Pattam" available in Tamilnadu with suitable crops.

The following flow chart gives the systematic process MCDM:

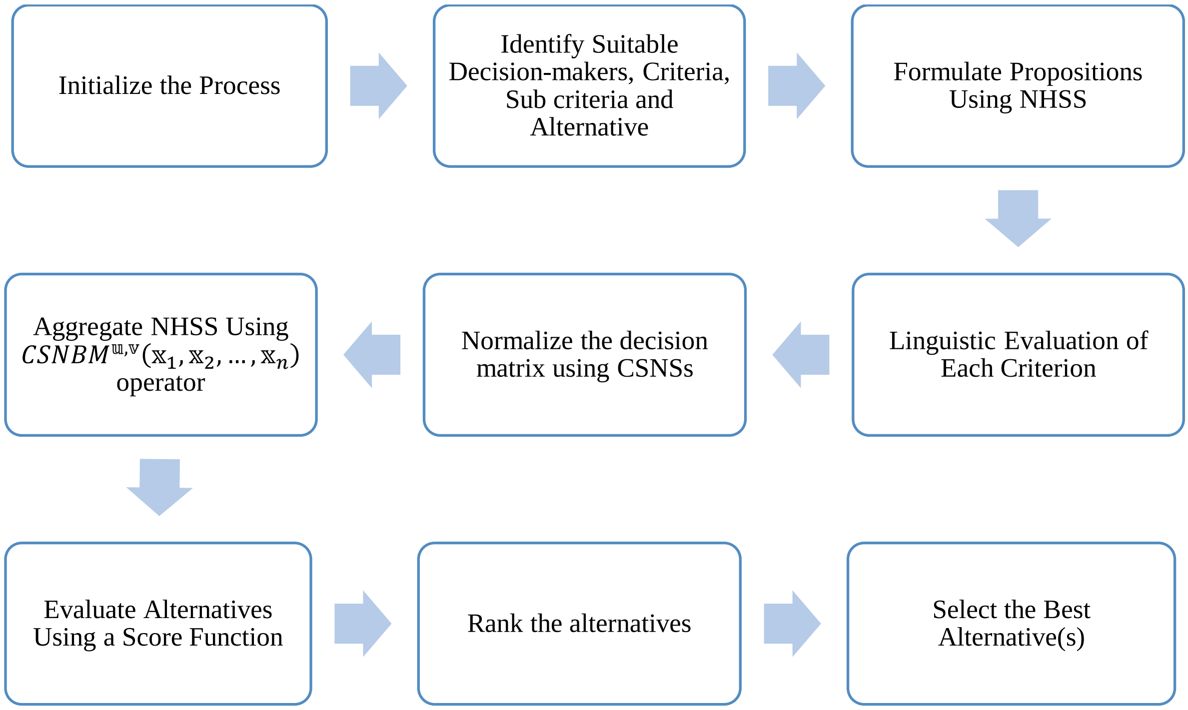

Figure 2. The systematic process MCDM

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3.1 Numerical Example for Multi Criteria Decision-Making Using Neutrosophic Hyper Soft Sets

We define a hyperactive soft set over universe: X = {`ά$_1$, `ά$_2$, `ά$_3$, `ά$_4$, `ά$_5$}, where

- Small Onion or Brinjal or Tomato- `ά$_1$
- Winter Jasmine-`ά$_2$
- Sesame-`ά$_3$
- Ragi- `ά$_4$
- Bush Lablab, Blackgram- `ά$_5$

P(X)= { ∅, {`ά1}, {`ά2}, {`ά3}, {`ά4}, {`ά5}, {`ά1, `ά2}, {`ά1, `ά3}, {`ά1, `ά4}, {`ά1, `ά5}, {`ά2, `ά3}, {`ά2, `ά4 }, {`ά2, `ά5}, {`ά3, `ά4}, {`ά3, `ά5}, {`ά4, `ά5}, {`ά1, `ά2, `ά3}, {`ά1, `ά2, `ά4}, {`ά1, `ά2, `ά5}, {`ά1, `ά3, `ά4}, {`ά1, `ά3, `ά5}, {`ά1, `ά4, `ά5}, {`ά2, `ά3, `ά4}, {`ά2, `ά3, `ά5}, {`ά2, `ά4, `ά5}, {`ά3, `ά4, `ά5}, {`ά1, `ά2, `ά3, `ά4 }, {`ά1, `ά2, `ά3, `ά5}, {`ά1, `ά2, `ά4, `ά5}, {`ά1, `ά3, `ά4, `ά5}, {`ά2, `ά3, `ά4, `ά5}, {`ά1 `,ά2, `ά3, `ά4, `ά5} }

Criteria/Sub-Criteria as attributes:

- `ΰ1 = {Initial Investment Low- `ΰ11, Initial Investment Medium- `ΰ12, Initial Investment High- `ΰ13}
- `ΰ2 = {Yield Low -`ΰ21, Yield Medium- `ΰ22, Yield High- `ΰ23}
- `ΰ3 = {Market Requirement Stable -`ΰ31, Market Requirement Not Stable - `ΰ32}
- `ΰ4 = {Water Requirement Low- `ΰ41, Water Requirement Low-Medium - `ΰ42, Water Requirement Medium - `ΰ43}
- `ΰ5 = {Soils Loamy - `ΰ51, Soils Red - `ΰ52, Soils Black - `ΰ53}
- `ΰ6 = {Fertilizer Required Low- `ΰ61, Fertilizer Required Medium- `ΰ62, Fertilizer Required High- `ΰ63}
- `ΰ7 = {Affect by Pest Low- `ΰ71, Affect by Pest Medium- `ΰ72, Affect by Pest High- `ΰ73}

Then, the hyperactive soft set is defined as a mapping: F: ΰ1× ΰ2 × ΰ3 × ΰ4 × ΰ5 × ΰ6 × ΰ7 → P(X) where

$$
ΰ1 \times ΰ2 \times ΰ3 \times ΰ4 \times ΰ5 \times ΰ6 \times ΰ7 = \begin{cases}
(`ΰ11, `ΰ21, `ΰ31, `ΰ41, `ΰ51, `ΰ61, `ΰ71) \\
(`ΰ11, `ΰ21, `ΰ31, `ΰ41, `ΰ51, `ΰ61, `ΰ72) \\
(`ΰ11, `ΰ21, `ΰ31, `ΰ41, `ΰ51, `ΰ61, `ΰ73) \\
(`ΰ11, `ΰ21, `ΰ31, `ΰ41, `ΰ51, `ΰ62, `ΰ71) \\
(`ΰ11, `ΰ21, `ΰ31, `ΰ41, `ΰ51, `ΰ62, `ΰ72) \\
\ldots\ldots\ldots\ldots\ldots\ldots \\
\ldots\ldots\ldots\ldots\ldots\ldots \\
\ldots\ldots\ldots\ldots\ldots\ldots \\
(`ΰ13, `ΰ23, `ΰ32, `ΰ43, `ΰ53, `ΰ62, `ΰ72) \\
(`ΰ13, `ΰ23, `ΰ32, `ΰ43, `ΰ53, `ΰ62, `ΰ73) \\
(`ΰ13, `ΰ23, `ΰ32, `ΰ43, `ΰ53, `ΰ63, `ΰ71) \\
(`ΰ13, `ΰ23, `ΰ32, `ΰ43, `ΰ53, `ΰ63, `ΰ72) \\
(`ΰ13, `ΰ23, `ΰ32, `ΰ43, `ΰ53, `ΰ63, `ΰ73)
\end{cases}
$$

3.2 Neutrosophic HyperSoft sets in MCDM

The following table presents a linguistic scale designed to capture expert judgment using neutrosophic triplets, which express truth (T), indeterminacy (I), and falsity (F) components. Each linguistic term is associated with a specific neutrosophic value and a symbolic representation to facilitate systematic evaluation of influence levels in decision-making or modelling processes.

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Table 2: Linguistic terms, corresponding linguistic neutrosophic numbers and symbolic representation

S. No.	Linguistic Term	Linguistic Value (T, I, F)	Symbolic Representation
	Very High Influence	‘(0.999, 0.111, 0.111)	‘$Ӂ_1$
	High Influence	‘(0.777, 0.222, 0.333)	‘$Ӂ_2$
	Medium Influence	‘(0.555, 0.333, 0.444)	‘$Ӂ_3$
	Low Influence	(0.333, 0.666, 0.777)	‘$Ӂ_4$
	No Influence	‘(0.111, 0.888, 0.999)	‘$Ӂ_5$

The table below illustrates the linguistic evaluations provided by multiple experts for various criteria and their corresponding sub-criteria using neutrosophic symbolic representations. Each expert's opinion is encoded through linguistic terms previously defined ($Ӂ_1$ to $Ӂ_5$), which encapsulate the levels of influence using neutrosophic triplets. This structured representation facilitates a comprehensive comparative analysis of expert judgments across multiple decision-making dimensions, ensuring consistency and clarity in complex multi-criteria evaluations.

Table 3: Linguistic evaluation of decision makers

‘Experts	‘Criteria	‘Sub-Criteria	`ᾰ1	ᾰ2	`ᾰ3	`ᾰ4	`ᾰ5
‘E1	`ű1	ű11	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$
		ű12	‘$Ӂ_3$	‘$Ӂ_2$	‘$Ӂ_3$	‘$Ӂ_2$	‘$Ӂ_3$
		ű13	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$
	`ű2	ű21	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$
		ű22	‘$Ӂ_4$	‘$Ӂ_2$	‘$Ӂ_4$	‘$Ӂ_2$	‘$Ӂ_4$
		ű23	‘$Ӂ_2$	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$	‘$Ӂ_2$
	`ű3	ű31	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$
		ű32	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$
	`ű4	ű41	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$
		ű42	‘$Ӂ_3$	‘$Ӂ_2$	‘$Ӂ_3$	‘$Ӂ_2$	‘$Ӂ_3$
		ű43	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$
	`ű5	ű51	‘$Ӂ_2$	‘$Ӂ_2$	‘$Ӂ_5$	‘$Ӂ_2$	‘$Ӂ_4$
		ű52	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$
		ű53	‘$Ӂ_5$	‘$Ӂ_2$	‘$Ӂ_4$	‘$Ӂ_2$	‘$Ӂ_4$
	`ű6	ű61	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$
		ű62	‘$Ӂ_4$	‘$Ӂ_2$	‘$Ӂ_4$	‘$Ӂ_2$	‘$Ӂ_4$
		ű63	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$
	`ű7	ű71	‘$Ӂ_2$	‘$Ӂ_1$	‘$Ӂ_2$	‘$Ӂ_1$	‘$Ӂ_2$
		ű72	‘$Ӂ_2$	‘$Ӂ_5$	‘$Ӂ_2$	‘$Ӂ_5$	‘$Ӂ_2$
		ű73	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$	‘$Ӂ_4$	‘$Ӂ_5$

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‘E2	`ΰ1	**ΰ11**	‘Ӂ$_2$	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_1$	‘Ӂ$_1$
		ΰ12	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_2$
		ΰ13	‘Ӂ$_4$	‘Ӂ$_3$	‘Ӂ$_3$	‘Ӂ$_2$	‘Ӂ$_4$
	`ΰ2	**ΰ21**	‘Ӂ$_4$	‘Ӂ$_5$	‘Ӂ$_4$	‘Ӂ$_5$	‘Ӂ$_4$
		ΰ22	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_2$
		ΰ23	‘Ӂ$_1$	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_1$	‘Ӂ$_3$
	ΰ3	**ΰ31**	‘Ӂ$_3$	‘Ӂ$_4$	‘Ӂ$_4$	‘Ӂ$_5$	‘Ӂ$_2$
		ΰ32	‘Ӂ$_4$	‘Ӂ$_2$	‘Ӂ$_4$	‘Ӂ$_4$	‘Ӂ$_2$
	`ΰ4	**ΰ41**	‘Ӂ$_1$	‘Ӂ$_2$	‘Ӂ$_1$	‘Ӂ$_2$	‘Ӂ$_1$
		ΰ42	‘Ӂ$_1$	‘Ӂ$_3$	‘Ӂ$_1$	‘Ӂ$_4$	‘Ӂ$_4$
		ΰ43	‘Ӂ$_4$	‘Ӂ$_5$	‘Ӂ$_1$	‘Ӂ$_2$	‘Ӂ$_3$
	`ΰ5	**ΰ51**	‘Ӂ$_1$	‘Ӂ$_3$	‘Ӂ$_2$	‘Ӂ$_4$	‘Ӂ$_5$
		ΰ52	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_3$	‘Ӂ$_4$	‘Ӂ$_4$
		ΰ53	‘Ӂ$_5$	‘Ӂ$_3$	‘Ӂ$_2$	‘Ӂ$_5$	‘Ӂ$_2$
	`ΰ6	**ΰ61**	‘Ӂ$_2$	‘Ӂ$_4$	‘Ӂ$_5$	‘Ӂ$_4$	‘Ӂ$_1$
		ΰ62	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_2$
		ΰ63	‘Ӂ$_3$	‘Ӂ$_4$	‘Ӂ$_2$	‘Ӂ$_4$	‘Ӂ$_4$
	`ΰ7	**ΰ71**	‘Ӂ$_3$	‘Ӂ$_4$	‘Ӂ$_5$	‘Ӂ$_5$	‘Ӂ$_2$
		ΰ72	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_2$
		ΰ73	‘Ӂ$_1$	‘Ӂ$_2$	‘Ӂ$_3$	‘Ӂ$_3$	‘Ӂ$_4$

The following table provides a detailed representation of the neutrosophic values assigned by each expert to the various sub-criteria across multiple alternatives. These values are expressed in the form of triplets (T, I, F), where T denotes the degree of truth, I indicate the level of indeterminacy, and F reflects the degree of falsity.

Table 4: Provides a detailed representation of the neutrosophic values assigned

Experts	**Sub-Criteria**	**`α̈$_1$**	**α̈$_2$**	**`α̈$_3$**	**`α̈$_4$**	**`α̈$_5$**
	ΰ11	‘(0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)
	ΰ12	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)
	ΰ13	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)
	ΰ21	‘ (0.111, 0.888, 0.999)	(0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)
‘E1	**ΰ22**	‘ (0.333,	‘ (0.777,	‘ (0.333,	‘ (0.777,	‘ (0.333,

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		0.666, 0.777)	0.222, 0.333)	0.666, 0.777)	0.222, 0.333)	0.666, 0.777)
	ṹ23	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)
	ṹ31	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)
	ṹ32	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)
	ṹ41	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)
	ṹ42	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, ‘ 0.333)	‘ (0.555, 0.333, 0.444)
	ṹ43	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	(0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)
	ṹ51	‘ (0.777, 0.222, 0.333)	‘ (0.777, 0.222, 0.333)	‘ (0.111, 0.888, 0.999)	‘ (0.777, 0.222, 0.333)	‘ (0.333, 0.666, 0.777)
	ṹ52	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)
	ṹ53	‘ (0.111, 0.888, 0.999)	‘ (0.777, 0.222, 0.333)	‘ (0.333, 0.666, 0.777)	‘ (0.777, 0.222, 0.333)	‘ (0.333, 0.666, 0.777)
	ṹ61	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)
	ṹ62	‘ (0.333, 0.666, 0.777)	‘ (0.777, 0.222, 0.333)	‘ (0.333, 0.666, 0.777)	‘ (0.777, 0.222, 0.333)	‘ (0.333, 0.666, 0.777)
	ṹ63	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)
	ṹ71	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)
	ṹ72	‘ (0.777, 0.222, 0.333)	‘ (0.111, 0.888, 0.999)	‘ (0.777, 0.222, 0.333)	‘ (0.111, 0.888, 0.999)	‘ (0.777, 0.222, 0.333)
	ṹ73	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)
	ṹ11	‘ (0.777, 0.222, 0.333)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.999, 0.111, 0.111)	‘ (0.999, 0.111, 0.111)
	ṹ12	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)
	ṹ13	‘ (0.333, 0.666, 0.777)	‘ (0.555, 0.333, 0.444)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)	‘ (0.333, 0.666, 0.777)
	ṹ21	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)

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‘E2	**ΰ22**	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)
	ΰ23	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.999, 0.111, 0.111)	‘ (0.555, 0.333, 0.444)
	ΰ31	‘ (0.555, 0.333, 0.444)	‘ (0.333, 0.666, 0.777)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.777, 0.222, 0.333)
	ΰ32	‘ (0.333, 0.666, 0.777)	‘ (0.777, 0.222, 0.333)	‘ (0.333, 0.666, 0.777)	‘ (0.333, 0.666, 0.777)	‘ (0.777, 0.222, 0.333)
	ΰ41	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.999, 0.111, 0.111)
	ΰ42	‘ (0.999, 0.111, 0.111)	‘ (0.555, 0.333, 0.444)	‘ (0.999, 0.111, 0.111)	‘ (0.333, 0.666, 0.777)	‘ (0.333, 0.666, 0.777)
	ΰ43	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)
	ΰ51	‘ (0.999, 0.111, 0.111)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)
	ΰ52	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.555, 0.333, 0.444)	‘ (0.333, 0.666, 0.777)	‘ (0.333, 0.666, 0.777)
	ΰ53	‘ (0.111, 0.888, 0.999)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)	‘ (0.111, 0.888, 0.999)	‘ (0.777, 0.222, 0.333)
	ΰ61	‘ (0.777, 0.222, 0.333)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.333, 0.666, 0.777)	‘ (0.999, 0.111, 0.111)
	ΰ62	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)
	ΰ63	‘ (0.555, 0.333, 0.444)	‘ (0.333, 0.666, 0.777)	‘ (0.777, 0.222, 0.333)	‘ (0.333, 0.666, 0.777)	‘ (0.333, 0.666, 0.777)
	ΰ71	‘ (0.555, 0.333, 0.444)	‘ (0.333, 0.666, 0.777)	‘ (0.111, 0.888, 0.999)	‘ (0.111, 0.888, 0.999)	‘ (0.777, 0.222, 0.333)
	ΰ72	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.777, 0.222, 0.333)
	ΰ73	‘ (0.999, 0.111, 0.111)	‘ (0.777, 0.222, 0.333)	‘ (0.555, 0.333, 0.444)	‘ (0.555, 0.333, 0.444)	‘ (0.333, 0.666, 0.777)

Proposition 1: Maximizing Profitability through Intensive Management

This strategy targets farmers aiming for the highest possible returns by investing significantly in resources and management. It emphasizes high yielding, potentially high-value crops that demand substantial inputs like water and fertilizer, and require proactive pest control. Farmers opting for this proposition are prepared for higher initial investments and ongoing operational costs, seeking to capitalize on strong and stable market demands. The cubic spherical neutrosophic representation of decision maker’s decision matrix given in Table 5.

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Table 5: Cubic spherical neutrosophic representation of decision values

`Sub-Criteria	`ἀ1	`ἀ2	`ἀ3	`ἀ4	`ἀ5
ΰ11	(0.222,0.777, 0.888;0.192)	(0.444,0.500, 0.611;0.260)	(0.333,0.611, 0.722;0.451)	(0.555,0.444, 0.555;0.385)	(0.222,0.777, 0.888;0.192)
ΰ23	(0.888,0.167, 0.222;0.167)	(0.888,0.167, 0.222;0.167)	(0.666,0.278, 0.389;0.136)	(0.999,0.111, 0.111;0.000)	(0.666,0.278, 0.389;0.136)
`ΰ31	(0.777,0.222, 0.278;0.299)	(0.555,0.444, 0.555;0.385)	(0.666,0.389, 0.444;0.547)	(0.444,0.555, 0.666;0.577)	(0.888,0.167, 0.222;0.167)
ΰ42	(0.222,0.777, 0.888;0.192)	(0.222,0.777, 0.888;0.192)	(0.555,0.500, 0.555;0.738)	(0.555,0.444, 0.555;0.385)	(0.333,0.611, 0.722;0.451)
`ΰ51	(0.888,0.167, 0.222;0.167)	(0.666,0.278, 0.389;0.136)	(0.444,0.555, 0.666;0.577)	(0.555,0.444, 0.555;0.385)	(0.222,0.777, 0.888;0.192)
`ΰ63	(0.333,0.611, 0.722;0.451)	(0.333,0.666, 0.777;0.000)	(0.444,0.555, 0.666;0.5777)	(0.333,0.666, 0.777;0.000)	(0.222,0.777, 0.888;0.192)
ΰ73	(0.555,0.500, 0.555;0.738)	(0.555,0.444, 0.555;0.385)	(0.333,0.611, 0.722;0.451)	(0.444,0.500, 0.611;0.260)	(0.222,0.777, 0.888;0.192)

Cubic spherical neutrosophic representation:

Taking two decision maker values (0.111, 0.888, 0.999) and (0.333, 0.666, 0.777)

Center = $\left(\frac{0.111+0.333}{2}, \frac{0.888+0.666}{2}, \frac{0.999+0.777}{2}\right) = (0.222, 0.777, 0.888)$

Radius $= max\left(\begin{array}{c}\sqrt{(0.222-0.111)^2+(0.777-0.888)^2+(0.888-0.999)^2},\\ \sqrt{(0.222-0.333)^2+(0.777-0.666)^2+(0.888-0.777)^2}\end{array}\right) = 0.192$

Cubic spherical neutrosophic set $= (0.222, 0.777, 0.888; 0.192)$

We calculate `$CSNBM^{\mathbb{u},\mathbb{v}}(\mathbb{x}_1, \mathbb{x}_2, \dots, \mathbb{x}_n)$ and the values are given in Table 6.

Table 6: Cubic spherical neutrosophic Bonferroni values

`Crop/ Method	`$CSNBM^{1,1}$	`$CSNBM^{1,2}$	`$CSNBM^{2,1}$
ἀ1	(0.461,0.565,0.629;0.241)	(0.521,0.510,0.569;0.318)	(0.534,0.496,0.553;0.275)
ἀ2	(0.408,0.589,0.671;0.160)	(0.455,0.543,0.624;0.199)	(0.474,0.529,0.608;0.193)
ἀ3	(0.367,0.628,0.702;0.375)	(0.402,0.595,0.670;0.425)	(0.418,0.577,0.652;0.422)
ἀ4	(0.496,0.579,0.648;0.216)	(0.554,0.541,0.605;0.262)	(0.579,0.515,0.576;0.273)
ἀ5	(0.321,0.678,0.755;0.158)	(0.383,0.623,0.690;0.188)	(0.401,0.600,0.675;0.188)

Using the $CSNBM^{\mathbb{u},\mathbb{v}}(\mathbb{x}_1, \mathbb{x}_2, \dots, \mathbb{x}_n)$ operator, let take the parameter value is 1,1

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First consider the truth term in alternative $\ddot{\alpha}_{1,}$

The true values are (0.222, 0.888, 0.777, 0.222, 0.888, 0.333, 0.555)

$$T=\left(1-\left(\begin{array}{c}(1-(0.222\times0.222))^{\frac{1}{56}}\times(1-(0.222\times0.888))^{\frac{1}{56}}(1-(0.222\times0.777))^{\frac{1}{56}}\times\\ (1-(0.222\times0.222))^{\frac{1}{56}}\times(1-(0.222\times0.888))^{\frac{1}{56}}\times(1-(0.222\times0.333))^{\frac{1}{56}}\times\\ (1-(0.222\times0.555))^{\frac{1}{56}}\times(1-(0.888\times0.888))^{\frac{1}{56}}\times(1-(0.888\times0.777))^{\frac{1}{56}}\times\\ (1-(0.888\times0.222))^{\frac{1}{56}}\times(1-(0.888\times0.888))^{\frac{1}{56}}\times(1-(0.888\times0.333))^{\frac{1}{56}}\times\\ (1-(0.888\times0.555))^{\frac{1}{56}}\times(1-(0.777\times0.777))^{\frac{1}{56}}\times(1-(0.777\times0.222))^{\frac{1}{56}}\times\\ (1-(0.777\times0.888))^{\frac{1}{56}}\times(1-(0.777\times0.333))^{\frac{1}{56}}\times(1-(0.777\times0.555))^{\frac{1}{56}}\times\\ (1-(0.222\times0.222))^{\frac{1}{56}}\times(1-(0.222\times0.888))^{\frac{1}{56}}\times(1-(0.222\times0.333))^{\frac{1}{56}}\times\\ (1-(0.222\times0.555))^{\frac{1}{56}}\times(1-(0.888\times0.888))^{\frac{1}{56}}\times(1-(0.888\times0.333))^{\frac{1}{56}}\times\\ (1-(0.888\times0.555))^{\frac{1}{56}}\times(1-(0.333\times0.333))^{\frac{1}{56}}\times(1-(0.333\times0.555))^{\frac{1}{56}}\times\\ (1-(0.555\times0.555))^{\frac{1}{56}}\\ .\end{array}\right)\right)^{\frac{1}{2}}$$

$$=(1-0.78765)^{\frac{1}{2}}=0.461$$

The indeterminacy values are (0.777, 0.167, 0.222, 0.777, 0.167, 0.611, 0.500)

$$I=1-\left(1-\left(\begin{array}{c}\left(1-((1-0.777)\times(1-0.777))\right)^{\frac{1}{56}}\times\left(1-((1-0.777)\times(1-0.167))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.777)\times(1-0.222))\right)^{\frac{1}{56}}\times\left(1-((1-0.777)\times(1-0.777))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.777)\times(1-0.167))\right)^{\frac{1}{56}}\times\left(1-((1-0.777)\times(1-0.611))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.777)\times(1-0.500))\right)^{\frac{1}{56}}\times\left(1-((1-0.167)\times(1-0.167))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.167)\times(1-0.222))\right)^{\frac{1}{56}}\times\left(1-((1-0.167)\times(1-0.777))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.167)\times(1-0.167))\right)^{\frac{1}{56}}\times\left(1-((1-0.167)\times(1-0.611))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.167)\times(1-0.500))\right)^{\frac{1}{56}}\times\left(1-((1-0.222)\times(1-0.222))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.222)\times(1-0.777))\right)^{\frac{1}{56}}\times\left(1-((1-0.222)\times(1-0.167))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.222)\times(1-0.611))\right)^{\frac{1}{56}}\times\left(1-((1-0.222)\times(1-0.500))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.777)\times(1-0.777))\right)^{\frac{1}{56}}\times\left(1-((1-0.777)\times(1-0.167))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.777)\times(1-0.611))\right)^{\frac{1}{56}}\times\left(1-((1-0.777)\times(1-0.500))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.167)\times(1-0.167))\right)^{\frac{1}{56}}\times\left(1-((1-0.167)\times(1-0.611))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.167)\times(1-0.500))\right)^{\frac{1}{56}}\times\left(1-((1-0.611)\times(1-0.611))\right)^{\frac{1}{56}}\times\\ \left(1-((1-0.611)\times(1-0.500))\right)^{\frac{1}{56}}\times\left(1-((1-0.500)\times(1-0.500))\right)^{\frac{1}{56}}\times\end{array}\right)\right)^{\frac{1}{2}}$$

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$= 1 - (1 - (0.81108))^{\frac{1}{2}} = 0.565$

The false values are (0.888, 0.222, 0.278, 0.888, 0.222, 0.722, 0.555)

$$F = 1 - \left(1 - \left(\begin{array}{c} \left(1-\left((1-0.888)\times(1-0.888)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.888)\times(1-0.278)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.888)\times(1-0.888)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.888)\times(1-0.222)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.888)\times(1-0.722)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.888)\times(1-0.555)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.222)\times(1-0.222)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.222)\times(1-0.278)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.222)\times(1-0.888)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.222)\times(1-0.222)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.222)\times(1-0.722)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.222)\times(1-0.555)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.278)\times(1-0.278)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.278)\times(1-0.888)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.278)\times(1-0.222)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.278)\times(1-0.722)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.278)\times(1-0.555)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.888)\times(1-0.888)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.888)\times(1-0.222)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.888)\times(1-0.722)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.888)\times(1-0.555)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.222)\times(1-0.222)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.222)\times(1-0.722)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.222)\times(1-0.555)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.722)\times(1-0.722)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.722)\times(1-0.555)\right)\right)^{\frac{1}{56}} \times \\ \left(1-\left((1-0.888)\times(1-0.222)\right)\right)^{\frac{1}{56}} \times \left(1-\left((1-0.555)\times(1-0.555)\right)\right)^{\frac{1}{56}} \end{array} \right) \right)^{\frac{1}{2}}$$

$= 1 - (1 - 0.86233)^{\frac{1}{2}} = 0.629$

The radius values are (0.192, 0.167, 0.299, 0.192, 0.167, 0.451, 0.738)

$$r = \left(1 - \left(\begin{array}{c} (1-(0.192\times0.192))^{\frac{1}{56}} \times (1-(0.192\times0.167))^{\frac{1}{56}} (1-(0.192\times0.299))^{\frac{1}{56}} \times \\ (1-(0.192\times0.192))^{\frac{1}{56}} \times (1-(0.192\times0.167))^{\frac{1}{56}} \times (1-(0.192\times0.451))^{\frac{1}{56}} \times \\ (1-(0.192\times0.738))^{\frac{1}{56}} \times (1-(0.167\times0.167))^{\frac{1}{56}} \times (1-(0.167\times0.299))^{\frac{1}{56}} \times \\ (1-(0.167\times0.192))^{\frac{1}{56}} \times (1-(0.167\times0.167))^{\frac{1}{56}} \times (1-(0.167\times0.451))^{\frac{1}{56}} \times \\ (1-(0.167\times0.738))^{\frac{1}{56}} \times (1-(0.299\times0.299))^{\frac{1}{56}} \times (1-(0.299\times0.192))^{\frac{1}{56}} \times \\ (1-(0.299\times0.167))^{\frac{1}{56}} \times (1-(0.299\times0.451))^{\frac{1}{56}} \times (1-(0.299\times0.738))^{\frac{1}{56}} \times \\ (1-(0.192\times0.192))^{\frac{1}{56}} \times (1-(0.192\times0.167))^{\frac{1}{56}} \times (1-(0.192\times0.451))^{\frac{1}{56}} \times \\ (1-(0.192\times0.738))^{\frac{1}{56}} \times (1-(0.167\times0.167))^{\frac{1}{56}} \times (1-(0.167\times0.451))^{\frac{1}{56}} \times \\ (1-(0.167\times0.738))^{\frac{1}{56}} \times (1-(0.451\times0.451))^{\frac{1}{56}} \times (1-(0.451\times0.738))^{\frac{1}{56}} \times \\ (1-(0.738\times0.738))^{\frac{1}{56}} \end{array} \right) \right)^{\frac{1}{2}}.$$

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$= (1 - 0.94173)^{\frac{1}{2}} = 0.241$

(T, I, F; r) = (0.461, 0.565, 0.629 ; 0.241)

Using Score function, we calculate the crisp value for each crop and shown in Table 7.

Table 7: Crisp values of each crop

`Method/ Crop	`ά1	`ά2	`ά3	`ά4	`ά5
$CSNBM^{1,1}$	0.507	0.497	0.416	0.513	0.433
$CSNBM^{1,2}$	0.531	0.522	0.428	0.537	0.471
$CSNBM^{2,1}$	0.553	0.536	0.442	0.554	0.485

Taking the term (0.461, 0.565, 0.629; 0.241)

SF= $\frac{3+0.461-0.565-0.629-0.241}{4} = 0.507$

Similarly, calculate other values.

The ranking of crops is given in Table 8.

Table 8: Ranking of Crop

`Method	`Results	`Best Crop
`$CSNBM^{1,1}$	`ά4> `ά1 >`ά2> `ά5> `ά3	`ά4
`$CSNBM^{1,2}$	`ά4> `ά1 >`ά2> `ά5> `ά3	`ά4
`$CSNBM^{2,1}$	`ά4> `ά1 >`ά2> `ά5> `ά3	`ά4

The final ranking of crops based on these calculations confirms that ά4 -crop is the best choice among the evaluated options.

Proposition 2: Resource-Efficient and Low-Risk Cultivation

This approach is tailored for farmers with limited access to capital and water. It prioritizes resilient, low-input crops that offer reliable yields with minimal external dependency. The focus is on conserving resources, reducing financial risk, and ensuring consistent, stable market returns, even if individual crop values are not exceptionally high. This strategy favors crops that are naturally hardy against pests and diseases, and thrive with minimal fertilizer and primarily rainfed conditions. The cubic spherical neutrosophic representation of decision maker's decision matrix given in Table 9.

Table 9: Cubic spherical neutrosophic representation of decision values

`Sub-Criteria	`ά1	`ά2	`ά3	`ά4	`ά5
ΰ11	(0.888,0.167, 0.222;0.167)	(0.777,0.222, 0.333;0.000)	(0.777,0.222, 0.278;0.299)	(0.888,0.167, 0.222;0.167)	(0.999,0.111, 0.111;0.000)
`ΰ22	(0.555,0.444, 0.555;0.385)	(0.666,0.278, 0.289;0.136)	(0.555,0.444, 0.555;0.385)	(0.666,0.278, 0.389;0.136)	(0.555,0.444, 0.555;0.385)
`ΰ31	(0.777,0.222, 0.278;0.299)	(0.555,0.444, 0.555;0.385)	(0.666,0.389, 0.444;0.547)	(0.444,0.555, 0.666;0.577)	(0.888,0.167, 0.222;0.167)

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ΰ42	(0.777,0.222, 0.278;0.299)	(0.666,0.278, 0.389;0.136)	(0.777,0.222, 0.278;0.299)	(0.555,0.444, 0.555;0.385)	(0.444,0.500, 0.611;0.260)
`ΰ52	(0.888,0.167, 0.222;0.167)	(0.666,0.278, 0.389;0.136)	(0.777,0.222, 0.278;0.299)	(0.555,0.444, 0.555;0.385)	(0.666,0.389, 0.444;0.547)
`ΰ62	(0.888,0.167, 0.222;0.167)	(0.555,0.444, 0.555;0.385)	(0.555,0.500, 0.555;0.738)	(0.555,0.444, 0.555;0.385)	(0.888,0.167, 0.222;0.167)
`ΰ61	(0.666,0.278, 0.389;0.136)	(0.666,0.389, 0.444;0.547)	(0.444,0.555, 0.666;0.577)	(0.555,0.500, 0.555;0.738)	(0.777,0.222, 0.333;0.000)

We calculate $`CSNBM^{\mathbb{u},\mathbb{v}}(\mathbb{x}_1, \mathbb{x}_2, \ldots, \mathbb{x}_n)$ and the values are given in Table 10.

Table 10: Cubic spherical neutrosophic Bonferroni values

`Crop/ Method	$`CSNBM^{1,1}$	$`CSNBM^{1,2}$	$`CSNBM^{2,1}$
`ά₁	(0.626,0.398,0.465;0.167)	(0.669,0.356,0.421;0.189)	(0.669,0.358,0.421;0.201)
`ά₂	(0.493,0.490,0.580;0.186)	(0.529,0.456,0.545;0.246)	(0.540,0.441,0.534;0.225)
`ά₃	(0.500,0.511,0.572;0.337)	(0.539,0.472,0.531;0.397)	(0.552,0.459,0.518;0.372)
`ά₄	(0.463,0.547,0.625;0.301)	(0.493,0.517,0.596;0.368)	(0.526,0.486,0.564;0.340)
`ά₅	(0.649,0.433,0.494;0.167)	(0.690,0.389,0.451;0.221)	(0.710,0.383,0.436;0.216)

Using Score function, we calculate the crisp value for each crop and shown in Table 111.

Table 11: Crisp Values of Each Crop

`Method/ Crop	**$`ά_1$**	**$`ά_2$**	**$`ά_3$**	**$`ά_4$**	**$`ά_5$**
$`CSNBM^{1,1}$	0.649	0.557	0.520	0.498	0.639
$`CSNBM^{1,2}$	0.676	0.571	0.535	0.503	0.657
$`CSNBM^{2,1}$	0.672	0.585	0.551	0.534	0.669

The ranking of Crops is given in Table

Table 12: Ranking of Crop

`Method	**`Results**	**`Best Crop**
$`CSNBM^{1,1}$	$`ά_1 > `ά_5 > `ά_2 > `ά_3 > `ά_4$	$`ά_1$
$`CSNBM^{1,2}$	$`ά_1 > `ά_5 > `ά_2 > `ά_3 > ά_4$	$`ά_1$
$`CSNBM^{2,1}$	$`ά_1 > `ά_5 > `ά_2 > `ά_3 > `ά_4$	$`ά_1$

3.3 Sensitive Analysis:

Sensitivity analysis was conducted to evaluate the robustness and reliability of the proposed MCDM framework by examining how variations in input parameters, particularly the truth, indeterminacy, falsity values, and radius of cubic spherical neutrosophic sets, influence the final crop ranking. By systematically altering these values and observing the corresponding changes in the crisp scores and rankings, it was found that the decision outcome - especially the consistent selection of crop ά1 as the optimal choice- remained stable across multiple scenarios. This

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consistency highlights the strength and resilience of the integrated CSNS and NHSS-based model, confirming its suitability for practical decision-making under uncertain and dynamic agricultural conditions.

Table 13: Ranking of Crops for different values of $\mathbb{u}$ and $\mathbb{v}$ in $CSNBM$ operator

`Method	$\mathbb{u}$ & $\mathbb{v}$	`Results	`Best Crop
`$CSNBM^{0,0.25}$	$\mathbb{u}=0$	`$\ddot{\alpha}_1$> `$\ddot{\alpha}_5$ > `$\ddot{\alpha}_2$ > `$\ddot{\alpha}_3$ > `$\ddot{\alpha}_4$	`$\ddot{\alpha}_1$
`$CSNBM^{0,0.5}$		`$\ddot{\alpha}_1$ > `$\ddot{\alpha}_5$ > `$\ddot{\alpha}_2$ > `$\ddot{\alpha}_3$ >$\ddot{\alpha}_4$	`$\ddot{\alpha}_1$
`$CSNBM^{0,0.75}$		`$\ddot{\alpha}_1$> `$\ddot{\alpha}_5$ > `$\ddot{\alpha}_2$ > `$\ddot{\alpha}_3$ > `$\ddot{\alpha}_4$	`$\ddot{\alpha}_1$
`$CSNBM^{0.25,0}$	$\mathbb{v}=0$	`$\ddot{\alpha}_1$> `$\ddot{\alpha}_5$ > `$\ddot{\alpha}_2$ > `$\ddot{\alpha}_3$ > `$\ddot{\alpha}_4$	`$\ddot{\alpha}_1$
`$CSNBM^{0.5,0}$		`$\ddot{\alpha}_1$ > `$\ddot{\alpha}_5$ > `$\ddot{\alpha}_2$ > `$\ddot{\alpha}_3$ >$\ddot{\alpha}_4$	`$\ddot{\alpha}_1$
`$CSNBM^{0.75,0}$		`$\ddot{\alpha}_1$> `$\ddot{\alpha}_5$ > `$\ddot{\alpha}_2$ > `$\ddot{\alpha}_3$ > `$\ddot{\alpha}_4$	`$\ddot{\alpha}_1$
`$CSNBM^{1,1}$		`$\ddot{\alpha}_1$> `$\ddot{\alpha}_5$ > `$\ddot{\alpha}_2$ > `$\ddot{\alpha}_3$ > `$\ddot{\alpha}_4$	`$\ddot{\alpha}_1$
`$CSNBM^{1,2}$		`$\ddot{\alpha}_1$ > `$\ddot{\alpha}_5$ > `$\ddot{\alpha}_2$ > `$\ddot{\alpha}_3$ >$\ddot{\alpha}_4$	`$\ddot{\alpha}_1$
`$CSNBM^{2,1}$		`$\ddot{\alpha}_1$> `$\ddot{\alpha}_5$ > `$\ddot{\alpha}_2$ > `$\ddot{\alpha}_3$ > `$\ddot{\alpha}_4$	`$\ddot{\alpha}_1$

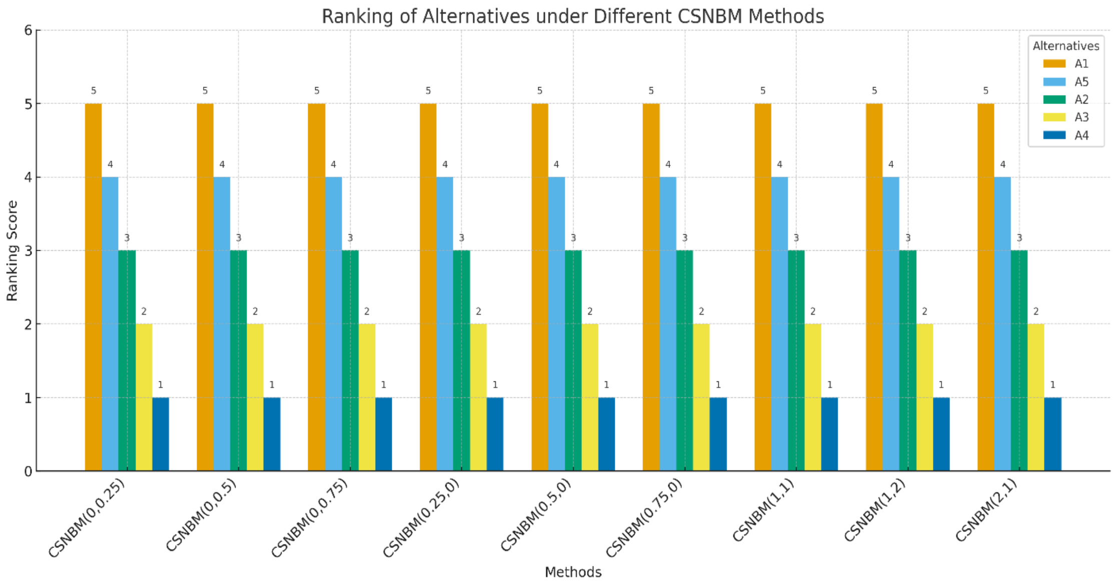

Figure 4. Ranking of Crops for different values of $\mathbb{u}$ and $\mathbb{v}$ in $CSNBM$ operator

4. Conclusion

This study introduces a robust Multi-Criteria Decision-Making (MCDM) framework that integrates Cubic Spherical Neutrosophic Sets (CSNS) with Neutrosophic Hyper Soft Sets (NHSS) to assist small-scale farmers in selecting optimal short-term crops across Tamil Nadu's diverse agricultural seasons (Pattams). By capturing the degrees of truth, indeterminacy, and falsity in a spherical structure with interval-valued logic, CSNS effectively models expert evaluations that involve uncertainty and subjectivity. Combined with the flexibility of NHSS in handling complex, multi-attribute decision environments, the model enables systematic analysis of conflicting criteria such as investment cost, yield, market stability, water and soil requirements, fertilizer needs, and pest resistance. Expert linguistic assessments are mapped to neutrosophic values and aligned with seasonal cropping patterns (Navarai, Chithiraipattam, Aadi, and Purattaasi), resulting in a context-aware, data-driven tool that enhances decision-making, improves resource use, and promotes sustainable agricultural practices.

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