A Predictive Analytics for Customer Lifetime Value Estimation in Digital Banking using Interval-Valued Neutrosophic Set with Fine Tuning Approach

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Abstract

As a generality of fuzzy sets (FS) and intuitionistic FS (IFS), neutrosophic sets (NS) was progressed by F. Smarandache for signifying incomplete, inaccurate, and uneven data present in the real world. Neutrosophic Logic (NL) is a neonate research field in which every proposition was projected to have the proportion of truth in a sub-set T, I, and F. Neutrosophic sets (NS) have been well employed for indeterminate information handling, and determine benefits to tackle indeterminate data. A NS is categorized by indeterminacy-, truth-, and a falsity-membership functions. Atanassov as a major simplification of FS presented the notion of IFS. IFS are very beneficial in conditions when problem description by linguistic variables, assumed with only a membership function, appears to be difficult. In recent times, IFS have been employed to numerous areas like medical diagnosis, logic programming, decision-making issues, etc. An interval NS (INS) is an example of NS, which is employed in real engineering and scientific applications. Owing to the competition in the banking industry and the importance, access to customer information is vital to establish a successful relationship that benefits both parties. Representing longer-term customer relationships and building brand equity are essential in modern banking, and therefore increasing relationship quality plays a significant part in the development of new services and customer lifetime value (CLV) approximation. CLV is an estimated profit that can be achieved by the organization from a customer for some time. Presently, the development of Machine Learning (ML) methods has resulted in better precision and effectiveness. Therefore, by utilizing ML methods of real-time customer data, predictions of a more precise future value of the customer are gained by businesses, which helps in establishing a more personal marketing approach. In this manuscript, we propose a Customer Lifetime Value Estimation using Interval-Valued Neutrosophic Set and Parameter Optimization Algorithms (CLVE-IVNSPOA). The foremost main of this paper is to progress a predictive analytics model for estimating customer lifetime value in digital banking utilizing advanced optimization methods. Initially, the data pre-processing phase was employed by using the Z-score method. Moreover, the pelican optimization algorithm (POA) is mainly executed by the feature subset selection in order to select the most optimal features from a dataset. For CLV prediction, the Interval-Valued Neutrosophic Set (IVNS)

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technique is exploited. At last, the model parameter adjustment process is performed through improved shark optimization (ISHO) algorithm for improving the prediction performance. The experimental evaluation of the CLVE-IVNSPOA occurs using benchmark database. The experimental outcomes indicated out an improved performance of CLVE-IVNSPOA compared to existing systems.

Keywords: Customer Lifetime Value; Neutrosophic set; Digital Banking; Interval-Valued Neutrosophic Set; Improved Shark Optimization Algorithm; Neutrosophic Logic

1. Introduction

The neutrosophic set (NS) concept from a philosophical standpoint is a generalization notion of FS and IFS [1]. An NS is marked by truth membership function (MF), an indeterminacy MF, and a falsity MF (FMF), and is a real standard or non-standard sub-set of non-standard component within the range of] −0, 1+[. In contrast to IFS, there is no limitation on MFs in NS, and the grade of hesitancy was incorporated in NS [2]. However, NSs are difficult to use in real-time issues because the truth, indeterminacy, and falsity values of MFs reside in] −0, 1+[. Hence, this idea is prolonged to several NSs whose truth, indeterminacy, and FMFs take merely one value from closed intervals [0, 1]. Another mathematical method presented to the literature as an outcome of effort to state uncertainty conditions in a perfect method is the NS theory suggested by Smarandache. Obviously; the notion of neutrosophy, a philosophy branch analyses the nature, source and range of neutralities. Neutrosophy reflects a concept, theory, event, proposition, or entity < A > in relation to its opposed < antiA >, and with < neutA >. Furthermore, neutrosophy is the foundation of neutrosophic probability, neutrosophic logic (NL) neutrosophic statistics, and NS. Here, NL represents a general architecture for union of numerous logics. NL goals is to describe every logical statement in a space of 3D-neutrosophic, whereas every space dimension signifies truth (T), the falsehood (F), and the indeterminacy (I) of statement, while T, I, F are not essentially any connection amongst them. After the NL and NS theory, an indeterminacy and a falsity was recommended by Smarandache, it was supposed that it might be employed as an effectual device in the contest against uncertainty by numerous scholars. The significant motive for this is that NSs are a generalization of mathematical methods like conventional FS, classical sets, interval-valued FS, IFS, and interval-valued IFS. Over the last 10 years, the retail-banking sector has begun to encounter a range of unprecedented obstacles that have had a significant negative influence on industry margins and profitability [3]. In many cases, these obstacles are triggered by progress in current information and telecommunication technologies (ICT), which eventually led to advanced expenditure disclosure and an increase in customers migrating between brands. This escalating rivalry has resulted in the core banking offerings, namely accepting deposits, home loans, and credit extensions, becoming standardized [4]. This trend has been additionally powered by a growing influx of newcomers in retail banking, coming from different sectors like insurance and automobile production. Several organizations have seen the loss of customer loyalty with the increase of e-commerce and its resultant aids to consumers, such as lower prices, increased choices, and ease of brand switching [5]. Nowadays, customers are the heart of business in any industry, and organizations have to efficiently handle their interactions with customers to continue functioning in an extremely competitive environment. Customer relationship management (CRM) offers a 360-degree customer view and their needs and preferences, enabling organizations to handle customer interactions, raise their productivity, and compute customer lifetime value (CLV) [6]. A change to customer-centric approaches is vital, highlighting CLV as a measure for advancing sustainable growth and aligning tactics with progressing customer expectations for profitability [7].

CLV measures a customer's value, assisting resource allocation, tailored marketing, and tactical decision-making. This is especially important in industries such as insurance and banking, assisting in identifying high-value customers, optimizing marketing, and deepening customer relationships for long-term success [8]. In finance, CLV provides portfolio optimization, tailored offerings, and better loyalty than the competition. Concentrating on high-value customers, utilizing data-driven segmentation increases returns and maintains effectiveness. Nevertheless, conventional CLVs depend solely on single-entity data, missing insights from customer activities in many firms [9]. Due to the intricate and adaptive nature of ecommerce data, conventional techniques for forecasting CLV often have drawbacks. One potential method to improve these estimations is using deep learning (DL) approaches, which are well known for identifying complex associations in large datasets [10]. DL systems can infer the intricacy and temporal dependencies affecting customer purchase behaviour through hierarchical representations learned at numerous abstraction levels.

This article proposes a Customer Lifetime Value Estimation using Interval-Valued Neutrosophic Set and Parameter Optimization Algorithms (CLVE-IVNSPOA). The data-reprocessing step is initially applied by utilizing the Z-score technique. The pelican optimization algorithm (POA) is implemented by using feature subset selection. For CLV prediction, the Interval-Valued Neutrosophic Set (IVNS) was used. The model parameter adjustment model is finally executed through improved shark optimization (ISHO) model. The empirical outcomes emphasized the superior performance of the CLVE-IVNSPOA approach compared to recent techniques.

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2. Existing Studies on Customer Lifetime Value Estimation in Digital Banking

Khan et al. [11] presented a method that implements dual sequential stages to analyse CLV depending on their purchase behaviours. To manage variety and imbalance in customer data, to examine 3 decision tree selection approaches to construct a general, effective ensemble. Decision tree selection is performed depending on individual tree performance on out-of-bag, independent, and subsamples for their incorporation in the final ensemble. The 1st phase forecasts whether a customer will purchase in a future window, whereas the 2nd phase predicts the monetary value of their future purchases. Saturi et al. [12] examined a quantum-enhanced predictive analytical approach focused on enhancing customer retention tactics and enhancing supply chain management. By examining past customer data to discover indicators and trends connected with attrition, this investigation combines various information, including customer involvement, satisfaction, and transactional history, to improve prediction precision. This method utilizes sophisticated ML methodologies like ANNs for modelling customer behaviour, while utilizing the K-means algorithm for customer segmentation. This enables businesses to modify retention tactics to different customer groups, tackling diverse necessities and preferences. Gaidhani et al. [13] introduced a broad AI-driven predictive analytical technique focused on refining customer satisfaction, engagement, and retention through sophisticated ML and NLP approaches. By employing BERT-based models for sentiment analysis and XGBoost for predicting churn, this technique effectively manages both structured and unstructured customer data.

Umezurike et al. [14] offered a thorough literature-driven examination into the approaches, opportunities, and challenges surrounding CLV's predictive modelling in subscription environments. Utilizing reports from ML, marketing analytics, and behavioural economics, the authors examine how current techniques from logistic regression to DL models improve predictive accuracy. Additionally, the authors discussed the tactical importance of incorporating predictive CLV methods into personalization, pricing, and service delivery. Kaluarachchi and Sedera [15] analysed how AI-driven customer engagement improves processes and modifies solutions. AI techniques assist banks learn client behaviours and preferences by examining extensive data, aiding a customer-centric approach that endorses loyalty and happiness.

The authors [16] presented an adaptable ML architecture to predict CLV in the B2B Software-as-a-Service (SaaS) infrastructure. The practical and modelling issues are related to highly heterogeneous populations, more nuanced customer relationship, manifold product offerings, and time-based data limitations. Bose et al. [17] explored the combination of predictive analytics and reinforcement learning (RL) as a new method to enhance CLV in several sectors. By employing sophisticated ML frameworks, this investigation solves the restrictions of conventional CLV techniques that frequently depend on static, rule-based architectures that fall short of capturing the adaptive nature of customer interactions and preferences. This method employs RL to dynamically tailor marketing tactics, making an iterative loop in which consumer responses to market activities are incessantly examined and enhanced.

3. Materials and Methods

In this article, we have proposed a CLVE-IVNSPOA model. The main intention is to progress a predictive analytics technique to estimate customer lifetime value in digital banking. To obtain this, the presented CLVE-IVNSPOA model has Z-score method, POA-based FS, prediction, and parameter fine-tuning. Fig. 1 specifies the complete process of CLVE-IVNSPOA system.

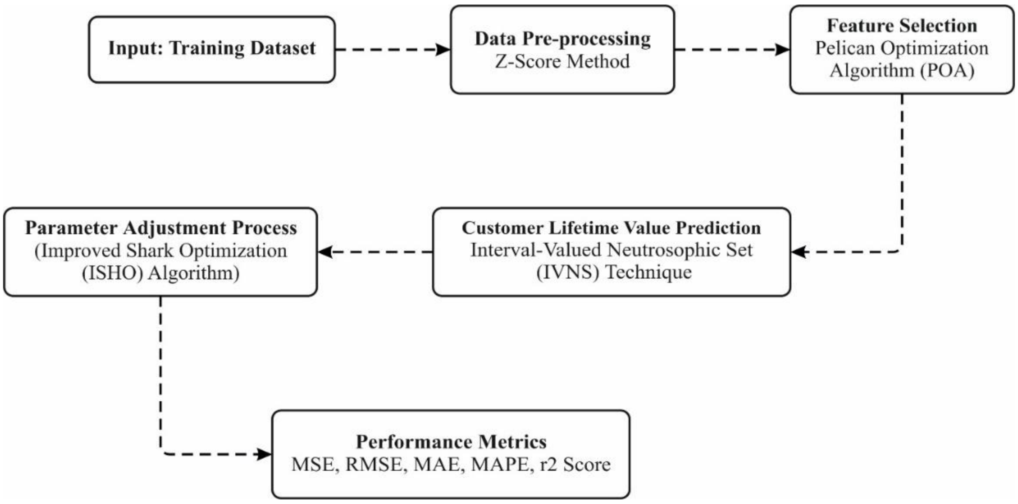

Figure 1. Complete Process of CLVE-IVNSPOA System

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A. Stage I: Application of Z-Score Normalization

At first, the data-reprocessing phase was employed by utilizing the Z-score method. It regulates data by changing values to have mean and Standard deviation (SD) [18]. This pre-processing phase extracts unit disparities, improves comparison through features, and increases the solution of methodology to stabilize convergence and decrease bias from outliers. To assure every each one of the features has an average of 0 and equal variation, this method standardizes data to deduce the mean and divide it by SD. This approach is useful while minimal and maximal values of data are unknown, specified in Eq. (1).

$$W_{new}=\frac{W-\mu}{\sigma}=\frac{W-Mean(W)}{StdDev(W)} \tag{1}$$

Here, W signifies preceding value, W_{ew} represents novel value has been computed from normalized findings, σ denotes the SD value and μ depicts mean of population.

B. Stage II: POA based Dimensionality Reduction

Additionally, the POA is mostly used by the feature subset selection to pick the best features from a dataset. The PO is an innovative stochastic optimization technique that derives from nature [19]. It is renowned for its exceptional capability of examining and utilizing the search space to find the global optimum. In recent times, there has been a lot of interest in swarm-inspired models, and the PO is specifically affected by behaviour of pelican and searching approach. While searching in the wild, pelicans often work together and take a multi-step technique. Once they have recognized where their prey is, they make a co-ordinated descent and then spread their wings. By forcing their prey to surface and migrating into shallower waters, the pelicans are catching their meal more simply.

The PO is a population-based model, initialize each group member of pelican at arbitrary the beginning of optimization procedure, utilizing Eq. (2). The size of population, counts of problematic variables, intervals, superior and inferior boundaries are depicted by $W_{j,i}$.

$$W_{j,i} = KA_i + rand * (UB_i - LB_i) j = 1,2, \dots M \tag{2}$$

Exploration:

It entails pelicans foraging for food resources, similar to pelicans searching for prey. The capability of PO to generate the position of prey at arbitrary angles enhances its exploration skills. The target function for i_{th} candidate solution of pelicans novel position has been specified in Eq. (3), here $W_{j,i}^{01}$ signifies j_{th} pelican's most recent location and 0_i represents the location of prey. The J parameter significantly affects the ability of PO's for discovering and methodically searching.

$$W_{j,i}^{01} = \begin{cases} W_{j,i} + rand.\left(O_i - J.W_{j,i}\right), E_0 < E_j, \\ W_{j,i} - rand.\left(O_i - J.W_{j,i}\right), else \end{cases} \tag{3}$$

$$W_j = \begin{cases} W_{j'}^{O1} E_j^{O1} < E_{j'}. \\ W_j, else \end{cases} \tag{4}$$

Exploitation:

Pelicans employ their wings to drag fish upward, gathering them in their neck pouch. This approach improves fish catch in specified region. PO enhances convergence toward advantageous position, improving efficacy in exploitation and local search. The mathematical model for this pretending behaviour, here s_{Max} indicates the maximal iteration counts, s signifies the existing iteration. $W_{j,i}^{02}$ indicates the j_{th} most recent position of pelican and Q represents a constant.

$$W_{j,i}^{02} = W_{j,i} + Q.\left(1 - \frac{s}{s_{\text{Max}}}\right).(2.rand - 1).W_{j,i} \tag{5}$$

The succeeding Eq. (6) then represents how the outcome is altered in light of novel location.

$$W_j = \begin{cases} W_j^{02}, E_j^{02} < E_j; \\ W_j, else \end{cases} \tag{6}$$

In Exploitation, W_j^{02} signifies j_{th} most recent position of pelican and E_j^{02} indicates the value of objective function.

The fitness function (FF) executed was intended to have an equilibrium among the chosen feature amount in every performance (least) and the classifier precision (highest) gained by exploiting these preferred attributes.

$$Fitness = \alpha\gamma_R(D) + \beta\frac{|R|}{|C|} \qquad (7)$$

Whereas $\gamma_R(D)$ signifies the classification error values of providing classifier. $|R|$ denote cardinality of chosen subset, and $|C|$ signifies complete feature counts from dataset. α and β are 2 parameters.

C. Stage III: Model Prediction using IVNS-based CLV Technique

For CLV prediction, the IVNS technique has been utilized. We provide the basic explanations of INS theory, NS theory, and soft set theory, which are beneficial for following discussions [20].

Definition1.1 Assume U as a space of points, with a general part in U signified by u. In U, a N-sets A is considered by I_A, T_A, and a F_A. $T_A(u)$; $I_A(x)$ and $F_A(u)$ denotes real standard or non-standard sub-sets of [0,1].

There is no constraint on total of $T_A(u)$; $I_A(u)$ and $F_A(u)$, so $0 \leq supT_A(u) + supI_A(u) + supF_A(u) \leq 3$.

Definition1.2 Assume U as space of points, where U signified by u. An IVN-sets A in U is described by I_A, T_A, and F_A. For every points $u \in U; T_A, I_A$ and $F_A subseteq[0,1]$.

Therefore, a IVN-set over U is signified by set of

$$A = \{\langle T_A, (u), I_A(u), F_A(u)\rangle / u : u \in U\}$$

Where, $(T_A(u), I_A(u), F_A(u))$ is named IVN-sets number for every $u \in U$ and each IVN-sets over U is signified by $IVN(U)$.

Example 1.3 Let universe of discourse $U = \{u_1, u_2\}$ while u_1 characterizes the excellence, u_2 specifies an objects prices. It might be more expected that u_1 and u_2 values are sub-set of [0,1] and attained from. The expert build an INV set as per to I_A, T_A, and a F_A as below;

$$A = \{\langle[0.1,1.0], [0.1,0.4], [0.4,0.7]\} / u_1, \langle[0.6,0.9], [0.8,1.0], [0.4,0.6]\} / u_2\}$$

Definition1.4 Assume that A a IVN-sets. Next, for every $u \in U$,

1. A is empty, signified $A = \widetilde{\emptyset}$, is expressed as

$$\widetilde{\emptyset} = \{< [0,0], [1,1], [1,1] > / u : u \in U\}$$

2. A is universal represented $A = \tilde{E}$, is definite as

$$\tilde{E} = \{< [1,1], [0,0], [0,0] > / u : u \in U\}$$

3. The A complement is represented by $\overline{A}$ and was expressed as

$$\overline{A} = \{< [infF_A(u), supF_A(u)], [1 - supI_A(u), 1 - infI_A(u)],$$
$$[infT_A(u), \sup T_A(u)] > / u : u \in U\}$$

Definition1.5 An INS A is restricted in other INS $B, A \mathrel{\tilde{\subseteq}} B$, if and only if

in $fT_A(u) \leq infT_B(u) \quad supI_A(u) \geq supI_B(u)$

$supT_A(u) \leq supT_B(u) \quad infF_A(u) \geq infF_B(u)$

$infI_A(u) \geq infI_B(u) \quad supF_A(u) \geq supF_B(u)$

for every $u \in U$.

Definition1.6 An IN numeral $X = (T_X, I_X, F_X)$ is greater than the other numeral $Y = (T_Y, I_Y, F_Y)$, signified $X \lesssim Y$, if and only if

$\inf T_X \leq \inf T_Y \quad supI_X \geq supI_Y$

$supT_X \leq supT_Y \quad infF_X \geq infF_Y$

$infI_X \geq infI_Y \quad supF_X \geq supF_Y$

Definition 1.7 Let A and B be dual INS. Next, for every $u \in U, a \in R^+$,

1. A and B Intersection , signified by $A \mathbin{\tilde{\cap}} B$, was definite as

$$A \mathbin{\tilde{\cap}} B = \{< \left[\min\left(\inf T_A(u), infT_b(u)\right), \min\left(\sup T_A(u), \sup T_B(u)\right)\right],$$
$$\left[\max\left(infI_A(u), infI_B(u)\right), \max\left(supIA(x), supI_B(u)\right)\right],$$

$$\left[\max\left(infF_A(u), infF_B(u)\right), \max\left(supF_A(u), supF_B(u)\right)\right] >/u : u \in U\}$$

2. A and B Union , symbolized by $A \tilde{\cup} B$, was definite as

$$A \tilde{\cup} B = \{< \left[\max\left(infT_A(u), infT_B(u)\right), \max\left(supT_A(u), supT_B(u)\right)\right],$$

$$[\min\left(infI_A(u), infI_B(u)\right), \min(supI_A(u), supI_B(u))],$$

$$\left[\min\left(infF_A(u), infF_B(u)\right), \min\left(supF_A(u), supF_B(u)\right)\right] >/u : u \in U\}$$

3. A and B Difference , represented by $A\tilde{\mp}B$, was expressed below

$$A\tilde{\mp}B = \{< \left[\min\left(infT_A(u), infF_B(u)\right), \min\left(supT_A(u), supF_B(x)\right)\right],$$

$$\left[\max\left(infI_A(u), 1 - supI_B(u)\right), \max\left(supI_A(u), 1 - infI_B(u)\right)\right],$$

$$\left[\max\left(infF_A(u), infT_B(u)\right), \max\left(supF_A(u), supT_B(u)\right)\right] >/u : u \in U\}$$

4. A and B Addition , signified as $A \mp B$, was demonstrated below

$$A\tilde{\mp}B = \{< [\min(\mathrm{inf}T_A(u) + \mathrm{inf}T_B(u), 1), \min(\mathrm{sup}T_A(u) + supT_B(u), 1$$

$$[\min(infI_A(u) + infI_B(u), 1), \min(supI_A(u) + supI_B(u), 1$$

$$[\min(infF_A(u) + infF_B(u), 1), \min(supF_A(u) + supF_B(u), 1)] >/u: u \in U\}$$

5. Scalar multiplication of A , signified by $A\tilde{.}a$, was stated as

$$A\tilde{.}a = \{< [\min(infT_A(u).a, 1), \min(supT_A(u).a, 1$$

$$[\min(infI_A(u).a, 1), \min(supI_A(u).a, 1$$

$$[\min(infF_A(u).a, 1), \min(supF_A(u).a, 1)] >/u: u \in U\}$$

6. Scalar division of A , meant by $A\overline{/}a$, was outlined as follows

$$A\tilde{/}a = \{< [\min(infT_A(u)/a, 1), \min(supT_A(u)/a, 1$$

$$[\min(infI_A(u)/a, 1), \min(supI_A(u)/a, 1$$

$$[\min(infF_A(u)/a, 1), \min(supF_A(u)/a, 1)] >/u: u \in U\}$$

7. Truth-Favorite of A, signified by $\tilde{\triangle} A$, is outlined by

$$\tilde{\triangle} A -= \{< [\min(\mathrm{inf}T_A(u) + infI_A(u), 1), \min(\mathrm{sup}T_A(u) + supI_A(u), 1\ [0,0],$$

$$[infF_A(u), supF_A(u)] >/u : u \in U\}$$

8. False-Favorite of A, represented by $\tilde{\nabla}A$, was expressed below

$$\tilde{\nabla}A = \{< [infT_A(u), supT_A(u)], [0,0],$$

$$[\min(infF_A(u) + infI_A(u), 1), \min(supF_A(u) + supI_A(u), 1)] >/u: u \in U\}$$

Definition1.8 Assume U as an universe, $P(U)$ is a power set of U,

E is a set of every parameter and $X \subseteq E$. Next a soft set F_X is outlined by a function demonstrating a map

$f_X: E \rightarrow P(U)$ such that $f_X(x) = \emptyset$ if $x \notin X$

While, f_X was named approximate function, and $f_X(x)$ is named x-element of soft set for each $x \in E$. It is worth observing that $f_X(x)$ might be random. Few may be empty and some might hold intersection of non-empty. Consequently, a U was signified by the set of

$$F_X = \left\{\left(x, f_X(x)\right): x \in E, f_X(x) \in P(U)\right\}$$

Example 1.9 Assume that $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ as a universe covers 6 house under concern in an real agent and $E = \{x_1 =$ cheap, $x_2 =$ beati ful, $x_3 =$greensurroundings, $x_4 =$costly, $x_5 = large\}$.

Let that $f_X(x_1) = \{u_1, u_2\}, f_X(x_2) = \{u_1\}, f_X(x_3) = \emptyset, f_X(x_4) = U, \{u_1, u_2, u_3, u_4, u_5\}$ then the F_X was transcribed as

$$F_X = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_4, u_5, u_6\})\ (x_4, U), (x_5, \{u_1, u_2, u_3, u_4, u_5\})\}$$

Definition1.10 Assume $U = \{u_1, u_2, \dots, u_k\}$ as an initial universe, $E = \{x_1, x_2, \dots, x_m\}$ is a group of parameters and F_X is a soft set. The $x_j \in E, f_X(x_j)$ is a sub-set of U. Next, $u_i \in U$ is c_i, assumed by $c_i = \sum_j u_{ij}$, while u_{ij} are the items. It is expressed below,

$$u_{ij} = \begin{cases} 1, u_i \in f_X(x_j) \\ 0, u_i \notin f_X(x_j) \end{cases}$$

Example1.11 Let take the abovementioned Example 1.9. Noticeably,

$c_1 = \sum_{j=1}^{5} u_{1j} = 4,$

$c_3 = c_6 = \sum_{j=1}^{5} u_{3j} = \sum_{j=1}^{5} u_{6j} = 2,$

$c_2 = c_4 = c_5 = \sum_{j=1}^{5} u_{2j} = \sum_{j=1}^{5} u_{4j} = \sum_{j=1}^{5} u_{5j} = 3$

Definition 1.12 Assume F_X and F_Y be dual sets of soft. Next,

1. F_X Complement is meant as $F_X^{\tilde{C}}$. Its estimated function $f_{X^c}(x) = U \setminus f_X(x)$ for every $x \in E$

2. F_X and F_Y Union is represented as $F_X \tilde{\cup} F_Y$. Its estimated function $f_{X\tilde{\cup}F_Y}$ was definite below

$f_{X\tilde{\cup}Y}(x) = f_X(x) \cup f_Y(x)$ for every $x \in E$.

3. F_X and F_Y Intersection is signified as $F_X \tilde{\cap} F_Y$. Its estimated function $f_{X\overline{\cap}Y}$ was well-defined as follows

$f_{X\tilde{\cap}Y}(x) = f_X(x) \cap f_Y(x)$ for all $x \in E$.

In this work, we review the basic principles of NS, SVNS, IVNS, and the present IVNN act as the basis for this paper and inform these upcoming investigations [21].

Definition2.1:

Described the NS A as shown: $A = x{:}TA(x), lA(x), FA(x) >, x \in X$, while $T(x), I(x)$, and $F(x)$ signify the functions of false, truth, and indeterminate MS, correspondingly, and they encounter the following condition: $T(x) = x{:}TA(x), l(x) = x{:}TA(x)$

$$-0 \leq TA(x) + lA(x) + FA(x) \leq 3 +.$$

Get the values for the 3 MS functions by plugging $in]\ O, 1 +$ [. On that account, they presented the usage of the SVNS, which is a basic version of an NS that might be used in some real-time condition.

Definition2.2:

The set defines A, the SVNS in X.

Dual conditions are encountered by the term $A \dots = x$: TA (x), IA $(x), FA(x), x = X$: wherein TA (x), IA $(x), FA(x) \in$ [0,1] and $x \in X$.

$$0 \leq TA(x) + IA(x) + FA(x) \leq 3.$$

Definition2.3:

An Intravenous Nutritional Supported System in X, signified by

$$A = \{\langle x{:}T \sim A(x) I \sim A(x), F \sim A(x) \rangle x \in X\}$$

$A = \{\langle x{:}[TLA(x)TUA(x)],$

$$[lLA(x)IUA(x)],$$

$$[FLA(x)FUA(x)]\rangle x \in X\}$$

Whereas the values in the interval $[TLA(x)TUA(x)]$,

$$[ILA(x), lIUA(x)],$$

$$[FLA(x), FUA(x)] \in [0,1]$$

encounter the state:

$$0 \leq supTA(x) + suplA(x) + supFA(x) \leq 3.$$

Now looked at some numerical processes that might be implemented on IVNNs, or IVN numbers.

Definition2.4:

Assume $A = \langle [TLa, TUa], [IL\text{a}, IUa], [FL\text{a}, FUa] \rangle$

$$andB = \langle [TLb, TUb], [IL\text{b}, IUb], [FL\text{b}, FUb] \rangle$$

be 2 lVNNs and $\eta > 0$. Then

$$A \oplus B = \langle [TLa + TLb - TLaTLb, TUa + TUb - TUaTUb], [ILalLb, IUalUb],$$

$$[FLaFLb, FUaFUb] \rangle$$

$$A \otimes B = \langle [TLaTLb, TUaTUb],$$

$$[ILa + ILb - ILaILb, IUa + IUb \ - IUaIUb]$$

$$[FLa + FLb - FLaFLb, FUa + FUb - FUaFUb] \rangle$$

$$\eta A = \langle [1 - (1 - TLa)\eta, 1 - (1 - TUa)\eta], [(lLa)\eta, (IUa)\eta], [(FLa)\eta, (FUa)\eta] \rangle$$

$$A\eta = \langle [(TLa)\eta, (TUa)\eta],$$

$[1 - (1 - ILa)\eta, 1 - (1 - IUa)\eta], [1 - (1 - FLa)\eta, 1 - (1 - FUa)\eta] \rangle$, where $\eta > 0$.

Definition2.5:

$$x \ominus y = \langle [TLx - FUy, TUx - FLy],$$

$$[\text{Max}(IL\text{x}, ILy), \text{Max}(IU\text{x}, lUy)], \times\ [FLx - TUy, FUx - TLy] \rangle$$

whereas $\chi = \langle [TLx, TUx], [IL\text{x}, IUx][FLx, FUx] \rangle$

and $y = \langle [TLy, TUy], [IL\text{y}, IUy], [FLy, FUy$

Definition2.6:

Deneutrosophic IVNSs as:

$$D = \frac{\left[\frac{TL + TU}{2}\right] + \beta \left[\frac{IL + lU}{2}\right] + (1 + \beta) \left[\frac{FL + FU}{2}\right]}{2}$$

Definition2.7:

$$T(x) = \begin{cases} TL(x) & a^1 \leq x \leq b^1 \\ \alpha & b^1 \leq x \leq c^1 \\ TU\ (x) & c^1 \leq x \leq d^1 \\ 0 & orherwise \end{cases}$$

$$I(x) = \begin{cases} IL(x)\ a^2 \leq x \leq b^2 \\ \beta\ b^2 \leq x \leq c^2 \\ IU(x)\ c^2 \leq x \leq d^2 \\ 1\ orherwise \end{cases}$$

$$F(x) = \begin{cases} FL(x) & a^3 \leq x \leq b^3 \\ \gamma & b^3 \leq x \leq c^3 \\ FU(x) & c^3 \leq x \leq d^3 \\ 1 & orherwise \end{cases}$$

D. Stage IV: Model Optimization Process

Eventually, the model parameter fine-tuning procedure is executed through ISHO model to improve the prediction performance. The SHO system is a strong meta-heuristic method stimulated by sharks' foraging behavior [22]. This model imitates the hunting tactics of sharks to discover and use the searching region successfully, making it mostly suited for tuning the parameters of ML methods. The model has certain norms, which is provided below:

The "wounded fish" is considered victim. The sharks identify these "blood traces" and directs them towards the target. Here, "wounded fish" signifies the best performance. The numerical representation of SHO is described as shown:

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1)-Initially, the primary sites and velocity of sharks were outlined below:

$$X_i^1 = \left[x_{i,1}^1, x_{i,2}^1, \dots, x_{i,n_d}^1\right] \tag{8}$$

$$V_i^1 = \left[v_{i,1}^1, v_{i,2}^1, \dots, v_{i,n_d}^1\right] \tag{9}$$

Whereas X_i^1 denote group of locations of the sharks. $x_{i,1}^1$ and $x_{i,2}^1$ refers to place in the first and second size of ith shark, nd means dimension counts. y_j^1 signifies shark's group of velocities, and $v_{i,1}^1$ denote velocity of ith shark.

2)-Every shark's velocity is upgraded to move towards the victim according to the fitness values. Eq. (10) signifies the updating velocity rule in the SHO model. The velocity limiter avoids unnecessary movement that may result in divergence or instability in the searching procedure. By limiting the velocity, the model keeps a balance among exploitation and exploration, safeguarding smoother convergence towards the best solution,

$$\left|v_{i,j}^k\right| = \min\left[\left|\beta_k R_1' \frac{\partial(OF)}{\partial(x_l)}\right| \mathrm{x}_{\mathrm{i,j}}, +\varepsilon_k R_2' v_{i,j}^{k-1}\right|, \left|\beta_k v_{i,j}^k\right|\right] \tag{10}$$

Whereas β_k denote gradient coefficient, R_1' and R_2' means randomly generated numerals, OF signifies objective function. ε_k means velocity limiter. $v_{i,j}^{k-1}$ symbolize velocity of ith shark in jth size, k stands for step counts.

3)-The sharks upgrade their places after upgrading their speeds. This method permits the model to iteratively improve the searching and enhance an excellence over the following iteration. Eq. (11) denotes the regulation of upgrading the position of sharks.

$$Y_i^{k+1} = X_i^k + V_i^k \Delta t \tag{11}$$

Whereas Δt denotes time period for kth phase.

4)- Sharks apply rotational movements to improve their capability to find target. This rotating effort is demonstrated as the local searching method, which permits sharks to discover the locality of advantageous results. The rotating movement presents diversity into the searching procedure, avoiding early convergence. Eq. (12) characterizes the rotational movement that can be applied to fine-tune the shark's position.

$$Z_j^{k+1,m'} = Y_i^{k+1} + RX_i^{k+1} \tag{12}$$

Whereas R_3' denote a randomly generated number, $Z_i^{k+1,m'}$ means new place of shark, and m stands for phase counts.

5- The shark's last place is established according to succeeding Eq. (13)

$$X_i^{k+1} = \mathrm{argmax}\left\{OF\left(Y_i^{k+1}\right), OF\left(Z_i^{k+1,1}\right), OF\left(Z_i^{k+1,2}\right), \dots, \left(Z_i^{k+1,M}\right)\right\} \tag{13}$$

Whereas M represents rotational movement counts and X_i^{k+1}signifies final position of the sharks. argmax Detects the input value (s) that maximize the functional outcome. Then, SHO may get stuck in local bests, which can lower the complete effectiveness and efficiency of the optimizer procedure.

The ISHO model utilizes a Gaussian mutation operator for overcoming the limits of original SHO. It includes arbitrary values, allowing the model to escape from local bests.

$$X_i^{mutated} = X_i^{k+1} + N(0, \sigma) \tag{14}$$

$$\sigma = \sigma_{initial} \left(\frac{\max(iteration) - t}{\max(iteration)}\right)^{n'} \tag{15}$$

Whereas $X_i^{mutated}$ refers to upgraded place of, $N(0, \sigma)$ signifies arbitrary value with σ and 0. $\sigma_{initial}$ denote primary value of σ, max (iteration) signifies maximal no. of iteration, t symbolizes no. of iteration, and γ stands for controller parameter that establishes the decay value of SD in time. Here, the ISHO can be employed to fix the hyperparameter involved in IVNS method. The MSE measures the objective function as below.

$$MSE = \frac{1}{T} \sum_{j=1}^{L} \sum_{i=1}^{M} \left(y_j^i - d_j^i\right)^2 \tag{16}$$

Here, M and L epitomize the resultant value of layer and data. d_j^i and y_j^i specifies the appropriate and accomplished sizes for j^{th} unit in tth time , respectively.

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4. Model Assessment and Discussion

The performance validation of CLVE-IVNSPOA system is verified under FinTech Customer Life Time Value (LTV) dataset [23]. This dataset aids to predict the LTV for consumers of digital wallets. It holds 19 features in total but only 16 are selected.

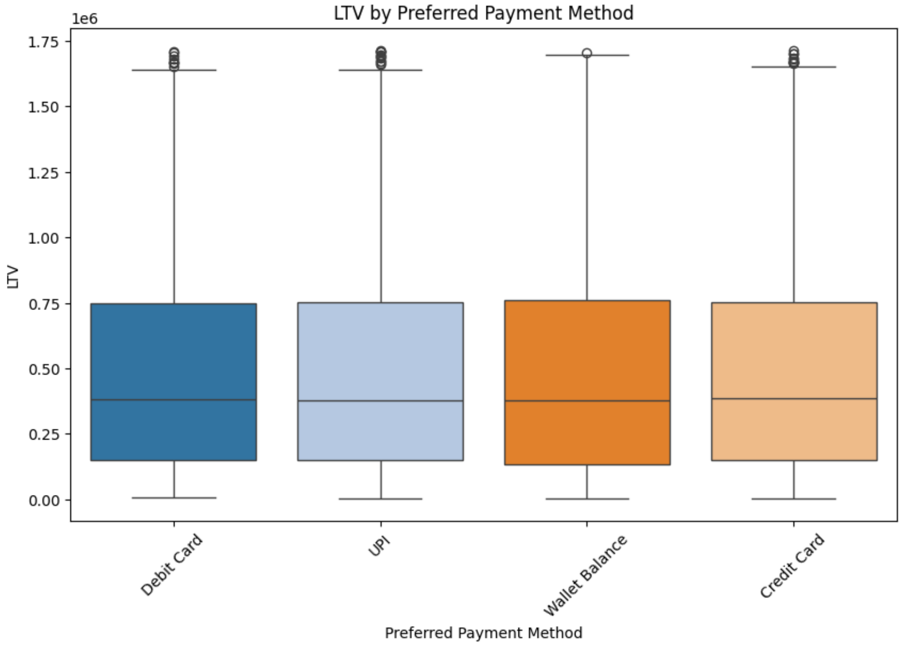

Figure 2. LTV graph under preferred payment method

Fig. 2 exemplifies the distribution of LTV across diverse preferred payment methods on Debit Card, UPI, Wallet Balance, and Credit Card. Across all types, the LTV distributions appear consistent, with similar medians and interquartile ranges, denoting that the selection of payment method does not significantly affect LTV. All groups display a wide spread of values and several high outliers, indicating that some customers contribute much more value irrespective of payment preference.

Table 1 depict the training phase (TRAPH) and testing phase (TESPH) of CLVE-IVNSPOA model with distinct metrics. On TRAPH, the proposed CLVE-IVNSPOA model got MSE of 0.000054, RMSE of 0.007382, MAE of 0.00575, MAPE of 0.102552, and r2-Score of 0.998923. Besides, at TESPH, the proposed CLVE-IVNSPOA model got MSE of 0.000055, RMSE of 0.007433, MAE of 0.005719, MAPE of 0.098022, and r2-Score of 0.998888.

Table 1: TRAPH and TESPH outcome of CLVE-IVNSPOA model under various metrics

Matrices	Training Phase	Testing Phase
MSE	0.000054	0.000055
RMSE	0.007382	0.007433
MAE	0.00575	0.005719
MAPE	0.102552	0.098022
r2 Score	0.998923	0.998888

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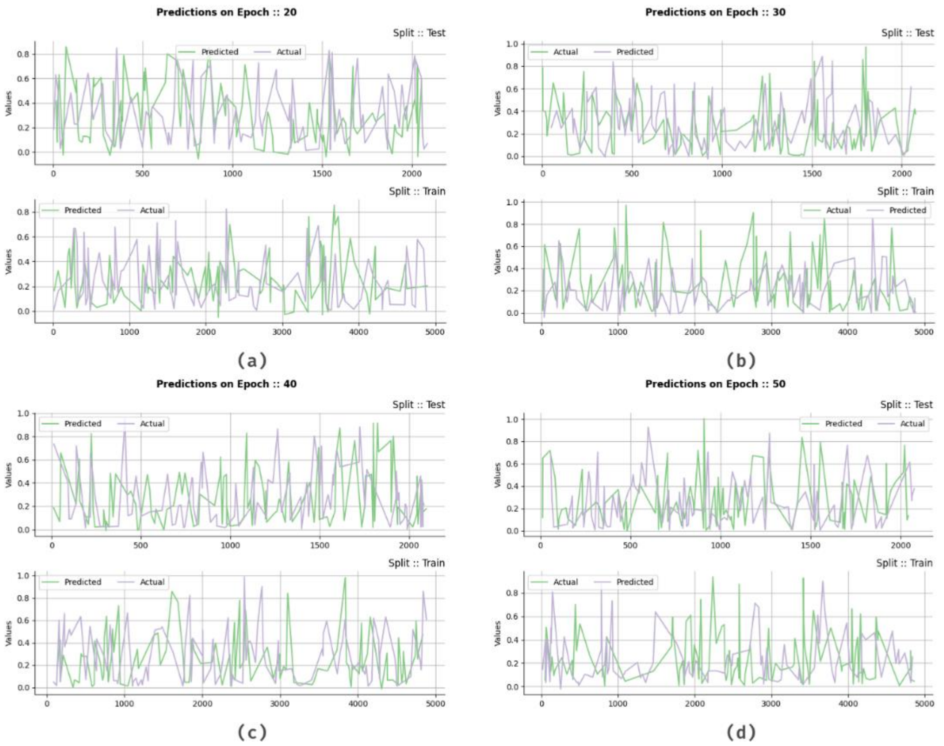

Predictions on Epoch :: 20

Predictions on Epoch :: 30

(a)

(b)

Predictions on Epoch :: 40

Predictions on Epoch :: 50

(c)

(d)

Figure 3. Result Analysis for Actual vs Predicted of CLVE-IVNSPOA model (a-d) Epochs 20-50

Fig. 3 displays the result analysis for the actual vs predicted CLVE-IVNSPOA model under various epochs. The figure denotes that the CLVE-IVNSPOA model accurately predicted the outcome. It is also observed that the values predicted by the CLVE-IVNSPOA model are closer to the actual values.

Table 2 present experimental values that indicate the CLVE-IVNSPOA model has better performances regarding MSE, RMSE, MAE, and MAPE [11, 24-25]. Whereas, on MSE, the proposed CLVE-IVNSPOA model got a lower MSE of 0.000054, while the current methodologies, such as LightGBM, ANN, Ensemble Model, OTE-SUB, Random Forest, kNN, and Catboosting, have obtained the highest MSE of 0.004904, 0.004374, 0.003574, 0.002764, 0.002214, 0.001664, and 0.000774, correspondingly. Besides, at MAPE, the developed CLVE-IVNSPOA model got a minimum MAPE of 0.102552, while the LightGBM, ANN, Ensemble Model, OTE-SUB, Random Forest, kNN, and Catboosting methodologies got a maximum MAPE of 0.150652, 0.145352, 0.136352, 0.130752, 0.123852, 0.116152, and 0.110452, respectively.

Table 2: Comparative study of CLVE-IVNSPOA model with recent techniques

Techniques	MSE	RMSE	MAE	MAPE
LightGBM	0.004904	0.049882	0.05565	0.150652
ANN	0.004374	0.044882	0.05005	0.145352
Ensemble Model	0.003574	0.038982	0.04445	0.136352
OTE-SUB	0.002764	0.032182	0.03555	0.130752
Random Forest	0.002214	0.027182	0.02895	0.123852
kNN	0.001664	0.020782	0.02175	0.116152
Catboosting	0.000774	0.015682	0.01395	0.110452
CLVE-IVNSPOA	0.000054	0.007382	0.00575	0.102552

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5. Conclusion

In this study, we have proposed a CLVE-IVNSPOA technique. The main intention is to progress a predictive analytics technique to estimate customer lifetime value in digital banking. At first, the data pre-processing stage is employed by utilizing the Z-score method. Additionally, the POA is mostly used by the feature subset selection to pick the best features from a dataset. For CLV prediction, the IVNS technique has been utilized. Eventually, the model parameter fine-tuning procedure is executed through the ISHO model to improve the prediction performance. The performance assessment of the CLVE-IVNSPOA model occurs utilizing benchmark database. The empirical outcomes indicated the greater performance of the CLVE-IVNSPOA model when equated to existing approaches.

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