



On Division of Symbolic n-Plithogenic Numbers

P. Arulpandy¹, S. Kalaiselvan^{2,*}, M. Sundar³, G. Govindharaj⁴, P. Sugapriya⁵

¹Department of Mathematics, Christ University, Bengaluru - 560029, India

²Department of Mathematics, Bannari Amman Institute of Technology, Erode - 638401, Tamil Nadu, India

³Department of Mathematics, Thiagarajar College of Engineering, Madurai - 625015, Tamilnadu, India

⁴Department of Mathematics, M. Kumarasamy College of Engineering, Karur - 639113, Tamilnadu, India

⁵Department of Mathematics, K. Ramakrishnan College of Engineering, Trichy - 621112, Tamilnadu, India

Emails: arulpandy.p@christuniversity.in; priankalai@gmail.com; msrmat@tce.edu;
govindharaj88@gmail.com; sugapriya.mat10@gmail.com

Abstract

The main goal of this article is to study the division of symbolic n-plithogenic numbers using the identification method and n-plithogenic AH-isometry. In particular, we discuss the division of symbolic 2-plithogenic numbers and 3-plithogenic numbers, and we generalize these divisions. Additionally, we prove the validity of the formulas using AH-isometry and provide four worked examples to enhance understanding.

Keywords: Symbolic 2-plithogenic numbers; 3-plithogenic numbers; Symbolic n-plithogenic numbers; Division

1 Introduction

Several researchers have explored the idea of refined neutrosophic structures.¹⁻⁴ Smarandache introduced symbolic plithogenic algebraic structures, which closely resemble refined neutrosophic structures, though they differ somewhat in the way the multiplication operation is defined.¹⁰

In,⁹ the algebraic characteristics of symbolic 2-plithogenic rings formed by combining symbolic plithogenic sets with algebraic rings are examined, including foundational properties and substructures such as AH-ideals, AH-homomorphisms, and AHS-isomorphisms. Subsequently, various researchers have introduced and investigated symbolic 3-plithogenic algebraic structures, including symbolic 3-plithogenic rings, vector spaces, and modules.^{5,6,8}

Recently, Alhasan et, al.⁷ studied the division of 3-Refined neutrosophic numbers by using the identification method and the 3-refined AH-isometry. Motivated by this work, in this article we studied the algorithm for finding the division of symbolic 2-plithogenic numbers, 3-plithogenic numbers and n-plithogenic numbers using the identification method and AH-isometry. Further, we proved that the results by all of these methods remains same through examples.

2 Division of Symbolic 2-Plithogenic Numbers using Identification Method

Let $l_1 = a_1 + b_1P_1 + c_1P_2$ and $l_2 = a_2 + b_2P_1 + c_2P_2$ be any two Symbolic 2-plithogenic numbers.

To find $\frac{l_1}{l_2}$:

Let $\frac{l_1}{l_2} = x + yP_1 + zP_2$ (say), where x, y, z real unknowns. Then

$$\frac{a_1 + b_1P_1 + c_1P_2}{a_2 + b_2P_1 + c_2P_2} = x + yP_1 + zP_2$$

$$\begin{aligned} \Rightarrow a_1 + b_1P_1 + c_1P_2 &= (x + yP_1 + zP_2)(a_2 + b_2P_1 + c_2P_2) \\ &= a_2x + a_2yP_1 + a_2zP_2 + b_2xP_1 + b_2yP_1^2 + b_2zP_1P_2 + c_2P_2x \\ &\quad + c_2P_2yP_1 + c_2zP_2^2 \\ &= a_2x + a_2yP_1 + a_2zP_2 + b_2xP_1 + b_2yP_1 + b_2zP_2 + c_2xP_2 + c_2yP_2 + c_2zP_2 \\ a_1 + b_1P_1 + c_1P_2 &= a_2x + (a_2y + b_2x + b_2y)P_1 + (a_2z + b_2z + c_2x + c_2y + c_2z)P_2 \end{aligned}$$

Equating the coefficients of 2-plithogenic numbers, we get

$$a_2x = a_1 \tag{1}$$

$$b_2x + (a_2 + b_2)y = b_1 \tag{2}$$

$$c_2x + c_2y + (a_2 + b_2 + c_2)z = c_1 \tag{3}$$

The system of equations (1), (2), (3) has a unique solution if $\Delta \neq 0$

$$\begin{vmatrix} a_2 & 0 & 0 \\ b_2 & a_2 + b_2 & 0 \\ c_2 & c_2 & a_2 + b_2 + c_2 \end{vmatrix} \neq 0$$

$$a_2(a_2 + b_2)(a_2 + b_2 + c_2) \neq 0$$

$$a_2 \neq 0, a_2 + b_2 \neq 0, a_2 + b_2 + c_2 \neq 0$$

Equation (1) implies

$$x = \frac{a_1}{a_2}$$

Substituting the value of x in Equation (2), we get

$$b_2 \left(\frac{a_1}{a_2} \right) + (a_2 + b_2)y = b_1$$

$$\Rightarrow y = \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}$$

Substituting the value of x and y in Equation (3), we get

$$c_2x + c_2y + (a_2 + b_2 + c_2)z = c_1$$

$$c_2 \left(\frac{a_1}{a_2} \right) + c_2 \left(\frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)} \right) + (a_2 + b_2 + c_2)z = c_1$$

$$\Rightarrow z = \frac{c_1(a_2 + b_2) - c_2(a_1 + b_1)}{(a_2 + b_2)(a_2 + b_2 + c_2)}$$

Hence the algorithm for the division of two symbolic 2-plithogenic numbers is

$$\frac{a_1 + b_1P_1 + c_1P_2}{a_2 + b_2P_1 + c_2P_2} = \frac{a_1}{a_2} + \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}P_1 + \frac{c_1(a_2 + b_2) - c_2(a_1 + b_1)}{(a_2 + b_2)(a_2 + b_2 + c_2)}P_2$$

exists only if $a_2 \neq 0, a_2 + b_2 \neq 0, a_2 + b_2 + c_2 \neq 0$

3 Division of Symbolic 3-Plithogenic Numbers using Identification Method

Let $l_1 = a_1 + b_1P_1 + c_1P_2 + d_1P_3$ and $l_2 = a_2 + b_2P_1 + c_2P_2 + d_2P_3$ be any two Symbolic 3-plithogenic numbers.

To find $\frac{l_1}{l_2}$:

Let $\frac{l_1}{l_2} = x + yP_1 + zP_2 + wP_3$ (say), where x, y, z, w real unknowns. Then

$$\frac{a_1 + b_1P_1 + c_1P_2 + d_1P_3}{a_2 + b_2P_1 + c_2P_2 + d_2P_3} = x + yP_1 + zP_2 + wP_3$$

This implies that

$$a_1 + b_1P_1 + c_1P_2 + d_1P_3 = a_2x + (a_2y + b_2x + b_2y)P_1 + (a_2z + b_2z + c_2x + c_2y + c_2z)P_2 + (a_2w + b_2w + c_2w + d_2x + d_2y + d_2z + d_2w)P_3$$

Equating the coefficients of 3-plithogenic numbers, we get

$$a_2x = a_1 \tag{4}$$

$$b_2x + (a_2 + b_2)y = b_1 \tag{5}$$

$$c_2x + c_2y + (a_2 + b_2 + c_2)z = c_1 \tag{6}$$

$$d_2x + d_2y + d_2z + (a_2 + b_2 + c_2 + d_2)w = d_1 \tag{7}$$

The system of equations (4), (5), (6)&(7) has a unique solution if $\Delta \neq 0$

$$\begin{vmatrix} a_2 & 0 & 0 & 0 \\ b_2 & a_2 + b_2 & 0 & 0 \\ c_2 & c_2 & a_2 + b_2 + c_2 & 0 \\ d_2 & d_2 & d_2 & a_2 + b_2 + c_2 + d_2 \end{vmatrix} \neq 0$$

$$a_2(a_2 + b_2)(a_2 + b_2 + c_2)(a_2 + b_2 + c_2 + d_2) \neq 0$$

$$a_2 \neq 0, a_2 + b_2 \neq 0, a_2 + b_2 + c_2 \neq 0, a_2 + b_2 + c_2 + d_2 \neq 0$$

Equation (4) implies

$$x = \frac{a_1}{a_2}$$

Substituting the value of x in Equation (5), we get

$$b_2 \left(\frac{a_1}{a_2} \right) + (a_2 + b_2)y = b_1$$

$$y = \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}$$

Substituting the value of x and y in Equation (6), we get

$$c_2x + c_2y + (a_2 + b_2 + c_2)z = c_1$$

$$c_2 \left(\frac{a_1}{a_2} \right) + c_2 \left(\frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)} \right) + (a_2 + b_2 + c_2)z = c_1$$

$$(a_2 + b_2 + c_2)z = c_1 - \frac{c_2a_1}{a_2} - \frac{c_2(a_2b_1 - a_1b_2)}{a_2(a_2 + b_2)}$$

$$(a_2 + b_2 + c_2)z = \frac{c_1a_2(a_2 + b_2) - c_2a_1(a_2 + b_2) - c_2a_2b_1 + c_2a_1b_2}{a_2(a_2 + b_2)}$$

$$\begin{aligned}
 (a_2 + b_2 + c_2)z &= \frac{a_2^2c_1 + a_2b_2c_1 - a_2a_1c_2 - a_1b_2c_2 - a_2b_1c_2 + a_1b_2c_2}{a_2(a_2 + b_2)} \\
 (a_2 + b_2 + c_2)z &= \frac{a_2c_1 + b_2c_1 - a_1c_2 - b_1c_2}{a_2 + b_2} \\
 z &= \frac{c_1(a_2 + b_2) - c_2(a_1 + b_1)}{(a_2 + b_2)(a_2 + b_2 + c_2)}
 \end{aligned}$$

Substituting the value of x , y and z in Equation (7), we get

$$\begin{aligned}
 d_2x + d_2y + d_2z + (a_2 + b_2 + c_2 + d_2)w &= d_1 \\
 (a_2 + b_2 + c_2 + d_2)w &= d_1 - d_2x - d_2y - d_2z \\
 &= d_1 - d_2 \left(\frac{a_1}{a_2} \right) - d_2 \left(\frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)} \right) - d_2 \left(\frac{c_1(a_2 + b_2) - c_2(a_1 + b_1)}{(a_2 + b_2)(a_2 + b_2 + c_2)} \right) \\
 &= \frac{d_1a_2[a_2^2 + 2a_2b_2 + b_2^2 + a_2c_2 + b_2c_2] - d_2[a_2^2(a_1 + b_1 + c_1) + a_1a_2b_2 + a_2b_2b_1 + a_2b_2c_1]}{a_2(a_2 + b_2)(a_2 + b_2 + c_2)} \\
 &= \frac{d_1[(a_2 + b_2)^2 + c_2(a_2 + b_2)] - d_2[a_2(a_1 + b_1 + c_1) + b_2(a_1 + b_1 + c_1)]}{(a_2 + b_2)(a_2 + b_2 + c_2)} \\
 &= \frac{d_1(a_2 + b_2)(a_2 + b_2 + c_2) - d_2(a_1 + b_1 + c_1)(a_2 + b_2)}{(a_2 + b_2)(a_2 + b_2 + c_2)} \\
 &= \frac{d_1(a_2 + b_2 + c_2) - d_2(a_1 + b_1 + c_1)}{a_2 + b_2 + c_2} \\
 w &= \frac{d_1(a_2 + b_2 + c_2) - d_2(a_1 + b_1 + c_1)}{(a_2 + b_2 + c_2)(a_2 + b_2 + c_2 + d_2)}
 \end{aligned}$$

Hence the algorithm for the division of two symbolic 3-plithogenic numbers is

$$\frac{a_1 + b_1P_1 + c_1P_2 + d_1P_3}{a_2 + b_2P_1 + c_2P_2 + d_2P_3} = \frac{a_1}{a_2} + \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}P_1 + \frac{c_1(a_2 + b_2) - c_2(a_1 + b_1)}{(a_2 + b_2)(a_2 + b_2 + c_2)}P_2 + \frac{d_1(a_2 + b_2 + c_2) - d_2(a_1 + b_1 + c_1)}{(a_2 + b_2 + c_2)(a_2 + b_2 + c_2 + d_2)}P_3$$

exists only if

$$a_2 \neq 0, \quad a_2 + b_2 \neq 0, \quad a_2 + b_2 + c_2 \neq 0, \quad a_2 + b_2 + c_2 + d_2 \neq 0$$

4 Division of Symbolic n-Plithogenic Numbers using Identification Method

From the division algorithm of symbolic 2-plithogenic and 3-plithogenic, we can generalize the division algorithm for symbolic n-plithogenic numbers.

Let $l_1 = a_0 + a_1P_1 + a_2P_2 + a_3P_3 + \dots + a_nP_n$ and $l_2 = b_0 + b_1P_1 + b_2P_2 + b_3P_3 + \dots + b_nP_n$ be any two Symbolic n-plithogenic numbers. Then the division algorithm for two n-plithogenic numbers is defined by

$$\begin{aligned}
 \frac{l_1}{l_2} &= \frac{a_0 + a_1P_1 + a_2P_2 + a_3P_3 + \dots + a_nP_n}{b_0 + b_1P_1 + b_2P_2 + b_3P_3 + \dots + b_nP_n} = \frac{a_0}{b_0} + \frac{b_0a_1 - a_0b_1}{b_0(b_0 + b_1)}P_1 + \frac{a_2(b_0 + b_1) - b_2(a_0 + a_1)}{(b_0 + b_1)(b_0 + b_1 + b_2)}P_2 \\
 &\quad + \frac{a_3(b_0 + b_1 + b_2) - b_3(a_0 + a_1 + a_2)}{(b_0 + b_1 + b_2)(b_0 + b_1 + b_2 + b_3)}P_3 + \frac{a_4(b_0 + b_1 + b_2 + b_3) - b_4(a_0 + a_1 + a_2 + a_3)}{(b_0 + b_1 + b_2 + b_3)(b_0 + b_1 + b_2 + b_3 + b_4)}P_4 \\
 &\quad + \dots + \frac{a_n(b_0 + b_1 + b_2 + b_3 + \dots + b_{n-1}) - b_n(a_0 + a_1 + a_2 + a_3 + \dots + a_{n-1})}{(b_0 + b_1 + b_2 + b_3 + \dots + b_{n-1})(b_0 + b_1 + b_2 + b_3 + b_4 + \dots + b_n)}P_n
 \end{aligned}$$

exists only if

$$b_0 \neq 0, \quad b_0 + b_1 \neq 0, \quad b_0 + b_1 + b_2 \neq 0, \quad b_0 + b_1 + b_2 + b_3 \neq 0, \quad b_0 + b_1 + b_2 + b_3 + b_4 \neq 0, \dots, \quad b_0 + b_1 + b_2 + b_3 + \dots + b_{n-1} \neq 0, \quad b_0 + b_1 + b_2 + b_3 + \dots + b_n \neq 0$$

Proof. We prove this result by mathematical induction method.

Let $\frac{l_1}{l_2} = Q \Rightarrow Ql_2 = l_1$,

where $Q = q_0 + q_1P_1 + q_2P_2 + q_3P_3 + \dots + q_nP_n$ is a quotient.

$$q_0 = \frac{a_0}{b_0}, q_1 = \frac{b_0a_1 - a_0b_1}{b_0(b_0 + b_1)}, q_2 = \frac{a_2(b_0 + b_1) - b_2(a_0 + a_1)}{(b_0 + b_1)(b_0 + b_1 + b_2)}, \dots$$

Step 1: (Base case $n = 2$)

$l_1 = a_0 + a_1P_1 + a_2P_2, l_2 = b_0 + b_1P_1 + b_2P_2$ and $Q = q_0 + q_1P_1 + q_2P_2$

$$\begin{aligned} Ql_2 &= (q_0 + q_1P_1 + q_2P_2)(b_0 + b_1P_1 + b_2P_2) \\ &= b_0q_0 + (b_1q_0 + b_0q_1 + b_1q_1)P_1 + (b_2q_0 + b_2q_1 + b_0q_2 + b_1q_2 + b_2q_2)P_2 \\ &= b_0 \left(\frac{a_0}{b_0} \right) + \left(b_1 \left(\frac{a_0}{b_0} \right) + (b_0 + b_1) \left(\frac{b_0a_1 - a_0b_1}{b_0(b_0 + b_1)} \right) \right) P_1 \\ &\quad + \left(b_2 \left(\frac{a_0}{b_0} \right) + b_2 \left(\frac{b_0a_1 - a_0b_1}{b_0(b_0 + b_1)} \right) + (b_0 + b_1 + b_2) \left(\frac{a_2(b_0 + b_1) - b_2(a_0 + a_1)}{(b_0 + b_1)(b_0 + b_1 + b_2)} \right) \right) P_2 \\ &= a_0 + a_1P_1 + a_2P_2 \\ &= l_1 \end{aligned}$$

Hence, the base case is true.

Step 2: Assume the formula is true for $n = k$.

$$l_1^{(k)} = \sum_{i=0}^k a_iP_i, \quad l_2^{(k)} = \sum_{i=0}^k b_iP_i,$$

we have:

$$\frac{l_1^{(k)}}{l_2^{(k)}} = Q^{(k)} \Rightarrow Q^{(k)}l_2^{(k)} = l_1^{(k)}$$

where $Q^{(k)} = \sum_{i=0}^k q_iP_i$

Step 3: We have to prove the formula holds for $n = k + 1$.

$$\begin{aligned} l_1^{(k+1)} &= l_1^{(k)} + a_{k+1}P_{k+1} \\ l_2^{(k+1)} &= l_2^{(k)} + b_{k+1}P_{k+1} \\ Q^{(k+1)} &= Q^{(k)} + q_{k+1}P_{k+1} \end{aligned}$$

We have to show that

$$Q^{(k+1)}l_2^{(k+1)} = l_1^{(k+1)}$$

$$\begin{aligned} Q^{(k+1)}l_2^{(k+1)} &= (Q^{(k)} + q_{k+1}P_{k+1}) (l_2^{(k)} + b_{k+1}P_{k+1}) \\ &= Q^{(k)}l_2^{(k)} + b_{k+1}Q^{(k)}P_{k+1} + q_{k+1}l_2^{(k)}P_{k+1} + q_{k+1}b_{k+1}P_{k+1} \end{aligned} \tag{8}$$

$$l_1^{(k+1)} = l_1^{(k)} + a_{k+1}P_{k+1} \tag{9}$$

Equating the coefficients of P_{k+1} in (8) and (9)

$$\begin{aligned} a_{k+1} &= b_{k+1}Q^{(k)} + q_{k+1}l_2^{(k)} + q_{k+1}b_{k+1} \\ q_{k+1} &= \frac{a_{k+1} - b_{k+1}Q^{(k)}}{l_2^{(k)} + b_{k+1}} \end{aligned}$$

Which matches the recursive structure in the general formula. Thus, assuming the formula holds for k , it also holds for $k + 1$ under the conditions that $l_2^{(k)} + b_{k+1} \neq 0$.

□

5 Division of Symbolic 2-Plithogenic numbers using 2-Plithogenic AH-Isometry

Let $f : R[P_1, P_2] \times R[P_1, P_2] \rightarrow R[P_1, P_2]$ be a literal 2-Plithogenic function of two variables. Then

$$\begin{aligned} f(a_1 + b_1P_1 + c_1P_2, a_2 + b_2P_1 + c_2P_2) &= f(a_1, a_2) + [f(a_2 + b_2, a_1 + b_1) - f(a_2, a_1)]P_1 \\ &\quad + [f(a_2 + b_2 + c_2, a_1 + b_1 + c_1) - f(a_2 + b_2, a_1 + b_1)]P_2 \\ &\equiv \frac{a_1}{a_2} + \left[\frac{a_1 + b_1}{a_2 + b_2} - \frac{a_1}{a_2} \right] P_1 + \left[\frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} - \frac{a_1 + b_1}{a_2 + b_2} \right] P_2 \end{aligned}$$

Proof. Here, we derive the division algorithm of 2-plithogenic AH-isometry from identification method. From the division algorithm of two 2-plithogenic numbers by identification method, we have

$$\begin{aligned} \frac{a_1 + b_1P_1 + c_1P_2}{a_2 + b_2P_1 + c_2P_2} &= \frac{a_1}{a_2} + \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)} P_1 + \frac{c_1(a_2 + b_2) - c_2(a_1 + b_1)}{(a_2 + b_2)(a_2 + b_2 + c_2)} P_2 \\ &= \frac{a_1}{a_2} + \frac{a_2b_1 + a_2a_1 - a_2a_1 - a_1b_2}{a_2(a_2 + b_2)} P_1 \\ &\quad + \frac{c_1a_2 + c_1b_2 + a_2a_1 + a_2b_1 + a_1b_2 + b_1b_2 - c_2a_1 - c_2b_1 - a_2a_1 - a_2b_1 - a_1b_2 - b_1b_2}{(a_2 + b_2)(a_2 + b_2 + c_2)} P_2 \\ &= \frac{a_1}{a_2} + \frac{a_2(a_1 + b_1) - a_1(a_2 + b_2)}{a_2(a_2 + b_2)} P_1 \\ &\quad + \frac{a_2(a_1 + b_1 + c_1) + b_2(a_1 + b_1 + c_1) - a_1((a_2 + b_2 + c_2) - b_1(a_2 + b_2 + c_2))}{(a_2 + b_2)(a_2 + b_2 + c_2)} P_2 \\ &= \frac{a_1}{a_2} + \frac{a_2(a_1 + b_1) - a_1(a_2 + b_2)}{a_2(a_2 + b_2)} P_1 + \frac{(a_1 + b_1 + c_1)(a_2 + b_2) - (a_2 + b_2 + c_2)(a_1 + b_1)}{(a_2 + b_2)(a_2 + b_2 + c_2)} P_2 \\ &= \frac{a_1}{a_2} + \left[\frac{a_1 + b_1}{a_2 + b_2} - \frac{a_1}{a_2} \right] P_1 + \left[\frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} - \frac{a_1 + b_1}{a_2 + b_2} \right] P_2 \\ &= f(a_1, a_2) + [f(a_2 + b_2, a_1 + b_1) - f(a_2, a_1)]P_1 \\ &\quad + [f(a_2 + b_2 + c_2, a_1 + b_1 + c_1) - f(a_2 + b_2, a_1 + b_1)]P_2 \end{aligned}$$

□

6 Division of Symbolic 3-Plithogenic numbers using 3-Plithogenic AH-Isometry

Let $f : R[P_1, P_2, P_3] \times R[P_1, P_2, P_3] \rightarrow R[P_1, P_2, P_3]$ be a literal 3-Plithogenic function of three variables. Then

$$\begin{aligned} &f(a_1 + b_1P_1 + c_1P_2 + d_1P_3, a_2 + b_2P_1 + c_2P_2 + d_2P_3) \\ &= f(a_1, a_2) + [f(a_2 + b_2, a_1 + b_1) - f(a_2, a_1)]P_1 + [f(a_2 + b_2 + c_2, a_1 + b_1 + c_1) - f(a_2 + b_2, a_1 + b_1)]P_2 \\ &\quad + [f(a_2 + b_2 + c_2 + d_2, a_1 + b_1 + c_1 + d_1) - f(a_2 + b_2 + c_2, a_1 + b_1 + c_1)]P_3 \\ &\equiv \frac{a_1}{a_2} + \left[\frac{a_1 + b_1}{a_2 + b_2} - \frac{a_1}{a_2} \right] P_1 + \left[\frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} - \frac{a_1 + b_1}{a_2 + b_2} \right] P_2 + \left[\frac{a_1 + b_1 + c_1 + d_1}{a_2 + b_2 + c_2 + d_2} - \frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} \right] P_3 \end{aligned}$$

Proof. Let

$$X = \frac{a_1 + b_1P_1 + c_1P_2 + d_1P_3}{a_2 + b_2P_1 + c_2P_2 + d_2P_3}$$

Here, we derive the division algorithm of 3-plithogenic AH-isometry from identification method. From the

division algorithm of two 3-plithogenic numbers by identification method, we have

$$\begin{aligned}
 X &= \frac{a_1}{a_2} + \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}P_1 + \frac{c_1(a_2 + b_2) - c_2(a_1 + b_1)}{(a_2 + b_2)(a_2 + b_2 + c_2)}P_2 + \frac{d_1(a_2 + b_2 + c_2) - d_2(a_1 + b_1 + c_1)}{(a_2 + b_2 + c_2)(a_2 + b_2 + c_2 + d_2)}P_3 \\
 &= \frac{a_1}{a_2} + \left[\frac{a_1 + b_1}{a_2 + b_2} - \frac{a_1}{a_2} \right] P_1 + \left[\frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} - \frac{a_1 + b_1}{a_2 + b_2} \right] P_2 + \left[\frac{a_1 + b_1 + c_1 + d_1}{a_2 + b_2 + c_2 + d_2} - \frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} \right] P_3 \\
 &= f(a_1, a_2) + [f(a_2 + b_2, a_1 + b_1) - f(a_2, a_1)]P_1 + [f(a_2 + b_2 + c_2, a_1 + b_1 + c_1) - f(a_2 + b_2, a_1 + b_1)]P_2 \\
 &\quad + [f(a_2 + b_2 + c_2 + d_2, a_1 + b_1 + c_1 + d_1) - f(a_2 + b_2 + c_2, a_1 + b_1 + c_1)]P_3
 \end{aligned}$$

□

7 Division of Symbolic n-Plithogenic Numbers using n-Plithogenic AH-Isometry

From the division of symbolic 2-plithogenic and 3-plithogenic numbers using AH-Isometry, we can generalize the division algorithm for symbolic n-plithogenic AH-Isometry.

Let $l_1 = a_0 + a_1P_1 + a_2P_2 + a_3P_3 + \dots + a_nP_n$ and $l_2 = b_0 + b_1P_1 + b_2P_2 + b_3P_3 + \dots + b_nP_n$ be any two Symbolic n-plithogenic numbers. Then the division algorithm for two n-plithogenic numbers is defined by

Let $f : R[P_1, P_2, P_3, \dots, P_n] \times R[P_1, P_2, P_3, \dots, P_n] \rightarrow R[P_1, P_2, P_3, \dots, P_n]$ be a literal n-Plithogenic function of n-variables. Then

$$\begin{aligned}
 &f(a_0 + a_1P_1 + a_2P_2 + a_3P_3 + \dots + a_nP_n, b_0 + b_1P_1 + b_2P_2 + b_3P_3 + \dots + b_nP_n) \\
 &= f(a_0, b_0) + [f(b_0 + b_1, a_0 + a_1) - f(b_0, a_0)]P_1 + [f(b_0 + b_1 + b_2, a_0 + a_1 + a_2) - f(b_0 + b_1, a_0 + a_1)]P_2 \\
 &\quad + [f(b_0 + b_1 + b_2 + b_3, a_0 + a_1 + a_2 + a_3) - f(b_0 + b_1 + b_2, a_0 + a_1 + a_2)]P_3 \\
 &\quad + \dots + [f(b_0 + b_1 + b_2 + b_3 + \dots + b_n, a_0 + a_1 + a_2 + a_3 + \dots + a_n) \\
 &\quad - f(b_0 + b_1 + b_2 + \dots + b_{n-1}, a_0 + a_1 + a_2 + \dots + a_{n-1})]P_n \\
 &\equiv \frac{a_0}{b_0} + \left[\frac{a_0 + a_1}{b_0 + b_1} - \frac{a_0}{b_0} \right] P_1 + \left[\frac{a_0 + a_1 + a_2}{b_0 + b_1 + b_2} - \frac{a_0 + a_1}{b_0 + b_1} \right] P_2 + \left[\frac{a_0 + a_1 + a_2 + a_3}{b_0 + b_1 + b_2 + b_3} - \frac{a_0 + a_1 + a_2}{b_0 + b_1 + b_2} \right] P_3 + \dots \\
 &\quad + \left[\frac{a_0 + a_1 + a_2 + a_3 + \dots + a_n}{b_0 + b_1 + b_2 + b_3 + \dots + b_n} - \frac{a_0 + a_1 + a_2 + \dots + a_{n-1}}{b_0 + b_1 + b_2 + \dots + b_{n-1}} \right] P_n
 \end{aligned}$$

8 Examples

In this section, we discuss some examples regarding division of two n-plithogenic numbers using Inversion method, Identification method and AH-Isometry function and to prove that the value remains same in all methods.

Example 8.1. Let

$$X = \frac{1 - 3P_1 + P_2}{10 - P_1 - 7P_2}$$

Inversion method:

$$\begin{aligned}
 X &= (1 - 3P_1 + P_2) \left(\frac{1}{10} + \left(\frac{1}{9} - \frac{1}{10} \right) P_1 + \left(\frac{1}{2} - \frac{1}{9} \right) P_2 \right) \\
 &= (1 - 3P_1 + P_2) \left(\frac{1}{10} + \frac{1}{90}P_1 + \frac{7}{18}P_2 \right) \\
 &= \frac{1}{10} + \frac{1}{90}P_1 + \frac{7}{18}P_2 - \frac{3}{10}P_1 - \frac{3}{90}P_1 - \frac{21}{18}P_2 + \frac{1}{10}P_2 + \frac{1}{90}P_2 + \frac{7}{18}P_2 \\
 &= \frac{1}{10} + \left(\frac{1}{90} - \frac{3}{10} - \frac{3}{90} \right) P_1 + \left(\frac{7}{18} - \frac{21}{18} + \frac{1}{10} + \frac{1}{90} + \frac{7}{18} \right) P_2 \\
 &= \frac{1}{10} + \left(\frac{1 - 27 - 3}{90} \right) P_1 + \left(\frac{-7}{18} + \frac{1}{9} \right) P_2 \\
 &= \frac{1}{10} - \frac{29}{90}P_1 - \frac{5}{18}P_2
 \end{aligned}$$

Identification method:

$$\begin{aligned} X &= \frac{1}{10} + \left(\frac{-30+1}{10(9)}\right) P_1 + \left(\frac{1(9)+7(-2)}{9(2)}\right) P_2 \\ &= \frac{1}{10} - \frac{29}{90} P_1 - \frac{5}{18} P_2 \end{aligned}$$

AH-Isometry method:

$$\begin{aligned} X &= \frac{1}{10} + \left(\frac{-2}{9} - \frac{1}{10}\right) P_1 + \left(\frac{-1}{2} + \frac{2}{9}\right) P_2 \\ &= \frac{1}{10} + \left(\frac{-20-9}{90}\right) P_1 + \left(\frac{-9+4}{18}\right) P_2 \\ &= \frac{1}{10} - \frac{29}{90} P_1 - \frac{5}{18} P_2 \end{aligned}$$

Hence,

$$X = \frac{1}{10} - \frac{29}{90} P_1 - \frac{5}{18} P_2$$

Example 8.2. Let

$$X = \frac{-5 + 4P_1 + P_2 - 2P_3}{3 + 2P_1 + P_2 - P_3}$$

Inversion method:

$$\begin{aligned} X &= (-5 + 4P_1 + P_2 - 2P_3) \left(\frac{1}{3} + \left(\frac{1}{5} - \frac{1}{3}\right) P_1 + \left(\frac{1}{6} - \frac{1}{5}\right) P_2 + \left(\frac{1}{5} - \frac{1}{6}\right) P_3\right) \\ &= (-5 + 4P_1 + P_2 - 2P_3) \left(\frac{1}{3} - \frac{2}{15} P_1 - \frac{1}{30} P_2 + \frac{1}{30} P_3\right) \\ &= \frac{-5}{3} + \frac{2}{3} P_1 + \frac{1}{6} P_2 - \frac{1}{6} P_3 + \frac{4}{3} P_1 - \frac{8}{15} P_1 - \frac{2}{15} P_2 + \frac{2}{15} P_3 \\ &\quad + \frac{1}{3} P_2 - \frac{2}{15} P_2 - \frac{1}{30} P_2 + \frac{1}{30} P_3 - \frac{2}{3} P_3 + \frac{4}{15} P_3 + \frac{1}{15} P_3 - \frac{1}{15} P_3 \\ &= -\frac{5}{3} + \left(\frac{2}{3} + \frac{4}{3} - \frac{8}{15}\right) P_1 + \left(\frac{1}{6} - \frac{1}{30} + \frac{1}{3}\right) P_2 + \left(\frac{-1}{6} + \frac{1}{30} - \frac{2}{3} + \frac{6}{15}\right) P_3 \\ &= -\frac{5}{3} + \left(\frac{30-8}{15}\right) P_1 + \left(\frac{15-1-8}{30}\right) P_2 + \left(\frac{-5-20+12+1}{30}\right) P_3 \\ &= -\frac{5}{3} + \frac{22}{15} P_1 + \frac{1}{5} P_2 - \frac{2}{5} P_3 \end{aligned}$$

Identification method:

$$\begin{aligned} X &= -\frac{5}{3} + \left(\frac{12+10}{3(5)}\right) P_1 + \left(\frac{1(5)-1(-1)}{5(6)}\right) P_2 + \left(\frac{-2(6)+1(0)}{6(5)}\right) P_3 \\ &= -\frac{5}{3} + \frac{22}{15} P_1 + \frac{1}{5} P_2 - \frac{2}{5} P_3 \end{aligned}$$

AH-Isometry method:

$$\begin{aligned} X &= -\frac{5}{3} + \left(\frac{-1}{5} + \frac{5}{3}\right) P_1 + \left(\frac{0}{6} - \frac{-1}{5}\right) P_2 + \left(\frac{-2}{5} - \frac{0}{6}\right) P_3 \\ &= -\frac{5}{3} + \left(\frac{-3+25}{15}\right) P_1 + \left(\frac{1}{5}\right) P_2 + \left(\frac{-2}{5}\right) P_3 \\ &= -\frac{5}{3} + \frac{22}{15} P_1 + \frac{1}{5} P_2 - \frac{2}{5} P_3 \end{aligned}$$

Hence,

$$X = -\frac{5}{3} + \frac{22}{15} P_1 + \frac{1}{5} P_2 - \frac{2}{5} P_3$$

Example 8.3. Let

$$X = \frac{1 - 2P_1 + 3P_2 + 4P_3 - 5P_4}{3 - P_1 + 2P_2 - P_3 + 4P_4}$$

Inversion method:

$$\begin{aligned} X &= (1 - 2P_1 + 3P_2 + 4P_3 - 5P_4) \left(\frac{1}{3} + \left(\frac{1}{2} - \frac{1}{3} \right) P_1 + \left(\frac{1}{4} - \frac{1}{2} \right) P_2 + \left(\frac{1}{3} - \frac{1}{4} \right) P_3 + \left(\frac{1}{7} - \frac{1}{3} \right) P_4 \right) \\ &= (1 - 2P_1 + 3P_2 + 4P_3 - 5P_4) \left(\frac{1}{3} + \frac{1}{6}P_1 - \frac{1}{4}P_2 + \frac{1}{12}P_3 - \frac{4}{21}P_4 \right) \\ &= \frac{1}{3} + \left(\frac{1}{6} - \frac{2}{3} - \frac{1}{3} \right) P_1 + \left(\frac{-1}{4} + \frac{1}{2} + 1 + \frac{1}{2} - \frac{3}{4} \right) P_2 + \left(\frac{-1}{12} - \frac{3}{4} + \frac{7}{3} \right) P_3 + \left(\frac{-4}{21} - \frac{15}{6} + \frac{10}{12} \right) P_4 \\ &= \frac{1}{3} - \frac{5}{6}P_1 + P_2 + \frac{3}{2}P_3 - \frac{13}{7}P_4 \end{aligned}$$

Identification method:

$$\begin{aligned} X &= \frac{1}{3} + \left(\frac{-6+1}{3(2)} \right) P_1 + \left(\frac{3(2) - 2(-1)}{2(4)} \right) P_2 + \left(\frac{4(4) + 1(2)}{4(3)} \right) P_3 + \left(\frac{-5(3) - 4(6)}{3(7)} \right) P_4 \\ &= \frac{1}{3} - \frac{5}{6}P_1 + P_2 + \frac{3}{2}P_3 - \frac{13}{7}P_4 \end{aligned}$$

AH-Isometry method:

$$\begin{aligned} X &= \frac{1}{3} + \left(\frac{-1}{2} - \frac{1}{3} \right) P_1 + \left(\frac{2}{4} - \frac{-1}{2} \right) P_2 + \left(\frac{6}{3} - \frac{2}{4} \right) P_3 + \left(\frac{1}{7} - \frac{6}{3} \right) P_4 \\ &= \frac{1}{3} - \frac{5}{6}P_1 + P_2 + \frac{3}{2}P_3 - \frac{13}{7}P_4 \end{aligned}$$

Hence,

$$X = \frac{1}{3} - \frac{5}{6}P_1 + P_2 + \frac{3}{2}P_3 - \frac{13}{7}P_4$$

Example 8.4. Let

$$X = \frac{3P_1 - 4P_3 + 11P_4 - P_5 + 7P_6 + 2P_7 - P_8}{15 - P_1 + 3P_2 - P_5 + 6P_6 + 2P_8}$$

Inversion method:

$$\begin{aligned} X &= (3P_1 - 4P_3 + 11P_4 - P_5 + 7P_6 + 2P_7 - P_8) \left(\frac{1}{15} + \frac{1}{210}P_1 - \frac{3}{238}P_2 + \frac{1}{272}P_5 - \frac{3}{176}P_6 - \frac{1}{264}P_8 \right) \\ &= \frac{3}{14}P_1 - \frac{9}{238}P_2 - \frac{4}{17}P_3 + \frac{11}{17}P_4 - \frac{7}{272}P_5 + \frac{29}{176}P_6 + \frac{1}{11}P_7 - \frac{29}{264}P_8 \end{aligned}$$

Identification method:

$$\begin{aligned} X &= \frac{0}{15} + \left(\frac{45+0}{15(14)} \right) P_1 + \left(\frac{0(14) - 3(3)}{14(17)} \right) P_2 + \left(\frac{-4(17) - 0(3)}{17(17)} \right) P_3 + \left(\frac{11(17) - 0(-1)}{17(17)} \right) P_4 \\ &\quad + \left(\frac{-1(17) + 1(10)}{17(16)} \right) P_5 + \left(\frac{7(16) - 6(9)}{16(22)} \right) P_6 + \left(\frac{2(22) - 0(16)}{22(22)} \right) P_7 + \left(\frac{-1(22) - 2(18)}{22(24)} \right) P_8 \\ &= \frac{3}{14}P_1 - \frac{9}{238}P_2 - \frac{4}{17}P_3 + \frac{11}{17}P_4 - \frac{7}{272}P_5 + \frac{29}{176}P_6 + \frac{1}{11}P_7 - \frac{29}{264}P_8 \end{aligned}$$

AH-Isometry method:

$$\begin{aligned} X &= \frac{0}{15} + \left(\frac{3}{14} - \frac{0}{15} \right) P_1 + \left(\frac{3}{17} - \frac{3}{14} \right) P_2 + \left(\frac{-1}{17} - \frac{3}{17} \right) P_3 + \left(\frac{10}{17} + \frac{1}{17} \right) P_4 \\ &\quad + \left(\frac{9}{16} - \frac{10}{17} \right) P_5 + \left(\frac{16}{22} - \frac{9}{16} \right) P_6 + \left(\frac{18}{22} - \frac{16}{22} \right) P_7 + \left(\frac{17}{24} - \frac{18}{22} \right) P_8 \\ &= \frac{3}{14}P_1 - \frac{9}{238}P_2 - \frac{4}{17}P_3 + \frac{11}{17}P_4 - \frac{7}{272}P_5 + \frac{29}{176}P_6 + \frac{1}{11}P_7 - \frac{29}{264}P_8 \end{aligned}$$

Hence,

$$X = \frac{3}{14}P_1 - \frac{9}{238}P_2 - \frac{4}{17}P_3 + \frac{11}{17}P_4 - \frac{7}{272}P_5 + \frac{29}{176}P_6 + \frac{1}{11}P_7 - \frac{29}{264}P_8$$

9 Conclusion

This article presents a comprehensive study on the division of symbolic n -plithogenic numbers through the identification method and AH-isometry. We specifically focused on the division of symbolic 2-plithogenic and 3-plithogenic numbers. The validity of the formulas was rigorously proven using AH-isometry, and four worked examples were provided to facilitate a deeper understanding of the topic. The findings contribute to the advancement of mathematical theories related to symbolic n -plithogenic numbers.

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