Some Special Types of Neutrosophic Domains

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Abstract

The neutrosophic ring cannot be an integral domain, but the pseudo-neutrosophic ring could be an integral domain. The main objective of this paper is to present and study some special types of neutrosophic domains, which has not been studied before, such as integral domains.

Keywords: Neutrosophic Ring; Pseudo Neutrosophic Ring; Neutrosophic Polynomial Ring; Neutrosophic Euclidean Ring; Neutrosophic Unique Factorization Domains; Neutrosophic Jacobson Radical; Neutrosophic Noetherian Ring; Neutrosophic Artinian Ring

1. Introduction

The Neutrosophic ring is a new algebraic structure introduced by Smarandache and Kandasamy in 2006 [1, 2, 3]. In recent years, other scholars interested in studying the neutrosophic ring theory, such as Agboola, Akinola, and Oyebola, have been in [4]. They studied some elementary properties of neutrosophic rings and the structure of neutrosophic polynomial rings.

Where they defined a neutrosophic ring with some properties of the structure of its elements, it may not be easy to generalize all of classical algebraic structures using neutrosophic algebraic structures. There are some solved problems in the classical ring, but here in the neutrosophic ring, they need to be studied and explained, for example, neutrosophic integral domains. In this paper, we briefly examine some of the neutrosophic special domains: Neutrosophic Euclidean Ring, Neutrosophic Unique Factorization Domains, Neutrosophic Jacobson Radical, Neutrosophic Noetherian Ring, Neutrosophic Artinian Ring, Neutrosophic Polynomial Rings, and Factorizations in Neutrosophic Polynomial Rings.

2. Preliminaries

In this section, we introduce a review of a neutrosophic ring, a neutrosophic subring, and a neutrosophic Ideal with some key points that will be used in the following sections.

Definition 2.1 [21] Let RBe any ring. The neutrosophic ring is also a ring generated by R and I under the operations of R, and denoted by $N(R)$, such that: $N(R) = \langle R \cup I \rangle = \{a + bI : a, b \in R\}$.

Theorem 2.1 [21] Let $\langle R \cup I \rangle$ be a Neutrosophic ring. Then $\langle R \cup I \rangle$ is ring.

Definition 2.2 [5-14] Let T be a non-empty set with two binary operations $+$ and •, we say that T to be a pseudo-neutrosophic ring if the following conditions are satisfied:
1. T contains an indeterminate element I, at least one element,
2. $(T, +)$ is an abelian group,

DOI: https://doi.org/10.54216/PAMDA.050103
Received: December 10, 2024 Revised: January 03, 2025 Accepted: January 30, 2025

3. $(T, \bullet)$ is a semigroup.
4. $+$ and $\bullet$ satisfy the distributive laws,

Definition 2.3 Let T be a pseudo-neutrosophic commutative ring. Then a nonzero element $x \in T$ is said to be a zero divisor if there exists a nonzero $y \in T$ such that $xy = 0$.

Definition 2.4 A pseudo-neutrosophic commutative ring T is an integral domain if it has no zero divisors.

3. Neutrosophic Polynomial Rings

Definition 3.1 [25] Let $\langle R \cup I \rangle$ be a Neutrosophic ring. the Neutrosophic polynomial with coefficients in $\langle R \cup I \rangle$ is a formal expression: $a_0 + a_1 x + \cdots + a_n x^n$, where n can be any non-negative integer, and the coefficients $a_0, a_1, \ldots, a_n$ are all in $\langle R \cup I \rangle$. then the neutrosophic polynomial ring in the variate x consists of the set for all neutrosophic polynomials with coefficients in $\langle R \cup I \rangle$, and is denoted by $\langle R \cup I \rangle[x]$.

Definition 3.2 [15] Addition and multiplication operations of Neutrosophic polynomials with coefficients in $\langle R \cup I \rangle$ are defined in a way familiar to us, where if $f(x) = a_0 + a_1 x + \ldots + a_n x^n$,
And $g(x) = b_0 + b_1 x + \ldots + b_m x^m$, then for neutrosophic polynomial addition is
$f(x) + g(x) = c_0 + c_1 x + \cdots + c_t x^t$, where $c_i = a_i + b_i$, $t = \max\{n, m\}$ and for neutrosophic polynomial multiplication is $f(x)g(x) = c_0 + c_1 x + \cdots + c_k x^k$, where $k = n + m$, and
$c_k = \sum_{i=o}^{k} a_i b_{k-i}$.

Definition 3.3 Let be $f(x) = \sum_{i=o}^{n} a_i x^i$ a neutrosophic polynomial in$\langle R \cup I \rangle[x]$, then
1. $f(x)$ is called a strong neutrosophic polynomial if a_i is of the form $a + bI$ for every $i \geq 0$, where $a, b \in R, b \neq 0$,
2. $f(x)$ is called a mixed neutrosophic polynomial if some $a_i \in R$ and some a_i of the form $a + bI$, $b \neq 0$,
3. $f(x)$ is called a polynomial if $a_i \in R$ for all i,

Proposition 3.1 [25] If $\langle R \cup I \rangle$ is a neutrosophic ring, then $\langle R \cup I \rangle[x]$ so is ring.

Example 3.1 $\langle Z \cup I \rangle[x]$, $\langle Q \cup I \rangle[x]$, $\langle R \cup I \rangle[x]$, $\langle C \cup I \rangle[x]$, are neutrosophic polynomial rings of integers, rational, real, and complex numbers, respectively. Also $\langle Z_n \cup I \rangle[x]$ is a neutrosophic polynomial ring of integers modulo n.

Proposition 3.2 [14-18] The neutrosophic ring $\langle R \cup I \rangle$ is a subring of $\langle R \cup I \rangle[x]$, namely the subring of Neutrosophic constant polynomials.

Proposition 3.3 Let $\langle R \cup I \rangle$ is a neutrosophic ring. Then
1. if $\langle R \cup I \rangle$ has an identity, then so does $\langle R \cup I \rangle[x]$, and
2. if $\langle R \cup I \rangle$ is commutative, then so is $\langle R \cup I \rangle[x]$.

Definition 3.4 If $f(x) = a_0 + a_1 x + \cdots + a_n x^n$ is a neutrosophic polynomial such that$a_n \neq 0$, then the degree of $f(x)$ is n, and written as $\deg f(x) = n$, where n is a nonnegative integer.

Proposition 3.4 [19] Let $\langle R \cup I \rangle$ be a neutrosophic ring, and let $f(x)$ and $g(x)$ be a nonzero neutrosophic polynomials in $\langle R \cup I \rangle[x]$ of degree n and m respectively, such that $f(x) = a_0 + a_1 x + \cdots + a_n x^n$, $g(x) = b_0 + b_1 x + \cdots + b_n x^n$, Then
1. We say that $f(x)$ is equal $g(x)$ and written $f(x) = g(x)$ if and only if $a_i = b_i$ for all integers $i \geq 0$, and $n = m$.
2. $\deg(f(x) + g(x)) \leq \max\{n, m\}$, or $f(x) + g(x) = 0$; and
3. $\deg(f(x)g(x)) \leq (n + m)$ or $f(x)g(x)$=0,

4. Factorizations in Neutrosophic Polynomial Rings

Definition 4.1 Let $\langle R \cup I \rangle$ be a neutrosophic ring, and
let $f(x) = a_0 + a_1 x + \cdots + a_n x^n \in \langle R \cup I \rangle[x]$, Further, we suppose that. $a_m \neq 0$ but $a_n = 0$ for all $n > m$, Then the degree of $f(x)$ is m, and we write $\deg f(x) = m$, The leading term of $f(x)$ is $a_m x^m$, and the leading

DOI: https://doi.org/10.54216/PAMDA.050103
Received: December 10, 2024 Revised: January 03, 2025 Accepted: January 30, 2025

coefficient is a_m, Note that the neutrosophic zero polynomial. 0 has no degree, leading term, or leading coefficient. A neutrosophic constant polynomial has degree. 0 or is the neutrosophic zero polynomial, if $\langle R \cup I \rangle$ has an identity 1, then $f(x)$ is called a neutrosophic polynomial monic if its leading coefficient is 1.

Definition 4.2 (Division Algorithm)**.** Let $\langle R \cup I \rangle$ be a neutrosophic ring with identity, $f(x)g(x) \in \langle R \cup I \rangle[x]$ and $g(x) \neq 0$, Assume that $f(x), g(x)$ are monic. Then there is a unique $q(x), r(x) \in \langle R \cup I \rangle[x]$ such that $f(x) = q(x)g(x) + r(x)$ where $r(x) = 0$ or $\deg r(x) < \deg g(x)$.

Definition 4.3 Let $\langle R \cup I \rangle[x]$ be a neutrosophic commutative polynomial ring, if $f(x), g(x) \in \langle R \cup I \rangle[x]$, then a greatest common divisor (gcd) of $f(x), g(x)$ is a common divisor $d(x)$ which is divisible by every common divisor $h(x)$; that is, if $h(x)|f(x)$ and $h(x)|g(x)$, then $h(x)|d(x)$.

Example 4.1 Let $f(x) = x2 - Ix - 2I, g(x) = x3 - 7Ix + 6I$ are two neutrosophic polynomials in a neutrosophic polynomial ring $\langle Q \cup I \rangle[x]$. Then the greatest common divisor of $(f(x), g(x))$ is a neutrosophic polynomial $d(x)$ where $d(x) = x - 2I$,

Definition 4.4 [20-22] Let $f(x)$ a neutrosophic polynomial in a neutrosophic polynomial ring $\langle R \cup I \rangle[x]$,then

1. $f(x)$ is called a neutrosophic polynomial reducible if $f(x)$ is neither 0 nor 1, and
   $f(x) = g(x)q(x)$ such that $g(x), q(x)$ are neutrosophic polynomials in $\langle R \cup I \rangle[x]$,
2. $f(x)$ is called a neutrosophic polynomial semi-reducible if $f(x)$ is neither 0 nor 1, such that $f(x) = g(x)q(x)$, but only one of $g(x)$ or $q(x)$ is a neutrosophic polynomial in $\langle R \cup I \rangle[x]$,
3. $f(x)$ is called a neutrosophic polynomial irreducible if $f(x)$ is neither 0 nor 1, such that $f(x) = g(x)q(x)$, but either $g(x)$ or $q(x)$ is I or 1.

Example 4.2 The neutrosophic polynomial $f(x) = 7Ix2 - 3I$ over a neutrosophic polynomial ring $\langle Q \cup I \rangle[x]$ is a neutrosophic polynomial irreducible, but it is a neutrosophic polynomial reducible over a neutrosophic polynomial ring $\langle R \cup I \rangle[x]$.

Definition 4.5 Let $\langle R \cup I \rangle$ be a neutrosophic ring with unit, and we suppose that
$f(x) = a_0 + a_1 x + \cdots + a_n x^n$
is a neutrosophic nonzero polynomial in $\langle R \cup I \rangle[x]$,

1. The neutrosophic content of $f(x)$ is the gcd of $(a_0, a_1, \dots, a_n)$,
2. We say that $f(x)$ is a neutrosophic primitive if the content of $f(x)$ is a unit or I in $\langle R \cup I \rangle$, this means that $(a_0, a_1, \dots, a_n)$ have no common factor except the unit.

Example 4.3 The neutrosophic content of a neutrosophic polynomial $f(x) = x5 + 13Ix2 + (1 + I)x - 5I$ in $\langle R \cup I \rangle[x]$ is 1, therefore the neutrosophic polynomial $f(x)$ is a neutrosophic primitive.

Definition 4.6 The element x in $\langle R \cup I \rangle$ is a root of the neutrosophic polynomial $f(x) \in \langle R \cup I \rangle[x]$ if $f(x) = 0$,

Example 4.4 The neutrosophic polynomial $f(x) = x2 + I$ has roots in $\langle C \cup I \rangle[x]$ they are $f_1 = -iI$ and $f_2 = +iI$. But has no roots in $\langle R \cup I \rangle[x]$,

5. Neutrosophic Euclidean Ring

Definition 5.1 A Pseudo Neutrosophic integral domain T is said to be a Neutrosophic Euclidean Ring if there is a degree function d from the nonzero elements of T to the nonnegative integers
$(d: T/\{0\} \longrightarrow N)$, That satisfies:

1. $d(x) \leq d(xy)$ for all $x, y \in T$, and $x \neq 0, y \neq 0$, and
2. Given $x \neq 0, y \neq 0$ .there is $q, r \in T$, such that $x = yq + r$ where $r = 0$ *or* $d(r) < d(y)$ (Division Algorithm).

Example 5.1 We have $T = \{\dots, -3I, -2I, -I, 0, I, 2I, 3I, \dots\}$ is a pseudo-neutrosophic ring of integer numbers. Clear that T is an integral domain, define the degree function on T as following:

$d: T/\{0\} \longrightarrow N$ via

$$d(x = aI) = \begin{cases} |a| & , if\ x \neq I \\ 1 & , if\ x = I \end{cases} \quad \text{, where } a \in \mathbb{Z},$$

DOI: https://doi.org/10.54216/PAMDA.050103
Received: December 10, 2024 Revised: January 03, 2025 Accepted: January 30, 2025

, then $|xy| = |x||y| \geq |x|$ for all x and y are nonzero in T, further,

for all $x \in T$, then

1. Either exists $q, r \in T$ such that $x = yq + r$; $r = 0, or\ d(r) < d(y)$
2. Or, we can write: $x = x\,I,\ I \in T$.

Theorem 5.1 Let T be a Pseudo Neutrosophic Euclidean domain.

1. Two nonzero elements $x, y\ \in T$, have a greatest common divisor which is contained in the ideal $Tx\ +\ Ty$, The greatest common divisor is unique up to multiplication by a unit.
2. Two elements $x, y\ \in T$ are relatively prime if, and only if $1 \in (Tx\ +\ Ty)$.
3. Every ideal in T is principal.
4. Every irreducible element is prime.
5. Every nonzero, non-unit element has a factorization into irreducibles, which is unique (up to units and up to order of the factors).

6. Neutrosophic Unique Factorization Domains

Definition 6.1 A Pseudo Neutrosophic integral domain T is a unique factorization domain if

1. Every nonzero element of T that is not a unit that can be written as a product of irreducible elements of T; and
2. The factorization into irreducible factors in part (1) is unique up to associates and the order in which the factors appear.

Proposition 6.1 [23-30] If T is a pseudo-neutrosophic unique factorization domain, then a greatest common divisor of any finite set of elements $x_1, x_2, \dots, x_n$ of T is exists ($\gcd(x_1, x_2, \dots, x_n)$ is exists).

Proposition 6.2 [25](T.lee, page 185)**.** Every pseudo-neutrosophic principal ideal domain is a pseudo-neutrosophic unique factorization domain

Example 6.1 [31-37](T.Lee,page 186 and 2-abstract algebrea,3d,page 285 +page 289)**.** Consider a pseudo-neutrosophic ring of integers $T = \{\dots, -3I, -2I, -I, 0, I, 2I, 3I, \dots\}$ As a principal ideal domain, it is also a pseudo-neutrosophic unique factorization domain.

Proposition 6.3 [17,28] Every pseudo-neutrosophic Euclidean domain is a unique factorization domain.

Definition 6.2 [38-41] In a Pseudo Neutrosophic Unique Factorization Domain, a nonzero element is a prime if and only if it is irreducible.

7. Neutrosophic Jacobson Radical

Definition 7.1 Let $\langle R \cup I \rangle$ be a neutrosophic ring. The Neutrosophic left Jacobson Radical of $\langle R \cup I \rangle$ is defined to be the intersection of all the neutrosophic maximal left ideals in $\langle R \cup I \rangle$, and is denoted by $J'(\langle R \cup I \rangle)$

Definition 7.2 Let $\langle R \cup\ I \rangle$ be a neutrosophic ring. The Neutrosophic right Jacobson Radical of $\langle R \cup I \rangle$ is defined to be the intersection of all the neutrosophic maximal right ideals in $\langle R \cup I \rangle$, and is denoted by $J''(\langle R \cup I \rangle)$

Definition 7.3 Let $\langle R \cup I \rangle$ be a neutrosophic ring. The Neutrosophic Jacobson Radical of $\langle R \cup I \rangle$ is

defined to be the intersection of all the neutrosophic maximal ideals in $\langle R \cup I \rangle$, and is denoted by $J(\langle R \cup I \rangle)$

Proposition 7.1 Let $\langle R \cup I \rangle$ be a neutrosophic commutative ring, then

$J(\langle R \cup I \rangle) = J''(\langle R \cup I \rangle) = J'(\langle R \cup I \rangle)$.

Example 7.1 Consider $\langle Z \cup I \rangle$ is the neutrosophic ring of integers, and $\langle nZ \cup I \rangle$ is a maximal ideal for all p, such that p is a prime number, then $J(\langle Z \cup I \rangle) = 0$,

8. Neutrosophic Noetherian Ring

Definition 8.1 A Pseudo-neutrosophic commutative ring T satisfies the ascending chain condition (ACC), if every ascending chain of ideals

DOI: https://doi.org/10.54216/PAMDA.050103
Received: December 10, 2024 Revised: January 03, 2025 Accepted: January 30, 2025

$P_1 \subseteq P_2 \subseteq \cdots \subseteq P_n \subseteq \ldots$ Stops; that is, there is an integer N with $P_N = P_{N+1} = P_{N+2} = \ldots$, this means that every strictly increasing sequence of ideals in T has finite length.

Definition 8.2 A Pseudo-neutrosophic commutative ring T satisfies the ascending chain condition (ACC) for ideals if every strictly increasing sequence of ideals has finite length.

Proposition 8.1 For a pseudo-neutrosophic commutative ring T the following are equivalent:

1. Every ideal of T is finitely generated.

2. T satisfies the ascending chain condition for ideals.

Definition 8.3 A Pseudo neutrosophic commutative ring is said to be Neutrosophic Noetherian if every ideal is finitely generated, or equivalently, if it satisfies the ACC for ideals.

Example 8.1 Consider $T = \{\ldots, -3I, -2I, -I, 0, I, 2I, 3I, \ldots\}$ is a neutrosophic Noetherian ring, where every increasing sequence of ideals in T is stops, for example: the increasing sequence of ideals $12T \subseteq 10T \subseteq 8T \subseteq 6T \subseteq 4T \subseteq 2T \subseteq T$ is stops,

9. Neutrosophic Artinian Ring

Definition 9.1 A Pseudo Neutrosophic commutative ring T is said to be A Neutrosophic Artinian

Ring (satisfy the descending chain condition on ideals or D.C.C. on ideals) if there is no infinite decreasing chain of ideals in T, this means, whenever$P_1 \supseteq P_2 \supseteq P_3 \supseteq \ldots$ is a decreasing chain of ideals of T, then there is a positive integer n such that $P_n = P_m$ every all $m \geq n$.

Example 9.1 We have $T = \{\ldots, -3I, -2I, -I, 0, I, 2I, 3I, \ldots\}$ is a neutrosophic Noetherian ring, but it is not a neutrosophic Artinian ring, because the decreasing sequence of ideals

$T \supseteq 2T \supseteq 4T \supseteq 6T \supseteq \ldots$ is no stops,

Proposition 9.1 Every Neutrosophic finite commutative ring is a Neutrosophic Artinian Ring.

Proposition 9.2 [26] If T is A Neutrosophic Artinian ring and P is an ideal in T, then

T/P is also Artinian.

Proposition 9.3 Every Neutrosophic Artinian Ring is a Neutrosophic Noetherian Ring, However, the converse may not be accurate.

10. Conclusion

In this paper, we presented some neutrosophic special domains that may only be applicable over the field of complex numbers. Additionally, there are properties of these domains that require further study, which will be the focus of our future research.

Funding: This paper received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

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Received: December 10, 2024 Revised: January 03, 2025 Accepted: January 30, 2025

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DOI: https://doi.org/10.54216/PAMDA.050103
Received: December 10, 2024 Revised: January 03, 2025 Accepted: January 30, 2025