Solution of Intuitionistic Fuzzy System of Linear Volterra Integro-differential Equations by a Novel Hybrid Method

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Abstract

Our study addresses the intuitionistic fuzzy system of linear Volterra-integro-differential equations of the second kind. Intuitionistic fuzzy General Transform (I-F-G-transform) method has been used to find the exact solution of these systems. We present two numerical examples for illustrating the applicability of the Intuitionistic fuzzy General integral transform method.

Keywords: Intuitionistic fuzzy number; Generalized Hukuhara differentiability; Intuitionistic fuzzy General integral transform; Parametric form; Volterra integral equation; Intuitionistic Fuzzy volterra Integro-differential Equation

1 introduction

The field of intuitionistic fuzzy integro-differential equations has grown rapidly. It is one of the most important topics in research due to its applicability in mathematics, physics, and other science as well as engineering subjects. The concept of fuzzy sets, was originally introduced by Zadeh.[19] Later, Chang and Zadeh[5] introduced the concept of fuzzy numbers in 1972. The generalization of fuzzy sets is the intuitionistic fuzzy set (IFS). This concept was first introduced by Atanassov.[3] This theory plays a major key role in diferent domains such as industry, audiovisual systems, robotics. A fuzzy number plays an important role in representation of such unknown quantity. Following this concept, the generalized concept of intuitionistic Fuzzy Number (IFN) introduced by Grzegrorzewski[9] in 2003.

The study of analytical and numerical methods for solving intuitionistic fuzzy integro-fifferentiel equations has been rapidly growing in recent years. Some researchers for solving systems of Volterra integro-differential equations have used several techniques. Integral transforms in the class of Laplace tansform such as (Sumudu, Elzaki, Natural, Aboodh, Pourreza, Mohand, G-transform, Sawi and Kamal transforms,[6],[7],[1],[2],[14],[18][11]) have been applied to determined exact solutions of fuzzy integral equations, fuzzy ordinary differential equations (ODEs), and fuzzy integro-differential equations. Hossein Jafari in[10] gives a new General integral transform which is covered all class of integral transform in the class of Laplace transform and he applied it to solve integral equations.

The objective of this research is to detremine exact solutions to Intuitionistic fuzzy system of Volterra integro-differential equations by using Intuitionistic fuzzy General integral transform.

The structure of this paper is organized as follows: Section 2 introduces some basic definitions and theorems about Intuitionistic fuzzy sets, Intuitionistic fuzzy numbers, Intuitionistic fuzzy integral and differentiation and Intuitionistic fuzzy systems of Volterra-integro differential equations. Section 3 is dedicated to present the New Intuitionistic fuzzy General integral transform with its properties. Section 4 gives the New Intuitionistic fuzzy General integral transform method for solving Intuitionistic fuzzy system of Volterra integro-differential equations and illustrates examples to demonstrate the effectiveness of this method. Finally, Section 5 is a brief conclusion.

2 Basic concepts

Definition 2.1 (Intuitionistic fuzzy set[3]**).** Let a set X be fixed. An intuitionistic fuzzy set (IFS) $A \subset X$ is defined by

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\},$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ respectively represent the degrees of membership and non-membership of the element $x \in X$ to the set A such that $0 \leq \mu_A + \nu_A \leq 1$.

Let $X = \mathbb{R}$, we denote by

$$\mathbb{IF}_1 = \mathbb{IF}(\mathbb{R}) = \{\langle \mu, \nu \rangle : \mathbb{R} \to [0,1], 0 \leq \mu_A + \nu_A \leq 1\}$$

Definition 2.2 (Intuitionistic fuzzy number(IFN)[15]**).** An element $\langle \mu, \nu \rangle \in \mathbb{IF}_1$ is called an intuitionistic fuzzy number if it satisfies the following conditions:

- $\langle \mu, \nu \rangle$ is normal i.e there exist a real number $x_0 \in \mathbb{R}$ such that, $\mu(x_0) = 1$, and $\nu(x_0) = 1$,
- the membership function μ is convex, i.e, $\mu(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu(x_1), \mu_A(x_2)) : \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]$,
- the non-membership function ν is concave, i.e, $\nu(\lambda x_1 + (1-\lambda)x_2) \leq \max(\nu(x_1), \nu_A(x_2)) : \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]$,
- μ is upper semi-continuous and ν is lower semi-continuous,
- $Supp\langle \mu, \nu \rangle = \overline{\{x \in \mathbb{R} : \nu_A(x) < 1\}}$ is bounded.

The set of all intuitionistic fuzzy number is denoted by $\mathbb{IF}_1$.

Definition 2.3 ($\alpha-$cute[15]**).** For all $\alpha \in [0,1]$ and $\langle \mu, \nu \rangle \in \mathbb{IF}_1$, the upper and lower $\alpha-$cuts of $\langle \mu, \nu \rangle$ are defined by

$$[\langle \mu, \nu \rangle]^\alpha = \{x \in \mathbb{R} : \nu(x) \leq 1 - \alpha\}$$

and

$$[\langle \mu, \nu \rangle]_\alpha = \{x \in \mathbb{R} : \nu(x) \geq \alpha\}.$$

We define $0_{(0,1)} \in \mathbb{IF}_1$ as

$$0_{(0,1)}(t) = \begin{cases} (1,0) & if \quad t = 0, \\ (0,1) & if \quad t \neq 0. \end{cases}$$

Let $\langle \mu, \nu \rangle, \langle \mu', \nu' \rangle \in \mathbb{IF}_1$ and $\lambda \in \mathbb{R}$, we define the following operations by

$$[\langle \mu, \nu \rangle \oplus \langle \mu', \nu' \rangle]^\alpha = [\langle \mu, \nu \rangle]^\alpha \oplus [\langle \mu', \nu' \rangle]^\alpha,$$

$$\lambda[\langle \mu, \nu \rangle]^\alpha = [\lambda \langle \mu, \nu \rangle]^\alpha,$$

$$[\langle \mu, \nu \rangle \oplus \langle \mu', \nu' \rangle]_\alpha = [\langle \mu, \nu \rangle]^\alpha \oplus [\langle \mu', \nu' \rangle]_\alpha,$$

$$\lambda[\langle \mu, \nu \rangle]_\alpha = [\lambda \langle \mu, \nu \rangle]_\alpha.$$

Definition 2.4. Let $\langle \mu, \nu \rangle \in \mathbb{IF}_1$ and $\alpha \in [0,1]$, we define the following sets:

$$[\langle \mu, \nu \rangle]_l^+(\alpha) = \inf\{x \in \mathbb{R} : \mu(x) \geq \alpha\},$$

$$[\langle \mu, \nu \rangle]_r^+(\alpha) = \sup\{x \in \mathbb{R} : \mu(x) \geq \alpha\},$$

$$[\langle \mu, \nu \rangle]_l^-(\alpha) = \inf\{x \in \mathbb{R} : \mu(x) \leq 1 - \alpha\},$$

$$[\langle \mu, \nu \rangle]_r^-(\alpha) = \sup\{x \in \mathbb{R} : \mu(x) \leq 1 - \alpha\}.$$

Remark 2.5.

$$[\langle\mu,\nu\rangle]_\alpha = [[\langle\mu,\nu\rangle]_l^+(\alpha), [\langle\mu,\nu\rangle]_r^+(\alpha)]$$

and

$$[\langle\mu,\nu\rangle]^\alpha = [[\langle\mu,\nu\rangle]_l^-(\alpha), [\langle\mu,\nu\rangle]_r^-(\alpha)]$$

Definition 2.6. The parameric form of an intuitionistic fuzzy number is a pair of functions

$$\langle\mu,\nu\rangle = ((\underline{\langle\mu,\nu\rangle}^+, \overline{\langle\mu,\nu\rangle}^+), (\underline{\langle\mu,\nu\rangle}^-, \overline{\langle\mu,\nu\rangle}^-))$$

wich satisfy the following statements:

(i) $\underline{\langle\mu,\nu\rangle}^+(\alpha)$ is a bounded monotonic increasing continuous function in the interval $[0,1]$,

(ii) $\overline{\langle\mu,\nu\rangle}^+(\alpha)$ is a bounded monotonic decreasing continuous function in the interval $[0,1]$,

(iii) $\underline{\langle\mu,\nu\rangle}^-(\alpha)$ is a bounded monotonic increasing continuous function in the interval $[0,1]$,

(iv) $\overline{\langle\mu,\nu\rangle}^-(\alpha)$ is a bounded monotonic decreasing continuous function in the interval $[0,1]$,

(v) $\underline{\langle\mu,\nu\rangle}^+(\alpha) \leq \overline{\langle\mu,\nu\rangle}^+(\alpha)$ and $\underline{\langle\mu,\nu\rangle}^-(\alpha) \leq \overline{\langle\mu,\nu\rangle}^-(\alpha)$ for all $\alpha \in [0,1]$.

We denote by W^n the set of intouitionistic fuzzy numbers in $\mathbb{R}^n$ and E^n the set of fuzzy numbers in $\mathbb{R}^n$. On the space $\mathbb{IF}_1$, we will consider the following metric for $1 \leq p < \infty$:[15]

$$\begin{aligned} d_p\left(\langle\mu,\nu\rangle, \langle\mu',\nu'\rangle\right) &= (\frac{1}{4}\int_0^1 |[\langle\mu,\nu\rangle]_r^+(\alpha) - [\langle\mu',\nu'\rangle]_r^+(\alpha)|^p d\alpha \\ &+ \frac{1}{4}\int_0^1 |[\langle\mu,\nu\rangle]_l^+(\alpha) - [\langle\mu',\nu'\rangle]_l^+(\alpha)|^p d\alpha \\ &+ \frac{1}{4}\int_0^1 |[\langle\mu,\nu\rangle]_r^-(\alpha) - [\langle\mu',\nu'\rangle]_r^-(\alpha)|^p d\alpha \\ &+ \frac{1}{4}\int_0^1 |[\langle\mu,\nu\rangle]_l^-(\alpha) - [\langle\mu',\nu'\rangle]_l^-(\alpha)|^p d\alpha) \end{aligned}$$

and

$$\begin{aligned} d_\infty\left(\langle\mu,\nu\rangle, \langle\mu',\nu'\rangle\right) &= \frac{1}{4}\sup_{0\leq\alpha\leq1}\left|[\langle\mu,\nu\rangle]_r^+(\alpha) - [\langle\mu',\nu'\rangle]_r^+(\alpha)\right| \\ &+ \frac{1}{4}\sup_{0\leq\alpha\leq1}\left|[\langle\mu,\nu\rangle]_l^+(\alpha) - [\langle\mu',\nu'\rangle]_l^+(\alpha)\right| \\ &+ \frac{1}{4}\sup_{0\leq\alpha\leq1}\left|[\langle\mu,\nu\rangle]_r^-(\alpha) - [\langle\mu',\nu'\rangle]_r^-(\alpha)\right| \\ &+ \frac{1}{4}\sup_{0\leq\alpha\leq1}\left|[\langle\mu,\nu\rangle]_l^-(\alpha) - [\langle\mu',\nu'\rangle]_l(\alpha)\right| \end{aligned}$$

Proposition 2.7. $(\mathbb{IF}_1, d_p)$ *is metric space.*

Proof. .[15]
From the definition of d_p and intuitionistic fuzzy numbers we conclude that $d_p\left(\langle\mu,\nu\rangle, \langle\mu',\nu'\rangle\right) < \infty$ and $supp(\langle\mu,\nu\rangle)$, $supp(\langle\mu',\nu'\rangle)$ are bounded for all $\langle\mu,\nu\rangle, \langle\mu',\nu'\rangle \in \mathbb{IF}_1$.
To prove that d_p is an intuitionistic fuzzy metric on $\mathbb{IF}_1$ it suffices to prove that for all $\langle\mu,\nu\rangle, \langle\mu',\nu'\rangle, \langle\mu'',\nu''\rangle \in \mathbb{IF}_1$

1. $d_p\left(\langle\mu,\nu\rangle, \langle\mu',\nu'\rangle\right) \geq 0$,

2. $d_p\left(\langle\mu,\nu\rangle,\langle\mu^{'},\nu^{'}\rangle\right)=0, \quad \text{iff} \quad \langle\mu,\nu\rangle=\langle\mu^{'},\nu^{'}\rangle$
3. $d_p\left(\langle\mu,\nu\rangle,\langle\mu^{'},\nu^{'}\rangle\right)=d_p\left(\langle\mu^{'},\nu^{'}\rangle,\langle\mu,\nu\rangle\right)$
4. $d_p\left(\langle\mu,\nu\rangle,\langle\mu^{''},\nu^{''}\rangle\right)\leq d_p\left(\langle\mu,\nu\rangle,\langle\mu^{'},\nu^{'}\rangle\right)+d_p\left(\langle\mu^{'},\nu^{'}\rangle,\langle\mu^{''},\nu^{''}\rangle\right)$

From the defintion of d_p we find that

- $d_p\left(\langle\mu,\nu\rangle,\langle\mu^{'},\nu^{'}\rangle\right)\geq 0$
- If $d_p\left(\langle\mu,\nu\rangle,\langle\mu^{'},\nu^{'}\rangle\right)=0$, implies

  $$[\langle\mu,\nu\rangle]_\alpha=\left[\langle\mu^{'},\nu^{'}\rangle\right]_\alpha, \text{and} \quad [\langle\mu,\nu\rangle]^\alpha=\left[\langle\mu^{'},\nu^{'}\rangle\right]^\alpha$$

  So $\langle\mu,\nu\rangle=\langle\mu^{'},\nu^{'}\rangle$
- It is easy to verify the triangle inequality and symmetry of d_p.
  So $(\mathbb{IF}_1,d_p)$ is an intuitionistic fuzzy metric space.

□

Definition 2.8. Let $\langle\mu,\nu\rangle,\langle\mu^{'},\nu^{'}\rangle\in\mathbb{IF}_1$

1. If there exists $\langle\Psi,\chi\rangle\in\mathbb{IF}_1$ such that

   $$\langle\mu,\nu\rangle=\langle\mu^{'},\nu^{'}\rangle+\langle\Psi,\chi\rangle$$

   then $\langle\Psi,\chi\rangle$ is called Hukuhara difference of $\langle\mu,\nu\rangle$ and $\langle\mu^{'},\nu^{'}\rangle$ denote by $\langle\Psi,\chi\rangle\ominus_H\langle\mu^{'},\nu^{'}\rangle$
2. The generalized Hukuhara difierence of two fuzzy numbers $\langle\mu,\nu\rangle,\langle\mu^{'},\nu^{'}\rangle\in\mathbb{IF}_1$ is defined by

   $$\begin{aligned}\langle\mu,\nu\rangle\ominus_{gH}\langle\mu^{'},\nu^{'}\rangle &= \langle\Psi,\chi\rangle\\ &\Leftrightarrow \begin{cases}\quad\langle\mu,\nu\rangle=\langle\mu^{'},\nu^{'}\rangle+\langle\Psi,\chi\rangle,\\ or\quad \langle\mu^{'},\nu^{'}\rangle=\langle\mu,\nu\rangle+(-1)\langle\Psi,\chi\rangle.\end{cases}\end{aligned}$$

Definition 2.9. .[8] Let $f:[a,b]\to\mathbb{IF}_1$ be an intuitionistic fuzzy valued mapping and $t_0\in[a,b]$. Then f is called intuitionistic fuzzy continuous in t_0 if an only if:

$$\forall\varepsilon{>}0\ \ \exists\delta{>}0\ \ \forall t\in[a,b]:|t-t_0|\leq\delta\Rightarrow d_p(f(t),f(t_0)\leq\varepsilon.$$

Definition 2.10. .[16] f is called intuitionistic fuzzy continuous iff f is intuitionistic fuzzy continuous in every point of $[a,b]$.

Definition 2.11. .[4] Let $f:[a,b]\to\mathbb{IF}_1$ and $x_0\in[a,b]$. f is called strongly generalized differentiable on x_0 if $\exists(f^+)^{'}(x_0)\in E^{'}$, such that:

(i) for all $h{>}0$ sufficiently small, $\exists f^+(x_0+h)\ominus_{gh}f^+(x_0)$, $f^+(x_0)\ominus_{gh}f^+(x_0-h)$ and the limits (in the metric D)

$$\lim_{h\to0}\frac{f^+(x_0+h)\ominus_{gh}f^+(x_0)}{h}=\lim_{h\to0}\frac{f^+(x_0)\ominus_{gh}f^+(x_0-h)}{h}=(f^+)^{'}(x_0),$$

(ii) for all $h{>}0$ sufficiently small, $\exists f^+(x_0)\ominus_{gh}f^+(x_0+h)$, $f^+(x_0-h)\ominus_{gh}f^+(x_0)$, and the limits (in the metric D)

$$\lim_{h\to0}\frac{f^+(x_0)\ominus_{gh}f^+(x_0+h)}{-h}=\lim_{h\to0}\frac{f^+(x_0-h)\ominus_{gh}f^+(x_0)}{-h}=(f^+)^{'}(x_0),$$

(iii) for all $h>0$ sufficiently small, $\exists f^+(x_0+h)\ominus_{gh} f^+(x_0)$, $f^+(x_0-h)\ominus_{gh} f^+(x_0)$ and the limits (in the metric D)

$$\lim_{h\to 0}\frac{f^+(x_0+h)\ominus_{gh} f^+(x_0)}{h}=\lim_{h\to 0}\frac{f^+(x_0-h)\ominus_{gh} f^+(x_0)}{-h}=\left(f^+\right)'(x_0),$$

Or

(iv) for all $h>0$ sufficiently small, $\exists f^+(x_0)\ominus_{gh} f^+(x_0+h)$, $f^+(x_0)\ominus_{gh} f^+(x_0-h)$ and the limits (in the metric D)

$$\lim_{h\to 0}\frac{f^+(x_0)\ominus_{gh} f^+(x_0+h)}{-h}=\lim_{h\to 0}\frac{f^+(x_0)\ominus_{gh} f^+(x_0-h)}{h}=\left(f^+\right)'(x_0),$$

where, h and $(-h)$ at denominators mean $\frac{1}{h}\odot$ and $-\frac{1}{h}\odot$, respectively

Results similar to (i) to (iv) can be defined to the function $\left(f^-\right)'(x_0)\in E^1$.

Theorem 2.12. .[17] *Let $f(x)$ be an intuitionistic fuzzy-valued function on $[a,\infty)$ represented by $(\underline{f}(x,\alpha),\overline{f}(x,\alpha),$ $\underline{\underline{f}}(x,\alpha),\overline{\overline{f}}(x,\alpha))$ for all $\alpha\in[0,1]$. Assume that $\underline{f}(x,\alpha),\overline{f}(x,\alpha),\underline{\underline{f}}(x,\alpha),\overline{\overline{f}}(x,\alpha)$ are Riemann integrabl on $[a,b]$ for every $b\geq a$, and assume that there are four positive constant $\underline{N}(\alpha),\overline{N}(\alpha),\underline{\underline{N}}(\alpha),\overline{\overline{N}}(\alpha)$ satisfying*

$$\int_\alpha^\infty|\underline{f}(x,\alpha)|dt\leq\underline{N}(\alpha),$$

$$\int_\alpha^\infty|\overline{f}(x,\alpha)|dt\leq\overline{N}(\alpha),$$

$$\int_\alpha^\infty|\underline{\underline{f}}(x,\alpha)|dt\leq\underline{\underline{N}}(\alpha),$$

$$\int_\alpha^\infty|\overline{\overline{f}}(x,\alpha)|dt\leq\overline{\overline{N}}(\alpha),$$

Then $f(x,t)$ is an improper intuitionistic fuzzy Riemann$-$integrable on $[a,\infty)$ and the improper intuitionistic fuzzy Riemann integrale is an intuitionistic fuzzy number and Furthermore, we have for all $\alpha\in[0,1]$

$$\int_\alpha^\infty f(x,\alpha)dt=\left(\int_\alpha^\infty\underline{f}(x,\alpha)dt,\int_\alpha^\infty\overline{f}(x,\alpha)dt,\int_\alpha^\infty\underline{\underline{f}}(x,\alpha)dt,\int_\alpha^\infty\overline{\overline{f}}(x,\alpha)dt\right).$$

Definition 2.13. **(Intuitionistic Fuzzy system of linear Volterra integro-differential equations)**
Let f be an intuitionistic fuzzy valued finction. For all $i=1,2,..,n$, the linear intuitionistic fuzzy system of Volterra integro-differential equations of the second kind (IFSVIDE-2) is defined by

$$\begin{cases} f_1'(t)=g_1(t)+\sum\limits_{j=1}^{n}\lambda_{1j}\int_0^t k_{1j}(t-x)f_j(x)dx,\\ \\ f_2'(t)=g_2(t)+\sum\limits_{j=1}^{n}\lambda_{2j}\int_0^t k_{2j}(t-x)f_j(x)dx,\\ \vdots\\ f_n'(t)=g_n(t)+\sum\limits_{j=1}^{n}\lambda_{nj}\int_0^1 k_{nj}(t-x)f_j(x)dx.\end{cases}\tag{1}$$

With initial conditions

$$\begin{cases} f_1(t_0)=\beta_1,\\ \\ f_2(t_0)=\beta_2,\\ \vdots\\ f_n(t_0)=\beta_n.\end{cases}$$

Where $g_i(t)$ are intuitionistic fuzzy valued functions, $k_{ij}(t,x)$ are crisp Kernels, and $\lambda_{ij}\neq 0,\beta_i$ are real constant.

The parametric form of the IFSVIDE-2 (1) is given by

$$\begin{cases} \underline{f'}_1(t,\alpha) = \underline{g}_1(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{1j} \int_0^t k_{1j}(t-x)\underline{f_j(x,\alpha)}dx, \\ \underline{f'}_2(t,\alpha) = \underline{g}_2(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{2j} \int_0^t k_{2j}(t-x)\underline{f_j(x,\alpha)}dx, \\ \vdots \\ \underline{f'}_n(t,\alpha) = \underline{g}_n(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{nj} \int_0^1 k_{nj}(t-x)\underline{f_j(x,\alpha)}dx. \end{cases}$$

,

$$\begin{cases} \overline{f}'_1(t,\alpha) = \overline{g}_1(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{1j} \int_0^t k_{1j}(t-x)\overline{f_j(x,\alpha)}dx, \\ \overline{f}'_2(t,\alpha) = \overline{g}_2(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{2j} \int_0^t k_{2j}(t-x)\overline{f_j(x,\alpha)}dx, \\ \vdots \\ \overline{f}'_n(t,\alpha) = \overline{g}_n(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{nj} \int_0^1 k_{nj}(t-x)\overline{f_j(x,\alpha)}dx. \end{cases}$$

,

$$\begin{cases} \underline{\underline{f'}}_1(t,\alpha) = \underline{\underline{g}}_1(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{1j} \int_0^t k_{1j}(t-x)\underline{\underline{f_j(x,\alpha)}}dx, \\ \underline{\underline{f'}}_2(t,\alpha) = \underline{\underline{g}}_2(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{2j} \int_0^t k_{2j}(t-x)\underline{\underline{f_j(x,\alpha)}}dx, \\ \vdots \\ \underline{\underline{f'}}_n(t,\alpha) = \underline{\underline{g}}_n(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{nj} \int_0^1 k_{nj}(t-x)\underline{\underline{f_j(x,\alpha)}}dx. \end{cases}$$

and

$$\begin{cases} \overline{\overline{f'}}_1(t,\alpha) = \overline{\overline{g}}_1(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{1j} \int_0^t k_{nj}(t-x)\overline{\overline{f_j(x,\alpha)}}dx, \\ \overline{\overline{f'}}_2(t,\alpha) = \overline{\overline{g}}_2(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{2j} \int_0^t k_{2j}(t-x)\overline{\overline{f_j(x,\alpha)}}dx, \\ \vdots \\ \overline{\overline{f'}}_n(t,\alpha) = \overline{\overline{g}}_n(t,\alpha) + \sum\limits_{j=1}^{n} \lambda_{nj} \int_0^1 k_{nj}(t-x)\overline{\overline{f_j(x,\alpha)}}dx. \end{cases}$$

With initial conditions

$$\begin{cases} (\underline{f}_1(t_0,\alpha), \overline{f}_1(t_0,\alpha), \underline{\underline{f_1}}(t_0,\alpha), \overline{\overline{f_1}}(t_0),\alpha) = (\underline{\beta_1}, \overline{\beta_1}\underline{\underline{\beta_1}}, \overline{\overline{\beta_1}}), \\ (\underline{f}_2(t_0,\alpha), \overline{f}_2(t_0,\alpha), \underline{\underline{f_2}}(t_0,\alpha), \overline{\overline{f_2}}(t_0,\alpha) = (\underline{\beta_2}, \overline{\beta_2}\underline{\underline{\beta_2}}, \overline{\overline{\beta_2}}), \\ \vdots \\ (\underline{f}_n(t_0,\alpha), \overline{f}_n(t_0,\alpha), \underline{\underline{f_n}}(t_0,\alpha), \overline{\overline{f_n}}(t_0,\alpha)) = (\underline{\beta_n}, \overline{\beta_n}\underline{\underline{\beta_n}}, \overline{\overline{\beta_n}}). \end{cases}$$

3 Intuitionistic fuzzy General integral transform (I-F-G transform)

Definition 3.1. Let $f(t)$ be a continuous intutionistic fuzzy valued function and let $q(s)$, $p(s) \neq 0$ be two positive real functions. Suppose that $f(t)e^{-q(s)t}$ is improper intuitionistic fuzzy Rimann-integrable on $[0,\infty)$. Then New intutionstic fuzzy General integral transforms of f denoted by (I-F-G-transform) is defined by:

$$G(s) = \mathcal{G}(f(t),s) = p(s)\int_0^\infty f(t)e^{-q(s)t}dt$$

Remark 3.2. For all $\alpha \in [0,1]$, we can present the definition of I-F-G-transform based on the $\alpha-$cut of the intuitionistic fuzzy valued function by

$$\begin{aligned} G(s,\alpha) &= \mathcal{G}(f(t,\alpha),s) \\ &= \left(p(s)\int_0^\infty \underline{f}(t,\alpha)e^{-q(s)t}dt, p(s)\int_0^\infty \overline{f}(t,\alpha)e^{-q(s)t}dt, p(s)\int_0^\infty \underline{\underline{f}}(t,\alpha)e^{-q(s)t}dt, p(s)\int_0^\infty \overline{\overline{f}}(t,\alpha)e^{-q(s)t}dt\right). \end{aligned}$$

Theorem 3.3. [10] *Let f be an integrable intuitionistic fuzzy valued function, such that its derivatives are up to $(n-1)$th order are continuous for all $t > 0$ and f^n exits. Then*

1. *If f is $(i)-$differentiable, we have*
    - $\mathcal{G}(f^{'}(t), s) = q(s)\mathcal{G}(f(t), s) \ominus p(s)f(0)$.
    - $\mathcal{G}(f^{''}(t), s) = q^2(s)\mathcal{G}(f(t), s) \ominus q(s)p(s)f(0) \ominus p(s)f^{'}(0)$.
    - $\mathcal{G}(f^n(t), s) = q^n(s)\mathcal{G}(f(t), s) \ominus p(s)\sum_{k=0}^{n-1} q^{n-1-k}(s)f^k(0)$.

Definition 3.4. (Intuitionistic fuzzy convolution)
Let f_1, f_2 are two peice-wise continuous intuitionistic fuzzy-valued functions. Then the intuitionistic fuzzy Convolution of f_1 and f_2 is defined by

$$(f_1 * f_2)(t) = \int_0^{\infty} f_1(s)f_2(t-v)dv.$$

Theorem 3.5. *Let f_1, f_2 are two piece-wise continuous intuitionistic fuzzy valued functions on $[0, \infty[$ and of exponential order. If $\mathcal{G}(f_1(t), s) = G_1(s)$ and $\mathcal{G}(f_2(t), s) = G_2(s)$ then the new intuitionistic fuzzy General integral transform of the Convolution of f_1 and f_2 is*

$$\mathcal{G}((f_1 * f_2)(t), s) = \mathcal{G}\left(\int_0^{\infty} f_1(t)f_2(t-v)dv\right) = \frac{G_1(s)G_2(s)}{p(s)}.$$

4 I-F-G transform method for solving intuitionistic fuzzy system of linear Volterra integro-differential equations

By applying I-F-G-transform on system 1, we obtain

$$\begin{cases} \mathcal{G}(f_1'(t), s) = \mathcal{G}(g_1(t), s) + \sum\limits_{j=1}^{n} \lambda_{1j}\mathcal{G}(k_{1j}(t, s) * f_j(t, s)), \\ \mathcal{G}(f_2'(t), s) = \mathcal{G}(g_2(t), s) + \sum\limits_{j=1}^{n} \lambda_{2j}\mathcal{G}((k_{2j}(t, s) * (f_j(t, s)), \\ \vdots \\ \mathcal{G}(f_n'(t), s) = \mathcal{G}(g_n(t), s) + \sum\limits_{j=1}^{n} \lambda_{nj}\mathcal{G}((k_{nj}(t, s) * (f_j(t, s)). \end{cases} \tag{2}$$

By using Convolution theorem 3.5 of I-F-G-transform and theorem 3.3 in system 2, we have

$$\begin{cases} q(s)\mathcal{G}(f_1(t), s) - p(s)f_1(t_0) = \mathcal{G}(g_1(t), s) + \left[\sum\limits_{j=1}^{n} \lambda_{1j}\mathcal{G}((k_{1j}(t), s)\mathcal{G}(f_j(t), s)) \times \dfrac{1}{p(s)}\right], \\ q(s)\mathcal{G}(f_2(t), s) - p(s)f_2(t_0) = \mathcal{G}(g_2(t), s) + \left[\sum\limits_{j=1}^{n} \lambda_{2j}\mathcal{G}(k_{2j}(t), s)\mathcal{G}(f_j(t), s) \times \dfrac{1}{p(s)}\right], \\ \vdots \\ q(s)\mathcal{G}(f_n(t), s) - p(s)f_n(t_0) = \mathcal{G}(g_n(t), s) + \left[\sum\limits_{j=1}^{n} \lambda_{nj}\mathcal{G}(k_{nj}(t), s)G(f_j(t), s) \times \dfrac{1}{p(s)}\right]. \end{cases} \tag{3}$$

The parametric form of the fuzzy system 3 is

$$\begin{cases} q(s)\mathcal{G}(\underline{f}_1(t, \alpha), s) - p(s)(\underline{f}_1(t_0, \alpha)) = \mathcal{G}(\underline{g}_1(t, \alpha), s) + \left[\sum\limits_{j=1}^{n} \lambda_{1j}\mathcal{G}(k_{1j}(t, \alpha), s)\underline{\mathcal{G}(f_j(t, \alpha), s)} \times \dfrac{1}{p(s)}\right], \\ \\ q(s)\mathcal{G}(\underline{f}_2(t, \alpha), s) - p(s)(\underline{f}_2(t_0, \alpha)) = \mathcal{G}(\underline{g}_2(t, \alpha), s) + \left[\sum\limits_{j=1}^{n} \lambda_{2j}\mathcal{G}(k_{2j}(t, \alpha), s)\underline{\mathcal{G}(f_j(t, \alpha), s)} \times \dfrac{1}{p(s)}\right], \\ \vdots \\ q(s)\mathcal{G}(\underline{f}_n(t, \alpha), s) - p(s)(\underline{f}_n(t_0, \alpha)) = \mathcal{G}(\underline{g}_n(t, \alpha), s) + \left[\sum\limits_{j=1}^{n} \lambda_{nj}\mathcal{G}(k_{nj}(t, \alpha), s)\underline{\mathcal{G}(f_j(t, \alpha), s)} \times \dfrac{1}{p(s)}\right]. \end{cases}$$

$$\begin{cases} q(s)\mathcal{G}(\overline{f}_1(t,\alpha),s)-p(s)\overline{f}_1(t_0,\alpha)=\mathcal{G}(\overline{g}_1(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)\overline{\mathcal{G}(f_j(t,\alpha),s)}\times\dfrac{1}{p(s)}\right], \\ q(s)\mathcal{G}(\overline{f}_2(t,\alpha),s)-p(s)\overline{f}'_2(t_0,\alpha)=\mathcal{G}(\overline{g}_2(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)\overline{\mathcal{G}(f_j(t,\alpha),s)}\times\dfrac{1}{p(s)}\right], \\ \vdots \\ q(s)\mathcal{G}(\overline{f}_n(t,\alpha),s)-p(s)\overline{f}_n(t_0,\alpha)=\mathcal{G}(\overline{g}_n(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)\overline{\mathcal{G}(f_j(t,\alpha),s)}\times\dfrac{1}{p(s)}\right]. \end{cases}$$

$$\begin{cases} q(s)\mathcal{G}(\underline{\underline{\underline{f_1}}}(t,\alpha),s)-p(s)(\underline{\underline{\underline{f_1}}}(t_0,\alpha))=\mathcal{G}(\underline{\underline{\underline{g_1}}}(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)\underline{\underline{\underline{\mathcal{G}(f_j(t,\alpha),s)}}}\times\dfrac{1}{p(s)}\right], \\ q(s)\mathcal{G}(\underline{\underline{\underline{f_2}}}(t,\alpha),s)-p(s)(\underline{\underline{\underline{f_2}}}(t_0,\alpha))=\mathcal{G}(\underline{\underline{\underline{g_2}}}(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)\underline{\underline{\underline{\mathcal{G}(f_j(t,\alpha),s)}}}\times\dfrac{1}{p(s)}\right], \\ \vdots \\ q(s)\mathcal{G}(\underline{\underline{\underline{f_n}}}(t,\alpha),s)-p(s)(\underline{\underline{\underline{f_n}}}(t_0,\alpha))=\mathcal{G}(\underline{\underline{\underline{g_n}}}(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)\underline{\underline{\underline{\mathcal{G}(f_j(t,\alpha),s)}}}\times\dfrac{1}{p(s)}\right]. \end{cases}$$

and

$$\begin{cases} q(s)\mathcal{G}(\overline{\overline{\overline{f_1}}}(t,\alpha),s)-p(s)(\overline{\overline{\overline{f_1}}}(t_0,\alpha))=\mathcal{G}(\overline{\overline{\overline{g_1}}}(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)\overline{\overline{\overline{\mathcal{G}(f_j(t,\alpha),s)}}}\times\dfrac{1}{p(s)}\right], \\ q(s)\mathcal{G}(\overline{\overline{\overline{f_2}}}(t,\alpha),s)-p(s)(\overline{\overline{\overline{f_2}}}(t_0,\alpha))=\mathcal{G}(\overline{\overline{\overline{g_2}}}(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)\overline{\overline{\overline{\mathcal{G}(f_j(t,\alpha),s)}}}\times\dfrac{1}{p(s)}\right], \\ \vdots \\ q(s)\mathcal{G}(\overline{\overline{\overline{f_n}}}(t,\alpha),s)-p(s)(\overline{\overline{\overline{f_n}}}(t_0,\alpha))=\mathcal{G}(\overline{\overline{\overline{g_n}}}(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)\overline{\overline{\overline{\mathcal{G}(f_j(t,\alpha),s)}}}\times\dfrac{1}{p(s)}\right]. \end{cases}$$

So

$$\begin{cases} q(s)\mathcal{G}(\underline{f}_1(t,\alpha),s)=p(s)(\underline{f}_1(t_0,\alpha))+\mathcal{G}(\underline{g}_1(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)\underline{\mathcal{G}(f_j(t,\alpha),s)}\times\dfrac{1}{p(s)}\right], \\ q(s)\mathcal{G}(\underline{f}_2(t,\alpha),s)=p(s)(\underline{f}_2(t_0,\alpha))+\mathcal{G}(\underline{g}_2(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)\underline{\mathcal{G}(f_j(t,\alpha),s)}\times\dfrac{1}{p(s)}\right], \\ \vdots \\ q(s)\mathcal{G}(\underline{f}_n(t,\alpha),s)=p(s)(\underline{f}_n(t_0,\alpha))+\mathcal{G}(\underline{g}_n(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)\underline{\mathcal{G}(f_j(t,\alpha),s)}\times\dfrac{1}{p(s)}\right]. \end{cases}$$

$$\begin{cases} q(s)\mathcal{G}(\overline{f}_1(t,\alpha),s)=p(s)\overline{f}_1(t_0,\alpha)+\mathcal{G}(\overline{g}_1(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)\overline{\mathcal{G}(f_j(t,\alpha),s)}\times\dfrac{1}{p(s)}\right], \\ q(s)\mathcal{G}(\overline{f}_2(t,\alpha),s)=p(s)\overline{f}_2(t_0,\alpha)+\mathcal{G}(\overline{g}_2(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)\overline{\mathcal{G}(f_j(t,\alpha),s)}\times\dfrac{1}{p(s)}\right], \\ \vdots \\ q(s)\mathcal{G}(\overline{f}_n(t,\alpha),s)=p(s)\overline{f}_n(t_0,\alpha)+\mathcal{G}(\overline{g}_n(t,\alpha),s)+\left[\sum\limits_{j=1}^{n}\lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)\overline{\mathcal{G}(f_j(t,\alpha),s)}\times\dfrac{1}{p(s)}\right]. \end{cases}$$

$$\begin{cases} q(s)\mathcal{G}(\underline{\underline{f_1}}(t,\alpha),s) = p(s)(\underline{\underline{f_1}}(t_0,\alpha)) + \mathcal{G}(\underline{\underline{g_1}}(t,\alpha),s) + \left[\sum\limits_{j=1}^{n} \lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)\underline{\underline{\mathcal{G}(f_j(t,\alpha),s)}} \times \dfrac{1}{p(s)}\right], \\ q(s)\mathcal{G}(\underline{\underline{f_2}}(t,\alpha),s) = p(s)(\underline{\underline{f_2}}(t_0,\alpha)) + \mathcal{G}(\underline{\underline{g_2}}(t,\alpha),s) + \left[\sum\limits_{j=1}^{n} \lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)\underline{\underline{\mathcal{G}(f_j(t,\alpha),s)}} \times \dfrac{1}{p(s)}\right], \\ \vdots \\ q(s)\mathcal{G}(\underline{\underline{f_n}}(t,\alpha),s) = p(s)(\underline{\underline{f_n}}(t_0,\alpha)) + \mathcal{G}(\underline{\underline{g_n}}(t,\alpha),s) + \left[\sum\limits_{j=1}^{n} \lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)\underline{\underline{\mathcal{G}(f_j(t,\alpha),s)}} \times \dfrac{1}{p(s)}\right]. \end{cases}$$

and

$$\begin{cases} q(s)\mathcal{G}(\overline{\overline{f_1}}(t,\alpha),s) = p(s)(\overline{\overline{f_1}}(t_0,\alpha)) + \mathcal{G}(\overline{\overline{g_1}}(t,\alpha),s) + \left[\sum\limits_{j=1}^{n} \lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)\overline{\overline{\mathcal{G}(f_j(t,\alpha),s)}} \times \dfrac{1}{p(s)}\right], \\ q(s)\mathcal{G}(\overline{\overline{f_2}}(t,\alpha),s) = p(s)(\overline{\overline{f_2}}(t_0,\alpha)) + \mathcal{G}(\overline{\overline{g_2}}(t,\alpha),s) + \left[\sum\limits_{j=1}^{n} \lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)\overline{\overline{\mathcal{G}(f_j(t,\alpha),s)}} \times \dfrac{1}{p(s)}\right], \\ \vdots \\ q(s)\mathcal{G}(\overline{\overline{f_n}}(t,\alpha),s) = p(s)(\overline{\overline{f_n}}(t_0,\alpha)) + \mathcal{G}(\overline{\overline{g_n}}(t,\alpha),s) + \left[\sum\limits_{j=1}^{n} \lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)\overline{\overline{\mathcal{G}(f_j(t,\alpha),s)}} \times \dfrac{1}{p(s)}\right]. \end{cases}$$

After simplification, we have

$$\begin{cases} (q(s) - \frac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)})\mathcal{G}(\underline{f}_1(t,\alpha),s) - \sum_{j=2}^{n} \frac{\lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)}{p(s)}\mathcal{G}(\underline{f}_j(t,\alpha),s) = p(s)\underline{\beta_1} + \mathcal{G}(\underline{g}_1(t,\alpha),s), \\ (q(s) - \frac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)})\mathcal{G}(\underline{f}_2(t,\alpha),s) - \sum_{j=1,j\neq 2}^{n} \frac{\lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)}{p(s)}\mathcal{G}(\underline{f}_j(t,\alpha),s) = p(s)\underline{\beta_2} + \mathcal{G}(\underline{g}_2(t,\alpha),s), \\ \vdots \\ \sum_{j=1,}^{n} \frac{\lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)}{p(s)}\mathcal{G}(\underline{f}_j(t,\alpha),s) + (q(s) - \frac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)})\mathcal{G}(\underline{f}_n(t,\alpha),s) = p(s)\underline{\beta_n} + \mathcal{G}(\underline{g}_n(t,\alpha),s). \end{cases} \tag{4}$$

$$\begin{cases} (q(s) - \frac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)})\mathcal{G}(\overline{f}_1(t,\alpha),s) - \sum_{j=2}^{n} \frac{\lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)}{p(s)}\mathcal{G}(\overline{f}_j(t,\alpha),s) = p(s)\overline{\beta_1} + \mathcal{G}(\overline{g}_1(t,\alpha),s), \\ (q(s) - \frac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)})\mathcal{G}(\overline{f}_2(t,\alpha),s) - \sum_{j=1,j\neq 2}^{n} \frac{\lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)}{p(s)}\mathcal{G}(\overline{f}_j(t,\alpha),s) = p(s)\overline{\beta_2} + \mathcal{G}(\overline{g}_2(t,\alpha),s), \\ \vdots \\ \sum_{j=1,}^{n} \frac{\lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)}{p(s)}\mathcal{G}(\overline{f}_j(t,\alpha),s) + (q(s) - \frac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)})\mathcal{G}(\overline{f}_n(t,\alpha),s) = p(s)\overline{\beta_n} + \mathcal{G}(\overline{g}_n(t,\alpha),s). \end{cases} \tag{5}$$

,

$$\begin{cases} (q(s) - \frac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)})\mathcal{G}(\underline{\underline{f_1}}(t,\alpha),s) - \sum_{j=2}^{n} \frac{\lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)}{p(s)}\mathcal{G}(\underline{\underline{f_j}}(t,\alpha),s) = p(s)\underline{\underline{\beta_1}} + \mathcal{G}(\underline{\underline{g_1}}(t,\alpha),s), \\ (q(s) - \frac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)})\mathcal{G}(\underline{\underline{f_2}}(t,\alpha),s) - \sum_{j=1,j\neq 2}^{n} \frac{\lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)}{p(s)}\mathcal{G}(\underline{\underline{f_j}}(t,\alpha),s) = p(s)\underline{\underline{\beta_2}} + \mathcal{G}(\underline{\underline{g_2}}(t,\alpha),s), \\ \vdots \\ \sum_{j=1,}^{n} \frac{\lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)}{p(s)}\mathcal{G}(\underline{\underline{f_j}}(t,\alpha),s) + (q(s) - \frac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)})\mathcal{G}(\underline{\underline{f_n}}(t,\alpha),s) = p(s)\underline{\underline{\beta_n}} + \mathcal{G}(\underline{\underline{g_n}}(t,\alpha),s). \end{cases} \tag{6}$$

,

$$\begin{cases} (q(s) - \frac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)})\mathcal{G}(\overline{\overline{f_1}}(t,\alpha),s) - \sum_{j=2}^{n} \frac{\lambda_{1j}\mathcal{G}(k_{1j}(t,\alpha),s)}{p(s)}\mathcal{G}(\overline{\overline{f_j}}(t,\alpha),s) = p(s)\overline{\overline{\beta_1}} + \mathcal{G}(\overline{\overline{g_1}}(t,\alpha),s), \\ (q(s) - \frac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)})\mathcal{G}(\overline{\overline{f_2}}(t,\alpha),s) - \sum_{j=1,j\neq 2}^{n} \frac{\lambda_{2j}\mathcal{G}(k_{2j}(t,\alpha),s)}{p(s)}\mathcal{G}(\overline{\overline{f_j}}(t,\alpha),s) = p(s)\overline{\overline{\beta_2}} + \mathcal{G}(\overline{\overline{g_2}}(t,\alpha),s), \\ \vdots \\ \sum_{j=1,}^{n} \frac{\lambda_{nj}\mathcal{G}(k_{nj}(t,\alpha),s)}{p(s)}\mathcal{G}(\overline{\overline{f_j}}(t,\alpha),s) + (q(s) - \frac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)})\mathcal{G}(\overline{\overline{f_n}}(t,\alpha),s) = p(s)\overline{\overline{\beta_n}} + \mathcal{G}(\overline{\overline{g_n}}(t,\alpha),s). \end{cases} \tag{7}$$

Matrix visualization of systems 4, 5, 6, and 7 respectively are

$$P\underline{F} = \underline{G}, P\overline{F} = \overline{G}, P\underline{\underline{F}} = \underline{\underline{G}}, P\overline{\overline{F}} = \overline{\overline{G}}$$

Where

$$P=\begin{bmatrix} (q(s)-\frac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}) & -\frac{\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & ... & -\frac{\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)} \\ \frac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & (q(s)-\frac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)}) & ... & -\frac{\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)} \\ \vdots & \vdots & ... & \vdots \\ \frac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & -\frac{\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & ... & (q(s)-\frac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}) \end{bmatrix}$$

$$\underline{F}=\begin{bmatrix} \mathcal{G}(\underline{f}_1(t,\alpha),s) \\ \mathcal{G}(\underline{f}_2(t,\alpha),s) \\ \vdots \\ \mathcal{G}(\underline{f}_n(t,\alpha),s) \end{bmatrix},\underline{G}=\begin{bmatrix} p(s)\underline{\beta_1}+\mathcal{G}(\underline{g}_1(t,\alpha),s) \\ p(s)\underline{\beta_2}+\mathcal{G}(\underline{g}_2(t,\alpha),s) \\ \vdots \\ p(s)\underline{\beta_n}+\mathcal{G}(\underline{g}_n(t,\alpha),s) \end{bmatrix},\overline{F}=\begin{bmatrix} \mathcal{G}(\overline{f}_1(t,\alpha),s) \\ \mathcal{G}(\overline{f}_2(t,\alpha),s) \\ \vdots \\ \mathcal{G}(\overline{f}_n(t,\alpha),s) \end{bmatrix},\overline{G}=\begin{bmatrix} p(s)\overline{\beta_1}+\mathcal{G}(\overline{g}_1(t,\alpha),s) \\ p(s)\overline{\beta_2}+\mathcal{G}(\overline{g}_2(t,\alpha),s) \\ \vdots \\ p(s)\overline{\beta_n}+\mathcal{G}(\overline{g}_n(t,\alpha),s) \end{bmatrix}$$

$$\underline{\underline{F}}=\begin{bmatrix} \mathcal{G}(\underline{\underline{f_1}}(t,\alpha),s) \\ \mathcal{G}(\underline{\underline{f_2}}(t,\alpha),s) \\ \vdots \\ \mathcal{G}(\underline{\underline{f_n}}(t,\alpha),s) \end{bmatrix},\underline{\underline{G}}=\begin{bmatrix} p(s)\underline{\underline{\beta_1}}+\mathcal{G}(\underline{\underline{g_1}}(t,\alpha),s) \\ p(s)\underline{\underline{\beta_2}}+\mathcal{G}(\underline{\underline{g_2}}(t,\alpha),s) \\ \vdots \\ p(s)\underline{\underline{\beta_n}}+\mathcal{G}(\underline{\underline{g_n}}(t,\alpha),s) \end{bmatrix},\overline{\overline{F}}=\begin{bmatrix} \mathcal{G}(\overline{\overline{f_1}}(t,\alpha),s) \\ \mathcal{G}(\overline{\overline{f_2}}(t,\alpha),s) \\ \vdots \\ \mathcal{G}(\overline{\overline{f_n}}(t,\alpha),s) \end{bmatrix},\overline{\overline{G}}=\begin{bmatrix} p(s)\overline{\overline{\beta_1}}+\mathcal{G}(\overline{\overline{g_1}}(t,\alpha),s) \\ p(s)\overline{\overline{\beta_2}}+\mathcal{G}(\overline{\overline{g_2}}(t,\alpha),s) \\ \vdots \\ p(s)\overline{\overline{\beta_n}}+\mathcal{G}(\overline{\overline{g_n}}(t,\alpha),s) \end{bmatrix}$$

Using Cramer rule, the solutions of systems 4,5,6 and 7 are given by

$$\mathcal{G}(\underline{f}_1(t,\alpha),s)=\frac{\begin{vmatrix} p(s)\underline{\beta_1}+\mathcal{G}(\underline{g}_1(t,\alpha),s) & \frac{-\lambda_{12}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)} & ... & \frac{-\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)} \\ \underline{\beta_2}+\mathcal{G}(\underline{g}_2(t,\alpha),s) & \left(q(s)-\frac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)}\right) & .... & \frac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)} \\ \vdots & & & \vdots \\ \underline{\beta_n}+\mathcal{G}(\underline{g}_n(t,\alpha),s) & \frac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & ... & \left(q(s)-\frac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right) \end{vmatrix}}{\begin{vmatrix} \left(q(s)-\frac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \frac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & ... & \frac{\lambda_{1m}\mathcal{G}(k_{1m}(t,\alpha),s)}{p(s)} \\ \frac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & \left(q(s)-\frac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)}\right) & ... & \frac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)} \\ \vdots & & & \vdots \\ \frac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \frac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & ... & \left(q(s)-\frac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right) \end{vmatrix}},$$

$$\mathcal{G}(\underline{f}_2(t,\alpha),s)=\frac{\begin{vmatrix} \left(q(s)-\frac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \underline{\beta_1}+\mathcal{G}(\underline{g}_1(t,\alpha),s) & ... & \frac{\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)} \\ \frac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & \underline{\beta_2}+\mathcal{G}(\underline{g}_2(t,\alpha),s) & ... & \frac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)} \\ \vdots & & & \vdots \\ \frac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \underline{\beta_2}+\mathcal{G}(\underline{g}_n(t,\alpha),s) & ... & \left(q(s)-\frac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right) \end{vmatrix}}{\begin{vmatrix} \left(q(s)-\frac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \frac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & ... & \frac{-\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)} \\ \frac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & q(s)-\frac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)} & ... & \frac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)} \\ \vdots & & & \vdots \\ \frac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \frac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & ... & \left(q(s)-\frac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right) \end{vmatrix}},$$

$$G(\underline{f}_n(t,\alpha),s)=\frac{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & ... & \underline{\beta_1}+\mathcal{G}(\underline{g}_1(t,\alpha),s)\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)}\right) & ... & \underline{\beta_2}+\mathcal{G}(\underline{g}_1(t,\alpha),s)\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & ... & \underline{\beta_n}+\mathcal{G}(\underline{g}_1(t,\alpha),s)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & ... & \dfrac{\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(\underline{k}_{21}(t,\alpha),s)}{p(s)} & q(s)-\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s) & ... & \dfrac{-\lambda_{2n}\mathcal{G}(\underline{k}_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ -\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s) & -\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s) & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}}.$$

and

$$\mathcal{G}(\overline{f}_1(t,\alpha),s)=\frac{\begin{vmatrix}p(s)\overline{\beta_1}+\mathcal{G}(\overline{g}_1(t,\alpha),s) & \dfrac{-\lambda_{12}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)} & ... & \dfrac{-\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)}\\ \overline{\beta_2}+\mathcal{G}(\overline{g}_2(t,\alpha),s) & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)}\right) & .... & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ \overline{\beta_n}+\mathcal{G}(\overline{g}_n(t,\alpha),s) & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & ... & \dfrac{\lambda_{1m}\mathcal{G}(k_{1m}(t,\alpha),s)}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)}\right) & ... & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}},$$

$$\mathcal{G}(\overline{f}_2(t,\alpha),s)=\frac{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \overline{\beta_1}+\mathcal{G}(\overline{g}_1(t,\alpha),s) & ... & \dfrac{\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & \overline{\beta_2}+\mathcal{G}(\overline{g}_2(t,\alpha),s) & ... & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \overline{\beta_2}+\mathcal{G}(\overline{g}_n(t,\alpha),s) & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & ... & \dfrac{-\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)} & ... & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}},$$

$$G(\overline{f}_n(t,\alpha),s)=\frac{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & \dots & \overline{\beta_1}+\mathcal{G}(\overline{g}_1(t,\alpha),s)\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)}\right) & \dots & \overline{\beta_2}+\mathcal{G}(\overline{g}_1(t,\alpha),s)\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & \dots & \overline{\beta_n}+\mathcal{G}(\overline{g}_1(t,\alpha),s)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & \dots & \dfrac{\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(\underline{k}_{21}(t,\alpha),s)}{p(s)} & q(s)-\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s) & \dots & \dfrac{-\lambda_{2n}\mathcal{G}(\underline{k}_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ -\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s) & -\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s) & \dots & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}}.$$

and

$$\mathcal{G}(\underline{\underline{f_1}}(t,\alpha),s)=\frac{\begin{vmatrix}p(s)\underline{\underline{\beta_1}}+\mathcal{G}(\underline{\underline{g_1}}(t,\alpha),s) & \dfrac{-\lambda_{12}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)} & \dots & \dfrac{-\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)}\\ \underline{\underline{\beta_2}}+\mathcal{G}(\underline{\underline{g_2}}(t,\alpha),s) & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)}\right) & \dots. & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ \underline{\underline{\beta_n}}+\mathcal{G}(\underline{\underline{g_n}}(t,\alpha),s) & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & \dots & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & \dots & \dfrac{\lambda_{1m}\mathcal{G}(k_{1m}(t,\alpha),s)}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)}\right) & \dots & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & \dots & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}},$$

$$\mathcal{G}(\underline{\underline{f_2}}(t,\alpha),s)=\frac{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \underline{\underline{\beta_1}}+\mathcal{G}(\underline{\underline{g_1}}(t,\alpha),s) & \dots & \dfrac{\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & \underline{\underline{\beta_2}}+\mathcal{G}(\underline{\underline{g_2}}(t,\alpha),s) & \dots & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \underline{\underline{\beta_2}}+\mathcal{G}(\underline{\underline{g_n}}(t,\alpha),s) & \dots & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & \dots & \dfrac{-\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)} & \dots & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & \dots & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}},$$

$$G(\underline{\underline{f_n}}(t,\alpha),s)=\frac{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & ... & \underline{\underline{\beta_1}}+\mathcal{G}(\underline{\underline{g_1}}(t,\alpha),s)\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha),s)}{p(s)} & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s)}{p(s)}\right) & ... & \underline{\underline{\beta_2}}+\mathcal{G}(\underline{\underline{g_1}}(t,\alpha),s)\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s)}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s)}{p(s)} & ... & \underline{\underline{\beta_n}}+\mathcal{G}(\underline{\underline{g_1}}(t,\alpha),s)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha),s)}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha),s)}{p(s)} & ... & \dfrac{\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha),s)}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(\underline{k}_{21}(t,\alpha),s)}{p(s)} & q(s)-\lambda_{22}\mathcal{G}(k_{22}(t,\alpha),s) & ... & \dfrac{-\lambda_{2n}\mathcal{G}(\underline{k}_{2n}(t,\alpha),s)}{p(s)}\\ \vdots & & & \vdots\\ -\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha),s) & -\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha),s) & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha),s)}{p(s)}\right)\end{vmatrix}}.$$

$$G(\overline{\overline{f_1}}(t,\alpha))=\frac{\begin{vmatrix}p(s)\overline{\overline{\beta_1}}+\mathcal{G}(\overline{\overline{g}}_1(t,\alpha)) & \dfrac{-\lambda_{12}\mathcal{G}(k_{1n}(t,\alpha))}{p(s)} & ... & \dfrac{-\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha))}{p(s)}\\ \overline{\overline{\beta_2}}+\mathcal{G}(\overline{\overline{g}}_2(t,\alpha)) & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha))}{p(s)}\right) & .... & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha))}{p(s)}\\ \vdots & & & \vdots\\ \overline{\overline{\beta_n}}+\mathcal{G}(\overline{\overline{g}}_n(t,\alpha)) & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha))}{p(s)} & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha))}{p(s)}\right)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha))}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha))}{p(s)} & ... & \dfrac{\lambda_{1m}\mathcal{G}(k_{1m}(t,\alpha))}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha))}{p(s)} & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha))}{p(s)}\right) & ... & \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha))}{p(s)}\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha))}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha))}{p(s)} & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha))}{p(s)}\right)\end{vmatrix}},$$

$$G(\overline{\overline{f_2}}(t,\alpha))=\frac{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha))}{p(s)}\right) & \overline{\overline{\beta_1}}+\mathcal{G}(\overline{\overline{g}}_1(t,\alpha)) & ... & \dfrac{\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha))}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha))}{p(s)} & \overline{\overline{\beta_2}}+\mathcal{G}(\overline{\overline{g}}_2(t,\alpha)) & ... & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha))}{p(s)}\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha))}{p(s)} & \overline{\overline{\beta_2}}+\mathcal{G}(\overline{\overline{g}}_n(t,\alpha)) & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha))}{p(s)}\right)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha))}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha))}{p(s)} & ... & \dfrac{-\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha))}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha))}{p(s)} & q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha))}{p(s)} & ... & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha))}{p(s)}\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha))}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha))}{p(s)} & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha))}{p(s)}\right)\end{vmatrix}},$$

$$G(\overline{\overline{f}}_n(t,\alpha),s)=\frac{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha))}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha))}{p(s)} & ... & \overline{\overline{\beta_1}}+\mathcal{G}(\overline{\overline{g}}_1(t,\alpha))\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha))}{p(s)} & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha))}{p(s)}\right) & ... & \overline{\overline{\beta}}_2+\mathcal{G}(\overline{\overline{g}}_1(t,\alpha))\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha))}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha))}{p(s)} & ... & \overline{\overline{\beta}}_n+\mathcal{G}(\overline{\overline{g}}_1(t,\alpha))\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{\lambda_{11}\mathcal{G}(k_{11}(t,\alpha))}{p(s)}\right) & \dfrac{-\lambda_{12}\mathcal{G}(k_{12}(t,\alpha))}{p(s)} & ... & \dfrac{\lambda_{1n}\mathcal{G}(k_{1n}(t,\alpha))}{p(s)}\\ \dfrac{-\lambda_{21}\mathcal{G}(k_{21}(t,\alpha))}{p(s)} & \left(q(s)-\dfrac{\lambda_{22}\mathcal{G}(k_{22}(t,\alpha))}{p(s)}\right) & ... & \dfrac{-\lambda_{2n}\mathcal{G}(k_{2n}(t,\alpha))}{p(s)}\\ \vdots & & & \vdots\\ \dfrac{-\lambda_{n1}\mathcal{G}(k_{n1}(t,\alpha))}{p(s)} & \dfrac{-\lambda_{n2}\mathcal{G}(k_{n2}(t,\alpha))}{p(s)} & ... & \left(q(s)-\dfrac{\lambda_{nn}\mathcal{G}(k_{nn}(t,\alpha))}{p(s)}\right)\end{vmatrix}}.$$

The inverse of I-F-G-transform of these functions give the required values $(\underline{f}(t,\alpha),\overline{f}(t,\alpha),\underline{\underline{f}}(t,\alpha),\overline{\overline{f}}(t,\alpha))$. To illustrate the effectiveness of this method we give the following examples.

Example 4.1. Consider the following Intuitionistic Fuzzy system of linear Volterra integro-differential equations

$$\begin{cases} f_1'(t,\alpha)=(\alpha+1,3-\alpha,\alpha,-\alpha)\dfrac{t^3}{3}+\int_0^t(t-x)^2f_1(x)dx+\int_0^t(t-x)^2f_2(x)dx,\\ \\ f_2'(t,\alpha)=(\alpha+1,3-\alpha,\alpha,-\alpha)(-t^3-\dfrac{t^7}{210})+\int_0^t(t-x)^2f_1(x)dx-\int_0^t(t-x)^2f_2(x)dx.\end{cases}\tag{8}$$

With,

$$\begin{cases} f_1(0,\alpha)=(\alpha+1,3-\alpha,\alpha,-\alpha),\\ \\ f_2(0,\alpha)=(\alpha+1,3-\alpha,\alpha,-\alpha).\end{cases}$$

For all $\alpha\in[0,1]$.
Operating the I-F-G-transform in system 8 and using theorem 3.3 with 3.5, we get

$$\begin{cases} \left(q(s)-\dfrac{2}{q^3(s)}\right)\mathcal{G}(\underline{f}_1(t,\alpha),s)-\dfrac{2}{q^3(s)}\mathcal{G}(\underline{f}_2(t,\alpha),s)=(\alpha+1)p(s)\left(1+\dfrac{2}{q^4(s)}\right),\\ -\dfrac{2}{q^3(s)}\mathcal{G}(\underline{f}_1(t,\alpha),s)+\left(q(s)+\dfrac{2}{q^3(s)}\right)\mathcal{G}(\underline{f}_2(t,\alpha),s)=(\alpha+1)p(s)\left(1-\dfrac{6}{q^4(s)}-\dfrac{24}{q^8(s)}\right).\end{cases}\tag{9}$$

,

$$\begin{cases} \left(q(s)-\dfrac{2}{q^3(s)}\right)\mathcal{G}(\overline{f}_1(t,\alpha),s)-\dfrac{2}{q^3(s)}\mathcal{G}(\overline{f}_2(t,\alpha),s)=(3-\alpha),s)p(s)\left(1+\dfrac{2}{q^4(s)}\right),\\ -\dfrac{2}{q^3(s)}\mathcal{G}(\overline{f}_1(t,\alpha),s)+\left(q(s)+\dfrac{2}{q^3(s)}\right)\mathcal{G}(\overline{f}_2(t,\alpha),s)=(3-\alpha)p(s)\left(1-\dfrac{6}{q^4(s)}-\dfrac{24}{q^8(s)}\right).\end{cases}\tag{10}$$

,

$$\begin{cases} \left(q(s)-\dfrac{2}{q^3(s)}\right)\mathcal{G}(\underline{\underline{f_1}}(t,\alpha),s)-\dfrac{2}{q^3(s)}\mathcal{G}(\underline{\underline{f_2}}(t,\alpha),s)=\alpha p(s)\left(1+\dfrac{2}{q^4(s)}\right),\\ -\dfrac{2}{q^3(s)}\mathcal{G}(\underline{\underline{f_1}}(t,\alpha),s)+\left(q(s)+\dfrac{2}{q^3(s)}\right)\mathcal{G}(\underline{\underline{f_2}}(t,\alpha),s)=\alpha p(s)\left(1-\dfrac{6}{q^4(s)}-\dfrac{24}{q^8(s)}\right).\end{cases}\tag{11}$$

,

$$\begin{cases} \left(q(s)-\dfrac{2}{q^3(s)}\right)\mathcal{G}(\overline{\overline{f_1}}(t,\alpha),s)-\dfrac{2}{q^3(s)}\mathcal{G}(\overline{\overline{f_2}}(t,\alpha),s)=-\alpha p(s)\left(1+\dfrac{2}{q^4(s)}\right), \\ -\dfrac{2}{q^3(s)}\mathcal{G}(\overline{\overline{f_1}}(t,\alpha),s)+\left(q(s)+\dfrac{2}{q^3(s)}\right)\mathcal{G}(\overline{\overline{f_2}}(t,\alpha),s)=-\alpha p(s)\left(1-\dfrac{6}{q^4(s)}-\dfrac{24}{q^8(s)}\right). \end{cases} \tag{12}$$

Using Cramer rule, the solutions of systems (9), (10), (11) and (12) are given by

$$\mathcal{G}(\underline{f}_1(t,\alpha),s)=\frac{\begin{vmatrix} (\alpha+1)p(s)\left(1+\dfrac{2}{q^4(s)}\right) & -\dfrac{2}{q^3(s)} \\ (\alpha+1)p(s)\left(1-\dfrac{6}{q^3(s)}-\dfrac{24}{q^8(s)}\right) & \left(q(s)+\dfrac{2}{q^3(s)}\right) \end{vmatrix}}{\begin{vmatrix} \left(q(s)-\dfrac{2}{q^3(s)}\right) & -\dfrac{2}{q^3(s)} \\ -\dfrac{2}{q^3(s)} & \left(q(s)+\dfrac{2}{q^3(s)}\right) \end{vmatrix}}=(\alpha+1)p(s)\left(\frac{1}{q(s)}+\frac{6}{q(s)^5}\right),$$

$$\mathcal{G}(\underline{f}_2(t,\alpha),s)=\frac{\begin{vmatrix} q(s)-\dfrac{2}{q^3(s)} & (\alpha+1)p(s)\left(1+\dfrac{2}{q^4(s)}\right) \\ -\dfrac{2}{q^3(s)} & (\alpha+1)p(s)\left(1-\dfrac{6}{q^3(s)}-\dfrac{24}{q^8(s)}\right) \end{vmatrix}}{\begin{vmatrix} \left(q(s)-\dfrac{2}{q^3(s)}\right) & -\dfrac{2}{q^3(s)} \\ -\dfrac{2}{q^3(s)} & \left(q(s)+\dfrac{2}{q^3(s)}\right) \end{vmatrix}}=(\alpha+1)p(s)\left(\frac{1}{q(s)}-\frac{6}{q(s)^5}\right),$$

,

$$\mathcal{G}(\overline{f}_1(t,\alpha),s)=\frac{\begin{vmatrix} (3-\alpha)p(s)\left(1+\dfrac{2}{q^4(s)}\right) & -\dfrac{2}{q^3(s)} \\ (3-\alpha)p(s)\left(1-\dfrac{6}{q^3(s)}-\dfrac{24}{q^8(s)}\right) & \left(q(s)+\dfrac{2}{q^3(s)}\right) \end{vmatrix}}{\begin{vmatrix} \left(q(s)-\dfrac{2}{q^3(s)}\right) & -\dfrac{2}{q^3(s)} \\ -\dfrac{2}{q^3(s)} & \left(q(s)+\dfrac{2}{q^3(s)}\right) \end{vmatrix}}=(3-\alpha)p(s)\left(\frac{1}{q(s)}+\frac{6}{q(s)^5}\right),$$

$$\mathcal{G}(\overline{f}_2(t,\alpha),s)=\frac{\begin{vmatrix} q(s)-\dfrac{2}{q^3(s)} & (3-\alpha)p(s)\left(1+\dfrac{2}{q^4(s)}\right) \\ -\dfrac{2}{q^3(s)} & (3-\alpha)p(s)\left(1-\dfrac{6}{q^3(s)}-\dfrac{24}{q^8(s)}\right) \end{vmatrix}}{\begin{vmatrix} \left(q(s)-\dfrac{2}{q^3(s)}\right) & -\dfrac{2}{q^3(s)} \\ -\dfrac{2}{q^3(s)} & \left(q(s)+\dfrac{2}{q^3(s)}\right) \end{vmatrix}}=(3-\alpha)p(s)\left(\frac{1}{q(s)}-\frac{6}{q(s)^5}\right),$$

,

$$\mathcal{G}(\underline{\underline{f_1}}(t,\alpha),s)=\frac{\begin{vmatrix}\alpha p(s)\left(1+\dfrac{2}{q^4(s)}\right) & -\dfrac{2}{q^3(s)}\\[2ex] \alpha p(s)\left(1-\dfrac{6}{q^3(s)}-\dfrac{24}{q^8(s)}\right) & \left(q(s)+\dfrac{2}{q^3(s)}\right)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{2}{q^3(s)}\right) & -\dfrac{2}{q^3(s)}\\[2ex] -\dfrac{2}{q^3(s)} & \left(q(s)+\dfrac{2}{q^3(s)}\right)\end{vmatrix}}=\alpha p(s)\left(\frac{1}{q(s)}+\frac{6}{q(s)^5}\right),$$

$$\mathcal{G}(\underline{\underline{f_2}}(t,\alpha),s)=\frac{\begin{vmatrix}q(s)-\dfrac{2}{q^3(s)} & \alpha p(s)\left(1+\dfrac{2}{q^4(s)}\right)\\[2ex] -\dfrac{2}{q^3(s)} & \alpha p(s)\left(1-\dfrac{6}{q^3(s)}-\dfrac{24}{q^8(s)}\right)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{2}{q^3(s)}\right) & -\dfrac{2}{q^3(s)}\\[2ex] -\dfrac{2}{q^3(s)} & \left(q(s)+\dfrac{2}{q^3(s)}\right)\end{vmatrix}}=\alpha p(s)\left(\frac{1}{q(s)}-\frac{6}{q(s)^5}\right),$$

$$\mathcal{G}(\overline{\overline{f_1}}(t,\alpha),s)=\frac{\begin{vmatrix}-\alpha p(s)\left(1+\dfrac{2}{q^4(s)}\right) & -\dfrac{2}{q^3(s)}\\[2ex] -\alpha p(s)\left(1-\dfrac{6}{q^3(s)}-\dfrac{24}{q^8(s)}\right) & \left(q(s)+\dfrac{2}{q^3(s)}\right)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{2}{q^3(s)}\right) & -\dfrac{2}{q^3(s)}\\[2ex] -\dfrac{2}{q^3(s)} & \left(q(s)+\dfrac{2}{q^3(s)}\right)\end{vmatrix}}=-\alpha p(s)\left(\frac{1}{q(s)}+\frac{6}{q(s)^5}\right),$$

$$\mathcal{G}(\overline{\overline{f_2}}(t,\alpha),s)=\frac{\begin{vmatrix}q(s)-\dfrac{2}{q^3(s)} & -\alpha p(s)\left(1+\dfrac{2}{q^4(s)}\right)\\[2ex] -\dfrac{2}{q^3(s)} & -\alpha p(s)\left(1-\dfrac{6}{q^3(s)}-\dfrac{24}{q^8(s)}\right)\end{vmatrix}}{\begin{vmatrix}\left(q(s)-\dfrac{2}{q^3(s)}\right) & -\dfrac{2}{q^3(s)}\\[2ex] -\dfrac{2}{q^3(s)} & \left(q(s)+\dfrac{2}{q^3(s)}\right)\end{vmatrix}}=-\alpha p(s)\left(\frac{1}{q(s)}-\frac{6}{q(s)^5}\right),$$

Operating the inverse of I-F-G-transform, we get

$$\begin{cases}\underline{f_1}=(\alpha+1)\left(1+\frac{1}{4}t^4\right), \quad \overline{f_1}=(3-\alpha)\left(1+\frac{1}{4}t^4\right), \quad \underline{\underline{f_1}}=\alpha\left(1+\frac{1}{4}t^4\right), \quad \overline{\overline{f_1}}=-\alpha\left(1+\frac{1}{4}t^4\right).\\ and\\ \underline{f_2}=(\alpha+1)\left(1-\frac{1}{4}t^4\right), \quad \overline{f_2}=(3-\alpha)\left(1-\frac{1}{4}t^4\right), \quad \underline{\underline{f_2}}=\alpha\left(1-\frac{1}{4}t^4\right), \quad \overline{\overline{f_2}}=-\alpha\left(1-\frac{1}{4}t^4\right).\end{cases}$$

Example 4.2. Consider the following Intuitionistic Fuzzy system of linear Volterra integro-differential equations

$$\begin{cases}f_1''(t,\alpha)=(2\alpha,\alpha-1,-\alpha,\alpha+2)\cos t+\int_0^t f_1(x)dx+\int_0^t f_2(x)dx,\\[2ex] f_2''(t,\alpha)=(-2\alpha,1-\alpha,\alpha,-2-\alpha)(t-\sin t)-\int_0^t f_1(x)dx-\int_0^t f_2(x)dx.\end{cases} \tag{13}$$

With,

$$\begin{cases} f_1(0,\alpha) = (2\alpha, \alpha - 1, -\alpha, \alpha + 2), \quad f_1^{'}(0,\alpha) = 0, \\ \\ f_2(0,\alpha) = (-2\alpha, 1-\alpha, \alpha, -2-\alpha), \quad f_2^{'}(0,\alpha) = 0. \end{cases} \tag{14}$$

Taking I-F-G-transform on system 13 and using theorem 3.3 with theorem 3.5 and initial conditions, we have

$$\begin{cases} \left(q(s)^2 - \dfrac{1}{q(s)}\right) \mathcal{G}(\underline{f_1}(t,\alpha),s) - \dfrac{1}{q(s)} \mathcal{G}(\underline{f_2}(t,\alpha),s) = 2\alpha q(s)p(s) + \dfrac{2\alpha q(s)p(s)}{q^2(s)+1}, \\ \\ \dfrac{1}{q(s)} \mathcal{G}(\underline{f_1}(t,\alpha),s) + \left(q(s)^2 + \dfrac{1}{q(s)}\right) \mathcal{G}(\underline{f_2}(t,\alpha),s) = -2\alpha q(s)p(s) - \dfrac{2\alpha p(s)}{q^2(s)} + \dfrac{2\alpha p(s)}{q^2(s)+1}. \end{cases} \tag{15}$$

Similarly, we obtain

$$\begin{cases} \left(q(s)^2 - \dfrac{1}{q(s)}\right) \mathcal{G}(\overline{f_1}(t,\alpha),s) - \dfrac{1}{q(s)} \mathcal{G}(\overline{f_2}(t,\alpha),s) = (\alpha-1)q(s)p(s) + \dfrac{(\alpha-1)q(s)p(s)}{q^2(s)+1}, \\ \\ \dfrac{1}{q(s)} \mathcal{G}(\overline{f_1}(t,\alpha),s) + \left(q(s)^2 + \dfrac{1}{q(s)}\right) \mathcal{G}(\overline{f_2}(t,\alpha),s) = (1-\alpha)q(s)p(s) + \dfrac{(1-\alpha)p(s)}{q^2(s)} - \dfrac{(1-\alpha)p(s)}{q^2(s)+1}. \end{cases} \tag{16}$$

,

$$\begin{cases} \left(q(s)^2 - \dfrac{1}{q(s)}\right) \mathcal{G}(\underline{\underline{f_1}}(t,\alpha),s) - \dfrac{1}{q(s)} \mathcal{G}(\underline{\underline{f_2}}(t,\alpha),s) = \alpha q(s)p(s) + \dfrac{\alpha q(s)p(s)}{q^2(s)+1}, \\ \\ \dfrac{1}{q(s)} \mathcal{G}(\underline{\underline{f_1}}(t,\alpha),s) + \left(q(s)^2 + \dfrac{1}{q(s)}\right) \mathcal{G}(\underline{\underline{f_2}}(t,\alpha),s) = -\alpha q(s)p(s) - \dfrac{\alpha p(s)}{q^2(s)} + \dfrac{\alpha p(s)}{q^2(s)+1}. \end{cases} \tag{17}$$

and

$$\begin{cases} \left(q(s)^2 - \dfrac{1}{q(s)}\right) \mathcal{G}(\overline{\overline{f_1}}(t,\alpha),s) - \dfrac{1}{q(s)} \mathcal{G}(\overline{\overline{f_2}}(t,\alpha),s) = (\alpha+2)q(s)p(s) + \dfrac{(\alpha+2)q(s)p(s)}{q^2(s)+1}, \\ \\ \dfrac{1}{q(s)} \mathcal{G}(\overline{\overline{f_1}}(t,\alpha),s) + \left(q(s)^2 + \dfrac{1}{q(s)}\right) \mathcal{G}(\overline{\overline{f_2}}(t,\alpha),s) = (-2-\alpha)q(s)p(s) + \dfrac{(-2-\alpha)p(s)}{q^2(s)} - \dfrac{(-2-\alpha)p(s)}{q^2(s)+1}. \end{cases} \tag{18}$$

The solutions of systems (15), (16), (17) and (18) are given by

$$\begin{aligned} \mathcal{G}(\underline{f}_1(t,\alpha),s) &= \frac{\begin{vmatrix} 2\alpha q(s)p(s) + \dfrac{2\alpha q(s)p(s)}{q^2(s)+1} & -\dfrac{1}{q(s)} \\ \\ -2\alpha q(s)p(s) - \dfrac{2\alpha p(s)}{q^2(s)} + \dfrac{2\alpha p(s)}{q^2(s)+1} & \left(q^2(s) + \dfrac{1}{q(s)}\right) \end{vmatrix}}{\begin{vmatrix} \left(q^2(s) - \dfrac{1}{q(s)}\right) & -\dfrac{1}{q(s)} \\ \\ \dfrac{1}{q(s)} & \left(q^2(s) + \dfrac{1}{q(s)}\right) \end{vmatrix}} \\ &= 2\alpha \left(\frac{3p(s)}{q(s)} - \frac{p(s)}{q^2(s)} - \frac{p(s)}{q^3(s)} + \frac{p(s)}{q^4(s)} + \frac{p(s)}{q^5(s)} - \frac{p(s)}{q^7(s)} - \frac{2p(s)q(s)}{q^2(s)+1} + \frac{p(s)}{q^2(s)+1} \right), \end{aligned}$$

,

$$\begin{aligned}\mathcal{G}(\underline{f}_2(t,\alpha),s) &= \frac{\begin{vmatrix} q^2(s)-\dfrac{1}{q(s)} & 2\alpha q(s)p(s)+\dfrac{2\alpha q(s)p(s)}{q^2(s)+1} \\[2ex] \dfrac{1}{q(s)} & -2\alpha q(s)p(s)-\dfrac{2\alpha p(s)}{q^2(s)}+\dfrac{2\alpha p(s)}{q^2(s)+1} \end{vmatrix}}{\begin{vmatrix} \left(q^2(s)-\dfrac{1}{q(s)}\right) & -\dfrac{1}{q(s)} \\[2ex] \dfrac{1}{q(s)} & \left(q^2(s)+\dfrac{1}{q(s)}\right) \end{vmatrix}} \\ &= -2\alpha\left(\frac{2p(s)}{q(s)}-\frac{2p(s)}{q^2(s)}-\frac{p(s)}{q^3(s)}+\frac{2p(s)}{q^4(s)}+\frac{p(s)}{q^5(s)}-\frac{p(s)}{q^7(s)}-\frac{p(s)q(s)}{q^2(s)+1}+\frac{2p(s)}{q^2(s)+1}\right),\end{aligned}$$

,

$$\begin{aligned}\mathcal{G}(\overline{f}_1(t,\alpha),s) &= \frac{\begin{vmatrix} 2(\alpha-1)q(s)p(s)+\dfrac{(\alpha-1)q(s)p(s)}{q^2(s)+1} & -\dfrac{1}{q(s)} \\[2ex] (1-\alpha)q(s)p(s)-\dfrac{(1-\alpha)p(s)}{q^2(s)}+\dfrac{(1-\alpha)p(s)}{q^2(s)+1} & \left(q^2(s)+\dfrac{1}{q(s)}\right) \end{vmatrix}}{\begin{vmatrix} \left(q^2(s)-\dfrac{1}{q(s)}\right) & -\dfrac{1}{q(s)} \\[2ex] \dfrac{1}{q(s)} & \left(q^2(s)+\dfrac{1}{q(s)}\right) \end{vmatrix}} \\ &= 2\alpha\left(\frac{3p(s)}{q(s)}-\frac{p(s)}{q^2(s)}-\frac{p(s)}{q^3(s)}+\frac{p(s)}{q^4(s)}+\frac{p(s)}{q^5(s)}-\frac{p(s)}{q^7(s)}-\frac{2p(s)q(s)}{q^2(s)+1}+\frac{p(s)}{q^2(s)+1}\right),\end{aligned}$$

,

$$\begin{aligned}\mathcal{G}(\overline{f}_2(t,\alpha),s) &= \frac{\begin{vmatrix} q^2(s)-\dfrac{1}{q(s)} & 2\alpha q(s)p(s)+\dfrac{2\alpha q(s)p(s)}{q^2(s)+1} \\[2ex] \dfrac{1}{q(s)} & -2\alpha q(s)p(s)-\dfrac{2\alpha p(s)}{q^2(s)}+\dfrac{2\alpha p(s)}{q^2(s)+1} \end{vmatrix}}{\begin{vmatrix} \left(q^2(s)-\dfrac{1}{q(s)}\right) & -\dfrac{1}{q(s)} \\[2ex] \dfrac{1}{q(s)} & \left(q^2(s)+\dfrac{1}{q(s)}\right) \end{vmatrix}} \\ &= (1-\alpha)\left(\frac{2p(s)}{q(s)}-\frac{2p(s)}{q^2(s)}-\frac{p(s)}{q^3(s)}+\frac{2p(s)}{q^4(s)}+\frac{p(s)}{q^5(s)}-\frac{p(s)}{q^7(s)}-\frac{p(s)q(s)}{q^2(s)+1}+\frac{2p(s)}{q^2(s)+1}\right),\end{aligned}$$

$$\begin{aligned}\mathcal{G}(\underline{\underline{f_1}}(t,\alpha),s) &= \frac{\begin{vmatrix} 2\alpha q(s)p(s)+\dfrac{2\alpha q(s)p(s)}{q^2(s)+1} & -\dfrac{1}{q(s)} \\[2ex] -2\alpha q(s)p(s)-\dfrac{2\alpha p(s)}{q^2(s)}+\dfrac{2\alpha p(s)}{q^2(s)+1} & \left(q^2(s)+\dfrac{1}{q(s)}\right) \end{vmatrix}}{\begin{vmatrix} \left(q^2(s)-\dfrac{1}{q(s)}\right) & -\dfrac{1}{q(s)} \\[2ex] \dfrac{1}{q(s)} & \left(q^2(s)+\dfrac{1}{q(s)}\right) \end{vmatrix}} \\ &= -\alpha\left(\frac{3p(s)}{q(s)}-\frac{p(s)}{q^2(s)}-\frac{p(s)}{q^3(s)}+\frac{p(s)}{q^4(s)}+\frac{p(s)}{q^5(s)}-\frac{p(s)}{q^7(s)}-\frac{2p(s)q(s)}{q^2(s)+1}+\frac{p(s)}{q^2(s)+1}\right),\end{aligned}$$

,

$$\begin{aligned}
\mathcal{G}(\underline{\underline{f_2}}(t,\alpha),s) &= \frac{\begin{vmatrix} q^2(s)-\dfrac{1}{q(s)} & 2\alpha q(s)p(s)+\dfrac{2\alpha q(s)p(s)}{q^2(s)+1} \\[2ex] \dfrac{1}{q(s)} & -2\alpha q(s)p(s)-\dfrac{2\alpha p(s)}{q^2(s)}+\dfrac{2\alpha p(s)}{q^2(s)+1} \end{vmatrix}}{\begin{vmatrix} \left(q^2(s)-\dfrac{1}{q(s)}\right) & -\dfrac{1}{q(s)} \\[2ex] \dfrac{1}{q(s)} & \left(q^2(s)+\dfrac{1}{q(s)}\right) \end{vmatrix}} \\
&= \alpha\left(\frac{2p(s)}{q(s)}-\frac{2p(s)}{q^2(s)}-\frac{p(s)}{q^3(s)}+\frac{2p(s)}{q^4(s)}+\frac{p(s)}{q^5(s)}-\frac{p(s)}{q^7(s)}-\frac{p(s)q(s)}{q^2(s)+1}+\frac{2p(s)}{q^2(s)+1}\right),
\end{aligned}$$

,

$$\begin{aligned}
\mathcal{G}(\overline{\overline{f_1}}(t,\alpha),s) &= \frac{\begin{vmatrix} 2(\alpha-1)q(s)p(s)+\dfrac{(\alpha-1)q(s)p(s)}{q^2(s)+1} & -\dfrac{1}{q(s)} \\[2ex] (1-\alpha)q(s)p(s)-\dfrac{(1-\alpha)p(s)}{q^2(s)}+\dfrac{(1-\alpha)p(s)}{q^2(s)+1} & \left(q^2(s)+\dfrac{1}{q(s)}\right) \end{vmatrix}}{\begin{vmatrix} \left(q^2(s)-\dfrac{1}{q(s)}\right) & -\dfrac{1}{q(s)} \\[2ex] \dfrac{1}{q(s)} & \left(q^2(s)+\dfrac{1}{q(s)}\right) \end{vmatrix}} \\
&= (\alpha+2)\left(\frac{3p(s)}{q(s)}-\frac{p(s)}{q^2(s)}-\frac{p(s)}{q^3(s)}+\frac{p(s)}{q^4(s)}+\frac{p(s)}{q^5(s)}-\frac{p(s)}{q^7(s)}-\frac{2p(s)q(s)}{q^2(s)+1}+\frac{p(s)}{q^2(s)+1}\right),
\end{aligned}$$

,

$$\begin{aligned}
\mathcal{G}(\overline{\overline{f_2}}(t,\alpha),s) &= \frac{\begin{vmatrix} q^2(s)-\dfrac{1}{q(s)} & 2\alpha q(s)p(s)+\dfrac{2\alpha q(s)p(s)}{q^2(s)+1} \\[2ex] \dfrac{1}{q(s)} & -2\alpha q(s)p(s)-\dfrac{2\alpha p(s)}{q^2(s)}+\dfrac{2\alpha p(s)}{q^2(s)+1} \end{vmatrix}}{\begin{vmatrix} \left(q^2(s)-\dfrac{1}{q(s)}\right) & -\dfrac{1}{q(s)} \\[2ex] \dfrac{1}{q(s)} & \left(q^2(s)+\dfrac{1}{q(s)}\right) \end{vmatrix}} \\
&= (-\alpha-2)\left(\frac{2p(s)}{q(s)}-\frac{2p(s)}{q^2(s)}-\frac{p(s)}{q^3(s)}+\frac{2p(s)}{q^4(s)}+\frac{p(s)}{q^5(s)}-\frac{p(s)}{q^7(s)}-\frac{p(s)q(s)}{q^2(s)+1}+\frac{2p(s)}{q^2(s)+1}\right),
\end{aligned}$$

Operating the inverse of I-F-G-transform, we obtain the required solution of the systems (13) with (14) as

$$\begin{cases}
\underline{f_1} = 2\alpha\left(3 - t - \frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{\Gamma(5)}t^4 - \frac{1}{\Gamma(7)}t^6 - 2\cos t + \sin t\right), \\
\underline{f_2} = -2\alpha\left(2 - 2t - \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{\Gamma(5)}t^4 - \frac{1}{\Gamma(7)}t^6 - \cos t + 2\sin t\right), \\
\overline{f_1} = (\alpha - 1)\left(3 - t - \frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{\Gamma(5)}t^4 - \frac{1}{\Gamma(7)}t^6 - 2\cos t + \sin t\right), \\
\overline{f_2} = (1 - \alpha)\left(2 - 2t - \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{\Gamma(5)}t^4 - \frac{1}{\Gamma(7)}t^6 - \cos t + 2\sin t\right), \\
\underline{\underline{f_1}} = \alpha\left(3 - t - \frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{\Gamma(5)}t^4 - \frac{1}{\Gamma(7)}t^6 - 2\cos t + \sin t\right), \\
\underline{\underline{f_2}} = -\alpha\left(2 - 2t - \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{\Gamma(5)}t^4 - \frac{1}{\Gamma(7)}t^6 - \cos t + 2\sin t\right), \\
\\
\overline{\overline{f_1}} = (2 + \alpha)\left(3 - t - \frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{\Gamma(5)}t^4 - \frac{1}{\Gamma(7)}t^6 - 2\cos t + \sin t\right), \\
\overline{\overline{f_2}} = (-2 - \alpha)\left(2 - 2t - \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{\Gamma(5)}t^4 - \frac{1}{\Gamma(7)}t^6 - \cos t + 2\sin t\right).
\end{cases}$$

Conclusion

In this paper we have presented intuitionistic fuzzy system of linear Volterra integro-differential equations of the second kind and we applied the Intuitionistic fuzzy General integral transform method to solve these systems. The methodology is completely explained by two numerical examples. The results of these examples show that the Intuitionistic fuzzy General integral transform is a very effective to determine the exact solution of fuzzy system of linear Volterra integro-differential equations of the second kind. Future research can explore the extension of this method to solve nonlinear intuitionistic fuzzy system of Volterra integro-differential equations and other types of Intuitionistic fuzzy integral equations. Additionally We can extend these method to solve integral, differential and integro-differential equations in Neutrosophic Environment.

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References

[1] K. S. Aboodh. (2013). The New Integral Transform Aboodh Transform, *Global Journal of pure and Applied Mathematics*, 9(1), 35-43.

[2] S. A. P. Ahmadi, H. Hosseinzadeh, A. Y. Cherati. (2019). A New Integral Transform for Solving Higher Order Linear Ordinary Differential Equations, *Nonlinear Dyn Syst Theory*, 19(2), 243-52.

[3] K.T. Atanassov. (1986). Intuitionistic fuzzy sets. *Fuzzy sets and systems*, 20, 87-96. https://doi.org/10.1016/S0165-0114(86)80034-3.

[4] B. Bede, I. J. Rudas, A. L. Bencsik. (2007). First order linear fuzzy differential equations under generalized di6erentiability, *Information Sciences*, 177(7), 1648-1662. https://doi.org/10.1016/j.ins.2006.08.021

[5] S. S. L. Chang, L. Zadeh. (1972). On fuzzy mapping and control, *IEEE Trans. Systems Man Cybernet*, 2, 30-34. https://doi.org/10.1109/TSMC.1972.5408553

[6] H. Eltayeb, A. Kiliman, and B. Fisher. (2010). A new integral transform and associated distributions, *Integral Transforms Special Funct*. 21(5), 367-379.

[7] T. M. Elzaki. (2011). The new integral transform Elzaki Transform. *Global Journal of Pure and Applied Mathematics*, 7(1), 57-64.

[8] R. Ettoussi, S. Melliani, M. Elomari and L. S. Chadli. (2015). Solution of Intuitionistic Fuzzy Differential Equations by successive approximation method, *Notes on Intuitionistic Fuzzy Sets*, 21(2), 51-62.

[9] P. Grzegorzewski. (2003). The hamming distance between intuitionistic fuzzy sets, *Proceedings of the 10th IFSA World Congress, Istanbul, Turkey*, 35-38.

[10] H. Jafari. (2021). A new general integral transform for solving integral equations, *Journal of Advanced Research*, 32, 133-138. https://doi.org/10.1016/j.jare.2020.08.016

[11] H. Kamal. (2016). A. Sedeeg, The new integral transform Kamal transform. *Adv Theoret Appl Mathe*, 11(4), 451-8.

[12] M. Matloka. (1987). On fuzzy integrals, *2nd Polish Symp.on Interval. Politechnika Poznansk*, 167-170.

[13] M. Mohand, A. Mahgoub. (2017). The new integral transform Mohand transfrom, *Adv. Theor. Appl. Math*, 12(2), 113–120.

[14] M. Mohand, M. Abdelrahim. (2019). The new integral transform Sawi Transform, *Adv Theoretical Appl Math*, 14(1), 81–87.

[15] S. Melliani, M. Elomari, L. S. Chadli and R. Ettoussi. (2015). Intuitionistic fuzzy metric space, *Notes on Intuitionistic Fuzzy Sets*, 21(1), 43-53.

[16] V. Nirmala, V. Parimala, P. Rajarajeswari. (2015). Modified Euler Method for Finding Numerical Solution of Intuitionistic Fuzzy Differential Equation under Generalised Differentiability Concept, *International Journal of Applied Engineering Research*, 10(72), 40-45.

[17] J.Y. Park, Y.C. Kwan, J.V. Jeong. (1995). Existence of solutions of fuzzy integral equations in Banach spaces, *Fuzzy Sets and System*, 72 (3), 373-378. https://doi.org/10.1016/0165-0114(94)00296-J

[18] K. Shah , M. Junaid , N. Ali. (2015). Extraction of laplace, sumudu, fourier and mellin transform from the natural transform. *Journal of Applied Environmental and Biological Sciences*, 5(9), 108-115.

[19] L.A. Zadeh. (1965). Fuzzy sets, *Information and Control*, 8(3), 338-353. https://doi.org/10.1016/S0019- 9958(65)90241-X.