Interval-Valued Picture Fuzzy Almost Ideals in Semigroups

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Abstract

An interval-valued neutrosophic set is a type of neutrosophic sets where the membership, indeterminacy, and non-membership degrees are represented by closed intervals within the unit interval $[0, 1]$. An interval-valued picture fuzzy set is one of special cases of interval-valued neutrosophic sets. In this paper, we apply interval-valued picture fuzzy sets on almost ideals of semigroups. Moreover, we study a relationship between each almost ideal in a semigroup and their interval-valued picture fuzzifications.

Keywords: Ideals; Almost ideals; Interval-valued fuzzy sets; Interval-valued picture fuzzy sets; Interval-valued picture fuzzy almost ideals

1 Introduction

A fuzzy subset of a set X was introduced by Zadeh[1] in 1965. The membership/truth (t) of for each element in a set X described with the membership function from X into the unit closed interval [0, 1]. Fuzzy set theory were applied in many algebraic structures. The concept of a fuzzy subset was applied to the elementary of groupoids and groups by Rosenfeld,[2] to that rings by Liu,[3] and to that semigroups by Kuroki.[4] Interval-valued fuzzy sets are a generalization of classical fuzzy sets where the membership values are simple closed intervals subsets of unit closed interval $[0, 1]$. In 2006, Narayanan and Manikantan[5] gave the notions of an interval-valued fuzzy subsemigroup and various interval-valued fuzzy ideals in semigroups. In 1983, Atanassov[6] extended a fuzzy subset to an intuitionistic fuzzy set. In intuitionistic fuzzy sets, an element is expressed by a degree of membership/truth (t) and a degree of nonmembership/falsehood (f), where the sum of these two degrees of membership is always less than or equal to one. Later, Atanassov and Gargov[7] proposed interval-valued intuitionistic fuzzy sets based on intuitionistic fuzzy sets and interval-valued fuzzy subsets. The neutrosophic set was introduced by Smarandache[8] who introduced the degree of indeterminancy/neuterality (i) and proposed the neutrosophic set on three components: (t, i, f) = (truth, indeterminacy, falsehood). Neutrosophic set theory is very important in many application fields since indeterminacy is quantified explicitly and the truth membership function, indeterminacy membership function and falsity membership functions are independent. Many years later, in 2013, a picture fuzzy set was first introduced by Cuong et al.,[9] which are extensions of the fuzzy subsets and the intuitionistic fuzzy sets. A picture fuzzy set is one of special cases of neutrosophic sets. In general, picture fuzzy sets are applicable in situations where human decisions demand more variety of responses: yes, abstain, no, and refusal. Additionally, a definition and properties of an interval-valued picture fuzzy set was simultaneously created. After a while, picture fuzzy sets have been redefined by Yang et al.[10] without mentioned the Cuong's defining of picture fuzzy sets. Later, Yiarayong[11] applied the concept of a picture fuzzy set on semigroups. A concept of an almost left [right] ideal and an almost ideal in a semigroup were lauched and studied by Grosek and Satko[12] in 1980.

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In 2014, Ye[13] given the concepts of similarity measures between interval neutrosophic sets showed some applications to multicriteria decision-making. The concepts of interval valued neutrosophic sets were applied on multi-attribute decision-making based on generalized weighted aggregation operator. In 2025, Yang, Niu and Li[14] showed applications of interval-valued neutrosophic sets in security evaluation in computer networks. Moreover, in 2025, Mukherjee and Das[15] given some generalization of interval-valued neutrosophic set and showed their applications.

In 2019, interval-valued picture fuzzy sets were studied in Liu et al.[16] and Khalil et al.[17] An interval-valued picture fuzzy set is one of special cases of interval-valued neutrosophic sets. Interval-valued picture fuzzy sets were applied to some process on decision making[16–18] and GRP method.[19]

In this paper, we apply interval-valued picture fuzzy sets on some subsets of semigroups. In Section 2, we will refer to almost ideals and fuzzy almost ideals of semigroups. In Section 3, we verify some basics of interval-valued picture fuzzy sets that are used in the main results of this paper. Main results of this paper will be contained in section 4, We introduce an interval-valued picture fuzzy almost left ideal, an interval-valued picture fuzzy almost right ideal, an interval-valued picture fuzzy almost ideal, an interval-valued picture fuzzy almost bi-ideal, an interval-valued picture fuzzy almost interior ideal, and an interval-valued picture fuzzy almost quasi-ideal in a semigroup. Next, we investigate their properties. Moreover, we give some real world problems related this paper in Section 5.

2 Almost ideals and fuzzy almost ideals of semigroups

Let S be a semigroup and let A and B be nonempty subsets of S. We define

$$AB = \{ab \mid a \in A, b \in B\}.$$

A nonempty subset A of S is called

1. an *almost left* [*right*] *ideal* of S if $SA \cap A \neq \emptyset$ [$AS \cap A \neq \emptyset$],
2. a *two-sided almost ideal* (*almost ideal*) of S, if A is both an almost left and an almost right ideal of S,
3. an *almost bi-ideal* of S if $ASA \cap A \neq \emptyset$,
4. an *almost interior ideal* of S if $SAS \cap A \neq \emptyset$,
5. an *almost quasi-ideal* of S if $AS \cap SA \cap A \neq \emptyset$.

A *fuzzy subset* of a set X is a function from X into a closed interval $[0, 1]$.

Let A be a nonempty subset of X. A *characteristic function* C_A of X is defined by

$$C_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

Let f be a nonzero fuzzy subset of a semigroup S. Then f is called

1. a *fuzzy almost left* [*right*] *ideal* of S if $(f \circ C_S) \cap f \neq 0 \Big[(C_S \circ f) \cap f \neq 0\Big]$,
2. a *fuzzy almost two-sided ideal* (*fuzzy almost ideal*) of S, if it is both a fuzzy almost left and a fuzzy almost right ideal of S,
3. a *fuzzy almost bi-ideal* of S if $(f \circ C_S \circ f) \cap f \neq 0$,
4. a *fuzzy almost interior ideal* of S if $(C_S \circ f \circ C_S) \cap f \neq 0$,
5. a *fuzzy almost quasi-ideal* of S if $(f \circ C_S) \cap (C_S \circ f) \cap f \neq 0$.

3 Basic concepts on interval picture fuzzy sets

Let $CI[0,1]$ be the set of all closed subintervals of $[0,1]$, that is,

$$CI[0,1] = \big\{[a,b] \mid a \leq b \text{ and } a,b \in [0,1]\big\}.$$

.

Let $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ be elements of $CI[0,1]$.

1. The *refined minimum* of I_1 and I_2, denoted by rmin$\{I_1, I_2\}$, is defined by
$$\text{rmin}\{I_1, I_2\} = \big[\min\{a_1, a_2\}, \min\{b_1, b_2\}\big].$$
2. The *refined maximum* of I_1 and I_2, denoted by rmax$\{I_1, I_2\}$, is defined by
$$\text{rmax}\{I_1, I_2\} = \big[\max\{a_1, a_2\}, \max\{b_1, b_2\}\big].$$
3. $I_1 \succeq I_2$ if and only if $a_1 \geq a_2$ and $b_1 \geq b_2$.
4. $I_1 \preceq I_2$ if and only if $a_1 \leq a_2$ and $b_1 \leq b_2$.
5. $I_1 = I_2$ if and only if $a_1 = a_2$ and $b_1 = b_2$.
6. $I_1 \succ I_2$ if and only if $I_1 \succeq I_2$ and $I_1 \neq I_2$.
7. $I_1 \prec I_2$ if and only if $I_1 \preceq I_2$ and $I_1 \neq I_2$.

Remark 3.1. Let I_1 and I_2 be element of $CI[0,1]$. Then

(1) If $I_1 \preceq I_2$ and $I_1 \neq [0,0]$, then $I_2 \neq [0,0]$.

(2) If $I_1 \preceq I_2$ and $I_2 \neq [1,1]$, then $I_1 \neq [1,1]$.

Let $\big\{[a_i, b_i] \mid i \in \Lambda\big\}$ be the collection of close subintervals of $[0,1]$. We define

$$\inf_{i\in\Lambda}[a_i, b_i] = \inf_{i\in\Lambda} a_i, \text{ and } \sup_{i\in\Lambda}[a_i, b_i] = \sup_{i\in\Lambda} b_i,$$
$$\text{rinf}_{i\in\Lambda}[a_i, b_i] = \big[\inf_{i\in\Lambda} a_i, \inf_{i\in\Lambda} b_i\big], \text{ and } \text{rsup}_{i\in\Lambda}[a_i, b_i] = \big[\sup_{i\in\Lambda} a_i, \sup_{i\in\Lambda} b_i\big].$$

For the rest of this paper, let X be a nonempty set and let S be a semigroup.

An *interval-valued fuzzy subset* of X is a function from X into $CI[0,1]$.

Let $\overline{f}$ and $\overline{g}$ be interval-valued fuzzy subsets of X. Then

1. $\overline{f} \subseteq \overline{g}$ if and only if $\overline{f}(x) \preceq \overline{g}(x)$ for all $x \in X$.
2. $\overline{f} = \overline{g}$ if and only if $\overline{f}(x) = \overline{g}(x)$ for all $x \in X$.
3. $(\overline{f} \cup \overline{g})(x) = \text{rmax}\{\overline{f}(x), \overline{g}(x)\}$ for all $x \in X$.
4. $(\overline{f} \cap \overline{g})(x) = \text{rmin}\{\overline{f}(x), \overline{g}(x)\}$ for all $x \in X$.

Proposition 3.2. [4] *Let $\overline{f}$, $\overline{g}$ and $\overline{h}$ be interval-valued fuzzy subsets of X. Then the following statements are true.*

(1) $\overline{f} \subseteq \overline{f} \cup \overline{g}$ and $\overline{g} \subseteq \overline{f} \cup \overline{g}$.

(2) $\overline{f} \cap \overline{g} \subseteq \overline{f}$ *and* $\overline{f} \cap \overline{g} \subseteq \overline{g}$.

(3) If $\overline{f} \subseteq \overline{g}$ *and* $\overline{g} \subseteq \overline{h}$, *then* $\overline{f} \subseteq \overline{h}$.

(4) If $\overline{f} \subseteq \overline{g}$, *then* $\overline{f} \cup \overline{h} \subseteq \overline{g} \cup \overline{h}$ *and* $\overline{h} \cup \overline{f} \subseteq \overline{h} \cup \overline{g}$.

(5) If $\overline{f} \subseteq \overline{g}$, *then* $\overline{f} \cap \overline{h} \subseteq \overline{g} \cap \overline{h}$ *and* $\overline{h} \cap \overline{f} \subseteq \overline{h} \cap \overline{g}$.

Let A be a nonempty subset of X. Define interval-valued fuzzy subsets $\overline{C}_A$ and $\overline{C}'_A$ of X by

$$\overline{C}_A(x) = \begin{cases} [1,1] & \text{if } x \in A, \\ [0,0] & \text{if } x \notin A, \end{cases} \text{ and } \overline{C}'_A(x) = \begin{cases} [0,0] & \text{if } x \in A, \\ [1,1] & \text{if } x \notin A. \end{cases}$$

Proposition 3.3. [4] *Let A and B be two nonempty subsets of X. The following properties are true.*

(1) $A \subseteq B$ *if and only if* $\overline{C}_A \subseteq \overline{C}_B$.

(2) $\overline{C}_A \cup \overline{C}_B = \overline{C}_{A\cup B}$.

(3) $\overline{C}_A \cap \overline{C}_B = \overline{C}_{A\cap B}$.

From the above proposition, we can construct the following proposition.

Proposition 3.4. *Let A and B be nonempty subsets of X. Then*

(1) $A \subseteq B$ *if and only if* $\overline{C}'_B \subseteq \overline{C}'_A$.

(2) $\overline{C}'_{A\cup B} \subseteq \overline{C}'_A \cup \overline{C}'_B$.

(3) $\overline{C}'_A \cap \overline{C}'_B \subseteq \overline{C}'_{A\cap B}$.

(4) $\overline{C}'_A \cup \overline{C}'_B = \overline{C}'_{A\cap B}$.

(5) $\overline{C}'_A \cap \overline{C}'_B = \overline{C}'_{A\cup B}$.

Let $\overline{f}$ and $\overline{g}$ be two interval-valued fuzzy subsets of S. Define interval-valued fuzzy subsets $\overline{f} \,\overline{\circ}\, \overline{g}$ and $\overline{f} \,\overline{\bullet}\, \overline{g}$ of S by

$$(\overline{f}\overline{\circ}\overline{g})(x) = \begin{cases} \operatorname{rsup}_{x=yz} \operatorname{rmin}\{\overline{f}(y), \overline{g}(z)\} & \text{if } x \in S^2, \\ [0,0] & \text{if } x \notin S^2, \end{cases}$$

and

$$(\overline{f}\overline{\bullet}\overline{g})(x) = \begin{cases} \operatorname{rinf}_{x=yz} \operatorname{rmax}\{\overline{f}(y), \overline{g}(z)\} & \text{if } x \in S^2, \\ [1,1] & \text{if } x \notin S^2. \end{cases}$$

It is well-known that the operations $\overline{\circ}$ and $\overline{\bullet}$ are associative.

The following two propositions are properties of $\overline{\circ}$ and $\overline{\bullet}$.

Proposition 3.5. *Let $\overline{f}$, $\overline{g}$ and $\overline{h}$ be interval-valued fuzzy subsets of S.*

(1) If $\overline{g} \subseteq \overline{h}$, *then* $\overline{f} \,\overline{\circ}\, \overline{g} \subseteq \overline{f} \,\overline{\circ}\, \overline{h}$ *and* $\overline{g} \,\overline{\circ}\, \overline{f} \subseteq \overline{h} \,\overline{\circ}\, \overline{f}$.

(2) $\overline{f} \,\overline{\circ}\, (\overline{g} \cup \overline{h}) = (\overline{f} \,\overline{\circ}\, \overline{g}) \cup (\overline{f} \,\overline{\circ}\, \overline{h})$.

(3) $\overline{f} \,\overline{\circ}\, (\overline{g} \cap \overline{h}) = (\overline{f} \,\overline{\circ}\, \overline{g}) \cap (\overline{f} \,\overline{\circ}\, \overline{h})$.

(4) $(\overline{f} \cup \overline{g}) \,\overline{\circ}\, \overline{h} = (\overline{f} \,\overline{\circ}\, \overline{h}) \cup (\overline{g} \,\overline{\circ}\, \overline{h})$.

(5) $(\overline{f} \cap \overline{g}) \mathbin{\bar{\circ}} \overline{h} = (\overline{f} \mathbin{\bar{\circ}} \overline{h}) \cap (\overline{g} \mathbin{\bar{\circ}} \overline{h})$.

Proposition 3.6. *Let $\overline{f}, \overline{g}$ and $\overline{h}$ be interval-valued fuzzy subsets of S.*

(1) *If $\overline{g} \subseteq \overline{h}$, then $\overline{f} \mathbin{\bar{\bullet}} \overline{g} \subseteq \overline{f} \mathbin{\bar{\bullet}} \overline{h}$ and $\overline{g} \mathbin{\bar{\bullet}} \overline{f} \subseteq \overline{h} \mathbin{\bar{\bullet}} \overline{f}$.*

(2) $\overline{f} \mathbin{\bar{\bullet}} (\overline{g} \cup \overline{h}) = (\overline{f} \mathbin{\bar{\bullet}} \overline{g}) \cup (\overline{f} \mathbin{\bar{\bullet}} \overline{h})$.

(3) $\overline{f} \mathbin{\bar{\bullet}} (\overline{g} \cap \overline{h}) = (\overline{f} \mathbin{\bar{\bullet}} \overline{g}) \cap (\overline{f} \mathbin{\bar{\bullet}} \overline{h})$.

(4) $(\overline{f} \cup \overline{g}) \mathbin{\bar{\bullet}} \overline{h} = (\overline{f} \mathbin{\bar{\bullet}} \overline{h}) \cup (\overline{g} \mathbin{\bar{\bullet}} \overline{h})$.

(5) $(\overline{f} \cap \overline{g}) \mathbin{\bar{\bullet}} \overline{h} = (\overline{f} \mathbin{\bar{\bullet}} \overline{h}) \cap (\overline{g} \mathbin{\bar{\bullet}} \overline{h})$.

Proposition 3.7. [4] *Let A and B be nonempty subsets of S. Then*

$$\overline{C}_A \mathbin{\bar{\circ}} \overline{C}_B = \overline{C}_{AB}.$$

Proposition 3.8. *Let A and B be nonempty subsets of S. Then*

$$\overline{C}'_A \mathbin{\bar{\bullet}} \overline{C}'_B = \overline{C}'_{AB}.$$

An *interval-valued picture fuzzy set*[10] of X is defined to be the set

$$\Big\{\big(x, \overline{\mu}(x), \overline{\eta}(x), \overline{\nu}(x)\big) \mid x \in X\Big\},$$

where $\overline{\mu}, \overline{\eta}, \overline{\nu}$ are interval-valued fuzzy subsets of X which satisfy the following conditions

$$0 \leq \sup\ \overline{\mu}(x) + \sup\ \overline{\nu}(x) \leq 1 \text{ and } 0 \leq \sup\ \overline{\mu}(x) + \sup\ \overline{\eta}(x) + \sup\ \overline{\nu}(x) \leq 2 \text{ for all } x \in X.$$

The interval-valued picture fuzzy set of X is briefly denoted by $(\overline{\mu}, \overline{\eta}, \overline{\nu})$.

Let $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1)$ and $(\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ be two interval-valued picture fuzzy sets of X. Then

1. $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1) \subseteq (\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ if and only if

$$\overline{\mu}_1(x) \preceq \overline{\mu}_2(x), \overline{\eta}_1(x) \succeq \overline{\eta}_2(x), \text{ and } \overline{\nu}_1(x) \succeq \overline{\nu}_2(x) \text{ for all } x \in X.$$

2. $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1) = (\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ if and only if

$$\overline{\mu}_1(x) = \overline{\mu}_2(x), \overline{\eta}_1(x) = \overline{\eta}_2(x), \text{ and } \overline{\nu}_1(x) = \overline{\nu}_2(x) \text{ for all } x \in X.$$

3. The *union* of $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1)$ and $(\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ is defined by

$$(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1) \cup (\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2) = \left\{\Big(x, (\overline{\mu}_1 \cup \overline{\mu}_2)(x), (\overline{\eta}_1 \cap \overline{\eta}_2)(x), (\overline{\nu}_1 \cap \overline{\nu}_2)(x)\Big) \mid x \in X\right\}.$$

4. The *intersection* of $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1)$ and $(\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ is defined by

$$(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1) \cap (\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2) = \left\{\Big(x, (\overline{\mu}_1 \cap \overline{\mu}_2)(x), (\overline{\eta}_1 \cup \overline{\eta}_2)(x), (\overline{\nu}_1 \cup \overline{\nu}_2)(x)\Big) \mid x \in X\right\}.$$

The next results are following from Proposition 3.2.

Proposition 3.9. *Let $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ be interval-valued picture fuzzy sets of X.*

(1) $\mathcal{A}_1 \subseteq \mathcal{A}_1 \cup \mathcal{A}_2$ *and* $\mathcal{A}_2 \subseteq \mathcal{A}_1 \cup \mathcal{A}_2$.

(2) $\mathcal{A}_1 \cap \mathcal{A}_2 \subseteq \mathcal{A}_1$ *and* $\mathcal{A}_1 \cap \mathcal{A}_2 \subseteq \mathcal{A}_2$.

(3) *If* $\mathcal{A}_1 \subseteq \mathcal{A}_2$ *and* $\mathcal{A}_2 \subseteq \mathcal{A}_3$, *then* $\mathcal{A}_1 \subseteq \mathcal{A}_3$.

(4) *If* $\mathcal{A}_1 \subseteq \mathcal{A}_2$, *then* $\mathcal{A}_1 \cup \mathcal{A}_3 \subseteq \mathcal{A}_2 \cup \mathcal{A}_3$ *and* $\mathcal{A}_3 \cup \mathcal{A}_1 \subseteq \mathcal{A}_3 \cup \mathcal{A}_2$.

(5) *If* $\mathcal{A}_1 \subseteq \mathcal{A}_2$, *then* $\mathcal{A}_1 \cap \mathcal{A}_3 \subseteq \mathcal{A}_2 \cap \mathcal{A}_3$ *and* $\mathcal{A}_3 \cap \mathcal{A}_1 \subseteq \mathcal{A}_3 \cap \mathcal{A}_2$.

Let A be a nonempty subset of X. The *characteristic interval-valued picture fuzzy set* of A in X is defined by

$$\left\{\left(x, \overline{C}_A(x), \overline{C}'_A(x), \overline{C}'_A(x)\right) \mid x \in X\right\}.$$

We denote the characteristic interval-valued picture fuzzy set of A in X by $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$.

Remark 3.10. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be interval-valued picture fuzzy sets of S.

(1) If $\mathcal{A}_1 \subseteq \mathcal{A}_2$ and $\mathcal{A}_1 \neq (\overline{C}'_S, \overline{C}_S, \overline{C}_S)$, then $\mathcal{A}_2 \neq (\overline{C}'_S, \overline{C}_S, \overline{C}_S)$.

(2) If $\mathcal{A}_1 \subseteq \mathcal{A}_2$ and $\mathcal{A}_2 \neq (\overline{C}_S, \overline{C}'_S, \overline{C}'_S)$, then $\mathcal{A}_1 \neq (\overline{C}_S, \overline{C}'_S, \overline{C}'_S)$.

The following proposition is a direct consequence of Proposition 3.3 and Proposition 3.4.

Proposition 3.11. *Let* $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ *and* $(\overline{C}_B, \overline{C}'_B, \overline{C}'_B)$ *be two characteristic interval-valued picture fuzzy sets of a set* A *and* B *of* X, *respectively. Then*

(1) $A \subseteq B$ *if and only if* $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A) \subseteq (\overline{C}_B, \overline{C}'_B, \overline{C}'_B)$.

(2) $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A) \cup (\overline{C}_B, \overline{C}'_B, \overline{C}'_B) = (\overline{C}_{A\cup B}, \overline{C}'_{A\cup B}, \overline{C}'_{A\cup B})$.

(3) $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A) \cap (\overline{C}_B, \overline{C}'_B, \overline{C}'_B) = (\overline{C}_{A\cap B}, \overline{C}'_{A\cap B}, \overline{C}'_{A\cap B})$.

The *interval-valued picture fuzzy product* of interval-valued picture fuzzy sets $\mathcal{A}_1 = (\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1)$ and $\mathcal{A}_2 = (\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ of S, written as $\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{A}_2$, is defined by

$$\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{A}_2 = \left\{\left(x, (\overline{\mu}_1 \,\bar{\circ}\, \overline{\mu}_2)(x), (\overline{\eta}_1 \,\bar{\bullet}\, \overline{\eta}_2)(x), (\overline{\nu}_1 \,\bar{\bullet}\, \overline{\nu}_2)(x)\right) \mid x \in S\right\}.$$

The Proposition 3.5 leads to the next proposition.

Proposition 3.12. *Let* $\mathcal{A}_1, \mathcal{A}_2$, *and* $\mathcal{A}_3$ *be interval-valued picture fuzzy sets of* S. *If* $\mathcal{A}_2 \subseteq \mathcal{A}_3$, *then* $\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{A}_2 \subseteq \mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{A}_3$ *and* $\mathcal{A}_2 \,\bar{\circ}_p\, \mathcal{A}_1 \subseteq \mathcal{A}_3 \,\bar{\circ}_p\, \mathcal{A}_1$.

The following results follow directly from Proposition 3.6.

Proposition 3.13. *Let* $\mathcal{A}_1$, $\mathcal{A}_2$ *and* $\mathcal{A}_3$ *be interval-valued picture fuzzy sets of* S.

(1) $\mathcal{A}_1 \,\bar{\circ}_p\, \big(\mathcal{A}_2 \cap \mathcal{A}_3\big) = \big(\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{A}_2\big) \cap \big(\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{A}_3\big)$.

(2) $\big(\mathcal{A}_1 \cap \mathcal{A}_2\big) \,\bar{\circ}_p\, \mathcal{A}_3 = \big(\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{A}_3\big) \cap \big(\mathcal{A}_2 \,\bar{\circ}_p\, \mathcal{A}_3\big)$.

(3) $\mathcal{A}_1 \,\bar{\circ}_p\, \big(\mathcal{A}_2 \cup \mathcal{A}_3\big) = \big(\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{A}_2\big) \cup \big(\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{A}_3\big)$.

(4) $\big(\mathcal{A}_1 \cup \mathcal{A}_2\big) \,\bar{\circ}_p\, \mathcal{A}_3 = \big(\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{A}_3\big) \cup \big(\mathcal{A}_2 \,\bar{\circ}_p\, \mathcal{A}_3\big)$.

Proposition 3.7 and Proposition 3.8 yield the following result.

Proposition 3.14. *Let* $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ *and* $(\overline{C}_B, \overline{C}'_B, \overline{C}'_B)$ *be two characteristic interval-valued picture fuzzy sets of* S. *Then*

$$(\overline{C}_A, \overline{C}'_A, \overline{C}'_A) \,\bar{\circ}_p\, (\overline{C}_B, \overline{C}'_B, \overline{C}'_B) = (\overline{C}_{AB}, \overline{C}'_{AB}, \overline{C}'_{AB}).$$

A *support* of an interval-valued picture fuzzy set $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ of X, written as $supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$, is defined by

$$supp(\overline{\mu}, \overline{\eta}, \overline{\nu}) = \left\{x \in X \mid \overline{\mu}(x) \neq [0,0] \text{ or } \overline{\eta}(x) \neq [1,1] \text{ or } \overline{\nu}(x) \neq [1,1]\right\}.$$

4 Main Results

First, we will define an interval-valued picture almost left ideal, an interval-valued picture almost right ideal, and an interval-valued picture almost ideal of a semigroup.

Definition 4.1. Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy subset of S such that $(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Then

(1) $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is called an *interval-valued picture fuzzy almost left ideal* of S, if

$$\Big((\overline{C}_S, \overline{C}'_S, \overline{C}'_S) \,\bar{\circ}_p\, (\overline{\mu}, \overline{\eta}, \overline{\nu})\Big) \cap (\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big).$$

(2) $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is called an *interval-valued picture fuzzy almost right ideal* of S, if

$$\Big((\overline{\mu}, \overline{\eta}, \overline{\nu}) \,\bar{\circ}_p\, (\overline{C}_S, \overline{C}'_S, \overline{C}'_S)\Big) \cap (\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big).$$

(3) $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is called an *interval-valued picture fuzzy almost ideal* of S if it is both an interval-valued picture fuzzy almost left ideal and an interval-valued picture fuzzy almost right ideal of S.

Theorem 4.2. *Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy set of S such that $(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Then*

(1) $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost left ideal of S if and only if there exists $x \in S$ such that

$$\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0], \ \textit{or} \ \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1], \ \textit{or} \ \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq [1,1].$$

(2) $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost right ideal of S if and only if there exists $x \in S$ such that

$$\big[(\overline{\mu} \,\bar{\circ}\, \overline{C}_S) \cap \overline{\mu}\big](x) \neq [0,0], \ \textit{or} \ \big[(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S) \cup \overline{\eta}\big](x) \neq [1,1], \ \textit{or} \ \big[(\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S) \cup \overline{\nu}\big](x) \neq [1,1].$$

(3) $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost ideal of S if and only if there exists $x \in S$ such that

$$\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0] \ \textit{or} \ \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1] \ \textit{or} \ \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq [1,1],$$
$$\textit{and}$$
$$\big[(\overline{\mu} \,\bar{\circ}\, \overline{C}_S) \cap \overline{\mu}\big](x) \neq [0,0] \ \textit{or} \ \big[(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S) \cup \overline{\eta}\big](x) \neq [1,1] \ \textit{or} \ \big[(\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S) \cup \overline{\nu}\big](x) \neq [1,1].$$

Proof. (1) We see that

$$\begin{aligned}\Big((\overline{C}_S, \overline{C}'_S, \overline{C}'_S) \,\bar{\circ}_p\, (\overline{\mu}, \overline{\eta}, \overline{\nu})\Big) \cap (\overline{\mu}, \overline{\eta}, \overline{\nu}) &= \Big(\overline{C}_S \,\bar{\circ}\, \overline{\mu}, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta}, \overline{C}'_S \,\bar{\bullet}\, \overline{\nu}\Big) \cap (\overline{\mu}, \overline{\eta}, \overline{\nu}) \\ &= \Big((\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}, (\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}, (\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\Big).\end{aligned}$$

Assume that $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost left ideal of S. So

$$\Big((\overline{C}_S, \overline{C}'_S, \overline{C}'_S) \,\bar{\circ}_p\, (\overline{\mu}, \overline{\eta}, \overline{\nu})\Big) \cap (\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big).$$

Then $\Big((\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}, (\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}, (\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\Big) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Thus there exists $x \in S$ such that

$$\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq \overline{C}'_S(x) = [0,0], \text{ or } \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq \overline{C}_S(x) = [1,1], \text{ or }$$
$$\big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq \overline{C}_S(x) = [1,1].$$

To prove the converse, assume that there exists $x \in S$ such that

$$\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0], \text{ or } \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1], \text{ or } \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq [1,1].$$

Then $\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq \overline{C}'_S(x)$, or $\big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq \overline{C}_S(x)$, or $\big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq \overline{C}_S(x)$. Thus

$$\Big((\overline{C}_S, \overline{C}'_S, \overline{C}'_S) \,\bar{\circ}_p\, (\overline{\mu}, \overline{\eta}, \overline{\nu})\Big) \cap (\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big).$$

Hence, $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost left ideal of S.

The proof of (2) is similar to the proof of (1), and (3) is a result from (1) and (2). □

Theorem 4.3. *Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be interval-valued picture fuzzy sets of S such that $\mathcal{A}_1 \subseteq \mathcal{A}_2$. Then the following statements hold:*

(1) If $\mathcal{A}_1$ is an interval-valued picture fuzzy almost left ideal of S, then $\mathcal{A}_2$ is an interval-valued picture fuzzy almost left ideal of S.

(2) If $\mathcal{A}_1$ is an interval-valued picture fuzzy almost right ideal of S, then $\mathcal{A}_2$ is an interval-valued picture fuzzy almost right ideal of S.

(3) If $\mathcal{A}_1$ is an interval-valued picture fuzzy almost ideal of S, then $\mathcal{A}_2$ is an interval-valued picture fuzzy almost ideal of S.

Proof. (1) Let $\mathcal{A}_1$ be an interval-valued picture fuzzy almost left ideal of S and $\mathcal{A}_2$ be an interval-valued picture fuzzy subset of S such that $\mathcal{A}_1 \subseteq \mathcal{A}_2$. Let $\mathcal{S} = (\overline{C}_S, \overline{C}'_S, \overline{C}'_S)$. Then $(\mathcal{S} \,\bar{\circ}_p\, \mathcal{A}_1) \cap \mathcal{A}_1 \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. By Proposition 3.9 and 3.12, $(\mathcal{S} \,\bar{\circ}_p\, \mathcal{A}_1) \cap \mathcal{A}_1 \subseteq (\mathcal{S} \,\bar{\circ}_p\, \mathcal{A}_2) \cap \mathcal{A}_2$. It follows from Remark 3.10 that $(\mathcal{S} \,\bar{\circ}_p\, \mathcal{A}_2) \cap \mathcal{A}_2 \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Hence, $\mathcal{A}_2$ is an interval-valued picture fuzzy almost left ideal of S.

The proof of (2) is similar to the proof of (1), and (3) is a result from (1) and (2). □

Corollary 4.4. *Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be two interval-valued picture fuzzy sets of S. Then the following properties hold:*

(1) If $\mathcal{A}_1$ and $\mathcal{A}_2$ are interval-valued picture fuzzy almost left ideals of S, then $\mathcal{A}_1 \cup \mathcal{A}_2$ is also an interval-valued picture fuzzy almost left ideal of S.

(2) If $\mathcal{A}_1$ and $\mathcal{A}_2$ are interval-valued picture fuzzy almost right ideals of S, then $\mathcal{A}_1 \cup \mathcal{A}_2$ is also an interval-valued picture fuzzy almost right ideal of S.

(3) If $\mathcal{A}_1$ and $\mathcal{A}_2$ are interval-valued picture fuzzy almost ideals of S, then $\mathcal{A}_1 \cup \mathcal{A}_2$ is also an interval-valued picture fuzzy almost ideal of S.

Example 4.5. *Consider the semigroup $\mathbb{Z}_5$ under the usual addition. Interval-valued picture fuzzy sets $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1)$ and $(\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ of $\mathbb{Z}_5$ are defined by*

$$\begin{aligned}(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1) = \Big\{ &\Big(\overline{0}, [0.3, 0.4], [0.2, 0.5], [0.1, 0.1]\Big), \Big(\overline{1}, [0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\Big),\\ &\Big(\overline{2}, [0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\Big), \Big(\overline{3}, [0.5, 0.8], [0.2, 0.2], [0.0, 0.0]\Big),\\ &\Big(\overline{4}, [0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\Big)\Big\}, \text{ and}\end{aligned}$$

$$\begin{aligned}(\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2) = \Big\{ &\Big(\overline{0}, [0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\Big), \Big(\overline{1}, [0.0, 0.3], [0.1, 0.4], [0.1, 0.1]\Big),\\ &\Big(\overline{2}, [0.1, 0.1], [0.4, 0.6], [0.1, 0.1]\Big), \Big(\overline{3}, [0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\Big),\\ &\Big(\overline{4}, [0.2, 0.2], [0.0, 0.0], [0.0, 0.0]\Big)\Big\}.\end{aligned}$$

We have that

$$\begin{aligned}
&\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}_1) \cap \overline{\mu}_1\big](\bar{0}) = [0.3, 0.4], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}_1) \cup \overline{\eta}_1\big](\bar{0}) = [0.2, 0.5], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}_1) \cup \overline{\nu}_1\big](\bar{0}) = [0.1, 0.1],\\
&\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}_1) \cap \overline{\mu}_1\big](\bar{1}) = [0.0, 0.0], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}_1) \cup \overline{\eta}_1\big](\bar{1}) = [1.0, 1.0], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}_1) \cup \overline{\nu}_1\big](\bar{1}) = [1.0, 1.0],\\
&\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}_1) \cap \overline{\mu}_1\big](\bar{2}) = [0.0, 0.0], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}_1) \cup \overline{\eta}_1\big](\bar{2}) = [1.0, 1.0], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}_1) \cup \overline{\nu}_1\big](\bar{2}) = [1.0, 1.0],\\
&\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}_1) \cap \overline{\mu}_1\big](\bar{3}) = [0.5, 0.8], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}_1) \cup \overline{\eta}_1\big](\bar{3}) = [0.2, 0.2], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}_1) \cup \overline{\nu}_1\big](\bar{3}) = [0.0, 0.0],\\
&\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}_1) \cap \overline{\mu}_1\big](\bar{4}) = [0.0, 0.0], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}_1) \cup \overline{\eta}_1\big](\bar{4}) = [1.0, 1.0], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}_1) \cup \overline{\nu}_1\big](\bar{4}) = [1.0, 1.0].
\end{aligned}$$

Then $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1)$ is an interval-valued picture fuzzy almost left ideal of $\mathbb{Z}_5$. Similarly, $(\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ is an interval-valued picture fuzzy almost left ideals of $\mathbb{Z}_5$.

We obtain that $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1) \cup (\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ is an interval-valued picture fuzzy almost left ideal of $\mathbb{Z}_5$ by Corollary 4.4 but $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1) \cap (\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ is not an interval-valued picture fuzzy almost left ideal of $\mathbb{Z}_5$ because $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1) \cap (\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2) = (\overline{C}'_S, \overline{C}_S, \overline{C}_S)$.

Theorem 4.6. *Let A be a nonempty subset of S. Then the following properties hold:*

(1) A is an almost left ideal of S if and only if $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ is an interval-valued picture fuzzy almost left ideal of S.

(2) A is an almost right ideal of S if and only if $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ is an interval-valued picture fuzzy almost right ideal of S.

(3) A is an almost ideal of S if and only if $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ is an interval-valued picture fuzzy almost ideal of S.

Proof. Let A be an almost left ideal of S. Then $SA \cap A \neq \emptyset$. Thus $x \in SA \cap A$ for some $x \in S$. Then

$$\Big[\big(\overline{C}_S \,\bar{\circ}\, \overline{C}_A\big) \cap \overline{C}_A\Big](x) = \big(\overline{C}_{SA} \cap \overline{C}_A\big)(x) = \overline{C}_{SA \cap A}(x) = [1, 1] \neq [0, 0].$$

By Theorem 4.2, $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ is an interval-valued picture fuzzy almost left ideal of S.

Conversely, let $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ be an interval-valued picture fuzzy almost left ideal of S. Thus there exists $x \in S$ such that $\overline{C}_{SA \cap A}(x) \neq [0, 0]$ or $\overline{C}'_{SA \cap A}(x) \neq [1, 1]$. Hence, $x \in SA \cap A$, that is, $SA \cap A \neq \emptyset$. Therefore, A is an almost left ideal of S.

(2) This proof is similar to (1).

(3) This proof follows from (1) and (2). □

Theorem 4.7. *Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy set S. The following statements hold:*

(1) If $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost left ideal of S, then $supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost left ideal of S.

(2) If $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost right ideal of S, then $supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost right ideal of S.

(3) If $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost ideal of S, then $supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost ideal of S.

Proof. (1) Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy almost left ideal of S. Then there exists $x \in S$ such that

$$\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0, 0], \text{ or } \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1, 1], \text{ or } \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq [1, 1].$$

Case 1: $\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0]$. Then $(\overline{C}_S \,\bar{\circ}\, \overline{\mu})(x) \neq [0,0]$ and $\overline{\mu}(x) \neq [0,0]$. Choose $b \in S$ such that $x = ab$ and $\overline{\mu}(b) = \underset{x=yz}{\text{rsup}}\,\overline{\mu}(z)$. Thus

$$\overline{\mu}(b) = \underset{x=yz}{\text{rsup}}\,\overline{\mu}(z) = \underset{x=yz}{\text{rsup}}\,\text{rmin}\big\{\overline{C}_S(y), \overline{\mu}(z)\big\} = (\overline{C}_S \,\bar{\circ}\, \overline{\mu})(x) \neq [0,0].$$

Hence, $x, b \in supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$. So that $x = ab \in S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$. That is $S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \emptyset$. Therefore, $supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost left ideal of S.

Case 2: $\big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1]$. Then $(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta})(x) \neq [1,1]$ and $\overline{\eta}(x) \neq [1,1]$. Choose $b \in S$ such that $x = ab$ and $\overline{\eta}(b) = \underset{x=yz}{\text{rinf}}\,\overline{\eta}(z)$. Thus

$$\overline{\eta}(b) = \underset{x=yz}{\text{rinf}}\,\overline{\eta}(z) = \underset{x=yz}{\text{rinf}}\,\text{rmax}\big\{\overline{C}'_S(y), \overline{\eta}(z)\big\} = (\overline{C}'_S \,\bar{\bullet}\, \overline{\eta})(x) \neq [1,1].$$

Hence, $x, b \in supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$. So that $x = ab \in S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$. That is $S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \emptyset$. Therefore, $supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost left ideal of S.

Case 3: $\big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq [1,1]$. This case we can be considered like Case 2.

(2) This proof is similar to (1) by changing the multiplication direction.

(3) This statement follows from (1) and (2). □

Example 4.8. *Consider the semigroup $S = \{a, b, c, d\}$ with multiplication table as follows:*

	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	b
d	a	a	b	c

An interval-valued picture fuzzy set $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ of S is defined by

$$(\overline{\mu}, \overline{\eta}, \overline{\nu}) = \Big\{\Big(a, [0,0], [1,1], [1,1]\Big), \Big(b, [0,0], [0,0], [1,1]\Big), \Big(c, [0,0], [1,1], [1,1]\Big), \Big(d, [0,0], [1,1], [0,0]\Big)\Big\}.$$

Then we get $supp(\overline{\mu}, \overline{\eta}, \overline{\nu}) = \{b, d\}$ and it is an almost left ideal of S because

$$S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu}) = \{a, b, c\} \cap \{b, d\} = \{b\} \neq \emptyset.$$

However,

$$\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](a) = [0,0], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](a) = [1,1], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](a) = [1,1],$$
$$\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](b) = [0,0], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](b) = [1,1], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](b) = [1,1],$$
$$\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](c) = [0,0], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](c) = [1,1], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](c) = [1,1],$$
$$\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](d) = [0,0], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](d) = [1,1], \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](d) = [1,1].$$

Hence, $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is not an interval-valued picture fuzzy almost left ideal of S.

Next, we will introduce an interval-valued picture fuzzy almost bi-ideal of S and study its properties.

Definition 4.9. Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy subset of S such that $(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Then $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is called an *interval-valued picture fuzzy almost bi-ideal* of S, if

$$\Big((\overline{\mu}, \overline{\eta}, \overline{\nu}) \,\bar{\circ}_p\, (\overline{C}_S, \overline{C}'_S, \overline{C}'_S) \,\bar{\circ}_p\, (\overline{\mu}, \overline{\eta}, \overline{\nu})\Big) \cap (\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big).$$

Theorem 4.10. *Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy set of S such that $(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Then $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost bi-ideal of S if and only if there exists $x \in S$ such that*

$$\big[(\overline{\mu} \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0] \;\; or \;\; \big[(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1] \;\; or \;\; \big[(\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq [1,1].$$

Proof. We see that

$$\begin{aligned}
&\Big((\overline{\mu},\overline{\eta},\overline{\nu}) \,\bar{\circ}_p\, (\overline{C}_S,\overline{C}'_S,\overline{C}'_S) \,\bar{\circ}_p\, (\overline{\mu},\overline{\eta},\overline{\nu})\Big) \cap (\overline{\mu},\overline{\eta},\overline{\nu})\\
&\qquad = \Big(\overline{\mu} \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{\mu}, \overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta}, \overline{\nu} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\nu}\Big) \cap (\overline{\mu},\overline{\eta},\overline{\nu})\\
&\qquad = \Big((\overline{\mu} \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}, (\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}, (\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\Big).
\end{aligned}$$

Assume that $(\overline{\mu},\overline{\eta},\overline{\nu})$ is an interval-valued picture fuzzy almost bi-ideal of S. Then

$$\Big((\overline{\mu},\overline{\eta},\overline{\nu}) \,\bar{\circ}_p\, (\overline{C}_S,\overline{C}'_S,\overline{C}'_S) \,\bar{\circ}_p\, (\overline{\mu},\overline{\eta},\overline{\nu})\Big) \cap (\overline{\mu},\overline{\eta},\overline{\nu}) \neq \big(\overline{C}'_S,\overline{C}_S,\overline{C}_S\big).$$

So $\Big((\overline{\mu} \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}, (\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}, (\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\Big) \neq \big(\overline{C}'_S,\overline{C}_S,\overline{C}_S\big)$. Thus there exists $x \in S$ such that

$$\big[(\overline{\mu} \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq \overline{C}'_S(x) = [0,0], \text{ or } \big[(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq \overline{C}_S(x) = [1,1], \text{ or } \big[(\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq \overline{C}'_S(x) = [1,1].$$

Conversely, assume that there exists $x \in S$ such that

$$\big[(\overline{\mu} \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0], \text{ or } \big[(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1], \text{ or } \big[(\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq [1,1].$$

Then $\big[(\overline{\mu} \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}(x) \neq \overline{C}'_S(x)$, or $\big[(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq \overline{C}_S(x)$, or $\big[(\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq \overline{C}_S(x)$. Thus

$$\Big((\overline{\mu},\overline{\eta},\overline{\nu}) \,\bar{\circ}_p\, (\overline{C}_S,\overline{C}'_S,\overline{C}'_S) \,\bar{\circ}_p\, (\overline{\mu},\overline{\eta},\overline{\nu})\Big) \cap (\overline{\mu},\overline{\eta},\overline{\nu}) \neq \big(\overline{C}'_S,\overline{C}_S,\overline{C}_S\big).$$

Hence, $(\overline{\mu},\overline{\eta},\overline{\nu})$ is an interval-valued picture fuzzy almost bi-ideal of S. □

Example 4.11. *Consider the semigroup $\mathbb{Z}_5$ under the usual addition, and an interval-valued picture fuzzy set of $\mathbb{Z}_5$ defined by*

$$\begin{aligned}
(\overline{\mu},\overline{\eta},\overline{\nu}) = \Big\{ &\Big(\overline{0}, [1,1], [0,0], [0,0]\Big), \Big(\overline{1}, [1,1], [0,0], [0,0]\Big), \Big(\overline{2}, [1,1], [0,0], [0,0]\Big),\\
&\Big(\overline{3}, [0,0], [1,1], [1,1]\Big), \Big(\overline{4}, [0,0], [1,1], [1,1]\Big)\Big\}.
\end{aligned}$$

Therefore, $(\overline{\mu},\overline{\eta},\overline{\nu})$ is an interval-valued picture fuzzy almost bi-ideal of $\mathbb{Z}_5$.

Theorem 4.12. *Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be interval-valued picture fuzzy subsets of S such that $\mathcal{A}_1 \subseteq \mathcal{A}_2$. If $\mathcal{A}_1$ is an interval-valued picture fuzzy almost bi-ideal of S, then $\mathcal{A}_2$ is an interval-valued picture fuzzy almost bi-ideal of S.*

Proof. Let $\mathcal{A}_1$ be an interval-valued picture fuzzy almost bi-ideal of S and $\mathcal{A}_2$ be an interval-valued picture fuzzy subset of S such that $\mathcal{A}_1 \subseteq \mathcal{A}_2$. Let $\mathcal{S} = (\overline{C}_S, \overline{C}'_S, \overline{C}'_S)$. Then $(\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{S} \,\bar{\circ}_p\, \mathcal{A}_1) \cap \mathcal{A}_1 \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Since $(\mathcal{A}_1 \,\bar{\circ}_p\, \mathcal{S} \,\bar{\circ}_p\, \mathcal{A}_1) \cap \mathcal{A}_1 \subseteq (\mathcal{A}_2 \,\bar{\circ}_p\, \mathcal{S} \,\bar{\circ}_p\, \mathcal{A}_2) \cap \mathcal{A}_2$, we obtain that $(\mathcal{A}_2 \,\bar{\circ}_p\, \mathcal{S} \,\bar{\circ}_p\, \mathcal{A}_2) \cap \mathcal{A}_2 \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Hence, $\mathcal{A}_2$ is an interval-valued picture fuzzy almost bi-ideal of S. □

Corollary 4.13. *The union of two interval-valued picture fuzzy almost bi-ideals of S is also an interval-valued picture fuzzy almost bi-ideal of S.*

Theorem 4.14. *Let A be a nonempty subset of S. Then A is an almost bi-ideal of S if and only if $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ is an interval-valued picture fuzzy almost bi-ideal of S.*

Proof. Let A be an almost bi-ideal of S. Then $ASA \cap A \neq \emptyset$. Thus $x \in ASA \cap A$ for some $x \in S$. Then

$$\Big[(\overline{C}_A \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{C}_A) \cap \overline{C}_A\Big](x) = (\overline{C}_{ASA} \cap \overline{C}_A)(x) = \overline{C}_{ASA\cap A}(x) = [1,1] \neq [0,0].$$

Hence, $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ is an interval-valued picture fuzzy almost bi-ideal of S.

Conversely, let $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ be an interval-valued picture fuzzy almost bi-ideal of S. Thus there exists $x \in S$ such that $\overline{C}_{ASA\cap A}(x) \neq [0,0]$ or $\overline{C}'_{ASA\cap A}(x) \neq [1,1]$. Hence, $x \in ASA \cap A$, that is, $ASA \cap A \neq \emptyset$. Therefore, A is an almost bi-ideal of S. □

From Example 4.11, since $\{\overline{0}, \overline{1}, \overline{2}\}$ is an almost bi-ideal of $\mathbb{Z}_5$ and $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is equal to $(\overline{C}_{\{\overline{0},\overline{1},\overline{2}\}}, \overline{C}'_{\{\overline{0},\overline{1},\overline{2}\}}, \overline{C}'_{\{\overline{0},\overline{1},\overline{2}\}})$, we get $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost bi-ideal of $\mathbb{Z}_5$ by Theorem 4.14.

Theorem 4.15. *Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy set of S. If $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost bi-ideal of S, then $supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost bi-ideal of S.*

Proof. Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy almost bi-ideal of S. Then there exists $x \in S$ such that $\big[(\overline{\mu} \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0]$, or $\big[(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1]$, or $\big[(\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq [1,1]$.
Case 1: $\big[(\overline{\mu} \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0]$. Then $(\overline{\mu} \,\bar{\circ}\, \overline{C}_S \,\bar{\circ}\, \overline{\mu})(x) \neq [0,0]$ and $\overline{\mu}(x) \neq [0,0]$. Choose $a, c \in S$ such that $x = abc$, $\overline{\mu}(a) = \operatorname*{rsup}_{x=uvw} \overline{\mu}(u)$, and $\overline{\mu}(c) = \operatorname*{rsup}_{x=uvw} \overline{\mu}(w)$. Thus

$$\overline{\mu}(a) = \operatorname*{rsup}_{x=uvw} \overline{\mu}(u) = \operatorname*{rsup}_{x=uvw} \operatorname{rmin}\big\{\overline{\mu}(u), \overline{C}_S(v), \overline{\mu}(w)\big\} = (\overline{C}_S \,\bar{\circ}\, \overline{\mu})(x) \neq [0,0],$$
$$\overline{\mu}(c) = \operatorname*{rsup}_{x=uvw} \overline{\mu}(w) = \operatorname*{rsup}_{x=uvw} \operatorname{rmin}\big\{\overline{\mu}(u), \overline{C}_S(v), \overline{\mu}(w)\big\} = (\overline{C}_S \,\bar{\circ}\, \overline{\mu})(x) \neq [0,0],$$

Hence, $x, a, c \in supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$. So that $x = abc \in \big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big)S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$. That is, $S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \emptyset$. Therefore, $supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost bi-ideal of S.
Case 2: $\big[(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1]$. Then $(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta})(x) \neq [1,1]$ and $\overline{\eta}(x) \neq [1,1]$. Choose $a, c \in S$ such that $x = abc$, $\overline{\eta}(a) = \operatorname*{rinf}_{x=uvw} \overline{\eta}(u)$, and $\overline{\eta}(c) = \operatorname*{rinf}_{x=uvw} \overline{\eta}(w)$. Thus

$$\overline{\eta}(a) = \operatorname*{rinf}_{x=uvw} \overline{\eta}(u) = \operatorname*{rinf}_{x=uvw} \operatorname{rmax}\big\{\overline{\eta}(u), \overline{C}'_S(v), \overline{\eta}(w)\big\} = (\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta})(x) \neq [1,1],$$
$$\overline{\eta}(c) = \operatorname*{rinf}_{x=uvw} \overline{\eta}(w) = \operatorname*{rinf}_{x=uvw} \operatorname{rmax}\big\{\overline{\eta}(u), \overline{C}'_S(v), \overline{\eta}(w)\big\} = (\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S \,\bar{\bullet}\, \overline{\eta})(x) \neq [1,1].$$

Hence, $x, a, c \in supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$. So that $x = abc \in \big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big)S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$. That is $\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big)S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \emptyset$. Therefore, $supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost bi-ideal of S.
Case 3: $\big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1]$. It is similar to Case 2. □

Next, we give a notion of an interval-valued picture fuzzy almost interior ideal of S and investigate some properties of it.

Definition 4.16. Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy subset of S such that $(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Then $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is called an *interval-valued picture fuzzy almost interior ideal* of S, if

$$\Big((\overline{C}_S, \overline{C}'_S, \overline{C}'_S) \,\bar{\circ}_p\, (\overline{\mu}, \overline{\eta}, \overline{\nu}) \,\bar{\circ}_p\, (\overline{C}_S, \overline{C}'_S, \overline{C}'_S)\Big) \cap (\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big).$$

Theorem 4.17. *Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy set of S such that $(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Then $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost interior ideal of S if and only if there exists $x \in S$ such that*

$$\big[(\overline{C}_S \,\bar{\circ}\, \overline{\mu} \,\bar{\circ}\, \overline{C}_S) \cap \overline{\mu}\big](x) \neq [0,0], \text{ or } \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta} \,\bar{\bullet}\, \overline{C}'_S) \cup \overline{\eta}\big](x) \neq [1,1], \text{ or } \big[(\overline{C}'_S \,\bar{\bullet}\, \overline{\nu} \,\bar{\bullet}\, \overline{C}'_S) \cup \overline{\nu}\big](x) \neq [1,1].$$

Proof. This proof is similar to the proof of Theorem 4.2. □

Example 4.18. *Define an interval-valued picture fuzzy set of the semigroup $\mathbb{Z}_5$ under the usual addition by*

$$(\overline{\mu},\overline{\eta},\overline{\nu}) = \Big\{\Big(\overline{0},[1,1],[0,0],[0,0]\Big),\Big(\overline{1},[0,0],[1,1],[1,1]\Big),\Big(\overline{2},[1,1],[0,0],[0,0]\Big), \Big(\overline{3},[0,0],[1,1],[1,1]\Big),\Big(\overline{4},[0,0],[1,1],[1,1]\Big)\Big\}.$$

Then $(\overline{\mu},\overline{\eta},\overline{\nu})$ is an interval-valued picture fuzzy almost interior ideal of $\mathbb{Z}_5$.

Theorem 4.19. *Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be interval-valued picture fuzzy subsets of S such that $\mathcal{A}_1 \subseteq \mathcal{A}_2$. If $\mathcal{A}_1$ is an interval-valued picture fuzzy almost interior ideal of S, then $\mathcal{A}_2$ is an interval-valued picture fuzzy almost interior ideal of S.*

Proof. This proof is similar to the proof of Theorem 4.3. □

Corollary 4.20. *Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be two interval-valued picture fuzzy almost interior ideals of S. Then $\mathcal{A}_1 \cup \mathcal{A}_2$ is also an interval-valued picture fuzzy almost interior ideal of S.*

Theorem 4.21. *Let A be a nonempty subset of S. Then A is an almost interior ideal of S if and only if $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ is an interval-valued picture fuzzy almost interior ideal of S.*

Proof. This proof is similar to proof of Theorem 4.14. □

Theorem 4.22. *Let $(\overline{\mu},\overline{\eta},\overline{\nu})$ be an interval-valued picture fuzzy set of S. If $(\overline{\mu},\overline{\eta},\overline{\nu})$ is an interval-valued picture fuzzy almost interior ideal of S, then $supp(\overline{\mu},\overline{\eta},\overline{\nu})$ is an almost interior ideal of S.*

Proof. This proof is similar to the proof of Theorem 4.15. □

At the end of this section, an interval-valued picture fuzzy almost quasi-ideal of S will be defined and studied.

Definition 4.23. Let $(\overline{\mu},\overline{\eta},\overline{\nu})$ be an interval-valued picture fuzzy subset of S such that $(\overline{\mu},\overline{\eta},\overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Then $(\overline{\mu},\overline{\eta},\overline{\nu})$ is called an *interval-valued picture fuzzy almost quasi-ideal* of S, if

$$\Big((\overline{\mu},\overline{\eta},\overline{\nu}) \,\overline{\circ}_p\, (\overline{C}_S, \overline{C}'_S, \overline{C}'_S)\Big) \cap \Big((\overline{C}_S, \overline{C}'_S, \overline{C}'_S) \,\overline{\circ}_p\, (\overline{\mu},\overline{\eta},\overline{\nu})\Big) \cap (\overline{\mu},\overline{\eta},\overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big).$$

Theorem 4.24. *Let $(\overline{\mu},\overline{\eta},\overline{\nu})$ be an interval-valued picture fuzzy set of S such that $(\overline{\mu},\overline{\eta},\overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big)$. Then $(\overline{\mu},\overline{\eta},\overline{\nu})$ is an interval-valued picture fuzzy almost quasi-ideal of S if and only if there exists $x \in S$ such that*

$$\big[(\overline{\mu} \,\overline{\circ}\, \overline{C}_S) \cap (\overline{C}_S \,\overline{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0], \textit{ or } \big[(\overline{\eta} \,\overline{\bullet}\, \overline{C}'_S) \cup (\overline{C}'_S \,\overline{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1], \textit{ or}$$
$$\big[(\overline{\nu} \,\overline{\bullet}\, \overline{C}'_S) \cup (\overline{C}'_S \,\overline{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq [1,1].$$

Proof. We see that

$$\begin{aligned}
&\Big((\overline{\mu},\overline{\eta},\overline{\nu}) \,\overline{\circ}_p\, (\overline{C}_S, \overline{C}'_S, \overline{C}'_S)\Big) \cap \Big((\overline{C}_S, \overline{C}'_S, \overline{C}'_S) \,\overline{\circ}_p\, (\overline{\mu},\overline{\eta},\overline{\nu})\Big) \cap (\overline{\mu},\overline{\eta},\overline{\nu}) \\
&\quad = \Big((\overline{\mu} \,\overline{\circ}\, \overline{C}_S), (\overline{\eta} \,\overline{\bullet}\, \overline{C}'_S), (\overline{\nu} \,\overline{\bullet}\, \overline{C}'_S)\Big) \cap \Big((\overline{C}_S \,\overline{\circ}\, \overline{\mu}), (\overline{C}'_S \,\overline{\bullet}\, \overline{\eta}), (\overline{C}'_S \,\overline{\bullet}\, \overline{\nu})\Big) \cap (\overline{\mu},\overline{\eta},\overline{\nu}) \\
&\quad = \Big((\overline{\mu} \,\overline{\circ}\, \overline{C}_S) \cap (\overline{C}_S \,\overline{\circ}\, \overline{\mu}) \cap \overline{\mu}, (\overline{\eta} \,\overline{\bullet}\, \overline{C}'_S) \cup (\overline{C}'_S \,\overline{\bullet}\, \overline{\eta}) \cup \overline{\eta}, (\overline{\nu} \,\overline{\bullet}\, \overline{C}'_S) \cup (\overline{C}'_S \,\overline{\bullet}\, \overline{\nu}) \cup \overline{\nu}\Big).
\end{aligned}$$

Assume that $(\overline{\mu},\overline{\eta},\overline{\nu})$ is an interval-valued picture fuzzy almost quasi-ideal of S. Then

$$\Big((\overline{\mu},\overline{\eta},\overline{\nu}) \,\overline{\circ}_p\, (\overline{C}_S, \overline{C}'_S, \overline{C}'_S)\Big) \cap \Big((\overline{C}_S, \overline{C}'_S, \overline{C}'_S) \,\overline{\circ}_p\, (\overline{\mu},\overline{\eta},\overline{\nu})\Big) \cap (\overline{\mu},\overline{\eta},\overline{\nu}) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big).$$

Thus

$$\Big((\overline{\mu} \,\overline{\circ}\, \overline{C}_S) \cap (\overline{C}_S \,\overline{\circ}\, \overline{\mu}) \cap \overline{\mu}, (\overline{\eta} \,\overline{\bullet}\, \overline{C}'_S) \cup (\overline{C}'_S \,\overline{\bullet}\, \overline{\eta}) \cup \overline{\eta}, (\overline{\nu} \,\overline{\bullet}\, \overline{C}'_S) \cup (\overline{C}'_S \,\overline{\bullet}\, \overline{\nu}) \cup \overline{\nu}\Big) \neq \big(\overline{C}'_S, \overline{C}_S, \overline{C}_S\big).$$

So there exists $x \in S$ such that

$$\big[(\overline{\mu}\,\bar{\circ}\,\overline{C}_S)\cap(\overline{C}_S\,\bar{\circ}\,\overline{\mu})\cap\overline{\mu}\big](x)\neq[0,0],\ \text{or}\ \big[(\overline{\eta}\,\bar{\bullet}\,\overline{C}'_S)\cup(\overline{C}'_S\,\bar{\bullet}\,\overline{\eta})\cup\overline{\eta}\big](x)\neq[1,1],\ \text{or}$$
$$\big[(\overline{\nu}\,\bar{\bullet}\,\overline{C}'_S)\cup(\overline{C}'_S\,\bar{\bullet}\,\overline{\nu})\cup\overline{\nu}\big](x)\neq[1,1].$$

To prove the converse, assume that there exists $x\in S$ such that

$$\big[(\overline{\mu}\,\bar{\circ}\,\overline{C}_S)\cap(\overline{C}_S\,\bar{\circ}\,\overline{\mu})\cap\overline{\mu}\big](x)\neq[0,0],\ \text{or}\ \big[(\overline{\eta}\,\bar{\bullet}\,\overline{C}'_S)\cup(\overline{C}'_S\,\bar{\bullet}\,\overline{\eta})\cup\overline{\eta}\big](x)\neq[1,1],\ \text{or}$$
$$\big[(\overline{\nu}\,\bar{\bullet}\,\overline{C}'_S)\cup(\overline{C}'_S\,\bar{\bullet}\,\overline{\nu})\cup\overline{\nu}\big](x)\neq[1,1].$$

Then $\big[(\overline{\mu}\,\bar{\circ}\,\overline{C}_S)\cap(\overline{C}_S\,\bar{\circ}\,\overline{\mu})\cap\overline{\mu}\big](x)\neq\overline{C}'_S(x)$, or $\big[(\overline{\eta}\,\bar{\bullet}\,\overline{C}'_S)\cup(\overline{C}'_S\,\bar{\bullet}\,\overline{\eta})\cup\overline{\eta}\big](x)\neq\overline{C}_S(x)$, or $\big[(\overline{\nu}\,\bar{\bullet}\,\overline{C}'_S)\cup(\overline{C}'_S\,\bar{\bullet}\,\overline{\nu})\cup\overline{\nu}\big](x)\neq\overline{C}_S(x)$. Thus

$$\Big((\overline{\mu},\overline{\eta},\overline{\nu})\,\bar{\circ}_p\,(\overline{C}_S,\overline{C}'_S,\overline{C}'_S)\Big)\cap\Big((\overline{C}_S,\overline{C}'_S,\overline{C}'_S)\,\bar{\circ}_p\,(\overline{\mu},\overline{\eta},\overline{\nu})\Big)\cap(\overline{\mu},\overline{\eta},\overline{\nu})\neq\big(\overline{C}'_S,\overline{C}_S,\overline{C}_S\big).$$

Hence, $(\overline{\mu},\overline{\eta},\overline{\nu})$ is an interval-valued picture fuzzy almost bi-ideal of S. □

Example 4.25. *Consider the semigroup $\mathbb{Z}_5$ under the usual addition. Define an interval-valued picture fuzzy sets of $\mathbb{Z}_5$ by*

$$(\overline{\mu}_1,\overline{\eta}_1,\overline{\nu}_1)=\Big\{\Big(\overline{0},[0.3,0.4],[0,0],[0,0]\Big),\Big(\overline{1},[0,0],[0.3,0.4],[1,1]\Big),\Big(\overline{2},[0,0],[0.3,0.4],[1,1]\Big),$$
$$\Big(\overline{3},[0,0],[0.3,0.4],[1,1]\Big),\Big(\overline{4},[0,0],[0.3,0.4],[1,1]\Big)\Big\},\ \textit{and}$$
$$(\overline{\mu}_2,\overline{\eta}_2,\overline{\nu}_2)=\Big\{\Big(\overline{0},[0.3,0.4],[0,0],[0,0]\Big),\Big(\overline{1},[0.3,0.4],[0,0],[0,0]]\Big),\Big(\overline{2},[0,0],[0.3,0.4],[1,1]\Big),$$
$$\Big(\overline{3},[0,0],[0.3,0.4],[1,1]\Big),\Big(\overline{4},[0,0],[0.3,0.4],[1,1]\Big)\Big\}.$$

Then $(\overline{\mu}_1,\overline{\eta}_1,\overline{\nu}_1)$ and $(\overline{\mu}_2,\overline{\eta}_2,\overline{\nu}_2)$ are interval-valued picture fuzzy almost quasi-ideals of $\mathbb{Z}_5$.

Theorem 4.26. *Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be interval-valued picture fuzzy subsets of S such that $\mathcal{A}_1\subseteq\mathcal{A}_2$. If $\mathcal{A}_1$ is an interval-valued picture fuzzy almost quasi-ideal of S, then $\mathcal{A}_2$ is an interval-valued picture fuzzy almost quasi-ideal of S.*

Proof. Let $\mathcal{A}_1$ be an interval-valued picture fuzzy almost quasi-ideal of S and $\mathcal{A}_2$ be an interval-valued picture fuzzy subset of S such that $\mathcal{A}_1\subseteq\mathcal{A}_2$. Let $\mathcal{S}=(\overline{C}_S,\overline{C}'_S,\overline{C}'_S)$. Then $(\mathcal{A}_1\,\bar{\circ}_p\,\mathcal{S})\cap(\mathcal{S}\,\bar{\circ}_p\,\mathcal{A}_1)\cap\mathcal{A}_1\neq\big(\overline{C}'_S,\overline{C}_S,\overline{C}_S\big)$. Since

$$(\mathcal{A}_1\,\bar{\circ}_p\,\mathcal{S})\cap(\mathcal{S}\,\bar{\circ}_p\,\mathcal{A}_1)\cap\mathcal{A}_1\subseteq(\mathcal{A}_2\,\bar{\circ}_p\,\mathcal{S})\cap(\mathcal{S}\,\bar{\circ}_p\,\mathcal{A}_2)\cap\mathcal{A}_2,$$

we obtain that $(\mathcal{A}_2\,\bar{\circ}_p\,\mathcal{S})\cap(\mathcal{S}\,\bar{\circ}_p\,\mathcal{A}_2)\cap\mathcal{A}_2\neq\big(\overline{C}'_S,\overline{C}_S,\overline{C}_S\big)$. Hence, $\mathcal{A}_2$ is an interval-valued picture fuzzy almost quasi-ideal of S. □

Corollary 4.27. *The union of two interval-valued picture fuzzy almost quasi-ideals of S is also an interval-valued picture fuzzy almost quasi-ideal of S.*

Theorem 4.28. *Let A be a nonempty subset of S. Then A is an almost quasi-ideal of S if and only if $(\overline{C}_A,\overline{C}'_A,\overline{C}'_A)$ is an interval-valued picture fuzzy almost quasi-ideal of S.*

Proof. Let A be an almost quasi-ideal of S. Then $AS\cap SA\cap A\neq\emptyset$. Thus $x\in AS\cap SA\cap A$ for some $x\in S$. Then

$$\begin{aligned}\Big[(\overline{C}_A\,\bar{\circ}\,\overline{C}_S)\cap(\overline{C}_S\,\bar{\circ}\,\overline{C}_A)\cap\overline{C}_A\Big](x)&=(\overline{C}_{AS}\cap\overline{C}_{SA}\cap\overline{C}_A)(x)\\&=\overline{C}_{AS\cap SA\cap A}(x)\\&=[1,1]\\&\neq[0,0].\end{aligned}$$

Therefore, $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ is an interval-valued picture fuzzy almost quasi-ideal of S.

Conversely, let $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ be an interval-valued picture fuzzy almost quasi-ideal of S. Thus there exists $x \in S$ such that $\overline{C}_{AS\cap SA\cap A}(x) \neq [0,0]$ or $\overline{C}'_{AS\cap SA\cap A}(x) \neq [1,1]$. Hence, $x \in AS \cap SA \cap A$. That is, $AS \cap SA \cap A \neq \emptyset$. Therefore, A is an almost quasi-ideal of S. □

Theorem 4.29. *Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy set of S. If $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an interval-valued picture fuzzy almost quasi-ideal of S, then supp$(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost quasi-ideal of S.*

Proof. Let $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ be an interval-valued picture fuzzy almost quasi-ideal of S. Then there exists $x \in S$ such that

$$\big[(\overline{\mu} \,\bar{\circ}\, \overline{C}_S) \cap (\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0], \text{ or } \big[(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S) \cup (\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1], \text{ or}$$
$$\big[(\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S) \cup (\overline{C}'_S \,\bar{\bullet}\, \overline{\nu}) \cup \overline{\nu}\big](x) \neq [1,1].$$

Case 1: $\big[(\overline{\mu} \,\bar{\circ}\, \overline{C}_S) \cap (\overline{C}_S \,\bar{\circ}\, \overline{\mu}) \cap \overline{\mu}\big](x) \neq [0,0]$. Then $(\overline{\mu} \,\bar{\circ}\, \overline{C}_S)(x) \neq [0,0]$, $(\overline{C}_S \,\bar{\circ}\, \overline{\mu})(x) \neq [0,0]$, and $\overline{\mu}(x) \neq [0,0]$. Choose $a, d \in S$ such that $x = ab = cd$, $\overline{\mu}(a) = \underset{x=yz}{\text{rsup}}\, \overline{\mu}(y)$, and $\overline{\mu}(d) = \underset{x=yz}{\text{rsup}}\, \overline{\mu}(z)$. Thus

$$\begin{aligned}\overline{\mu}(a) &= \underset{x=yz}{\text{rsup}}\, \overline{\mu}(y) = \underset{x=yz}{\text{rsup}}\, \text{rmin}\big\{\overline{\mu}(y), \overline{C}_S(z)\big\} = (\overline{\mu} \,\bar{\circ}\, \overline{C}_S)(x) \neq [0,0],\\ \overline{\mu}(d) &= \underset{x=yz}{\text{rsup}}\, \overline{\mu}(z) = \underset{x=yz}{\text{rsup}}\, \text{rmin}\big\{\overline{C}_S(y), \overline{\mu}(z)\big\} = (\overline{C}_S \,\bar{\circ}\, \overline{\mu})(x) \neq [0,0].\end{aligned}$$

Hence, $x, a, d \in$ *supp*$(\overline{\mu}, \overline{\eta}, \overline{\nu})$. So that

$$x = ab = cd \in \big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big)S \cap S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu}).$$

That is, $\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big)S \cap S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \emptyset$. Therefore, *supp*$(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost quasi-ideal of S.

Case 2: $\big[(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S) \cup (\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1]$. Then $(\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S)(x) \neq [1,1]$, $(\overline{C}'_S \,\bar{\bullet}\, \overline{\eta})(x) \neq [1,1]$, and $\overline{\eta}(x) \neq [1,1]$. Choose $a, d \in S$ such that $x = ab = cd$, $\overline{\eta}(a) = \underset{x=yz}{\text{rinf}}\, \overline{\eta}(y)$, and $\overline{\eta}(d) = \underset{x=yz}{\text{rinf}}\, \overline{\eta}(z)$. Thus

$$\begin{aligned}\overline{\eta}(a) &= \underset{x=yz}{\text{rinf}}\, \overline{\eta}(y) = \underset{x=yz}{\text{rinf}}\, \text{rmax}\big\{\overline{\eta}(y), \overline{C}'_S(z)\big\} = (\overline{\eta} \,\bar{\bullet}\, \overline{C}'_S)(x) \neq [1,1],\\ \overline{\eta}(d) &= \underset{x=yz}{\text{rinf}}\, \overline{\eta}(z) = \underset{x=yz}{\text{rinf}}\, \text{rmax}\big\{\overline{C}'_S(y), \overline{\eta}(z)\big\} = (\overline{C}'_S \,\bar{\bullet}\, \overline{\eta})(x) \neq [1,1].\end{aligned}$$

Hence, $x, a, d \in$ *supp*$(\overline{\mu}, \overline{\eta}, \overline{\nu})$. So that

$$x = ab = cd \in \big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big)S \cap S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu}).$$

That is, $\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big)S \cap S\big(supp(\overline{\mu}, \overline{\eta}, \overline{\nu})\big) \cap supp(\overline{\mu}, \overline{\eta}, \overline{\nu}) \neq \emptyset$. Therefore, *supp*$(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost quasi-ideal of S.

Case 3: $\big[(\overline{\nu} \,\bar{\bullet}\, \overline{C}'_S) \cup (\overline{C}'_S \,\bar{\bullet}\, \overline{\eta}) \cup \overline{\eta}\big](x) \neq [1,1]$. This proof is similar to Case 2. □

5 Real World Problems

Let S be all types of chicken breeds. For any two chicken breeds a and b in S, $ab = c$ means that breeding between a and b produces $c \in S$. Now, the idea of almost ideal A means that the subset of S is such that any chicken breed x, breeding between x and a for all $a \in A$ has at least one breed in A.

However, in the crossbreeding between a and b, there is still uncertainty, and moreover, in each breed, the in-depth data may be organized as neutrosophic sets. In this paper, we focus the data as an interval-valued picture fuzzy set that is a one of special cases of interval-valued neutrosophic sets.

6 Conclusion

An interval-valued picture fuzzy almost left ideal [right ideal, ideal, bi-ideal, interior ideal, quasi-ideal] in semigroups are defined. Some properties are investigated. If $(\overline{\mu}_1, \overline{\eta}_1, \overline{\nu}_1)$ is an interval-valued picture fuzzy almost left ideal [right ideal, ideal, bi-ideal, interior ideal, quasi-ideal] of a semigroup S and it is a subset of an interval-valued picture fuzzy set $(\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$, then $(\overline{\mu}_2, \overline{\eta}_2, \overline{\nu}_2)$ is also an interval-valued picture fuzzy almost left ideal [right ideal, ideal, bi-ideal, interior ideal, quasi-ideal] of S. Hence, we have that a union of interval-valued picture fuzzy almost left ideals [right ideals, ideals, bi-ideals, interior ideals, quasi-ideals] of S is an interval-valued picture fuzzy almost left ideal [right ideal, ideal, bi-ideal, interior ideal, quasi-ideal] of S but an intersection of interval-valued picture fuzzy almost left ideal [right ideal, ideal, bi-ideal, interior ideal, quasi-ideal] of S need not be an interval-valued picture fuzzy almost left ideal [right ideal, ideal, bi-ideal, interior ideal, quasi-ideal] of S. A nonempty subset A of a semigroup S is an almost left ideal [right ideal, ideal, bi-ideal, interior ideal, quasi-ideal] of S if and only if $(\overline{C}_A, \overline{C}'_A, \overline{C}'_A)$ is an interval-valued picture fuzzy almost left ideal [right ideal, ideal, bi-ideal, interior ideal, quasi-ideal] of S. Moreover, if an interval-valued picture fuzzy set $(\overline{\mu}, \overline{\eta}, \overline{\nu})$ of a semigroup S is an interval-valued picture fuzzy almost left ideal [right ideal, ideal, bi-ideal, interior ideal, quasi-ideal] of S, then $supp(\overline{\mu}, \overline{\eta}, \overline{\nu})$ is an almost left ideal [right ideal, ideal, bi-ideal, interior ideal, quasi-ideal] of S. These are the relationship of each almost ideal in semigroups and its interval-valued picture fuzzification.

In the future, we can apply neutrosophic sets or interval-valued picture fuzzy subsets on other subsets of semigroups. Moreover, we will use the concepts of this paper to study other algebraic structures.

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