Neutrosophic Approach in Route-Optimization of Traveling Salesman Problems

Udit Sharma[1], Tarun Kumar[1], Jahnvi[1], Kailash Dhanuk[2], M. K. Sharma[1,*]
[1]Department of Mathematics, Chaudhary Charan Singh University, Meerut, U.P., 250004, India
[2]Department of Mathematics, Dyal Singh College (University of Delhi), New Delhi, 110003, India
Email: kumarsharmau9@gmail.com; tkvats3@gmail.com; rathijhanvi@gmail.com;
kailash.dhanuk@dsc.du.ac.in; drmukeshsharma@gmail.com

Abstract

The Travelling Salesman Problem (TSP) possesses a significant challenge in optimization, complicated by real-world uncertainties such as fluctuating traffic conditions, weather variability and inconsistent travel durations. Traditional mathematical formulation fails to adequately incorporate these uncertainties, thus limiting their effectiveness. This paper introduces a modified approach to solving the TSP by employing Single-Valued Triangular Neutrosophic Sets (SVTNS), which effectively manages the indeterminate and ambiguous data. The proposed methodology to transform the neutrosophic fuzzy data into crisp numbers using a specifically modified score function. A stepwise procedure is introduced, encompassing crisp conversion, range evaluation and iterative optimization processes to attain an optimal and practically viable solution. The proposed methodology is validated through numerical computation to demonstrate its efficiency in determining the minimal crisp travelling costs and optimizing travelling schedules under the various weighting scenarios. This research advances the applicability of neutrosophic sets in decision-making to provide a reliable framework to address the uncertainties inherent in practical travelling Salesman issues.

Keywords: Fuzzy Set; Single-Valued Neutrosohic Fuzzy Set; Traveling Salesman Problem; Optimization; Score Function

1. Introduction

The background of the Traveling Salesman Problem (TSP) can be traced back to the 19th century, when Hamilton and Kirkman formulated the problem of finding a minimum-length route visiting each city precisely once before recurring to the initial point. Despite extensive studies, TSP formulations have traditionally neglected real-world variability such as traffic delays and weather disruptions.

However, the Travelling Salesman Problem overlooks uncertainties such as traffic, weather conditions, and customer availability. Alongside these elements, issues frequently occur because of inaccurate data and fluctuating travel durations.

In 1965, Zadeh [1] pioneered the idea of fuzzy sets (FS) to provide a mathematical tool to model uncertainty and vagueness more realistically. Later advancements by Florentin Smarandache introduced Neutrosophic Fuzzy Sets (NFS), incorporating indeterminacy into the traditional intuitionistic FS. This significantly improved the ability to handle inconsistent and inaccurate information, making it strong implement for complex decision-making processes.

The classical TSP has long served as a benchmark problem in combinatorial optimization. However, real-world routing scenarios are often riddled with uncertainty stemming from fluctuating traffic patterns, unpredictable weather conditions and imprecise travel estimates. Traditional deterministic and even fuzzy approaches fall short in fully capturing the indeterminate, inconsistent and incomplete nature of such data.

DOI: https://doi.org/10.54216/IJNS.270219

This study is motivated by the limitations of existing methods in handling uncertainty across three dimensions: truth, falsity and indeterminacy. While fuzzy and intuitionistic fuzzy sets address vagueness and hesitation, they fall short in addressing the indeterminacy present in practical data. SVTNS provide a more flexible framework by simultaneously representing acceptance, rejection and uncertainty. Conventional score functions often use fixed or equal weights for these components, which may not align with practical relevance. Therefore, a modified score function is proposed to adaptively balance truth, indeterminacy and falsity based on problem-specific needs. The paper also aims to develop a practical, stepwise methodology that integrates neutrosophic modelling with optimization to ensure the valid Hamiltonian cycles while addressing practical data ambiguity in transportation and logistics.

1.1 Objectives: There are three main objectives of this research

i) To study the TSP in Neutrosophic fuzzy set

ii) To modify the existing methodology to solve SVTNFTSP

iii) To Modify the existing score function for SVTNFS

1.2 Related Works

In 1965, Zadeh [1] laid the foundation for fuzzy logic by introducing the concept of FS, fuzzy systems, and fuzzy logic itself. Building upon this work, Atanasov [2] expanded the field by formulating the intuitionistic theory in 1986. Later, Smarandache [3] generalized intuitionistic fuzzy sets in 2005, clearly distinguishing neutrosophic sets from traditional IFS.

In the domain of optimization, Ismail et al. [4] applied a nature-inspired technique known as Magnetic Optimization to the Travelling Salesman Problem (TSP) in 2012. Subsequently, in 2014, Deli et al. [5] developed the weighted aggregation operator (SVTNWAO) to address multicriteria decision-making challenges. Biswas et al. [6] developed a ranking methodology based on value and ambiguity indices specifically for SVTNFS. Concurrently, Agatz et al. [7] reformulated the TSP as a mixed-integer programming model, proposing several heuristic strategies including “fast-route-first, cluster-second” approaches, integrating local search techniques and dynamic programming. Chen et al. [8] introduced two parallel optimization approaches in 2017: an improved genetic algorithm (IGA) and a hybrid PSO-ACO algorithm. In 2018, Prabha and Vimala [9] employed a methodology based on branch and bound technique to effectively solve triangular NFS assignment problems. In addition, in 2018, Chakraborty et al. [10] explored the various representations of the triangular neutrosophic numbers, developed deneutrosophication methods and discussed their practical applications.

In 2019, Subasri and Selvakumari [11] furthered neutrosophic methods by applying the branch and bound approach to a neutrosophic fuzzy TSP. Additionally, Dey et al. [12] solved problems on undirected neutrosophic weighted graphs, assigning single-valued neutrosophic numbers rather than conventional real or fuzzy numbers.

In 2020, Fan et al. [13] developed a method, which addresses the fuzzy multi-criteria group choice issue in the SVTNFS environment. Khalifa et al. [14] optimized the complex programming under neutrosophic environment. In 2021, Jangid and Kumar [15] established a few games theory theoretical facets in a neutrophilic setting. In 2021, Cakir et al. [16] presented a novel time-dependent Dijkstra’s and minimal vertex grade process on a sphere-shaped bipolar fuzzy weighted graph. Sheikh and Dutta [17] provided the solution to matrix games with SVTNFS payoffs in 2022. In 2022, Rajkumar and Richard [18] described the peculiar heptagonal neutrosophic number, and a score function for it was developed. In 2022, In order to design a car seat, Karasan et al. [19] designed and deployed neutrosophic value function deployment (QFD) technique based on neutrosophic AHP and neutrosophic DEMATEL. In 2023, In order to achieve a comprehensive ordering on neutrosophic triplets, Nayagam [20] developed a method for ordering single valued neutrosophic triplets (SVNT). In 2023, A novel Hungarian algorithm technique with a cost in the form of trapezoidal numbers was presented by Sinika and Ramesh [21] in an interval-valued neutrosophic environment. In 2023, In order to determine the shortest path (SP) between the start and end vertices, Raut et al. [22] used the fermatean NFS number as the network's proper edge weight. In 2023, Srinivas & Prabakaran [23] presented a methodology to solve the SVNTSP.

In 2024, Mishra et al. [24] introduced an integrated multi-attribute decision-making (MADM) framework, SVNSs to better manage uncertainty in decision processes. That same year, Mathew [25] presented new mathematical operations- scalar multiplication, addition, and subtraction - specifically designed for trapezoidal single-valued neutrosophic fuzzy numbers. Also in 2024, Dhouib et al. [26] solved the smallest spanning tree problem under the interval-valued Fermatean NFS framework. In another 2024 contribution, Jiang et al. [27] developed an Adaptive Neutrosophic Large Neighborhood Search (ANLNS) method to enhance problem-solving flexibility under neutrosophic environments.

Entering 2025, Kausar and Palanikumar [28] introduced new similarity measures tailored for Type-2 Diophantine neutrosophic interval-valued soft sets and established the fundamental operations for this structure. Yang et al. [29] analysed the neutrosophic data envelopment. Almutairi [30] proposed a novel Data Envelopment Analysis

DOI: https://doi.org/10.54216/IJNS.270219

(DEA) model capable of handling neutrosophic uncertainty within input and output variables, offering a means to assess whether a particular variable should be treated as deterministic or neutrosophic.

2. Preliminaries

2.1 Fuzzy Set (FS)

Let $Y \neq \varphi$ be crisp set. A fuzzy set B_{FS} on Y is specified by a single membership function: $\mu_{B_{FS}}: Y \to [0,1]$, where $\mu_{B_{FS}}(y)$ represents how strongly the element $y \in B_{FS}$. The set can be written

$$B_{FS} = \{\langle y, \mu_{B_{FS}}(y) \rangle \mid \mathrm{y} \in \mathrm{Y}\}$$

2.2 Intuitionistic Fuzzy Set (IFS)

Adding a complementary non-membership function to a fuzzy set leads to the formulation of an IFS. Alongside the membership degree $\mu_{B_{IFS}}(y)$, define a non-membership degree $\delta_{B_{IFS}}(y)$ subject to $0 \leq \mu_{B_{IFS}}(y) + \delta_{B_{IFS}}(y) \leq 1$ for every $y \in Y$.

Hence and IFS is represented as: $B_{IFS} = \{\langle y, \mu_{B_{IFS}}(y), \delta_{B_{IFS}}(y) \rangle \mid \mathrm{y} \in \mathrm{Y}\}$, capturing both acceptance and rejection of each element.

2.3 Neutrosophic Fuzzy Set (NFS)

Introducing a third component for uncertainty leads to a NFS. For each element $y \in Y$, assign:

- $\mu_{B_{NFS}}$, membership degree
- $\gamma_{B_{NFS}}$, indeterminacy degree
- $\delta_{B_{NFS}}$, non-membership degree

with $\mu_{B_{NFS}}, \gamma_{B_{IFS}}, \delta_{B_{NFS}} \to]0,1[^{+}$, $0 \leq \mu_{B_{NFS}}(y) + \gamma_{B_{NFS}} + \delta_{B_{NFS}}(y) \leq 3^{+}$

Thus, $B_{NFS} = \{\langle y, \mu_{B_{NFS}}(y), \gamma_{B_{NFS}}(y), \delta_{B_{NFS}}(y) \rangle \mid \mathrm{y} \in \mathrm{Y}\}$

2.3.1 Operations of Neutrosophic Fuzzy Set (NFS)

Let $A, B \in Y$. Then NFS operations on A and B are defined as,

(1) $\left(\mu_{A_{NFS}}(y), \gamma_{A_{IFS}}(y), \delta_{A_{NFS}}(y)\right) + \left(\mu_{B_{NFS}}(y), \gamma_{B_{NFS}}(y), \delta_{B_{NFS}}(y)\right) = \left(\mu_{A_{NFS}}(y) + \mu_{B_{NFS}}(y) - \mu_{A_{NFS}}(y)\mu_{B_{NFS}}(y), \gamma_{A_{IFS}}(y)\gamma_{B_{NFS}}(y), \delta_{A_{NFS}}(y)\delta_{B_{NFS}}(y)\right)$.

(2) $\left(\mu_{A_{NFS}}(y), \gamma_{A_{IFS}}(y), \delta_{A_{NFS}}(y)\right).\left(\mu_{B_{NFS}}(y), \gamma_{B_{NFS}}(y), \delta_{B_{NFS}}(y)\right) = \left(\mu_{A_{NFS}}(y)\mu_{B_{NFS}}(y), \gamma_{A_{IFS}}(y) + \gamma_{B_{NFS}}(y) - \gamma_{A_{NFS}}(y)\gamma_{B_{NFS}}(y), \delta_{A_{NFS}}(y) + \delta_{B_{NFS}}(y) - \delta_{A_{NFS}}(y)\delta_{B_{NFS}}(y)\right)$.

(3) $k\left(\mu_{A_{NFS}}(y), \gamma_{A_{IFS}}(y), \delta_{A_{NFS}}(y)\right) = \left(1 - \left(1 - \mu_{A_{NFS}}(y)\right)k, \gamma_{A_{IFS}}(y)k, \delta_{A_{NFS}}(y)k\right), (k \in R)$.

(4) $\left(\mu_{A_{NFS}}(y), \gamma_{A_{IFS}}(y), \delta_{A_{NFS}}(y)\right)k = \left(\mu_{A_{NFS}}(y)k, 1 - \left(1 - \gamma_{A_{IFS}}(y)\right)k, 1 - \left(1 - \delta_{A_{NFS}}(y)\right)k\right)(k \in R)$.

Figure 1 depicts the single-valued neutrosophic TFN

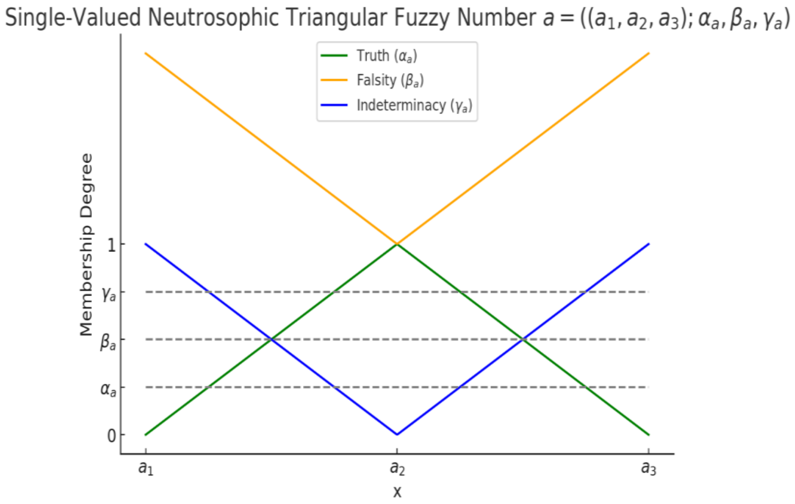

Figure 1. SVNTFN

2.3.2 Neutrosophic Score Function (NSF)

DOI: https://doi.org/10.54216/IJNS.270219

2.3.2.1 Existing Score Function

The following SF is employed to transform the neutrosophic data of the Neutrosophic Fuzzy Travelling Salesman Problem (NTSP) into its crisp form [23]:

$$S(\tilde{\mathrm{a}}) = \frac{1}{12}\left((\tilde{\mathrm{a}}_1 + \tilde{\mathrm{a}}_2 + \tilde{\mathrm{a}}_3)(2 + \mu_{\tilde{\mathrm{a}}}(y) - \gamma_{\tilde{\mathrm{a}}}(y), -\delta_{\tilde{\mathrm{a}}})\right), \tag{1}$$

here, $\tilde{\mathrm{a}} = \left((\tilde{\mathrm{a}}_1, \tilde{\mathrm{a}}_2, \tilde{\mathrm{a}}_3); \mu_{\tilde{\mathrm{a}}}, \gamma_{\tilde{\mathrm{a}}}, \delta_{\tilde{\mathrm{a}}}\right)$ is a SVTNF,

The parameters satisfy:, $\tilde{\mathrm{a}}_1, \tilde{\mathrm{a}}_2, \tilde{\mathrm{a}}_3 \in \mathbb{R}$, $\tilde{\mathrm{a}}_1 \leq \tilde{\mathrm{a}}_2 \leq \tilde{\mathrm{a}}_3$. $\mu_{\tilde{\mathrm{a}}}, \gamma_{\tilde{\mathrm{a}}}, \delta_{\tilde{\mathrm{a}}} \in [0,1]$

2.3.2.2 Modified Score Function

The modified SF is designed to reflect the contributions of the membership $\mu_{B_{NFS}}$, indeterminacy-membership $\gamma_{B_{NFS}}$ and non-membership $\delta_{B_{NFS}}$ values in the SVTNF representation. Unlike the existing score function (Eq.1), which assumes equal or fixed influence of these components, the modified function introduces weights w_1, w_2 and w_3 to flexibly balance their effects based on the problem-specific characteristics.

The modified SF applied to transform the neutrosophic data of the Neutrosophic Fuzzy Travelling Salesman Problem (NTSP) into crisp values is given below:

$$S(\tilde{\mathrm{a}}) = \frac{1}{12}\left((w_1\tilde{\mathrm{a}}_1 + w_2\tilde{\mathrm{a}}_2 + w_3\tilde{\mathrm{a}}_3)(2 + \mu_{\tilde{\mathrm{a}}}(y) - \gamma_{\tilde{\mathrm{a}}}(y), -\delta_{\tilde{\mathrm{a}}})\right), \tag{2}$$

where, $\tilde{\mathrm{a}} = \left((\tilde{\mathrm{a}}_1, \tilde{\mathrm{a}}_2, \tilde{\mathrm{a}}_3); \mu_{\tilde{\mathrm{a}}}, \gamma_{\tilde{\mathrm{a}}}, \delta_{\tilde{\mathrm{a}}}\right)$ is a SVTNF, $\mu_{\tilde{\mathrm{a}}}, \gamma_{\tilde{\mathrm{a}}}, \delta_{\tilde{\mathrm{a}}} \in [0,1]$, $\tilde{\mathrm{a}}_1, \tilde{\mathrm{a}}_2, \tilde{\mathrm{a}}_3 \in \mathbb{R}$, $\tilde{\mathrm{a}}_1 \leq \tilde{\mathrm{a}}_2 \leq \tilde{\mathrm{a}}_3$ and $w_1, w_2, w_3 \in [0,1]$.

2.3.2.3 Properties of Score Function:

Property 1: The modified SF $S(\tilde{\mathrm{a}})$ belongs to $0 \leq S(\tilde{\mathrm{a}}) \leq \frac{a_3}{4}$

Proof:

$$S(\tilde{\mathrm{a}}) = \frac{1}{12}\left((w_1\tilde{\mathrm{a}}_1 + w_2\tilde{\mathrm{a}}_2 + w_3\tilde{\mathrm{a}}_3)(2 + \mu_{\tilde{\mathrm{a}}}(y) - \gamma_{\tilde{\mathrm{a}}}(y), -\delta_{\tilde{\mathrm{a}}})\right),$$

where, $\tilde{\mathrm{a}} = \left((\tilde{\mathrm{a}}_1, \tilde{\mathrm{a}}_2, \tilde{\mathrm{a}}_3); \mu_{\tilde{\mathrm{a}}}, \gamma_{\tilde{\mathrm{a}}}, \delta_{\tilde{\mathrm{a}}}\right)$ is a SVTNF and $w_i \geq 0$, $w_1 + w_2 + w_3 = 1$.

The Triangular points satisfy $0 \leq \tilde{\mathrm{a}}_1 \leq \tilde{\mathrm{a}}_2 \leq \tilde{\mathrm{a}}_3$ (all distance and coasts are non-negative)

Let $P = w_1\tilde{\mathrm{a}}_1 + w_2\tilde{\mathrm{a}}_2 + w_3\tilde{\mathrm{a}}_3$ and $Q = 2 + \mu_{\tilde{\mathrm{a}}}(y) - \gamma_{\tilde{\mathrm{a}}}(y), -\delta_{\tilde{\mathrm{a}}}$

P is non-negative because all the a_i *and* w_i are non-negative.

The second factor satisfies $0 \leq Q \leq 3$.

Since $\mu_{\tilde{\mathrm{a}}}, \gamma_{\tilde{\mathrm{a}}}, \delta_{\tilde{\mathrm{a}}} \in [0,1]$. $1\text{-}\gamma_{\tilde{\mathrm{a}}} \geq 0$, $1\text{-}\delta_{\tilde{\mathrm{a}}} \geq 0, \mu_{\tilde{\mathrm{a}}} \geq 0. \Longrightarrow 2 + \mu_{\tilde{\mathrm{a}}}(y) - \gamma_{\tilde{\mathrm{a}}}(y), -\delta_{\tilde{\mathrm{a}}} \geq 0$; $Q \geq 0$

$1\text{-}\gamma_{\tilde{\mathrm{a}}} \leq 1$, $1\text{-}\delta_{\tilde{\mathrm{a}}} \leq 1$, $\mu_{\tilde{\mathrm{a}}} \leq 1$, $2 - \gamma_{\tilde{\mathrm{a}}} - \delta_{\tilde{\mathrm{a}}} \leq 2 \Longrightarrow 2 + \mu_{\tilde{\mathrm{a}}}(y) - \gamma_{\tilde{\mathrm{a}}}(y), -\delta_{\tilde{\mathrm{a}}} \leq 3$; $Q \leq 3$

Hence $P \geq 0$ and $Q \geq 0 \Longrightarrow S(\tilde{\mathrm{a}}) \geq 0$

Also, $\tilde{\mathrm{a}}_1, \tilde{\mathrm{a}}_2, \tilde{\mathrm{a}}_3 \in \mathbb{R}$, and $w_1 + w_2 + w_3 = 1 \Longrightarrow w_1\tilde{\mathrm{a}}_1 + w_2\tilde{\mathrm{a}}_2 + w_3\tilde{\mathrm{a}}_3 \leq \tilde{\mathrm{a}}_3$;

$$\Longrightarrow P \leq \tilde{\mathrm{a}}_3$$

Now, combining this with Q we have $S(\tilde{\mathrm{a}}) = \frac{PQ}{12} \leq \frac{\tilde{\mathrm{a}}_3 * 3}{12}$

Hence $0 \leq S(\tilde{\mathrm{a}}) = \frac{1}{12}\left((w_1\tilde{\mathrm{a}}_1 + w_2\tilde{\mathrm{a}}_2 + w_3\tilde{\mathrm{a}}_3)(2 + \mu_{\tilde{\mathrm{a}}}(y) - \gamma_{\tilde{\mathrm{a}}}(y), -\delta_{\tilde{\mathrm{a}}})\right) \leq \frac{\tilde{\mathrm{a}}_3}{4}$

Property 2: Comparison of two NTFN

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two NTFN then:

1. $S(\mathcal{T}_1) \leq S(\mathcal{T}_2)$, Then $\mathcal{T}_1 \leq \mathcal{T}_2$.
2. $S(\mathcal{T}_1) \geq S(\mathcal{T}_2)$, Then $\mathcal{T}_1 \geq \mathcal{T}_2$.
3. $S(\mathcal{T}_1) \approx S(\mathcal{T}_2)$, Then $\mathcal{T}_1 \approx \mathcal{T}_2$.

3. Mathematical Formulation

3.1 TSP Mathematical Formulation

Let $N = \{1,2, \ldots, n\}$ be the set of cities. Define binary decision variables

$$p_{ij} = \begin{cases} 1 \; if \; the \; travels \; directly \; from \; city \; i \; to \; city \; j, & \\ 0, \qquad\qquad\qquad\qquad\qquad\qquad Otherwise \end{cases} \quad i,j \in N.$$

Let d_{ij} denote the distance (or cost) associated with going from i to j.

The objective is to minimize the travel cost

$$Z = \sum_{(i,j)\in N\times N} d_{ij} p_{ij},$$

Degree (routing) constraints

$$\sum_{j\in N} p_{ij} = 1,\ \forall\, i \in N$$
$$\sum_{i\in N} p_{ij} = 1, \forall\, j \in N$$

$p_{ij} = 0$ or $1, i, j \in N$. d_{ij} represents the route distance from destination i to destination j.

3.2 NTSP Mathematical Formulation

TSP is expressed using a matrix known as the SVTNFS distance matrix, which is presented as follows.

$$S = \begin{pmatrix} \infty & s_{12} & \cdot & \cdot & \cdot & s_{1n} \\ s_{21} & \infty & \cdot & \cdot & \cdot & s_{2n} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ s_{n1} & s_{n2} & \cdot & \cdot & \cdot & \infty \end{pmatrix}$$

Here, each element is a of the SVTNF form:$\tilde{\alpha} = \left((\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3); \mu_{\tilde{\alpha}}, \gamma_{\tilde{\alpha}}, \delta_{\tilde{\alpha}}\right)$. Using the SF (Eq.2)

$$S(\tilde{\alpha}) = \frac{1}{12}\left((w_1\tilde{\alpha}_1 + w_2\tilde{\alpha}_2 + w_3\tilde{\alpha}_3)(2 + \mu_{\tilde{\alpha}}(y) - \gamma_{\tilde{\alpha}}(y), -\delta_{\tilde{\alpha}})\right),$$

Each element of the NTSP matrix, represented as a SVTNFS number, is defuzzified and transformed into its corresponding crisp value.

4. Proposed Methodology for Solving Neutrosophic Traveling Salesman Problem

The proposed method for solving the SVTFNTSP involves the following steps:

Step 1:

Begin by transforming all entries in the SVTNFS Distance Matrix into their crisp equivalents. This is done using the specified SF (as explained earlier):

$$S(\tilde{\alpha}) = \frac{1}{12}\left((w_1\tilde{\alpha}_1 + w_2\tilde{\alpha}_2 + w_3\tilde{\alpha}_3)(2 + \mu_{\tilde{\alpha}}(y) - \gamma_{\tilde{\alpha}}(y), -\delta_{\tilde{\alpha}})\right)$$

Step 2:

Next, calculate the range for each column in the matrix. The range is determined by subtracting the second-highest value from the highest value in that column. Record these range values beneath the respective columns.

Step 3:

Identify the column with the highest range value among all the columns calculated.

Step 4:

Within the selected column, find the minimum value. Divide all elements of the matrix by this minimum value. This process will result in some entries in the matrix becoming 1.

Step 5:

Attempt to select just one '1' from individual row and individual column. If successful, the optimal solution (OS) is achieved. If not, proceed by covering all the '1's with a minimum number of lines, and then identify the smallest uncovered element. Divide all uncovered elements by this minimum value and adjust the values at the intersections of the drawn lines accordingly, while keeping other covered elements unchanged.

Step 6:

After updating the matrix, again attempt to choose just one '1' from every row and column. If this is possible, move toward constructing the optimal solution. If not, repeat Step 5 until the condition is satisfied.

Step 7:

Once exactly one '1' is selected in every row and column, verify whether the resulting travel schedule forms a complete cycle: starting from the first city, visiting every city exactly once, and returning to the starting point.

If the schedule forms a valid cycle, the CTS is obtained, along with the TMCTC or TMCDT. These values represent the COS and the COTC or crisp optimal distance travelled (CODT). If a complete cycle is not formed, rearrange the rows appropriately to establish a cycle that meets the conditions mentioned, and then derive the result.

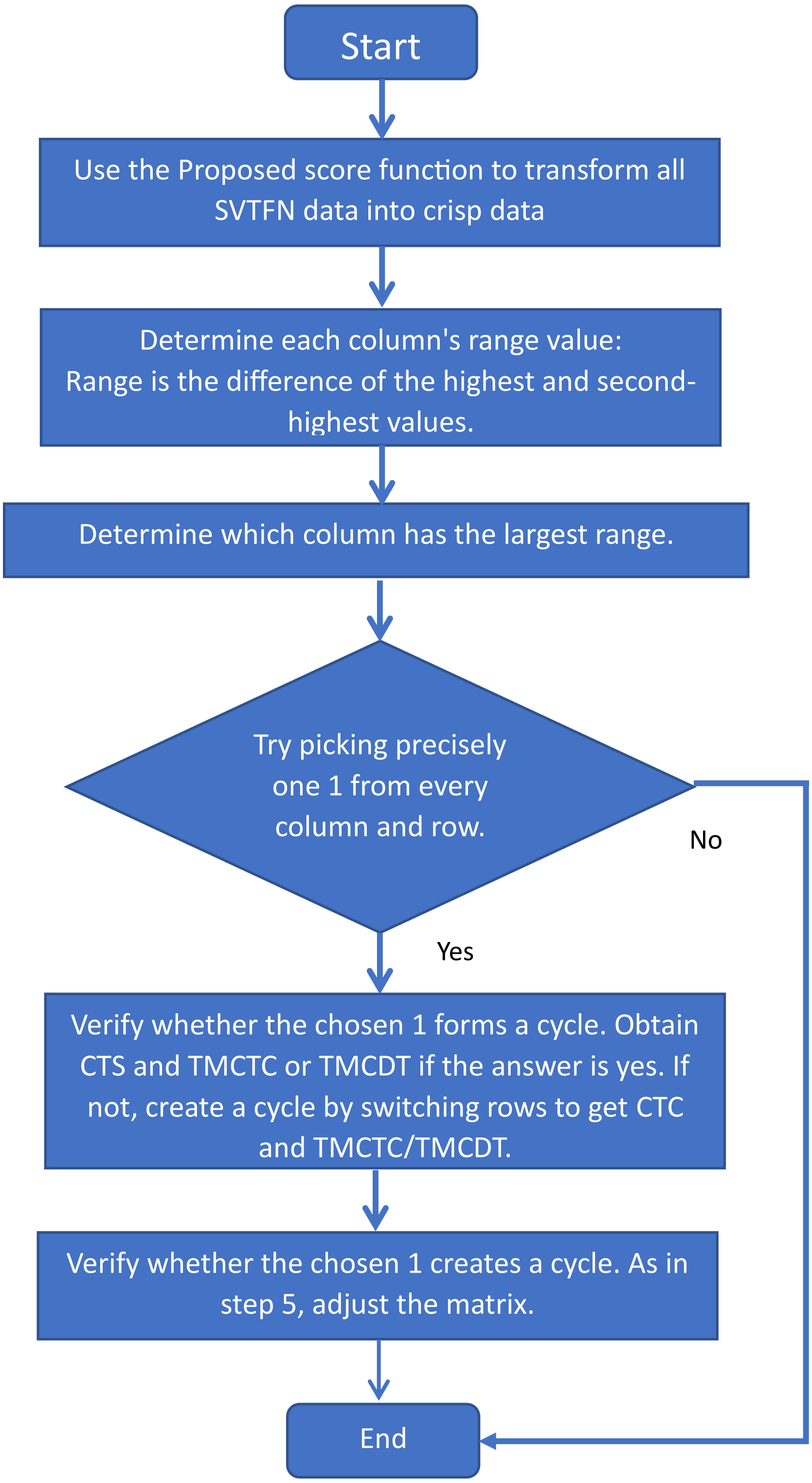

Figure 2. Flowchart of methodology

5. Numerical Computation

Problem 1: Consider a symmetric TSP represented by a SVTNFS Distance Matrix [23].

$$S = \begin{pmatrix} \infty & s_{12} & s_{13} & s_{14} \\ s_{21} & \infty & s_{23} & s_{24} \\ s_{31} & s_{32} & \infty & s_{34} \\ s_{41} & s_{42} & s_{43} & \infty \end{pmatrix}$$

where the elements are given as SVTNFS numbers:

$s_{12} = s_{21} = \langle(4,6,10); 0.8,0.4,0.2\rangle; s_{13} = s_{31} = \langle(2,5,9); 0.7,0.6,0.3\rangle;$
$s_{14} = s_{41} = \langle(4,7,9); 0.6,0.6,0.3\rangle; s_{23} = s_{32} = \langle(1,5,8); 0.8,0.5,0.2\rangle;$
$s_{24} = s_{42} = \langle(2,7,9); 0.8,0.5,0.4\rangle; s_{34} = s_{43} = \langle(1,5,10); 0.8,0.3,0.1\rangle.$

Problem 2: Let us consider a symmetric TSP represented by a SVTNFS distance matrix.

$$\mathrm{S} = \begin{pmatrix} \infty & s_{12} & s_{13} & s_{14} \\ s_{21} & \infty & s_{23} & s_{24} \\ s_{31} & s_{32} & \infty & s_{34} \\ s_{41} & s_{42} & s_{43} & \infty \end{pmatrix}$$

where the elements are given as SVTNFS numbers.:

- $s_{12} = s_{21} = \langle(2,3,5);\ 0.9, 0.1, 0.05\rangle$
- $s_{13} = s_{31} = \langle(1,2,4);\ 0.8, 0.2, 0.1\rangle$
- $s_{14} = s_{41} = \langle(3,4,6);\ 0.85, 0.15, 0.1\rangle$
- $s_{23} = s_{32} = \langle(3,5,7);\ 0.75, 0.2, 0.15\rangle$
- $s_{24} = s_{42} = \langle(2,3,4);\ 0.95, 0.05, 0.02\rangle$
- $s_{34} = s_{43} = \langle(4,6,8);\ 0.7, 0.25, 0.2\rangle$

Table 1: Problem 3, Let us consider a symmetric TSP represented by a SVTNF distance matrix

	City 1	City 2	City 3	City 4	City 5
City 1	∞	(3.51, 3.66, 3.85; 0.25, 0.11, 0.14)	(3.65, 3.80, 3.99; 0.26, 0.24, 0.26)	(3.41, 3.60, 3.75; 0.15, 0.24, 0.26)	(3.92, 4.06, 4.17; 0.18, 0.26, 0.11)
City 2	(3.42, 3.60, 3.79; 0.26, 0.25, 0.22)	∞	(3.94, 4.13, 4.29; 0.11, 0.21, 0.22)	(3.61, 3.74, 3.89; 0.18, 0.29, 0.25)	(3.18, 3.37, 3.50; 0.13, 0.15, 0.29)
City 3	(3.34, 3.52, 3.63; 0.19, 0.27, 0.13)	(3.46, 3.59, 3.77; 0.14, 0.21, 0.23)	∞	(3.53, 3.64, 3.82; 0.29, 0.28, 0.19)	(3.68, 3.87, 3.99; 0.18, 0.25, 0.24)
City 4	(3.30, 3.43, 3.55; 0.24, 0.23, 0.19)	(3.20, 3.32, 3.44; 0.28, 0.15, 0.15)	(3.82, 4.01, 4.17; 0.22, 0.21, 0.14)	∞	(3.87, 4.04, 4.17; 0.13, 0.18, 0.29)
City 5	(3.65, 3.79, 3.98; 0.12, 0.15, 0.11)	(3.51, 3.63, 3.81; 0.18, 0.11, 0.20)	(3.18, 3.35, 3.51; 0.13, 0.18, 0.27)	(3.21, 3.37, 3.51; 0.12, 0.11, 0.25)	∞

Table 2: Weight Combinations and Corresponding Crisp Travelling Cost (Problem 1, Problem 2 and Problem 3)

S. No.	**w_1**	**w_2**	**w_3**	**Total Crisp Cost (Problem 1)**	**Total Crisp Cost (Problem 2)**	**Total Crisp Cost (Problem 3)**
1	0.316	0.190	0.494	2.4865	2.5711	2.8527
2	0.013	0.656	0.331	2.4974	2.5830	2.8656
3	0.070	0.009	0.921	2.5385	2.6275	2.9304
4	0.002	0.516	0.482	2.5098	2.5965	2.8849
5	0.125	0.179	0.696	2.5168	2.6041	2.8974
6	0.445	0.472	0.083	2.4496	2.5281	2.8305
7	0.117	0.849	0.034	2.4892	2.5633	2.8402

DOI: https://doi.org/10.54216/IJNS.270219

S. No.	w_1	w_2	w_3	Total Crisp Cost (Problem 1)	Total Crisp Cost (Problem 2)	Total Crisp Cost (Problem 3)
8	0.030	0.837	0.133	2.4901	2.5675	2.8583
9	0.441	0.213	0.346	2.4676	2.5462	2.8287
10	0.234	0.446	0.320	2.4793	2.5585	2.8391
11	0.351	0.346	0.303	2.4704	2.5513	2.8624
12	0.103	0.308	0.589	2.5084	2.5986	2.8249
13	0.067	0.272	0.661	2.5203	2.6123	2.8373
14	0.681	0.110	0.209	2.4323	2.5083	2.8366
15	0.228	0.393	0.379	2.4846	2.5663	2.8489
16	0.062	0.728	0.210	2.4906	2.5673	2.8662
17	0.047	0.147	0.806	2.5321	2.6253	2.8411
18	0.351	0.255	0.394	2.4773	2.5548	2.8523
19	0.150	0.499	0.351	2.4909	2.5723	2.8224
20	0.119	0.312	0.569	2.5061	2.5938	2.8372

6. Result and Discussion

This section presents the computational results obtained by applying the proposed modified score function and methodology to three different numerical problems of the SVTFNTSP. The study validates the effectiveness of the methodology in handling uncertainty and indeterminacy by transforming neutrosophic fuzzy data into crisp values and solving the corresponding travelling schedules.

6.1 Problem 1 Analysis

Problem 1 involves a 4×4 neutrosophic distance matrix, transformed into crisp values using the proposed modified score function. Table 1 lists the total crisp travelling costs obtained under twenty different weight combinationsw_1, w_2, w_3. The minimal cost observed is 2.4496 when weights are set as w_1=0.445, w_2=0.472, w_3=0.083, indicating the sensitivity of the solution to the prioritization of membership, indeterminacy and non-membership values.

The optimization process follows the stepwise procedure of range evaluation, matrix adjustment and iterative refinement, culminating in an optimal travelling schedule that forms a valid Hamiltonian cycle. The graphical representation in Figure 3 demonstrates the stability of costs across weight variations, affirming the robustness of the proposed method for Problem 1.

6.2 Problem 2 Analysis

Similar to Problem 1, Problem 2 utilizes a distinct 4×4 neutrosophic matrix, with the defuzzification and optimization steps yielding minimal crisp costs as summarized in Table 1. Here, the lowest total cost is 2.5083 with w_1=0.681, w_2=0.110, w_3=0.209. The higher values compared to Problem 1 highlight the impact of different neutrosophic distance matrices on the travelling cost. The sensitivity analysis indicates consistent behaviour in cost fluctuations relative to the weight changes to confirm the method's flexibility to accommodate various decision-maker preferences and neutrosophic data characteristics. Figure 3 visually compares the cost trends between Problem 1 and Problem 2.

6.3 Problem 3 Analysis

Extending the study, Problem 3 introduces a more complex 5×5 neutrosophic matrix, with diagonal elements explicitly excluded to model realistic travelling constraints. The modified score function is applied to convert the neutrosophic numbers into crisp distances. The fixed travelling schedule considered is 1→3→5→2→4→1, consistent with the approach for previous problems. The results in Table 1 show the total minimal crisp costs under the same twenty weight scenarios. The minimal cost in Problem 3 is 2.8224 with w_1=0.150, w_2=0.499, w_3=0.351, reflecting the increasing complexity and scale of the problem. Despite this, the methodology remains computationally efficient and produces stable and interpretable results.

6.4 Comparative Analysis and Discussion

DOI: https://doi.org/10.54216/IJNS.270219

Figure 3 illustrates the comparative travelling costs across the three problems for the different weight sets to show the influence of the problem scale and neutrosophic data distribution on the overall costs. The modified SF successfully balances the components of neutrosophic data to allow a nuanced reflection of uncertainty effects on the travelling schedule.

Table 3 compares the proposed methodology with existing methods, emphasizing its superiority in generating valid Hamiltonian cycles, handling multiple weight scenarios, reducing optimization steps and providing clear interpretability.

The proposed approach's flexibility and robustness make it highly suitable for real-world applications where uncertainties and incomplete information are inherent, such as logistics, route planning and supply chain management.

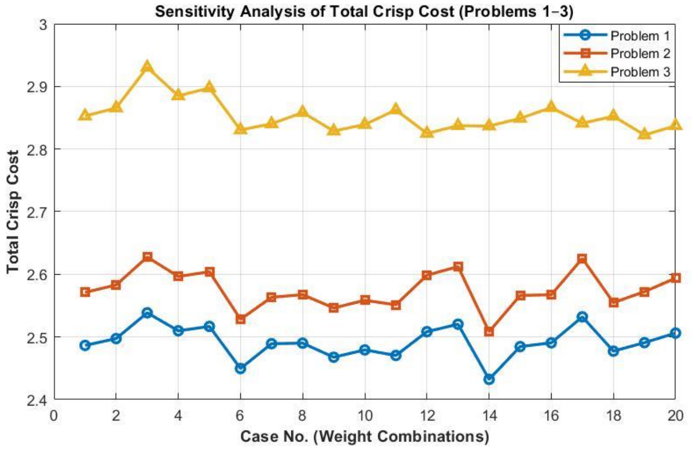

Figure 3. Evaluation of costs across different weight cases

Table 3: Comparison of methodology with existing method

S.no	Methodology used for solving SVTFNTSP	Limitation	Advantages
1	Traditional methods	Does not guarantee obtaining a proper Hamiltonian cycle. High computational burden.	Simple for very small-sized problems.
2	Method provided in the work of Srinivas and Prabakaran [23]	Row exchange criteria in Step 7 may not always result in a valid cycle. Can handle only a single weight scenario; lacks flexibility for different cases.	Improved initial structure compared to traditional methods.
3	MODIFIED METHODOLOGY	**None** (Successfully addresses both earlier limitations.)	Ensures a valid Hamiltonian cycle formation after solving SVTFNTSP. Capable of analyzing multiple weight scenarios. Reduces the number of optimization steps. Provides clear graphical interpretation. Increases computational efficiency and robustness under uncertainty.

7. Conclusion

DOI: https://doi.org/10.54216/IJNS.270219

In this study, a robust methodology based on a modified score function was proposed for solving the Single-Valued Triangular Fuzzy Neutrosophic TSP. The method effectively transforms neutrosophic-valued distances into crisp values while incorporating truth, indeterminacy and falsity components through flexible weight parameters. To evaluate the sensitivity and applicability of the approach, three problems of increasing complexity - a 4×4 TSP (Problem 1), another independent 4×4 TSP (Problem 2), and 5×5 TSP (Problem 3) - were examined under 20 different weight combinations. The results, concise in Table 1 and illustrated in Figure 3, highlight the influence of weight variation on the total crisp cost. The proposed methodology consistently generated feasible Hamiltonian cycles and revealed clear trends in cost sensitivity and thereby validate the robustness and adaptability of the score function. Compared to existing methods, the proposed technique demonstrates improved interpretability, computational simplicity and flexibility to handle diverse decision-maker preferences. Furthermore, the inclusion of Problem 3 confirms the scalability of the model to higher-order TSPs without compromising the methodological clarity.

This work opens avenues for further research on integrating Pythagorean or Fermatean Fuzzy set with neutrosophic frameworks to extend the model to multi-objective TSPs and exploring real-world logistics networks under uncertainty and indeterminacy.

Acknowledgements: The second author expresses gratitude to the University Grants Commission (UGC) for its financial support.

Interest Conflict Statement: The authors affirm that there are no conflicts of interest.

Author Credit Statement:

Udit Sharma: Methodology, design and analysis, result interpretation, writing and editing.

Tarun Kumar: Conceptualization, theoretical framework, original draft writing, critical revision.

M.K. Sharma: Supervision, conceptualization, methodology, result analysis, original draft review and editing.

Data Availability Statement: For additional information regarding the data used in this study, readers are encouraged to contact the authors. We are committed to providing further clarifications and details to support a better understanding and application of our research findings.

List of Abbreviation

Notations	Full Form
TSP	Travelling Salesman Problem
FS	Fuzzy Set
NFS	Neutrosophic Fuzzy Set
SF	Score Function
NTSP	Neutrosophic Travelling Salesman Problem
SVTFNTSP	Single-Valued Triangular Fuzzy Neutrosophic Travelling Salesman Problem
CTS	Crisp Travelling Schedule
TMCTC	Total Minimal Crisp Travelling Cost
TMCDT	Total Minimal Crisp Distance Travelled

References

[1] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, 1965, doi: 10.1016/S0019-9958(65)90241-X.

[2] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 87–96, 1986, doi: 10.1016/S0165-0114(86)80034-3.

[3] F. Smarandache, "Neutrosophic set—a generalization of the intuitionistic fuzzy set," *Int. J. Pure Appl. Math.*, vol. 24, no. 3, pp. 287–297, 2005. [Online]. Available: https://ijpam.eu/contents/2005-24-3/1/1.pdf

[4] M. M. Ismail *et al.*, "Magnetic optimization algorithm approach for travelling salesman problem," *World Acad. Sci., Eng. Technol.*, vol. 62, pp. 1393–1397, 2012. [Online]. Available: https://www.academia.edu/27356966/

DOI: https://doi.org/10.54216/IJNS.270219

[5] Deli and Y. Subas, "Single valued neutrosophic numbers and their applications to multicriteria decision making problem," *Neutrosophic Sets Syst.*, vol. 2, no. 1, pp. 1–13, 2014. [Online]. Available: https://fs.unm.edu/SN/Neutro-SingleValuedNeutroNumbers.pdf

[6] P. Biswas, S. Pramanik, and B. C. Giri, "Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making," *Neutrosophic Sets Syst.*, vol. 12, no. 1, pp. 127–138, 2016. [Online]. Available: https://digitalrepository.unm.edu/nss_journal/vol12/iss1/17/

[7] N. Agatz, P. Bouman, and M. Schmidt, "Optimization approaches for the traveling salesman problem with drone," ERIM Rep. Ser. Ref. No. ERS-2015-011-LIS, 2016. [Online]. Available: http://dx.doi.org/10.2139/ssrn.2639672

[8] J. Chen, F. Ye, and Y. Li, "Travelling salesman problem for UAV path planning with two parallel optimization algorithms," in *Proc. PIERS-FALL*, 2017, pp. 832–837, doi: 10.1109/PIERS-FALL.2017.8293250.

[9] S. K. Prabha and S. Vimala, "Neutrosophic assignment problem via BnB algorithm," in *Advances in Algebra and Analysis*. Springer, 2018, pp. 323–330, doi: 10.1007/978-3-030-01120-8_37.

[10] Chakraborty *et al.*, "Different forms of triangular neutrosophic numbers, de-neutrosophication techniques, and their applications," *Symmetry*, vol. 10, no. 8, p. 327, 2018, doi: 10.3390/sym10080327.

[11] S. Subasri and K. Selvakumari, "Neutrosophic travelling salesman problem in trapezoidal fuzzy number using branch and bound technique," *J. Phys., Conf. Ser.*, vol. 1362, no. 1, p. 012098, 2019, doi: 10.1088/1742-6596/1362/1/012098.

[12] Dey *et al.*, "A new algorithm for finding minimum spanning trees with undirected neutrosophic graphs," *Granular Comput.*, vol. 4, pp. 63–69, 2019, doi: 10.1007/s41066-018-0084-7.

[13] J. Fan, X. Jia, and M. Wu, "A new multi-criteria group decision model based on single-valued triangular neutrosophic sets and EDAS method," *J. Intell. Fuzzy Syst.*, vol. 38, no. 2, pp. 2089–2102, 2020, doi: 10.3233/JIFS-190811.

[14] H. A. E. W. Khalifa, P. Kumar, and F. Smarandache, "On optimizing neutrosophic complex programming using lexicographic order," *Infinite Study*, 2020, doi: 10.5281/zenodo.3723173.

[15] V. Jangid and G. Kumar, "Matrix games with single-valued triangular neutrosophic numbers as pay-offs," *Neutrosophic Sets Syst.*, vol. 45, no. 1, p. 14, 2021. [Online]. Available: https://digitalrepository.unm.edu/cgi/viewcontent.cgi?article=1915&context=nss_journal

[16] E. Çakir, Z. Ulukan, and T. Acarman, "Shortest fuzzy Hamiltonian cycle on transportation network using minimum vertex degree and time-dependent Dijkstra's algorithm," *IFAC-PapersOnLine*, vol. 54, no. 2, pp. 348–353, 2021, doi: 10.1016/j.ifacol.2021.06.048.

[17] M. R. Seikh and S. Dutta, "Solution of matrix games with payoffs of single-valued trapezoidal neutrosophic numbers," *Soft Comput.*, vol. 26, no. 3, pp. 921–936, 2022, doi: 10.1007/s00500-021-06559-7.

[18] Rajkumar and A. S. Richard, "A novel approach to solve the replacement problem using MATLAB programming in neutrosophic environment," in *AIP Conf. Proc.*, vol. 2529, no. 1, 2022, doi: 10.1063/5.0103739.

[19] Karasan *et al.*, "Customer-oriented product design using an integrated neutrosophic AHP & DEMATEL & QFD methodology," *Appl. Soft Comput.*, vol. 118, p. 108445, 2022, doi: 10.1016/j.asoc.2022.108445.

[20] V. Nayagam, "A total order on single valued and interval valued neutrosophic triplets," *Neutrosophic Sets Syst.*, vol. 55, no. 1, p. 23, 2023. [Online]. Available: https://digitalrepository.unm.edu/cgi/viewcontent.cgi?article=2301&context=nss_journal

[21] S. Sinika and G. Ramesh, "Effectiveness of new interval arithmetic operations in interval-valued neutrosophic assignment problem," in *AIP Conf. Proc.*, vol. 2852, no. 1, 2023, doi: 10.1063/5.0164620.

[22] P. K. Raut *et al.*, "Calculation of shortest path on Fermatean neutrosophic networks," *Neutrosophic Sets Syst.*, vol. 57, no. 1, p. 22, 2023. [Online]. Available: https://fs.unm.edu/NSS/ShortestPathNeutrosophic22.pdf

[23] S. Srinivas and K. Prabakaran, "Optimization of single-valued triangular neutrosophic fuzzy travelling salesman problem," *Neutrosophic Sets Syst.*, vol. 60, p. 366, 2023. [Online]. Available: https://digitalrepository.unm.edu/cgi/viewcontent.cgi?article=2468&context=nss_journal

[24] R. Mishra *et al.*, "Assessing the sustainable energy storage technologies using single-valued neutrosophic decision-making framework with divergence measure," *Expert Syst. Appl.*, vol. 238, p. 121791, 2024, doi: 10.1016/j.eswa.2023.121791.

[25] L. P. Mathew, L. Sebastian, and B. Thankachan, "Some operations on trapezoidal single valued neutrosophic fuzzy numbers," *Int. J. Neutrosophic Sci.*, vol. 23, no. 3, 2024. [Online]. Available: https://americaspg.com/article/pdf/2459

[26] S. Dhouib, S. Broumi, and M. Talea, "Solving the minimum spanning tree problem under interval-valued Fermatean neutrosophic domain," *Neutrosophic Sets Syst.*, vol. 67, no. 1, p. 2, 2024. [Online]. Available: https://fs.unm.edu/nss8/index.php/111/article/view/4422

[27] J. Jiang *et al.*, "An adaptive neutrosophic large neighborhood search and its application," *Neutrosophic Sets Syst.*, vol. 70, no. 1, p. 2, 2024. [Online]. Available: https://digitalrepository.unm.edu/nss_journal/vol70/iss1/2

[28] N. Kausar and M. Palanikumar, "Finding new similarities measures for Type-II Diophantine neutrosophic interval valued soft sets and its basic operations," *Int. J. Neutrosophic Sci.*, vol. 1, p. 505, 2025. [Online]. Available: https://www.americaspg.com/article/pdf/3075

[29] W. Yang *et al.*, "Triangular single valued neutrosophic data envelopment analysis: application to hospital performance measurement," *Symmetry*, vol. 12, no. 4, p. 588, 2020, doi: 10.3390/sym12040588.

[30] Almutairi, H. A. E. W. Khalifa, and B. E. El-Demerdash, "Incorporating neutrosophic uncertainty in data envelopment analysis," *Neutrosophic Sets Syst.*, vol. 76, pp. 241–252, 2025. [Online]. Available: https://fs.unm.edu/nss8/index.php/111/article/view/5177

DOI: https://doi.org/10.54216/IJNS.270219