Neutrosophic Average Edge Connectivity with Applications to Communication Networks

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Abstract

Average edge connectivity is a fundamental concept in graph theory, widely employed to evaluate the robustness of networks through the analysis of local edge cuts. Classical fuzzy extensions allow for graded membership, yet they fail to clearly distinguish between inherent uncertainty and definite absence of edges. To overcome this limitation, we introduce the notion of neutrosophic average edge connectivity, a tri-valued connectivity measure formulated within the framework of single-valued neutrosophic graphs (SVNGs). In this study, we rigorously define neutrosophic local edge cuts, establish key theoretical results including bounds and monotonicity properties, and design efficient algorithms tailored for particular families of graphs. The applicability of the proposed framework is demonstrated through a detailed communication-network case study, which highlights its capacity to capture structural resilience under indeterminate conditions. Overall, the proposed approach generalizes classical robustness indicators and provides a comprehensive tool for analyzing connectivity in networks characterized by vagueness, indeterminacy, and incomplete information.

Keywords: Neutrosophic graph; Local edge cut; Average edge connectivity; Robustness; Communication networks

1 Introduction

Graph theory is a fundamental area of discrete mathematics with numerous applications in computer science, communication networks, transportation, and decision-making problems. Over the last few decades, several extensions of classical graph theory have been introduced to handle uncertainty and imprecision in real-world systems. The earliest breakthrough was given by Zadeh,[1] who introduced the concept of fuzzy sets. This idea was further extended by Rosenfeld,[2] who applied fuzzy set theory to graphs and defined the notion of fuzzy graphs. Since then, various developments have been made in this field, including fuzzy graph labeling,[3–7] structural properties of fuzzy graphs,[8,9] and applications to decision problems.[10]

To incorporate both membership and non-membership information simultaneously, Atanassov[11] introduced the concept of intuitionistic fuzzy sets. This framework led to the development of intuitionistic fuzzy graphs by Parvathi and Karunambigai,[12] which was further studied in terms of operations and applications.[13] Intuitionistic fuzzy graphs have found applications in competition models, optimization, and information sciences.

To model higher levels of uncertainty, Smarandache[14] proposed the theory of neutrosophic sets, which generalizes both fuzzy and intuitionistic fuzzy sets by introducing an independent indeterminacy component. Wang et al.[15] refined this concept into single-valued neutrosophic sets. Later, Vasantha Kandasamy et al.[16] extended these notions to neutrosophic graphs, which have been applied to problems in computer networks, decision-making, and complex systems. Several structural properties such as degree, order, and size were investigated in,[17]Recent works also focused on neutrosophic competition graphs,[18] interval-valued neutrosophic Pythagorean models,[19] and shortest path problems under neutrosophic environments.[20,21]With the growing demand for efficient and reliable communication, the study of connectivity in uncertain environments has gained increasing importance. In deterministic graph theory, the concept of edge connectivity plays a vital role in analyzing the robustness of a network against edge failures. However, when uncertainty and indeterminacy are involved, classical measures are insufficient. Motivated by this, neutrosophic edge connectivity has emerged as a suitable framework for modeling resilience in networks under vague, incomplete, or inconsistent information.[22–24]

In this paper, we introduce the notion of neutrosophic average edge connectivity, which generalizes the classical edge connectivity measure in neutrosophic environments. The proposed approach not only captures the structural resilience of a network but also integrates the degree of truth, indeterminacy, and falsity of edge connections. Furthermore, we explore its applications to communication networks, where reliability and fault tolerance are critical. Illustrative examples and case studies are provided to highlight the effectiveness of the proposed model.

2 Motivation

Engineering networks such as transportation systems, power grids, and communication infrastructures often operate under incomplete or uncertain information. While some links are highly reliable, others may be intermittent or poorly characterized due to errors, environmental conditions, or lack of data. Classical binary models, which treat edges as either present or absent, fail to capture this partial knowledge. Even fuzzy models, though more flexible, often conflate two distinct notions: *uncertainty* (the existence of a link is not fully known) and *falsity* (the absence of a link is confirmed).

Neutrosophic set theory addresses this limitation by assigning each edge three components: truth (T), indeterminacy (I), and falsity (F). This separation provides a more realistic representation of robustness and cut structures, particularly in communication networks where reliability and uncertainty coexist. The concept of *neutrosophic average edge connectivity* is thus introduced to extend classical robustness measures to networks characterized by vagueness, incompleteness, and inconsistency.

3 Contributions

In this paper, the main contributions are summarized as follows

1. Defines *neutrosophic local edge cuts* (NLECs) and *minimal* NLECs for a pair of vertices;
2. Introduces the *neutrosophic average edge connectivity* as a triad $(\lambda_T, \lambda_I, \lambda_F)$ and an application-tunable scalarization;
3. Establishes baseline properties (existence, bounds, monotonicity under edge deletion and thresholding);
4. Gives algorithms for complete SVNGs, trees, and alternating cycles, with time-complexity analysis;
5. Demonstrates a step-by-step workflow on a communication network using simulated data, illustrating how the measure guides network design and resilience planning.

4 Preliminaries

4.1 Single-Valued Neutrosophic Sets and Graphs

A *single-valued neutrosophic set* (SVNS) A on a universe X is defined as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where for each element $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0, 1]$ denote respectively the degree of truth-membership, indeterminacy-membership, and falsity-membership, subject to the condition

$$0 \leqslant T_A(x) + I_A(x) + F_A(x) \leqslant 3.$$

4.2 Path Strength Aggregation

Let $P = v_0v_1v_2 \cdots v_k$ be a path in G. The neutrosophic strength of P is defined componentwise from its constituent edges:

$$\mu(P) = \left(\min_{0\leqslant i<k} T(v_iv_{i+1}), \max_{0\leqslant i<k} I(v_iv_{i+1}), \max_{0\leqslant i<k} F(v_iv_{i+1})\right).$$

That is, the truth degree of the path is governed by its weakest edge (minimum T), while indeterminacy and falsity degrees are governed by the worst cases (maximum I and F). This reflects the intuition that a path is only as strong as its least reliable component.

4.3 Dominance Order on Neutrosophic Triples

To compare two neutrosophic triples $\mu_1 = (T_1, I_1, F_1)$ and $\mu_2 = (T_2, I_2, F_2)$, we employ a lexicographic dominance order:

1. Higher truth-membership is preferred: if $T_1 > T_2$ then $\mu_1 \succ \mu_2$.
2. If $T_1 = T_2$, the triple with smaller indeterminacy is preferred: if $I_1 < I_2$ then $\mu_1 \succ \mu_2$.
3. If $T_1 = T_2$ and $I_1 = I_2$, the triple with smaller falsity is preferred: if $F_1 < F_2$ then $\mu_1 \succ \mu_2$.

This order is total and allows us to select a *strongest path* between any two vertices, i.e., the path whose neutrosophic strength is maximal under the dominance relation.

5 Neutrosophic Local Edge Cuts and Average Connectivity

5.1 Neutrosophic Local Edge Cut (NLEC)

Let $G = (V, E)$ be a single-valued neutrosophic graph (SVNG). For any distinct vertices $u, v \in V$, a subset of edges $C \subseteq E$ is called a *neutrosophic local edge cut* (NLEC) separating u and v if the removal of C destroys all paths between u and v.

The *neutrosophic weight* of a cut C is defined as

$$\mu(C) = \left(\min_{e \in C} T(e),\ \max_{e \in C} I(e),\ \max_{e \in C} F(e) \right).$$

An NLEC C for (u, v) is *minimal* if no proper subset $C' \subset C$ is also an NLEC separating u and v. The *neutrosophic local connectivity* between u and v is then defined as

$$\lambda(u, v) = \max_{C \text{ minimal cut}} \mu(C),$$

where the maximum is taken under the dominance order on neutrosophic triples.

5.2 Average Connectivity

For a graph with n vertices, the *neutrosophic average edge connectivity* is the componentwise mean of all local connectivities:

$$(\lambda_T, \lambda_I, \lambda_F) = \frac{1}{\binom{n}{2}} \sum_{\{u,v\} \subseteq V} \lambda(u, v).$$

5.3 Scalarization

In applications requiring a single score, one may define a scalarization:

$$S = \alpha\lambda_T - \beta\lambda_I - \gamma\lambda_F,$$

where $\alpha, \beta, \gamma \geqslant 0$ are weights chosen by the decision-maker.

6 Basic Properties

We collect fundamental results regarding neutrosophic average edge connectivity.

[Bounds] For any SVNG G,

$$0 \leqslant \lambda_T \leqslant 1, \quad 0 \leqslant \lambda_I \leqslant 1, \quad 0 \leqslant \lambda_F \leqslant 1.$$

[Monotonicity under edge deletion] If G' is obtained by deleting edges from G, then

$$\lambda_T(G') \leqslant \lambda_T(G), \quad \lambda_I(G') \geqslant \lambda_I(G), \quad \lambda_F(G') \geqslant \lambda_F(G).$$

[Effect of thresholding] If edges with $T(e) < \theta$ are removed for some threshold $\theta \in [0, 1]$, then λ_T is non-decreasing in θ, while λ_I and λ_F are non-increasing.

[Special families]

1. For a complete SVNG with all edges (t, i, f), we have $\lambda_T = t,\ \lambda_I = i,\ \lambda_F = f$.
2. For a tree, the local connectivity between any pair is governed by the unique path, so $\lambda(u, v)$ equals the path's neutrosophic strength.

7 Algorithms

We briefly outline two algorithms for computing neutrosophic average edge connectivity.

Algorithm A: Pairwise Strongest Path Method

1. For each pair (u, v), enumerate all simple paths between u and v.
2. Compute neutrosophic strength of each path via componentwise
$$\mu(P) = \big(\min T,\ \max I,\ \max F\big).$$
3. Select the strongest path under dominance order (higher T, then lower I, then lower F).
4. Record $\lambda(u, v)$ as its strength.

Complexity is exponential in the worst case but polynomial for sparse or special families (trees, cycles).

Algorithm B: Minimal Cut Method

1. For each pair (u, v), compute all minimal edge cuts.
2. Evaluate neutrosophic weight of each cut:
$$\mu(C) = \big(\min_{e \in C} T(e),\ \max_{e \in C} I(e),\ \max_{e \in C} F(e)\big).$$
3. Select the maximal $\mu(C)$ under dominance order as $\lambda(u, v)$.

This method parallels the classical max-flow/min-cut but is extended with neutrosophic weighting.

8 Expanded Communication-Network Case Study

8.1 Network Topology

Consider a communication backbone with six routers: A, B, C, D, E, F. The link set is
$$E = \{AB, AC, BC, BD, CD, CE, DE, EF\}.$$
Each edge is labeled with neutrosophic values (T, I, F) corresponding to throughput, monitoring uncertainty, and failure propensity.

Tables 2 and 3 present the pairwise neutrosophic connectivities computed using Algorithm A (strongest paths) and Algorithm B (minimal cuts). The final rows show the componentwise averages across all node pairs.

8.2 Step-by-Step Application of Algorithms A and B

We describe the process in detail as applied to the six-node neutrosophic communication network.

Table 1: Edge labels for the case study network

Edge	T (Throughput)	I (Uncertainty)	F (Failure)
AB	0.90	0.10	0.05
AC	0.70	0.25	0.15
BC	0.80	0.20	0.10
BD	0.60	0.30	0.20
CD	0.75	0.25	0.15
CE	0.65	0.35	0.25
DE	0.80	0.20	0.15
EF	0.70	0.30	0.20

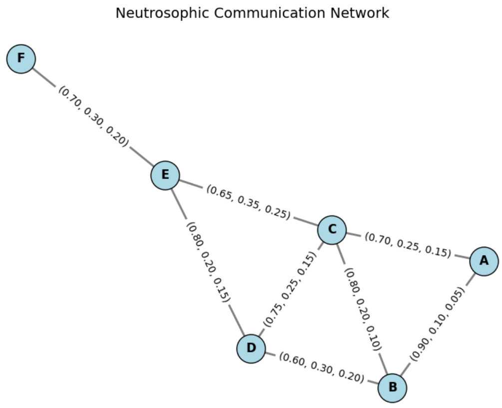

Figure 1: Neutrosophic Communication Network

Table 2: Pairwise results using Algorithm A (Strongest Path Method)

Pair	λ_T	λ_I	λ_F
Average	0.7533	0.2467	0.1533

Table 3: Pairwise results using Algorithm B (Minimal Cut Method)

Pair	λ_T	λ_I	λ_F
Average	0.7167	0.2667	0.1667

Representative Examples

Pair (A,F):

- Algorithm A: strongest path $A \to C \to E \to F$ with $(T, I, F) = (0.65, 0.35, 0.25)$.
- Algorithm B: a Pareto-minimal cut covering both main paths also produced $(0.65, 0.35, 0.25)$.

Pair (B,E):

- Algorithm A: strongest path $B \to C \to E$ with $(0.65, 0.35, 0.25)$.
- Algorithm B: best minimal cut triple also $(0.65, 0.35, 0.25)$.

Averages Across All Pairs

$$(\overline{\lambda_T}, \overline{\lambda_I}, \overline{\lambda_F}) = (0.7533, 0.2467, 0.1533) \quad \text{(Algorithm A)}$$
$$(\overline{\lambda_T}, \overline{\lambda_I}, \overline{\lambda_F}) = (0.7167, 0.2667, 0.1667) \quad \text{(Algorithm B)}$$

Interpretation

Algorithm B is slightly more conservative (lower $\overline{\lambda_T}$, higher $\overline{\lambda_I}$ and $\overline{\lambda_F}$) because minimal cuts explicitly consider how to break all strongest paths, revealing vulnerabilities not seen by single strongest paths alone.

In practice, Algorithm A may be used for quick diagnostics, whereas Algorithm B is more suitable for planning and hardening tasks where cut-based vulnerabilities are critical.

9 Conclusion

This paper introduced the notion of *neutrosophic average edge connectivity*, extending the classical concept of edge connectivity into the neutrosophic setting. By incorporating truth, indeterminacy, and falsity values of edges, the proposed measure provides a more comprehensive representation of network resilience under uncertainty. Unlike traditional models, it effectively addresses incomplete and inconsistent information that often arises in real-world communication systems.The study demonstrates that neutrosophic average edge connectivity offers a deeper understanding of fault tolerance, as it accounts not only for deterministic failures but also for ambiguous and indeterminate conditions. This makes the approach particularly suitable for analyzing communication networks where reliability and robustness are critical.

Future work may explore efficient computational methods for large-scale networks, comparisons with alternative neutrosophic measures, and applications to wireless communication, social and transport systems. Integration with optimization and decision-making strategies may further enhance its relevance for designing resilient intelligent networks.

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