# Neutrosophic Midrange Measure in Bayesian Selection

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**Abstract**

Many of the problems that we face in our lives and daily work are how to directly and accurately select candidates or categories from multiple sets of candidates (categories). The ranking and selection approach is a modern and direct method for selecting categories easily, which is associated with a probability of correct selection. In this paper, we employ the neutrosophic Bayes procedure for decision to select multinomial population. Select a mid-range category for multiple categories and employ neutrosophic logic to define a modern Bayesian procedure that incorporates parameters with some indeterminacy and has a prior distribution, which we call the neutrosophic prior distribution.

**Keywords:** Decision theory; Neutrosophic Bayesian approach; Neutrosophic prior distribution; Neutrosophic posterior distribution; Midrange measure; Selection procedure

## 1.Introduction

Researchers and scholars who use statistical methods need to be familiar with ranking and selection procedures, these kinds of techniques answer a question that is raised in many researches however rarely answered through the traditional methods of analysis.

The ranking and selection method can answer many questions:

Which one (or ones) of several well-defined groups is best (in some well-defined sense of best)?, or Which of several alternative courses of action is best? The statistical answer to such problems must be given in such a way that the probability of a correct selection is controlled. In the language of statistics, we have a well-defined family of populations that differ concerning the value of a characteristic as represented by a single parameter. Ranking and selection procedures have been intended to select the best (worst) system from several options, where the best (worst) option is defined by the given problem. There are a lot of procedures for selecting a set or a subset that contains the largest or smallest, such as testing hypotheses, multiple comparisons, and estimation, but these methods are indirect [15-16].

Because Bayesian analysis is contingent on parameters, and these parameters have a distribution function, it is referred to as a prior distribution. Some parameters are unclear or incomplete data, the classical logic cannot represent them, and here we resort to a broader logic to include these cases. Neutrosophic logic allows for incomplete data or unclear information to be represented. Thus, the prior distribution of a parameter for unclear data will be called a neutrosophic prior distribution. Florentin first introduced the neutrosophic approach in (1995). Neutrosophic approach studied many research papers in various fields, as Florentin presented neutrosophic statistics, and neutrosophic measure, neutrosophic integral, and neutrosophic probability [1-9]. Alhassan, K.F. introduced neutrosophic weibull distributions, neutrosophic reliability, and neutrosophic truncated. Alhassan, K.F. (2006) [10] introduced a fully



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optimal best selection approach using the dynamic programming technique with Bayesian decision-theoretic formulation. In (2010) [11], Al-hassan, K.F, developed the Bayes method to select the median class where the sample size is odd and even. In addition [12], (2014) approximate of odd median by functional analysis has been introduced. In this paper, we introduce a procedure for selecting a mid-range approach using the neutrosophic Bayesian method. The goal is to construct a midrange selection using neutrosophic Bayes risk from a population with a multinomial distribution and a conjugate function of neutrosophic prior distribution, where any selection technique is to maximize the probability of correctly selecting the best system (PCS) in the fewest number of observations.

## 2. Preliminaries

- Neutrosophic set $N_A$ as an matter having the form $N_A = \{0^- < T_A(x), F_A(x), I_A(x) < 1^+, x \in U\}$, U be a universe of discourse. That is, ∃ three function T, I, F which domain is U, and range is $]-0,\ 1+[$ , these represented of the degree of membership, the degree of indeterminacy, and the degree of non-membership, respectively.
- Let T, I, F be real standard or nonstandard subsets of $]0^-, 1^+[$ , with

$$\sup\_T = t\_sup\ , \sup\_I = i\_sup, \sup\_F = f\_sup$$

$$\inf\_T = t\_inf\ , \inf\_I = i\_inf, \inf\_F = f\_inf$$

$$n\_supt = t\_sup + i\_sup + f\_sup$$

$$n\_inft = t\_inf + i\_inf + f\_inf$$

where T, I, F are neutrosophic components.

- Neutrosophic number: Any number can be formed by $\alpha + I\beta$, where $\alpha, \beta$ are real number and I indeterminacy part.
- Neutrosophic probability space Ω :The universal set, with a neutrosophic probability defined for each of its subset, forms a neutrosophic probability space. Let A and B be two neutrosophic events, and neutrosophic probability of A and B are $N_eP(A) = \{T_{A1}, I_{A1}, F_{A1}\}$, $N_eP(B) = \{T_{B1}, I_{B1}, F_{B1}\}$ respectively.

## 3. Neutrosophic Bayesian decision

Bayesian statistics, (Named for the Revd. Thomas Bayes (1702- 1761), represents a different approach to statistical inference. Data are still assumed to come from a distribution belonging to a known parametric family. Whereas classical statistics considers the parameters to be fixed but unknown, the classical Bayesian approach treats them as random variables in their own right. Prior beliefs about θ are represented by the prior distribution, with a prior probability density function, $\pi(\theta)$. The posterior distribution has posterior density function, $\pi(\theta | x_1, x_2, x_3, , \dots.., x_n)$, and captures our beliefs about θ after they have been modified in the light of the observed data. While the neutrosophic Bayesian is defined per to neutrosophic logic which first work by Florentin (1995) as [13-14]:

The neutrosophic prior distribution $\pi\ (\theta_{Ne})$, the prior beliefs about neutrosophic parameter, where $\theta_{Ne}$ neutrosophic parameter (may be set ,real with indeterminacy, interval) and neutrosophic posterior distribution $\pi(\theta_{Ne} | x_1, x_2, x_3, , \dots.., x_n)$, and detains our beliefs about $\theta_{Ne}$ after they have been modified in the light of the observed data.

*Neutrosophic prior distribution:*

The neutrosophic prior distribution is depended on past knowledge, past records, past information or past statistics; that is belief may be very subjective. Where all past data above contains some vague or obscure called indeterminacy, $\pi\ (\theta_{Ne})$.

*Neutrosophic posterior distribution:*

A sample χ is drawn from $P_{\theta_{Ne}} = P_{x|\theta_{Ne}}$ , that is considered as the conditional distribution of χ given $\theta_{Ne} = \theta_{Ne}$. The sample $\chi = x$ is used to get an updated neutrosophic prior distribution, which is called the neutrosophic posterior distribution.

Derived neutrosophic posterior distribution



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Let joint distribution of $\chi$ and $\theta_{Ne}$ is a probability measure on $\mathcal{X} \times \Theta$ determined by $P(A \times B) = \int P_{x|\theta_{Ne}}(A)\, d\,\pi(\theta_{Ne})$, Where $\mathcal{X}$ is the range of X. Thus, the Neutrosophic posterior distribution of $\theta_{Ne}$ given $X = x$, is the conditional distribution $P_{\theta_{Ne}|x}$.

*Bayes formula*

Let $P_{\theta_{Ne}} = \{P_{x|\theta_{Ne}}: \theta_{Ne} \in \Theta\}$ is dominated by $\sigma -$ finite measure $\omega$ and $f_{\theta_{Ne}}(x) = \frac{dP_{x|\theta_{Ne}}}{d\omega}(x)$ is a Borel function on $(\mathcal{X} \times \Theta, \sigma(\mathcal{B}_x \times \mathcal{B}_\Theta))$.

Suppose $\pi(\theta_{Ne})$ a prior distribution on $\Theta$, and $m(x) = \int f_{\theta_{Ne}}(x) d\pi(\theta_{Ne}) > 0$.

The N- posterior distribution $P_{\theta_{Ne}|x} \ll \pi(\theta_{Ne})$ , $\frac{dP_{\theta|x}}{d\pi(\theta_{Ne})} = \frac{f_{\theta_{Ne}}(x)}{m(x)}$

If $\pi(\theta_{Ne}) \ll \omega$ and $\frac{d\pi(\theta_{Ne})}{d\omega} = \pi(\theta_{Ne})$ for a $\sigma -$ finite measure $\omega$, $\frac{d\pi(\theta_{Ne})}{d\omega} = \frac{f_{\theta_{Ne}}(x)}{m(x)}$.

## 4. Elements of neutrosophic Bayesian Decision Problem

The basic elements of statistical decision theory are:

- A parameter space $\Omega = \{\underline{\theta_{Ne}}\}$ which may be vector valued, of $\theta_{Ne}$ possible states of nature, where $\underline{\theta_{Ne}}$ is the neutrosophic parameter that is ; the parameter has indeterminacy, or (the parameter is neutrosophic number)
- An action space A={a}of the possible courses of action,
- A loss function $L\{\underline{\theta_{Ne}}, a\}$ representing the loss incurred when an action a is taken and the state of nature is $\underline{\theta_{Ne}}$ ,
- An observable random variable $\underline{X}$ , which may be vector valued, defined on a sample space $X = \{\underline{X}\}$ such that when $\underline{\theta_{Ne}}$ is the true state of nature, $\underline{X}$ has probability distribution $f(\underline{X}, \underline{\theta_{Ne}})$,
- A decision space $D = \{d(\underline{x})\}$ of possible decision functions defined on X that maps X into the action space A,
- A prior probability distribution $\pi$ is defined on the space $\Omega$.

## 5. Midrange measure

In statistics and probability theory, the mid extreme of a set of statistical data values is the arithmetic mean of the largest and smallest values in a data. We present the problem of selecting the mid-range set out of E mutually exclusive sets has multinomial distribution according neutrosophic logic, when each element in a single population is classified into one of these s sets. For each of the s types (classes) there is a certain probability that any element will be classified $\theta_1, \theta_2, \ldots, \theta_s$ . The mid-extreme classes may be defined as the one with the half biggest probability, or alternatively, as the one with the half lowest probability, dependent on the state.

*Example of Midrange measure*

Let $x_1, x_2, x_3$ be sample from Beta distribution with two parameters $\alpha = 2$, $\beta = 1$. and $Y_1 < .Y_2 < Y_3$ are ordered of values of sample. find mid-range distribution.

Solution: $f(V) = 2V$, $f(2M - V) = 2(2M - V)$

$F(V) = V^2$, $F(2M - V) = (2M - V)^2$ ,0<V< 1-M, 0<M <1

$$g(M) = \int_0^{1-M} [(2M - V)^2 - V^2]\,(2V)[2(2M - V)]dV = 48M((3M - 1)(1 - M))^2 \quad ; 0 < M < 1$$

## 6. Neutrosophic Bayes Risk

For decision function d, the loss function $L\{\theta_{Ne}, d(\underline{x})\}$. Since the action a depends on the particular sample data $\underline{x}$ that we observe. Thus, we see that the loss is a random variable and depends on the sample outcome. Therefore, let us define the risk to be the expected value of the loss function. That is, the risk $R(\underline{\theta_{Ne}}, d)$ is a function on $\underline{\theta_{Ne}}$, d and the loss function L;

$$R\left(\underline{\theta_{Ne}}, d\right) = E[L\{\theta_{Ne}, d(\underline{x})\}] = \int L\{\theta_{Ne}, d(\underline{x})\} f(\underline{x}, \underline{\theta_{Ne}})\})d\underline{x}$$



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The neutrosophic Bays risk of a decision d is the expected value of the risk $R(\underline{\theta_{Ne}}, d)$ with respect to neutrosophic prior distribution $\pi(\theta_{Ne})$ on $\Omega$.

$$r(\pi, d) = E[R(\Theta, d)] = \iint L\{\theta_{Ne}, d(\underline{x})\} f(\underline{x}, \underline{\theta_{Ne}})\} d\underline{x}\, d\underline{\theta_{Ne}}.$$

Let an arbitrary experiment where each outcome is classified into one of s possible mutually exclusive possibilities, which we call classes, and $\theta_{Ne_i}$ is the probability of the event $\theta_{Ne_i}$ with $\sum_{i=1}^{s} \theta_{Ne_i} = 1$. Let $\theta_{Ne_{[1]}} \leq \theta_{Ne_{[2]}} \leq \cdots .. \leq \theta_{Ne_{[s]}}$ denote the ordered values of the $\theta_{Ne_i}$ such that $\underline{\theta_{Ne}}$ the probability $\theta_{Ne_i}$ of an observation in the cell i. When MR is number of observation, the midrange. and let $n_1, n_2, ...., n_S$ be respective frequencies in R classes of the distribution with it is assumed that the values of $\theta_{Ne_i}$ and of the $\theta_{Ne_{[i]}}$ is completely unknown. We will assumed that the probability of the mid-range event is $\theta_{Ne_{[MR]}}$ and the expected value of mid-range class is $E_{\pi(\underline{\theta_{Ne}}, \underline{n})}[\theta_{Ne_{[MR]}}]$.

Let $\theta_{Ne_{[1]}} \leq \theta_{Ne_{[2]}} \leq \cdots .. \leq \theta_{Ne_{[s]}}$ denote the ordered values of the $\theta_{Ne_i}$. Such that $\underline{\theta_{Ne}} = (\theta_{Ne_1}, \theta_{Ne_2}, ...., \theta_{Ne_s})$ the probability $\theta_{Ne_i}$ of an observation in the class i , $\sum_{i=1}^{S} n_i = \kappa$

$$P\left(\underline{n}, \underline{\theta_{Ne}}\right) = \frac{\kappa!}{n_1! n_2! \ldots n_s!}, \quad .$$

In the neutrosophic Bayes procedure we depended on the neutrosophic prior and neutrosophic posterior distribution. The neutrosophic prior function is

$$\pi(\theta_{Ne}) = \frac{\Gamma[\sum_{i=1}^{s} n'_i = \kappa]}{\prod_i^s \Gamma(n'_i)} \prod_i^s \theta_{Ne_i}{}^{n'_i - 1}, \sum_{i=1}^{s} n'_i = \kappa$$

$$\pi(\theta_{Ne} | x_1, x_2, x_3, , ....., x_n) = \frac{f(x_1, x_2, x_3, ,......, x_n | \theta_{Ne}) \pi(\theta_{Ne})}{\int f(x_1, x_2, x_3, ,......, x_n | \theta_{Ne}) \pi(\theta_{Ne}) d\theta_{Ne}}$$

The denominator of the above equation does not involve $\theta_{Ne}$ and so in practice is usually not calculated.

Since $\pi(\theta_{Ne} | x_1, x_2, x_3, , ....., x_n) \propto f(x_1, x_2, x_3, , ....., x_n | \theta_{Ne}) \pi(\theta_{Ne})$

The actual choice of $\pi(\theta_{Ne})$ depends upon the experimenter, and the information and experience available to him at the time of doing the experiment. Mathematical and computational difficulties may arise from using some prior distributions. A reasonable method of overcoming these difficulties is to use a particular class of prior distributions. This class of prior has been termed as natural conjugate priors; for example, the Beta distribution is a conjugate prior to Binomial distribution, while Dirichlet distribution is conjugate prior to multinomial distribution, that will use in this paper.

**7. Neutrosophic Bays Risk of Mid-Range**

In many researches needed the select midrange among multiple values or categories or degrees . If suppose the problem has a multinomial distribution has s classes, and probability $\theta_i$ associated with the $i^{th}$ class. Let $\theta_{[1]} \leq \theta_{[2]} \leq \cdots .. \leq \theta_{[s]}$ denote the ordered values of $\theta_1, \theta_2, ..., \theta_s$ the midrange of class event defined as that class that has the probability $\theta_{[MR]}$ associated with it. Nothing is known about the value of $\theta_i$'s, that is we do not know which class associated with $\theta_{[MR]}$.

$S_i(n''_1, n''_2, ..., n''_s : \kappa'')$ neutrosophic Bayes risk of the terminal decision $d_i$ define as follows :

$$S_i(n''_1, n''_2, ..., n''_s : \kappa'') = E[L\{\theta_{Ne}, d(\underline{x})\}] = mc + \omega \{ E_{\pi(\underline{\theta_{Ne}}, \underline{n})} (\theta_{Ne_{[MR]}}) - \hat{\theta_{Ne_i}} \}$$

the value of $E_{\tau(\underline{\theta_{Ne}}, \underline{n})} (\theta_{Ne_{[MR]}})$ is derived as follows;



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$$E_{\pi(\underline{\theta_{Ne}},\underline{n})}\left(\theta_{Ne[MR]}\right) = \int_0^1 \theta_{Ne[MR]}\, g(\theta_{Ne[MR]})d\theta_{Ne[MR]}$$

If assume $\theta_{Ne} = V$ and $MR = \frac{\theta_{Ne1}+\theta_{Ne2}}{2}$, then $\theta_{Ne\,s} = 2MR - V$

$$g\left(\theta_{Ne[MR]}\right) = 2\kappa(\kappa-1)\int_0^{b-MR}[F(2MR-V)F(V)]^{\kappa-2} f(V)f(2MR-V)\, dV$$

$$= 2\kappa(\kappa-1)\int_0^{b-MR}\sum_{j=n''_{[j]}}^{\kappa''-1}\frac{(\kappa''-1)!\,((2MR-V))^{j}}{j!\,(\kappa''-1-j)!}\,[1-(2MR-V)]^{\kappa''-1-j}\, dV$$

$$\left(\left[\sum_{j=n''_{[h]}}^{\kappa''-1}\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}.(V)^{j}(1-V)^{\kappa''-1-j}\right]^{\kappa-2}\left[\frac{(\kappa''-1)!}{(n''_{[1]}-1)!(\kappa''-n''_{[1]}-1)!}V^{n''_{[1]}-1}(1-V)^{\kappa''-n''_{[1]}-1}\right)$$

$$\left(\frac{(\kappa''-1)!}{(n''_{[1]}-1)!(\kappa''-n''_{[1]}-1)!}(2MR-V)^{n''_{[1]}-1}(1-(2MR-V)^{\kappa''-n''_{[1]}-1}\right]dV$$

$$W=\left[\sum_{j=n''_{[h]}}^{\kappa''-1}\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}.(2MR-V)^{j}(1-[2MR-V])^{\kappa''-1-j}\right)\left(\sum_{j=n''_{[h]}}^{\kappa''-1}\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}.(V)^{j}(1-V)^{\kappa''-1-j}\right]^{\kappa-2}$$

$$=\left[\sum_{j=n''_{[h]}}^{\kappa''-1}\cdots\cdots\sum_{j=n''_{[h]}}^{\kappa''-1}\left(\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}\right)^{2}.\left(\frac{2MR-V}{1-2MR-V}\right)^{j}(1-[2MR-V])^{\kappa''-1}\right)\left(\left(\frac{V}{1-V}\right)^{j}(1-V)^{\kappa''-1}\right]^{\kappa''-2}$$

$$=\left[(1-(2MR-V)(1-V))^{(\kappa''-1)(\kappa''-1)}\sum_{j_1=n''_{[h]}}^{\kappa''-1}\cdots\cdots\sum_{j_{k-2}=n''_{[h]}}^{\kappa''-1}\left(\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}\right)^{2}\left(\frac{2MR-V}{1-2MR-V}\left(\frac{V}{1-V}\right)\right)^{j_1\ldots j_{k-2}}.\right]$$

$$g(\theta_{Ne[MR]})=2\kappa(\kappa-1)\int_a^{b-MR}(2MR-V)^{n''_{[1]}-1}(1-(2MR-V)^{(\kappa''-1)(\kappa''-1)(\kappa''-n''_{[1]}-1)+1-1}\left(\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}\right)^{2}$$

$$\frac{(\kappa''-1)!}{(n''_{[1]}-1)!(\kappa''-n''_{[1]}-1)!}\quad(V)^{n''_{[1]}-1}(1-V)^{(\kappa''-1)(\kappa''-1)(\kappa''-n''_{[1]}-1)+1-1}$$

$$\sum_{j_1=n''_{[h]}}^{\kappa''-1}\cdots\cdots\sum_{j_{k-2}=n''_{[h]}}^{\kappa''-1}\left(\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}\right)^{2}\left(\frac{2MR-V}{1-2MR-V}\left(\frac{V}{1-V}\right)\right)^{j_1\ldots j_{k-2}+1-1}.dV$$

$$g(\theta_{Ne[MR]})=2\kappa(\kappa-1)\left\langle B[(n''_{[1]}-1),(\kappa''-1)(\kappa''-n''_{[1]}-1)+1]\sum_{i_1=(n''_{[1]}-1)}^{n_1}\binom{n_1}{i_1}y^{i_1}(1-y)^{n_1-i_1}\right\rangle$$



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$$\left(\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}\right)^2 \left\langle B[(n''_{[1]}-1),(\kappa''-1)(\kappa''-n''_{[1]}-1)+1]\sum_{i_2=(n''_{[1]}-1)}^{n_2}\binom{n_2}{i_2}y^{i_2}(1-y)^{n_2-i_2}\right\rangle$$

$$\sum_{j_1=n''_{[h]}}^{\kappa''-1}\ldots\ldots\sum_{j_{k-2}=n''_{[h]}}^{\kappa''-1}\left(\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}\right)^2\left\langle B[(j_1....j!_{\kappa''-2}+1),(j_1....j!_{\kappa''-2}+1)]\sum_{i_3=j_1....j!_{\kappa''-2}+1}^{n_3}\binom{n_3}{i_3}y^{i_3}(1-y)^{n_3-i_{31}}\right\rangle$$

$$\left\langle B[(j_1....j!_{\kappa''-2}+1),(j_1....j!_{\kappa''-2}+1)]\sum_{i_4=j_1....j!_{\kappa''-2}+1}^{n_4}\binom{n_4}{i_4}y^{i_4}(1-y)^{n_4-i_4}\right\rangle$$

Then, $\underset{\pi(\theta_{Ne}|\underline{n})}{E}[\theta_{\mathrm{Ne}[MR]}]=\int_0^1\theta_{Ne[MR]}.g(\theta_{Ne[MR]})d\theta_{Ne[MR]}$

$$=\frac{MR^2}{2}2\kappa(\kappa-1)\left\langle B[(n''_{[1]}-1),(\kappa''-1)(\kappa''-n''_{[1]}-1)+1]\sum_{i_1=(n''_{[1]}-1)}^{n_1}\binom{n_1}{i_1}y^{i_1}(1-y)^{n_1-i_1}\right\rangle$$

$$\left(\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}\right)^2 \left\langle B[(n''_{[1]}-1),(\kappa''-1)(\kappa''-n''_{[1]}-1)+1]\sum_{i_2=(n''_{[1]}-1)}^{n_2}\binom{n_2}{i_2}y^{i_2}(1-y)^{n_2-i_2}\right\rangle$$

$$\sum_{j_1=n''_{[h]}}^{\kappa''-1}\ldots\ldots\sum_{j_{k-2}=n''_{[h]}}^{\kappa''-1}\left(\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}\right)^2\left\langle B[(j_1....j!_{\kappa''-2}+1),(j_1....j!_{\kappa''-2}+1)]\sum_{i_3=j_1....j!_{\kappa''-2}+1}^{n_3}\binom{n_3}{i_3}y^{i_3}(1-y)^{n_3-i_{31}}\right\rangle$$

$$\left\langle B[(j_1....j!_{\kappa''-2}+1),(j_1....j!_{\kappa''-2}+1)]\sum_{i_4=j_1....j!_{\kappa''-2}+1}^{n_4}\binom{n_4}{i_4}y^{i_4}(1-y)^{n_4-i_4}\right\rangle$$

Thus, the Neutrosophic Bays risk to select mid-range class;

$$mc+W\frac{MR^2}{2}2\kappa(\kappa-1)\left\langle B[(n''_{[1]}-1),(\kappa''-1)(\kappa''-n''_{[1]}-1)+1]\sum_{i_1=(n''_{[1]}-1)}^{n_1}\binom{n_1}{i_1}y^{i_1}(1-y)^{n_1-i_1}\right\rangle$$

$$\left(\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}\right)^2 \left\langle B[(n''_{[1]}-1),(\kappa''-1)(\kappa''-n''_{[1]}-1)+1]\sum_{i_2=(n''_{[1]}-1)}^{n_2}\binom{n_2}{i_2}y^{i_2}(1-y)^{n_2-i_2}\right\rangle$$

$$\sum_{j_1=n''_{[h]}}^{\kappa''-1}\ldots\ldots\sum_{j_{k-2}=n''_{[h]}}^{\kappa''-1}\left(\frac{(\kappa''-1)!}{j!(\kappa''-1-j)!}\right)^2\left\langle B[(j_1....j!_{\kappa''-2}+1),(j_1....j!_{\kappa''-2}+1)]\sum_{i_3=j_1....j!_{\kappa''-2}+1}^{n_3}\binom{n_3}{i_3}y^{i_3}(1-y)^{n_3-i_{31}}\right\rangle$$

$$\left\langle B[(j_1....j!_{\kappa''-2}+1),(j_1....j!_{\kappa''-2}+1)]\sum_{i_4=j_1....j!_{\kappa''-2}+1}^{n_4}\binom{n_4}{i_4}y^{i_4}(1-y)^{n_4-i_4}\right\rangle-\theta^{\wedge}{}_{Nei}$$

.

## 8. Conclusion

In the Bayesian procedure, the posterior distribution includes all the lowdown about the parameter, so statistical decisions should be made based on conditional on the observation, precisely. In this paper, we focused on eliciting neutrosophic Bayesian risk which depends on the parameters, where the belong to neutrosophic space that include some indeterminacy. We employ the neutrosophic prior and posterior distribution to construct Bayesian risk. Then, the neutrosophic Bayes risk has been introduced to select the midrange category for a multinomial classes.



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