Neutrosophic Boundary and Neutrosophic Semi-boundary on Fuzzy Setting

E. Poongothai[1,*], E. Kalaivani[2]

[1]Associate Professor, PG & Research Department of Mathematics, Shanmuga Industries Arts and Science College, Tiruvannamalai-606603, Tamilnadu, India

[2]Ph.D(Full-time) Research Scholar, PG & Research Department of Mathematics, Shanmuga Industries Arts and Science College, Tiruvannamalai-606 603, Tamilnadu, India

Emails: epoongothai5@gmail.com; kalai12feb@gmail.com

Abstract

The aim of this paper is to introduce the concept of fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary of a fuzzy neutrosophic topological space. Some characterization are discussed. Several examples and properties are obtained.The aim of this paper is to introduce the concept of fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary of a fuzzy neutrosophic topological space. Some characterization are dis- cussed. Several examples and properties are obtained.

Keywords: Fuzzy neutrosophic open; Fuzzy neutrosophic closed; Fuzzy neutrosophic semi-open; Fuzzzy neutrosophic semi-closed; Fuzzy neutrosophic semi-interior; Fuzzy neutrosophic semi-closure; Fuzzy neutrosophic boundary; Fuzzy neutrosophic semi-boundary

1. Introduction

After Zadeh's [27] introduction of fuzzy sets, Chang [8] defined and studied the notion of a fuzzy topological space in 1968. Since then, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed [1][2][4].

In recent years, fuzzy topology has been found to be very useful in solving many practical problems. Du et al [9] fuzzified the very successful 9-intersection Egenhofer model [9,10] for depicting topological relations in geographic information systems (GIS) query. In [11,12], El-Naschie showed that the notion of fuzzy topology might be relevant to quan- tum particle physics and quantum gravity in connection with string theory and e^{∞} theory. Tang [25] used a slightly changed version of Chang's fuzzy topological space to model spatial objects for GIS databases and structured query language (SQL) for GIS. Levine [20] introduced the concepts of semi-open sets and semi-continuous mappings in topological space. Interestingly, his work found applications in the field of digital topology [10]. for example, it was found that digital line is a $T1$-space [11], which is a weaker separation axiom based upon semi-open sets. Fuzzy digital topology [24] was introduced by Rosenfeld, which demonstrated the need for the fuzzification of weaker forms of notions of classical topology. Azad [6] carried out this fuzzification in 1981, and presented some general properties of fuzzy spaces. Several properties of fuzzy semi-open (resp., fuzzy semi-closed), fuzzy regular open (resp., closed) sets have been discussed. Moreover, he defined fuzzy semi-continuous (resp., semi-open, semi-closed) functions and studied the properties of fuzzy semi-continuous function in product related spaces. Finally, he defined and characterized fuzzy almost continuous mappings. In this direction, much work followed subsequently, for example [19-27].

Pu and Liu [23] defined the notion of fuzzy boundary in fuzzy topological spaces in 1980 and followed Ahmad and Athar [3] studied and discussed their properties. Florentin Smarandache [13] first introduced Neutrosophic sets in 1955, which is a generalization of the concepts of fuzzy sets and intuitionistic fuzzy

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sets. Neutrosophic sets allow for consideration of indeterminacy, neutrality, and inconsistency in a given space. In 2017, Veereswari [26] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields. Furthermore, the authors E.Poongothai and E.Padmavathi [22] introduced the concept of fuzzy neutrosophic semi-open (semi-closed) and fuzzy neutrosophic semi-interior (semi-closure) are introduced. In this paper, we have to introduce fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary and several properties with suitable examples are examined

2. Preliminaries

For the basic concepts and notations are given. The following definitions and lemmas are used in studying the properties of fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary.

Definition 2.1. [5] A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F : X \to [0, 1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$. With the condition $0 \le T_A*(x) + I_A*(x) + F_A*(x) \le 2$.

Definition 2.2. [5] A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B

(i.e.,) $A \subseteq B$ for all x if $T_A(x) \le T_B(x), I_A(x) \le I_B(x), F_A(x) \le F_B(x)$.

Definition 2.3. [5] Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, B =$

$\langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$$

$$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$$

Definition 2.4. [5] The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$

Definition 2.5. [5] A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0, I_A(x) = 0, F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N.

Definition 2.6. [5] A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N.

Definition 2.7. [5] The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = (T_A(x)$.

The complement of a fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8. [20] A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X

(i) $0_N, 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$. Satisfying the following axioms. In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9. [26] The complement A^C of a fuzzy neutrosophic set in A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10. [26] Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$int(A) = \cup\{G : G \text{ is a fuzzy neutrosophic open set in } X \text{ and } G \subseteq A\}$$

$$cl(A) = \cap\{G : G \text{ is a fuzzy neutrosophic closed set in } X \text{ and } G \subseteq G\}$$

Definition 2.11. [26] Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X, τ). Then A_N is said to be fuzzy neutrosophic semi-open in (X, τ_N) if there exists a fuzzy neutrosophic open set such that $fn0 \le A_N \le fncl(fn0)$.

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Lemma 2.12. [22] *Let A_N be any fuzzy neutrosophic set in the fuzzy neutrosophic topolog- ical space* (X_N, τ_N). *Then (i).* $(Sint(A_N))^C = Scl(A^C)$ *and* (ii). $(Scl(A))^C = Sint(A^C)$

Theorem 2.13. [22] *A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi-open set if only if $A_N \leq ((A_N)^+)^-$.*

Theorem 2.14. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then the union of two fuzzy neutrosophic semi-open sets is a fuzzy neutrosophic semi-open set in the fuzzy neutrosophic topological space (X, τ_N).*

Theorem 2.15. [22] *Let A_N be a fuzzy neutrosophic semi-open in the fuzzy neutrosophic topological space (X, τ_N). and suppose $A_N \leq B_N \leq (A_N)^-$. Then B_N is a fuzzy neutro- sophic semi-open set in (X, τ_N).*

Definition 2.16. [22] Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X, τ_N).Then A_N is said to be fuzzy neutrosophic semi-closed set in (X, τ_N) if there exists a fuzzy neutrosophic closed (fnC) set such that $fn(fnC)^+ \leq A_N \leq (fnC)$.

Theorem 2.17. [22] *A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi-closed set if only if $((A_N)^-)^+$.*

Theorem 2.18. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space and A_N be a fuzzy neutrosophic set of (X, τ_N). Then A_N is a fuzzy neutrosophic semi-closed set if and only if complement of $C(A_N)$ is fuzzy neutrosophic semi-open set in (X, τ_N).*

Theorem 2.19. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then intersec- tion of two fuzzy neutrosophic semi-closed sets is a fuzzy neutrosophic semi-closed set in fuzzy neutrosophic topological space (X, τ_N).*

Definition 2.20. [22] Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X, τ_N), the fuzzy neutrosophic semi-interior of A_N is the union of all fuzzy neutrosophic semi-open sets of (X, τ_N) contained in A_N.That is, $fnS(A_N)^+ = \vee\{G_N : G_N \text{ is a fuzzy neutrosophic semi} - \text{open set in } (X, \tau_N) \text{ and } G_N \leq A_N\}$

Definition 2.21. [22] Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X, τ_N), the fuzzy neutrosophic semi-closure of A_N [*$fnS\,cl(A_N)$ for short*] is the intersection of all fuzzy neutrosophic semi-closed sets of (X, τ_N) contained in A_N.That is, $fnS(A_N)^- = \wedge\{K_N : K_N \text{ isa } fnSC \text{ set in } (X, \tau_N) \text{ and } K_N \geq A_N\}$

Theorem 2.22. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have* (i) $fnS(A_N \vee B_N)^- = fnS(A_N)^- \vee fnS(B_N)^-$ *and* (ii) $fnS(A_N \wedge B_N)^- = fnS(A_N)^- \wedge fnS(B_N)^-$

Theorem 2.23. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic sets A_N and B_N of a (X, τ_N),*

(i) $C(fnS(A_N)^+) \leq fnS(C(A_N))^-$,
(ii) $C(fnS(A_N)^-) \leq fnS(C(A_N))^+$

Theorem 2.24. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have*

(i) $A_N \leq fnS(A_N)^-$,
(ii) A_N *is* $fnSC$ *set in* $(X, \tau_N) \Leftrightarrow fnS(A_N)^- = A_N$,
(iii) $fnS(fnS(A_N)^-)^- = fnS(A_N)^-$,
(iv) *If* $A_N \leq B_N$ *then* $fnS(A_N)^- \leq fnS(B_N)^-$.

Theorem 2.25. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have*

(i) $fnS(A_N)^- \geq A_N \vee fnS(fnS(A_N)^+)^-$,
(ii) $fnS(A_N)^+ \leq A_N \wedge fnS(fnS(A_N)^-)^+$,
(iii) $fn(fnS(A_N)^-)^+ \leq fn(fn(A_N)^-)^+$,
(iv) $fn(fnS(A_N)^-)^+ \geq fn(fnS(fnS(A_N)^+)^-)^+$

Theorem 2.26. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have*

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(i) $fnS(A_N \wedge B_N)^+ = fnS(A_N)^+ \wedge fnS(B_N)^+$,
(ii) $fnS(A_N \vee B_N)^+ \geq fnS(A_N)^+ \vee fnS(B_N)^+$.

Lemma 2.27. [26] *Let A_N and B_N be any fuzzy neutrosophic set in the fuzzy neutrosophic topological space (X_N, T_N). Then the following are true.*

(i) *$cl(A_N)$ is a fuzzy neutrosophic closed in X_N.*
(ii) *$int(A_N)$ is a fuzzy neutrosophic open in X_N.*
(iii) *$cl(A_N) \leq cl(B_N)$ if $A_N \leq B_N$.*
(iv) *$int(Int(A_N)) = int(A_N)$.*
(v) *$cl(cl(A_N)) = cl(A_N)$.*
(vi) *$int(A_N \wedge B_N) = int(A_N) \wedge int(B_N)$.*
(vii) *$int(A_N \vee B_N) = int(A_N) \vee int(B_N)$.*
(viii) *$cl(A_N \vee B_N) = cl(A_N) \vee cl(B_N)$.*
(ix) *$cl(A_N \wedge B_N) = cl(A_N) \wedge cl(B_N)$*

3. On fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary

Definition 3.1. Let A_N be a fuzzy neutrosophic set in an fuzzy neutrosophic topological space (X_N, T_N). Then the fuzzy neutrosophic boundary of A_N is defined as $Bdy(A_N) = C.(A_N) \wedge C.(1 - A_N)$. Obviously $Bdy(A_N)$ is a fuzzy neutrosophic closed set.

Remark 3.2. In fuzzy neutrosophic topology, we have $A_N \vee Bdy(A_N) \leq C.(A_N)$, for an arbitrary fuzzy neutrosophic set A_N in X_N, the equality need not hold as the following example.

Example 3.3. Let $X_N = \{a, b\}$ and $T_N = \{0, A_N, B_N, 1\}$. Then (X_N, T_N) be a fuzzy neutrosophic topological space. Let $A_N = \{a_{0.6}, b_{0.4}\}$, $B_N\{a_{0.3}, b_{0.4}\}$.Then $C.(A_N) =$

$\{a_{0.7}, b_{0.6}\}$ and $Bdy(A_N) = \{a_{0.6}, b_{0.4}\}$. It follows that $C.(A_N) \neq \{a_{0.6}, b_{0.6}\} = A_N \vee$

$Bdy(A_N)$

Proposition 3.4. *For a fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space. (X_N, T_N), then the following conditions hold.*

(1) *$Bdy(A_N) = Bdy(1 - A_N)$*
(2) *If A_N is fuzzy neutrosophic closed set, then $Bdy(A_N) \leq (A_N)$*
(3) *If A_N is fuzzy neutrosophic open set, then $Bdy(A_N) \leq (1 - A_N)$*
(4) *Let $A_N \leq B_N$ and $B_N \in c.(X_N)$ ($C.(X_N)$ denotes the class of fuzzy neutrosophic closed set). Then $Bdy(A_N) \leq B$(resp. $Bdy(A_N \leq 1 - B_N)$).*
(5) *$1 - Bdy(A_N) = I.(A_N) \vee I.(1 - A_N)$*
(6) *$Bdy(A_N) \leq Bdy(A_N)$*
(7) *$C.(Bdy(A_N)) \leq Bdy(A_N)$.*

Proof : (1). By Definition,3.1, $Bdy(A_N) = C.(A_N) \wedge C.(1 - A_N)$ and $Bdy(1 - A_N) = C.(1 - A_N) \wedge C.(A_N)$. Therefore $Bdy(A_N) = Bdy(1 - A_N)$. Hence, (1) proved.

(2). Let A_N be fuzzy neutrosophic closed set. By lemma 2.27, $C.(A_N) = A_N$. $Bdy(A_N) \leq C.(A_N) \wedge C.(1 - A_N) \leq C.(A_N) = A_N$ Hence (2) proved.

(3). Let A_N be fuzzy neutrosophic open set. By lemma 2.27, $I.(A_N) = A_N$. It follows that $Bdy(A_N) \leq C.(1 - A_N) = 1 - (I.(A_N)) = 1 - A_N$. Hence, (3) proved.

(4). Let $A_N \leq B_N$. Then by Lemma 2.27, $C.(A_N) \leq C.(B_N)$. Since $B \in c.(X_N)$, we have $C.(B_N) = B_N$. This implies that, $Bdy(A_N) = C.(A_N) \wedge C.(1 - A_N) \leq C.(B_N) \wedge C.(1 - B_N) \leq C.(B_N) = B$. That is, $Bdy(A_N) \leq B_N$. Let $B \in c.(X_N)$. Then $1 - B_N \in c.(X_N)$. Using the above, $Bdy(A_N) \leq 1 - B_N$. Hence, (4) proved.

(5). By definition 3.1, $Bdy(A_N) = C.(A_N) \wedge C.(1 - A_N)$ Taking complement on both sides, we get $1 - (Bdy(A_N)) = 1 - (C.(A_N) \wedge C.(1 - A_N)) = (1 - C.(A_N)) \vee 1 - (C.(1 - A_N)) = I.(1 - A_N) \vee I.(A_N)$. Hence, (5) proved.

(6). Since $C.(A_N) \leq C.(A_N)$ and $C.(1 - A_N) \leq C.(1 - A_N)$, we have $Bdy(A_N) = C.(A_N) \wedge C.(1 - A_N) \leq C.(A_N) \wedge C.(1 - A_N) = Bdy(A_N)$. Hence, (6) proved.

(7). $C.(Bdy(A_N)) = C.(C.(A_N) \wedge C.(1 - A_N)) \leq C.(C.(A_N)) \wedge C.(C.(1 - A_N)) = C.(A_N) \wedge C.(1 - A_N) = Bdy(A_N) \leq Bdy(A_N)$. Hence, (7) proved.

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Proposition 3.5. *Let A_N be a fuzzy neutrosophic set in an fuzzy neutrosophic topological space. (X_N, T_N). Then*

(1) $Bdy(A_N) = C.(A_N) \wedge (1 - I.(A_N))$
(2) $Bdy(I.(A_N)) \leq Bdy(A_N)$
(3) $Bdy(C.(A_N)) \leq Bdy(A_N)$
(4) $I.(A_N) \leq A_N \wedge 1 - (Bdy(A_N))$

Proof :(1).Since $C.(1-A_N) = 1-I.(A_N)$, we have $Bdy(A_N) = C.(A_N) \wedge C.(1-A_N) = C.(A_N) \wedge 1 - I.(A_N)$. This proves (1).

(2). By definition 3.1, $Bdy(I.(A_N)) = C.(I.(A_N)) \wedge C.(1 - I.(A_N)) = C.(I.(A_N)) \wedge C.(C.(1 - A_N)) = C.(I.(A_N)) \wedge C.(1 - A_N) = C.(I.(A_N)) \wedge 1 - (I.(A_N) \leq C.(A_N) \wedge (1 - I.(A_N)) = Bdy(A_N)$. Hence, (2) proved.

(3). $Bdy(C.(A_N)) = C.(C.(A_N)) \wedge C.(1 - C.(A_N)) = C.(C.(A_N)) \wedge 1 - (I.(C.(A_N))) \leq C.(A_N) \wedge (1 - I.(A_N)) = Bdy(A_N)$. Thus proves (3)

(4). $A_N \wedge 1 - (Bdy(A_N)) = A_N \wedge 1 - ((C.(A_N)) \wedge C.(1 - A_N)) = (A_N) \wedge (I.(1 - A_N)) \vee I.(A_N) = A_N \wedge I.(1 - A_N) \vee (A_N \wedge I.(A_N) = A_N \wedge I.(1 - A_N) \vee I.(A_N) \geq I.(A_N)$. Hence (4).

Proposition 3.6. *Let A_N and B_N be a fuzzy neutrosophic sets in an fuzzy neutrosophic topological space (X_N, T_N) then $Bdy(A_N \vee B_N) \leq Bdy(A_N) \vee Bdy(B_N)$*

Proof: We use Lemma 2.27, to prove this, $Bdy(A_N \vee B_N) = C.(A_N \vee B_N) \wedge C.(1 - (A_N \vee B_N)) = C.(A_N \vee B_N) \wedge C.((1-A_N) \wedge (1-B_N)) \leq C.(A_N) \vee C.(B_N) \wedge (C.(1-(A_N) \wedge C.(1 - B_N)) \leq (C.(A_N)) \wedge C.(1 - A_N)) \vee C.(B_N)) \wedge C.(1 - B_N)) = Bdy(A_N) \vee Bdy(B_N)$.Hence the proposition.

Example 3.7. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.3}, b_{0.4}\rangle\}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed sets of T_N is $1 - T_N = \{0, 1, \langle a_{0.7}, b_{0.6}\rangle\}$ and then let$A_N = \{a_{0.3}, b_{0.6}\}$, $B_N = \{a_{0.4}, b_{0.3}\}$. On computation, we see that $Bdy(A_N) = \{a_{0.4}, b_{0.5}\}$ and $Bdy(B_N) = \{a_{0.6}, b_{0.5}\}$. Now $A_N \vee B_N = \{a_{0.4}, b_{0.6}\}$ and $Bdy(A_N \vee B_N) = \{a_{0.5}, b_{0.5}\}$. This gives that $Bdy(A_N) \vee Bdy(B_N) = \{a_{0.6}, b_{0.5}\} \not\subseteq Bdy(A_N \vee B_N) = \{a_{0.5}, b_{0.5}\}$.

The following example shows that $Bdy(A_N \wedge B_N) \not\subseteq Bdy(A_N) \wedge Bdy(B_N)$ and $Bdy(A_N) \wedge Bdy(B_N) \not\subseteq Bdy(A_N \wedge B_N)$.

Example 3.8. Using the Example 3.7, $A_N = \{a_{0.3}, b_{0.6}\}$ and $B_N = \{a_{0.4}, b_{0.3}\}$. Then $Bdy(A_N) = \{a_{0.4}, b_{0.5}\}$ and $Bdy(B_N) = \{a_{0.6}, b_{0.5}\}$. Now $A_N \wedge B_N = \{a_{0.3}, b_{0.3}\}$ and $Bdy(A_N \wedge B_N) = \{a_{0.5}, b_{0.4}\}$.This implies that $Bdy(A_N) \wedge Bdy(B_N) = \{a_{0.4}, b_{0.5}\} \not\subseteq Bdy(A_N \wedge B_N)) = \{a_{0.5}, b_{0.4}\}$ and $Bdy(A_N \wedge B_N) \not\subseteq Bdy(A_N) \wedge Bdy(B_N)$.

Proposition 3.9. *For any fuzzy neutrosophic sets A_N and B_N in an fuzzy neutrosophic topological space (X_N, T_N), then $Bdy(A_N \wedge B_N) \leq (Bdy(A_N) \wedge C.(B_N)) \vee (Bdy(B_N) \wedge C.(A_N))$*

Proof : We use Lemma 2.27, to prove this $Bdy(A_N \wedge B_N) = C.(A_N \wedge B_N) \wedge C.(1 -(A_N \wedge B_N)) = C.(A_N \wedge B_N) \wedge C.((1 - A_N) \vee (1 - B_N)) \leq (C.(A_N) \wedge C.(B_N)) \wedge (C.(1- A_N) \vee C.(1 - B_N)) = (C.(A_N) \wedge C.(B_N) \wedge C.(1-A_N)) \vee (C.(A_N) \wedge C.(B_N) \wedge C.(1-B_N)) = (Bdy(A_N) \wedge C.(B_N)) \vee (Bdy(B_N) \wedge C.(A_N))$. Hence proved.

Proposition 3.10. *For any fuzzy neutrosophic set A_N in an fuzzy neutrosophic topological space then $Bdy(Bdy(A_N)) \leq Bdy(A_N)$ (2). $Bdy(Bdy(Bdy(A_N))) \leq Bdy(Bdy(A_N))$.*

Proof : We use Lemma 2.27 and Definition 3.1 to prove this $Bdy(Bdy(A_N)) = C.(Bdy(A_N)) \wedge C.(1 - Bdy(A_N)) \leq C.(Bdy(A_N)) = C.(C.(A_N) \wedge C.(1 - A_N)) = C.(C.(A_N)) \wedge C.(C.(1 - A_N)) = C.(A_N) \wedge C.(1 - A_N) = Bdy(A_N)$. This proves (1).$Bdy(Bdy(Bdy(A_N))) = C.(Bdy(Bdy(A_N))) \wedge C.(Bdy(1 - Bdy(A_N))) = Bdy(Bdy(A_N)) \wedge C.(Bdy(1 - Bdy(A_N))) \leq Bdy(Bdy(A_N))$. Hence the proposition.

This reverse inequality in proposition are not true as shown in the following example.

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Example 3.11. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.2}, b_{0.2}\rangle\}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed sets of T_N is $1 - T_N = \{0, 1, \langle a_{0.8}, b_{0.8}\rangle\}$. Let $A_N = \{a_{0.3}, b_{0.4}\}$. On computation, we see that $Bdy(A_N) = \{a_{0.7}, b_{0.7}\} \not\subset Bdy(Bdy(A_N)) = \{a_{0.6}, b_{0.7}\}$. Also, calculate $Bdy(Bdy(Bdy(A_N))) = \{a_{0.4}, b_{0.4}\}$. This shows that $Bdy(Bdy(A_N)) = \{a_{0.6}, b_{0.7}\} \not\subset Bdy(Bdy(Bdy(A_N))) = \{a_{0.4}, b_{0.4}\}$.

4. On fuzzy neutrosophic semi-boundary

In this section, we define fuzzy neutrosophic semi-boundary and discuss their properties with example.

Definition 4.1. [14] Let A_N be a fuzzy neutrosophic set in an fuzzy neutrosophic topolog- ical space (X_N, T_N). Then the fuzzy neutrosophic semi-boundary of A_N is defined as $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N)$. Obviously, $SBdy(A_N)$ is a fuzzy neutrosophic semi-closed set.

Remark 4.2. [15] In a fuzzy neutrosophic topology, we have $A_N \vee SBdy(A_N) \leq SC.(A_N)$ for an arbitrary fuzzy neutrosophic set A_N in X_N, the equality need not hlod as shown in the following example.

Example 4.3. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.8}, b_{0.8}\rangle, \langle a_{0.2}, b_{0.2}\rangle, \langle a_{0.3}, b_{0.7}\rangle\}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed sets of T_N is $1 - T_N = \{0, 1, \langle a_{0.2}, b_{0.2}\rangle, \langle a_{0.8}, b_{0.8}\rangle, \langle a_{0.7}, b_{0.3}\rangle\}$. Let $A_N = \{a_{0.6}, b_{0.9}\}$. Then $SC.(A_N) = \{a_{0.8}, b_{0.9}\}$ and $SBdy(A_N) = \{a_{0.4}, b_{0.3}\}$. It follows that $SC.(A_N) \vDash \{a_{0.6}, b_{0.9}\} = A_N \vee SBdy(A_N)$.

Proposition 4.4. *For a fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X_N, T_N), the following conditions hold.*

(1) $SBdy(A_N) = SBdy(1 - A_N)$.
(2) *If A_N is fuzzy neutrosophic semi-closed, then $SBdy(A_N) \leq A_N$.*
(3) *If A_N is fuzzy neutrosophic semi-open, then $SBdy(A_N) \leq 1 - A_N$.*
(4) *Let $A_N \leq B_N$ and $B_N \in Sc.(X_N)$ (resp.,$B_N \in So.(X_N)$). Then $SBdy(A_N) \leq B_N$ (resp.,$SBdy(A_N) \leq 1 - B_N$), where $Sc.(X_N)$ (resp., $So(X_N)$) denotes the class of family of fuzzy neutrosophic semi-closed (resp., fuzzy neutrosophic semi-open) sets in X_N.*
(5) $(1 - SBdy(A_N)) = SI.(A_N) \vee SI.(1 - A_N)$
(6) $SBdy(A_N) \leq Bdy(A_N)$.
(7) $SC.(SBdy(A_N)) \leq Bdy(A_N)$.

Proof: By definition 4.1, $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N)$ and $SBdy(1 - A_N) = SC.(1 - A_N) \wedge SC.(A_N)$. Therefore $SBdy(A_N) = SBdy(1 - A_N)$. Hence proved (1).

Let A_N be a fuzzy neutrosophic semi-closed. By lemma 2.12 $SC.(A_N) = A_N$. $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N) \leq SC.(A_N) = A_N$. Hence proved (2).

Let A_N be a fuzzy neutrosophic semi-open, by Lemma 2.12 $SI.(A_N) = A_N$. $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N) \leq SC.(1 - A_N) = (1 - SI.(A_N)) = 1 - A_N$. Hence proved (3).

Let $A_N \leq B_N$, Then by Lemma 2.12, $SC.(A_N) \leq SC.(B_N) = B_N$. That is, $SBdy(A_N) \leq B_N$. Let $B_N \in So.(X_N)$. Then $1 - B_N \in Sc.(X_N)$. Then by the above, $SBdy(A_N) \leq 1 - B_N$. Hence proved (4).

By definition 4.1, $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N)$. Taking complement on both sides, we get $(1 - SBdy(A_N)) = (1 - SC.(A_N) \wedge SC.(1 - A_N)) = (1 - SC.(A_N)) \vee (1 - SC.(1 - A_N)) = SI.(1 - A_N) \vee SI.(A_N)$. Hence proved (5).

Since $SC.(A_N) \leq C.(A_N)$ and $SC.(1 - A_N) \leq C.(1 - A_N)$, then we have $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N) \leq C.(A_N) \wedge C.(1 - A_N) = SBdy(A_N)$. Hence proved (6) $SC.(SBdy(A_N)) = SC.(SC.(A_N) \wedge SC.(1 - A_N)) \leq SC.(SC.(A_N)) \wedge SC.(SC.(1 - A_N)) = SC.(A_N) \wedge SC.(1 - A_N) = SBdy(A_N) \leq Bdy(A_N)$. Thus $SC.(SBdy(A_N)) \leq Bdy(A_N)$.

Hence proved (7).

The converse of (2) and (3) and reverse inequalities of (6) and (7) are not true as shown in the following example.

Example 4.5. Let $A_N = \{a_{0.6}, b_{0.9}\}$ and $B_N = \{a_{0.4}, b_{0.1}\}$ in the fuzzy neutrosophic topological space (X_N, T_N) be defined in Example 4.3 then $SBdy(A_N) = \{a_{0.4}, b_{0.3}\} \leq A_N$, but A_N is not fuzzy neutrosophic

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semi-closed. $SBdy(B_N) = \{a_{0.4}, b_{0.3}\} \leq 1 - B_N$, but B_N is not fuzzy neutrosophic semi-open. Then $Bdy(B_N) = \{a_{0.7}, b_{0.3}\} \not\subset SBdy(B_N) = \{a_{0.4}, b_{0.3}\}$ and $Bdy(B_N) \not\subset SC.(SBdy(A_N)) = \{a_{0.5}, b_{0.3}\}$.

Proposition 4.6. *Let A_N be a fuzzy neutrosophic set in a fuzzy neutrosophic topological space (X_N, T_N). Then*

(1) $SBdy(A_N) = SC.(A_N) \wedge (1 - SI.(A_N))$
(2) $SBdy(SI.(A_N)) \leq SBdy(A_N)$.
(3) $SBdy(SC.(A_N)) \leq SBdy(A_N)$.
(4) $SI.(A_N) \leq A_N \wedge (1 - SBdy(A_N))$

Proof [7]: Since $SC.(1-A_N) = (1-SI.(A_N))$, we have $SBdy(A_N) = SC.(A_N) \wedge SC.(1-A_N) = SC.(A_N) \wedge (1 - SI.(A_N))$. This proves (1).

$SBdy(SI.(A_N)) = SC.(SI.(A_N)) \wedge SC.(1 - SI.(A_N)) = SC.(SI.(A_N)) \wedge SC.(SC.(1 - A_N)) = SC.(SI.(A_N)) \wedge SC.(1-A_N) = SC.(SI.(A_N)) \wedge (1-SI.(A_N)) \leq SC.(A_N) \wedge (1-SI.(A_N)) = SBdy(A_N)$. Hence proves (2).

$SBdy(SC.(A_N)) = SC.(SC.(A_N)) \wedge SC.(1 - SC.(A_N)) = SC.(SC.(A_N)) \wedge (1 - SI.(SC.(A_N))) \leq SC.(A_N) \wedge (1 - SI.(A_N)) = SBdy(A_N)$. Hence proves (3).

$A_N \wedge (1 - SBdy(A_N)) = A_N \wedge SC.(A_N) \wedge (1 - SC.(1 - A_N)) = A_N \wedge (SI.(1 - A_N) \vee SI.(A_N)) = (A_N \wedge SI.(1 - A_N)) \vee (A_N \wedge SI.(A_N)) = (A_N \wedge SI.(1 - A_N)) \vee SI.(A_N) \geq SI.(A_N)$. Hence proves (4).

Proposition 4.7. [16] *Let A_N and B_N be a fuzzy neutrosophic sets in an fuzzy neutrosophic topological space (X_N, T_N). Then $SBdy(A_N \vee B_N) \leq SBdy(A_N) \vee SBdy(B_N)$.*

Proof : We use Theorem 2.22 to prove this $SBdy(A_N \vee B_N) = SC.(A_N \vee B_N) \wedge SC.(1-(A_N \vee B_N)) = SC.(A_N \vee B_N) \wedge (SC.(1 - A_N) \wedge (1 - B_N)) \leq (SC.(A_N) \vee SC.(B_N)) \wedge (SC.(1 - A_N) \wedge SC.(1 - B_N)) \leq (SC.(A_N) \wedge SC.(1 - A_N)) \vee SC.(B_N) \wedge SC.(1 - B_N) = SBdy(A_N) \vee SBdy(B_N)$. Hence the proposition.

The reverse inequality are not true as shown in the following example.

Example 4.8. [17] Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.8}, b_{0.8}\rangle, \langle a_{0.2}, b_{0.1}\rangle, \}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed set of T_N is $1 - T_N = \{0, 1, \langle a_{0.2}, b_{0.2}\rangle, \langle a_{0.8}, b_{0.9}\rangle, \}$. Let $A_N = \{a_{0.4}, b_{0.1}\}$ and $B_N = \{a_{0.3}, b_{0.4}\}$.

Then computation, we see that $SBdy(A_N) = \{a_{0.4}, b_{0.3}\}$ and $SBdy(B_N) = \{a_{0.3}, b_{0.6}\}$. Now $A_N \vee B_N = \{a_{0.4}, b_{0.4}\}$ and $SBdy(A_N \vee B_N) = \{a_{0.4}, b_{0.5}\}$. This gives that $SBdy(A_N) \vee SBdy(B_N) = \{a_{0.4}, b_{0.6}\} \not\subset SBdy(A_N \vee B_N) = \{a_{0.4}, b_{0.5}\}$.

The following example shows that $SBdy(A_N \wedge B_N) \not\subset SBdy(A_N) \wedge SBdy(B_N)$ and $SBdy(A_N) \wedge SBdy(B_N) \not\subset SBdy(A_N \wedge B_N)$.

Example 4.9. [18] Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.8}, b_{0.6}\rangle, \langle a_{0.2}, b_{0.3}\rangle, \}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed set of T_N is $1 - T_N = \{0, 1, \langle a_{0.2}, b_{0.4}\rangle, \langle a_{0.8}, b_{0.7}\rangle, \}$. Let $A_N = \{a_{0.3}, b_{0.4}\}$ and $B_N = \{a_{0.6}, b_{0.2}\}$.

Then the calculations give $SBdy(A_N) = \{a_{0.3}, b_{0.5}\}$ and $SBdy(B_N) = \{a_{0.5}, b_{0.4}\}$. Now $A_N \wedge B_N = \{a_{0.3}, b_{0.2}\}$ and $SBdy(A_N \wedge B_N) = \{a_{0.4}, b_{0.3}\}$. This gives that $SBdy(A_N) \wedge SBdy(B_N) = \{a_{0.3}, b_{0.4}\} \not\subset SBdy(A_N \wedge B_N) = \{a_{0.4}, b_{0.3}\}$. and $SBdy(A_N \wedge B_N) \not\subset SBdy(A_N) \wedge SBdy(B_N)$.

Proposition 4.10. *For any fuzzy neutrosophic sets A_N and B_N in an fuzzy neutro- sophic topological space (X_N, T_N) then $SBdy(A_N \wedge B_N) \leq (SBdy(A_N)) \wedge SC.(B_N)) \vee (SBdy(B_N) \wedge SC.(A_N))$.*

Proof : We use Theorem 2.22 to prove this, $SBdy(A_N \wedge B_N) = SC.(A_N \wedge B_N) \wedge SC.(1-(A_N \wedge B_N)) = SC.(A_N \wedge B_N) \wedge (SC.(1 - A_N) \vee (1 - B_N)) \leq (SC.(A_N) \wedge SC.(B_N)) \wedge (SC.(1 - A_N) \vee SC.(1 - B_N)) = (SC.(A_N) \wedge SC.(B_N) \wedge SC.(1 - A_N)) \vee (SC.(A_N) \wedge SC.(B_N) \wedge SC.(1 - B_N)) = (SBdy(A_N) \wedge SC.(B_N)) \vee (SBdy(B_N) \wedge SC.(A_N))$. Hence proved.

Remark 4.11. The reverse inequality in the above proposition not true as shown by the following example.

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Example 4.12. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.3}, b_{0.2}\rangle, \langle a_{0.6}, b_{0.8}\rangle, \}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed sets of T_N is $1 - T_N = \{0, 1, \langle a_{0.7}, b_{0.8}\rangle, \langle a_{0.4}, b_{0.2}\rangle, \}$. Let $A_N = \{a_{0.4}, b_{0.6}\}$ and $B_N = \{a_{0.5}, b_{0.3}\}$. Then the calculations, we see that $SBdy(A_N) = \{a_{0.5}, b_{0.4}\}, SC.(A_N) = \{a_{0.5}, b_{0.6}\}, SC.(B_N) = \{a_{0.5}, b_{0.4}\}$ and $SBdy(B_N) = \{a_{0.5}, b_{0.4}\}$. Now $A_N \wedge B_N = \{a_{0.4}, b_{0.3}\}$ and $SBdy(A_N \wedge B_N) = \{a_{0.4}, b_{0.4}\}$. This gives that $(SBdy(A_N) \wedge SBdy(B_N)) \vee (SBdy(B_N) \wedge SC.(A_N)) = \{a_{0.5}, b_{0.4}\} \not\subset SBdy(A_N \wedge B_N) = \{a_{0.4}, b_{0.4}\}$.

Proposition 4.13. *For any fuzzy neutrosophic set A_N in an fuzzy neutrosophic topological space (X_N, T_N) then*

(1) $SBdy(SBdy(A_N)) \leq SBdy(A_N)$.
(2) $SBdy(SBdy(SBdy(A_N))) \leq SBdy(SBdy(A_N))$.

Proof : $SBdy(SBdy(A_N)) = SC.(SBdy(A_N)) \wedge SC.(1 - SBdy(A_N)) \leq SC.(SBdy(A_N)) = SC.(SC.(A_N) \wedge SC.(1 - A_N)) = SC.(SC.(A_N)) \wedge SC.(SC.(1 - A_N)) = SC.(A_N) \wedge SC.(1 - A_N) = SBdy(A_N)$. This proves (1).

$SBdy(SBdy(SBdy(A_N))) = SC.(SBdy(SBdy(A_N))) \wedge SC.(Bdy(1 - SBdy(A_N))) = SBdy(SBdy(A_N)) \wedge SC.(SBdy(1 - SBdy(A_N))) \leq SBdy(SBdy(A_N))$. Hence proved (2).

Remark 4.14. The reverse inequality in the above proposition are not true as shown by the following example.

Example 4.15. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.3}, b_{0.2}\rangle, \langle a_{0.6}, b_{0.8}\rangle, \}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed sets of T_N is $1 - T_N = \{0, 1, \langle a_{0.7}, b_{0.8}\rangle, \langle a_{0.4}, b_{0.2}\rangle\}$. Let $A_N = \{a_{0.4}, b_{0.6}\}$. Then calculations gives $SBdy(A_N) = \{a_{0.5}, b_{0.7}\} \not\subset SBdy(SBdy(A_N)) = \{a_{0.5}, b_{0.6}\}$. It follows that $SBdy(SBdy(A_N)) = \{a_{0.5}, b_{0.6}\} \not\subset SBdy(SBdy(SBdy(A_N))) = \{a_{0.5}, b_{0.5}\}$.

5. Conclusion

In this paper, the concept of fuzzy neutrosophic boundary and fuzzy neutrosophic semi- boundary deals with fuzzy neutrosophic open, fuzzy neutrosophic closed, fuzzy neutrosophic semi-open, fuzzy neutrosophic semi-closed, fuzzy neutrosophic semi-interior, fuzzy neutrosophic semi-closure are introduced. Further, we examine certain characterizations and examples are discussed the concept of fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary. In addition, we investigate their interrelations between fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary on fuzzy neutrosophic topological spaces.

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