Group Message Prioritization Using Circular Bipolar Complex Dual Valued Fuzzy Linguistic Sets and Frank Aggregation Operators

M. Kaviyarasu[1], J. Angel[1], Prasanta Kumar Raut[2], Mana Donganont[3,*] , Said Broumi[4,5]
[1]Department of Mathematics, Vel Tech Rangarajan Dr Sagunthala R & D Institute of Science and Technology, Chennai, Avadi-600062, India
[2]Department of mathematics, Trident Academy of Technology, Bhubaneswar, Odisha, India
[3]Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand
[4] Department of Mathematics SIMATS Engineering ,Saveetha Institute of Medical and Technical Sciences Thandalam Chennai, Tamilnadu, India
[5]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University of Hassan II, Casablanca, Morocco
Emails: kavitamilm@gamil.com; angeljohn1206@gmail.com; prasantaraut95@gmail.com; mana.do@up.ac.th;broumisaid78@gmail.com

Abstract

This paper introduces a novel extension of the multi-attributive border approximation area comparison (MABAC) method based on circular bipolar complex dual valued fuzzy uncertain linguistic sets (CBCDVFULSs) using Frank power aggregation operators. In order to effectively integrate aspects of fuzzy set theory, bipolarity, complex-valued, and uncertain linguistic information, this paper presents a novel framework based on CBCDVFULSs. Frank power aggregation operators is used specifically for CBCDVFULSs in order to handle and aggregate such complex data. These operators maintain the circular and bipolar properties of the fuzzy linguistic data by utilizing the adaptability of Frank t-norms ($\mathcal{FTN}$) and t-conorms ($\mathcal{FTCN}$). In contrast to current approaches, the suggested method's superior handling of complex uncertain linguistic environments, flexibility, and applicability are demonstrated through a numerical example. A group message prioritization system for WhatsApp that involve deciding on the priority under complex, uncertain, and bipolar linguistic evaluations is used to demonstrate the efficacy of the suggested approach.

Keywords: Mlti-attributive border approximation area comparison; Frank power aggregation operator; Frank t-norms ($\mathcal{FTN}$) and t-conorms ($\mathcal{FTCN}$); Complex uncertain linguistic environments

1 Introduction

According to Zadeh's[1] introduction of classical fuzzy set (FS) theory, each membership is defined by a single degree. Atanassov[2] developed the idea of intuitionistic fuzzy sets (IFSs), which handle uncertainty more flexibly by including both membership and non-membership degrees. By adding more parameters to conventional fuzzy models that more fully capture uncertainty, T-spherical fuzzy sets provide a more expressive framework. Using this paradigm, Abid et al.[3] suggested new similarity metrics. Improved modeling of imprecise time-cost trade-offs in project management is made possible by trapezoidal intuitionistic fuzzy sets. Using this idea, Kasimoglu et al.[4] developed an optimization model that successfully crashes projects with limited funds under uncertainty. Under ambiguous and uncertain circumstances, generalized intuitionistic fuzzy soft sets. A novel theoretical model was presented by Khan et al.,[5] it is useful in a decision support system framework. A key component of unsupervised learning and soft data partitioning is fuzzy clustering. Ruspini et al.,[6] who also highlight significant advancements and turning points in the discipline. When making decisions, interval-valued IFS provide a better way to convey hesitancy and uncertainty. Under this framework, Zheng and Dong[7] presented a novel MADM method that makes use of bidirectional projection and entropy weights.

1.1 Complex Fuzzy Set

As an expansion of classical FSs, Complex Fuzzy Sets (CFSs) include a phase term in the conventional membership degree. Because of this, CFSs may be used to simulate periodic or oscillatory uncertainty, which makes them ideal for applications requiring ambiguous, dynamic, or time-dependent data. Compared to traditional fuzzy techniques, CFSs offer additional information by expressing membership grades as complex integers. By expanding conventional FSs with a complex-valued function that incorporates both magnitude and phase, Ramot et al.[8] developed CFSs. This breakthrough made it possible to simulate oscillatory and periodic uncertainty, which set the stage for future developments in fuzzy logic. Complex IFSs, which has values in the complex domain, were presented by Alkouri and Salleh[9] as an extension of CFSs. This approach improves the capacity to communicate hesitancy and disagreement in ambiguous situations. Ali et al.[10] combined the flexibility of T-spherical fuzzy sets with the complex domain and $\mathcal{FTN}$. This method works well for multifaceted and highly uncertain decision-making problems. A powerful aggregation technique for MADM issues, were introduced by Azeem et al.[11] Their model's actual implementation in hostel site selection served as validation, demonstrating its applicability in the real world.

1.2 Bipolar Fuzzy Set

Zhang[12] created a framework for representing both positive and negative information by being the first to propose the bipolar fuzzy sets(BFSs) to multi-agent decision-making systems and cognitive modeling. Zhang[13] built on bipolar fuzzy logic by introducing a transformative framework that connects BFS to fuzzy cryptography and quantum intelligence, theoretically establishing the foundation for intelligent, information-preserving systems. For stock portfolio selection, Sharma[14] suggested combining a hybrid optimization technique with a trapezoidal bipolar fuzzy VIKOR model. Decision-making under several competing criteria was improved by this method. Using novel entropy-based distance measurements and Einstein aggregation, Riaz et al.[15] presented a cubic bipolar fuzzy VIKOR technique to aid in renewable energy planning decision-making. Mahmood and Rehman[16] created a formal framework for bipolar complex fuzzy sets and used it to similarity metrics.

1.3 Circular Fuzzy Set

In order to improve the modeling of uncertainty in problems involving periodic, angular, or cyclic information, circular fuzzy sets were introduced. Circular fuzzy sets, which frequently use complex numbers on the unit circle, represent membership values in a circular or phase-based format, in contrast to classical fuzzy sets, which depend on real-valued membership functions. Circular fuzzy models are particularly helpful in domains like signal processing, pattern recognition, and circular decision-making because of this extension, which makes it possible to handle direction dependent uncertainty. The concept has since been extended to circular intuitionistic fuzzy sets (CIFSs), further enriching their expressive capacity. For CIFSs, Khan et al.[17] proposed new divergence metrics that accurately capture angular and periodic uncertainty. Their approach was used in real-world decision-making situations with cyclic or circular data. Chen[18] suggested a CIFSs assessment technique that enhances MCDM by utilizing distance from an average answer and also used in modeling angular or periodic uncertainty in intelligent decision systems, this method works very well. By CIFSs, which express membership and non-membership values in a circular (phase-based) domain, Atanassov[19] expanded upon the conventional intuitionistic fuzzy set framework. In complex decision environments, this model improves the management of cyclic or periodic uncertainty.

1.4 Uncertain Linguistic Fuzzy

A crucial component of MADM, particularly in group decision-making settings, is managing linguistic information uncertainty. By developing aggregation procedures for ambiguous language data, Xu[20] made it possible to simulate subjective human judgments more realistically. In order to capture both positive and negative assessments with varying degrees of uncertainty, academics have developed increasingly complex frameworks. This concept was expanded upon by Lan et al.[23] and Gao et al.,[21] who used techniques such as CODAS to assess practical issues like network security and risk assessment abroad. By adding phase and magnitude in uncertain linguistic settings, Mahmood et al.[22] significantly enhanced the expressive potential by integrating bipolar complex fuzzy uncertain linguistic sets into decision-making models. These advancements show a clear path for MADM frameworks that are more realistic, adaptable, and resilient in complex and uncertain contexts.

In this for decision-making that addresses complicated multi-criteria situations by combining the Frank power aggregation operator with CBCDVFULS. Bipolar, complex-valued uncertainty, the cyclical character of linguistic evaluations, and the reluctance inherent in human thinking are all captured by the suggested method. We introduce a group message prioritization, evaluating five options based on five competing criteria, to illustrate the model's usefulness. To deal with the special nature of CBCDVFUL data, a new set of Frank aggregation operators is created, enabling flexible and practical expert opinion aggregation. Additionally, a methodical algorithm is developed within the MABAC framework to rate the options. To confirm the usefulness, resilience, and interpretability of the suggested model in maintenance prioritization scenarios, a case study with real-world inspiration is included.

1.5 Research gap

The steady ranking procedure and mathematical simplicity of the MABAC approach have made it an effective tool in MADM. However, typical MABAC solutions are generally focused on real-valued fuzzy or crisp data, restricting their application in contexts requiring complicated, ambiguous, or contradicting information. Although the method's expressiveness has been improved by modifications that include intuitionistic, Pythagorean, and uncertain linguistic sets, these models are still unable to handle bipolar assessments and phase-based uncertainty that are present in many real-world choice situations. Exciting paths towards deeper information modeling are provided by recent advances in uncertain linguistic environments, CFS, and BFS. But there is still much to learn about how to incorporate these into the MABAC architecture. Specifically, the literature that combines CBCDVFULSs with MABAC to handle linguistic vagueness, periodic or direction-sensitive uncertainties, and dual-polarity evaluations at the same time is lacking.

1.6 Motivation

While CFSs combine phase and magnitude in the complex domain, allowing the representation of larger uncertainty structures, bipolar fuzzy models take into account dual-sided opinions (e.g., pros and cons, acceptance and rejection). Furthermore, in situations when precise numerical input is either unavailable or prohibitive, imprecise language variables offer a natural interface for human-centered assessments. The integration of all these elements circularity, bipolarity, complex-valued representation, dual valued, and linguistic uncertainty into a single framework for decision-making is still lacking, notwithstanding the individual benefits of these additions. In order to fully capture the range of human like, uncertain, periodic, and dual-nature evaluations, the CBCDVFULS model was developed. In situations where standard models are inadequate, the suggested model helps decision-makers to more effectively articulate and synthesize complicated judgments. The precision, adaptability, and interpretability of results in fields like expert systems, engineering design, social sciences, healthcare prioritization, and intelligent systems can be greatly improved by incorporating this sophisticated information representation into decision-making tools.

1.7 Advantages of CBCDVFULSs

Compared to other fuzzy models, the CBCDVFULS has a number of benefits. First, it combines five essential dimensions to capture a more comprehensive form of uncertainty namely bipolarity (dual-sided opinions), intuitionistic fuzziness (membership and non-membership), complex-valued representation (amplitude and phase), circularity (phase-based), and uncertain linguistic information (natural language inputs). More accurate modeling of human judgment is made possible by this integration, especially in fields where input is direction-sensitive or periodic. Furthermore, its complex and circular elements make it ideal for depicting rotational or time-dependent phenomena, and its bipolar and dual valued elements guarantee that both optimistic and pessimistic viewpoints are taken into consideration. In comparison to conventional fuzzy or linguistic approaches, the CBCDVFULS model offers a richer, more adaptable, and human-like decision-making environment.

1.8 Contribution

In the field of intelligent decision-making under high-order uncertainty, this work offers a number of significant advances. In order to provide a single, all-inclusive decision-making model, it first presents a CBCDVFULS framework that integrates circular, bipolar, complex, dual valued, and uncertain linguistic aspects. Second, the paper creates new Frank aggregation operators specifically for the CBCDVFULS context in order to process information in this enriched environment. By modeling different levels of interaction across criteria, these operators take use of the parametric flexibility of the $\mathcal{FTN}$, improving the aggregation process's accuracy and adaptability. Third, the study incorporates these operators into the framework of a well known MADM technique (like MABAC), allowing for a reliable and understandable ranking of options in challenging evaluation issues.

2 Preliminaries

The objective is to examine current concepts in order to assess the suggested hypothesis. Using the $\mathcal{FTCN}$ and $\mathcal{FTN}$ to create aggregation operators, are chosen for this. Additionally, we go over the $\mathcal{PA}$ and $\mathcal{PG}$ operator approach, which is used to aggregate data into a singleton set. In order to know new methods, we finally go through the BCFLSs methodology and its regulations.

Definition 2.1. [29,30] Any two positive numbers $\mathfrak{B}_1, \mathfrak{B}_2 \in [0,1]$, then $\mathcal{FTCN}$ and $\mathcal{FTN}$ is

$$\mathcal{FTN}^{\kappa}(\mathfrak{B}_1, \mathfrak{B}_2) = log_{\kappa}\Big(1 + \frac{(\kappa^{\mathfrak{B}_1} - 1)(\kappa^{\mathfrak{B}_2} - 1)}{(\kappa - 1)}\Big)$$

$$\mathcal{FTCN}^{\kappa}(\mathfrak{B}_1, \mathfrak{B}_2) = 1 - log_{\kappa}\Big(1 + \frac{(\kappa^{1-\mathfrak{B}_1} - 1)(\kappa^{1-\mathfrak{B}_2} - 1)}{(\kappa - 1)}\Big)$$

where $\kappa \geqslant 2$.

Definition 2.2. [1] For any set of positive integers $\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}$, the $\mathcal{PG}$ and $\mathcal{PA}$ operator's distance measure are defined as

$$\mathcal{PA}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \sum_{\iota=1}^{\mathfrak{R}} \frac{(1 + \aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1 + \aleph(\mathfrak{B}_\iota))}\mathfrak{B}_\iota$$

$$\mathcal{PG}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \prod_{\iota=1}^{\mathfrak{R}} \mathfrak{B}_\iota \frac{(1 + \aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1 + \aleph(\mathfrak{B}_\iota))}\mathfrak{B}_\iota$$

with $\aleph(\mathfrak{B}_\iota) = \sum_{i\neq\iota=1}^{\mathfrak{R}} S(\mathfrak{B}_{\mathrm{i}}, \mathfrak{B}_\iota)$, where $S(\mathfrak{B}_{\mathrm{i}}, \mathfrak{B}_\iota) = 1 - D(\mathfrak{B}_{\mathrm{i}}, \mathfrak{B}_\iota)$ with some conditions,

(1) $S(\mathfrak{B}_{\mathrm{i}}, \mathfrak{B}_\iota) = S(\mathfrak{B}_\iota, \mathfrak{B}_{\mathrm{i}})$

(2) $S(\mathfrak{B}_{\mathrm{i}}, \mathfrak{B}_\iota) \in [0,1]$

(3) $S(\mathfrak{B}_{\mathrm{i}}, \mathfrak{B}_\iota) > S(\mathfrak{B}_{\mathrm{k}}, \mathfrak{B}_{\mathrm{l}})$, then $D(\mathfrak{B}_{\mathrm{i}}, \mathfrak{B}_\iota) < D(\mathfrak{B}_{\mathrm{k}}, \mathfrak{B}_{\mathrm{l}})$

Definition 2.3. [31] For any set $\mathcal{V}$, BCFULS $\mathfrak{B}_{\aleph}$ is the form,

$$\mathfrak{B}_{\aleph} = \Big\{\Big([\mathfrak{T}_{I(\mathfrak{X}_o)}, \mathfrak{F}_{I(\mathfrak{X}_o)}], m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + im^{\mathfrak{I}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o), n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + in^{\mathfrak{I}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o)\Big)\backslash \mathfrak{X}_o \in \mathcal{V}\Big\}$$

where the term $[\mathfrak{T}_{I(\mathfrak{X}_o)}, \mathfrak{F}_{I(\mathfrak{X}_o)}], m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + im^{\mathfrak{I}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o), n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + in^{\mathfrak{I}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o)$ represents the linguistic term set,complex valued membership and non membership set, such as $[\mathfrak{T}, \mathfrak{F}] = \{\mathfrak{T}_{\iota(\mathfrak{X}_o)}, \mathfrak{F}_{\iota(\mathfrak{X}_o)} : \iota = 1, 2, \dots, \nu\}$ $m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph}}, m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph}} \in [0, 1]$ and $n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph}}, n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph}} \in [-1, 0]$. Then, the BCFLN is $\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + im^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}, n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + in^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\Big), \iota = 1, 2, \dots, \mathfrak{R}$.

Definition 2.4. [31] For any two BCFULN $\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + im^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}, n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + in^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\Big), \iota = 1, 2.$ the following operators are defined:

$$\mathfrak{B}_{\aleph_1} \oplus \mathfrak{B}_{\aleph_2} = \begin{pmatrix} [\mathfrak{T}_{\nu(\frac{I_1}{\nu}+\frac{I_2}{\nu}-\frac{I_1}{\nu}\frac{I_2}{\nu})}, \mathfrak{F}_{\nu(\frac{I_1}{\nu}+\frac{I_2}{\nu}-\frac{I_1}{\nu}\frac{I_2}{\nu})}], \Big(m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_1}} + m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_2}} - m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_1}} m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_2}}\Big) \\ +i\Big(m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_1}} + m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_2}} - m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_1}} m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_2}}\Big), -\Big(n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_1}} n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(-\Big(n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_1}} n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_2}}\Big)\Big) \end{pmatrix}$$

$$\mathfrak{B}_{\aleph_1} \otimes \mathfrak{B}_{\aleph_2} = \begin{pmatrix} [\mathfrak{T}_{\nu(\frac{I_1}{\nu}\frac{I_2}{\nu})}, \mathfrak{F}_{\nu(\frac{I_1}{\nu}\frac{I_2}{\nu})}], \Big(m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_1}} m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_1}} m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_1}} + n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_2}} + n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_1}} n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_1}} + n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_2}} + n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_1}} n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_2}}\Big) \end{pmatrix}$$

$$\dot{\mathfrak{A}}\mathfrak{B}_{\aleph_1} = \begin{pmatrix} [\mathfrak{T}_{\nu(1-(1-\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}, \mathfrak{F}_{\nu(1-(1-\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}], \Big(1 - (1 - m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) + i\Big(1 - (1 - m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), - \mid n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_1}} \mid^{\dot{\mathfrak{A}}} + i(- \mid n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_1}} \mid^{\dot{\mathfrak{A}}}) \end{pmatrix}$$

$$(\mathfrak{B}_{\aleph_1})^{\dot{\mathfrak{A}}} = \begin{pmatrix} [\mathfrak{T}_{\nu((\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}, \mathfrak{F}_{\nu((\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}], \Big((m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) + i\Big((m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), -1 + \Big(1 + n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_1}}\Big)^{\dot{\mathfrak{A}}} + i\Big(-1 + (1 + n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_1}}\Big)^{\dot{\mathfrak{A}}}\Big) \end{pmatrix}$$

Definition 2.5. [31] For any two BCFULNs $\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + im^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}, n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + in^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\Big), \iota = 1, 2.$

$$S(\mathfrak{B}_{\aleph_\iota}) = [\mathfrak{T}_{(\frac{I_\iota}{\nu})} + \mathfrak{F}_{(\frac{I_\iota}{\nu})}] * \frac{\Big(m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}} + n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\Big)}{2} \in [-\nu, \nu]$$

$$H(\mathfrak{B}_{\aleph_\iota}) = [\mathfrak{T}_{(\frac{I_\iota}{\nu})} + \mathfrak{F}_{(\frac{I_\iota}{\nu})}] * \frac{\Big(m^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + m^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}} - n^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} - n^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\Big)}{2} \in [\nu, \nu]$$

is the score & accuracy value.

3 Circular Bipolar Complex Dual Valued Fuzzy Uncertain Linguistic Sets

The definitions, notations, and aggregation processes that are essential to our MABAC method are presented in this section.

Definition 3.1. For any set $\mathcal{V}$, the BCFULS $\mathfrak{B}_{\aleph}$ is

$$\mathfrak{B}_{\aleph} = \left\{ \begin{array}{l} \Big([\mathfrak{T}_{I(\mathfrak{X}_o)}, \mathfrak{F}_{I(\mathfrak{X}_o)}], \mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + i\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o), \mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + i\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o), \\ \aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + i\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o), \aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + i\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o), R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + iR^{\mathfrak{I}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o)\Big)\backslash \mathfrak{X}_o \in \mathcal{V} \end{array} \right\}$$

where the term $[\mathfrak{T}_{I(\mathfrak{X}_o)}, \mathfrak{F}_{I(\mathfrak{X}_o)}], \mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + i\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o), \mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + i\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) and \aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + i\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o), \aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + i\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o)$ indicate uncertain linguistic set, positive (negative) complex membership value, and positive (negative) complex non membership value with a radius $R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o) + iR^{\mathfrak{I}}_{\mathfrak{B}_{\aleph}}(\mathfrak{X}_o)$ between both values,such as $[\mathfrak{T}, \mathfrak{F}] = \{\mathfrak{T}_{\iota(\mathfrak{X}_o)}, \mathfrak{F}_{\iota(\mathfrak{X}_o)} : \iota = 1, 2, \dots, \nu\}$. Additionally, the CBCDVFUL number (CBCDVFULN) represented as follows: $\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\Big) \iota = 1, 2, \dots, \mathfrak{R}$.

Definition 3.2. For any two CBCDVFULNs $\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\Re}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\Im}_{\mathfrak{B}_{\aleph_\iota}}\Big)\iota = 1, 2.$ we define the following operator

$$\mathfrak{B}_{\aleph_1} \oplus^{\text{tcn}} \mathfrak{B}_{\aleph_2} = \left(\begin{array}{l} [\mathfrak{T}_{\nu(\frac{I_1}{\nu}+\frac{I_2}{\nu}-\frac{I_1}{\nu}\frac{I_2}{\nu})}, \mathfrak{F}_{\nu(\frac{I_1}{\nu}+\frac{I_2}{\nu}-\frac{I_1}{\nu}\frac{I_2}{\nu})}], \\ \Big(\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}} + \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_2}} - \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}} + \mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_2}} - \mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_2}}\Big), \\ -\Big(\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(-\Big(\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_2}}\Big)\Big), \Big(\aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Re+}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Im+}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(\aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}} + \aleph^{\Re-}_{\mathfrak{B}_{\aleph_2}} + \aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Re-}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}} + \aleph^{\Im-}_{\mathfrak{B}_{\aleph_2}} + \aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Im-}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(R^{\Re}_{\mathfrak{B}_{\aleph_1}} + R^{\Re}_{\mathfrak{B}_{\aleph_2}} - R^{\Re}_{\mathfrak{B}_{\aleph_1}}R^{\Re}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(R^{\Im}_{\mathfrak{B}_{\aleph_1}} + R^{\Im}_{\mathfrak{B}_{\aleph_2}} - R^{\Im}_{\mathfrak{B}_{\aleph_1}}R^{\Im}_{\mathfrak{B}_{\aleph_2}}\Big) \end{array}\right)$$

$$\mathfrak{B}_{\aleph_1} \oplus^{\text{tn}} \mathfrak{B}_{\aleph_2} = \left(\begin{array}{l} [\mathfrak{T}_{\nu(\frac{I_1}{\nu}+\frac{I_2}{\nu}-\frac{I_1}{\nu}\frac{I_2}{\nu})}, \mathfrak{F}_{\nu(\frac{I_1}{\nu}+\frac{I_2}{\nu}-\frac{I_1}{\nu}\frac{I_2}{\nu})}], \\ \Big(\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}} + \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_2}} - \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}} + \mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_2}} - \mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_2}}\Big), \\ -\Big(\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(-\Big(\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_2}}\Big)\Big), \Big(\aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Re+}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Im+}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(\aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}} + \aleph^{\Re-}_{\mathfrak{B}_{\aleph_2}} + \aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Re-}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}} + \aleph^{\Im-}_{\mathfrak{B}_{\aleph_2}} + \aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Im-}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(R^{\Re}_{\mathfrak{B}_{\aleph_1}}R^{\Re}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(R^{\Im}_{\mathfrak{B}_{\aleph_1}}R^{\Im}_{\mathfrak{B}_{\aleph_2}}\Big) \end{array}\right)$$

$$\mathfrak{B}_{\aleph_1} \otimes^{\text{tcn}} \mathfrak{B}_{\aleph_2} = \left(\begin{array}{l} [\mathfrak{T}_{\nu(\frac{I_1}{\nu}\frac{I_2}{\nu})}, \mathfrak{F}_{\nu(\frac{I_1}{\nu}\frac{I_2}{\nu})}], \Big(\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}} + \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_2}} + \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}} + \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_2}} + \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(\aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}} + \aleph^{\Re+}_{\mathfrak{B}_{\aleph_2}} + \aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Re+}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}} + \aleph^{\Im+}_{\mathfrak{B}_{\aleph_2}} + \aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Im+}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(\aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Re-}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Im-}_{\mathfrak{B}_{\aleph_2}}\Big), \Big(R^{\Re}_{\mathfrak{B}_{\aleph_1}}R^{\Re}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(R^{\Im}_{\mathfrak{B}_{\aleph_1}}R^{\Im}_{\mathfrak{B}_{\aleph_2}}\Big) \end{array}\right)$$

$$\mathfrak{B}_{\aleph_1} \otimes^{\text{tn}} \mathfrak{B}_{\aleph_2} = \left(\begin{array}{l} [\mathfrak{T}_{\nu(\frac{I_1}{\nu}\frac{I_2}{\nu})}, \mathfrak{F}_{\nu(\frac{I_1}{\nu}\frac{I_2}{\nu})}], \Big(\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}} + \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_2}} + \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}} + \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_2}} + \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}}\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(\aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}} + \aleph^{\Re+}_{\mathfrak{B}_{\aleph_2}} - \aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Re+}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(\aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}} + \aleph^{\Im+}_{\mathfrak{B}_{\aleph_2}} - \aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Im+}_{\mathfrak{B}_{\aleph_2}}\Big), \\ \Big(-\aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Re-}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(-\Big(\aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}}\aleph^{\Im-}_{\mathfrak{B}_{\aleph_2}}\Big)\Big), \\ \Big(R^{\Re}_{\mathfrak{B}_{\aleph_1}} + R^{\Re}_{\mathfrak{B}_{\aleph_2}} - R^{\Re}_{\mathfrak{B}_{\aleph_1}}R^{\Re}_{\mathfrak{B}_{\aleph_2}}\Big) + i\Big(R^{\Im}_{\mathfrak{B}_{\aleph_1}} + R^{\Im}_{\mathfrak{B}_{\aleph_2}} - R^{\Im}_{\mathfrak{B}_{\aleph_1}}R^{\Im}_{\mathfrak{B}_{\aleph_2}}\Big) \end{array}\right)$$

$$\dot{\mathfrak{A}}_{\text{tcn}}\mathfrak{B}_{\aleph_1} = \left(\begin{array}{l} [\mathfrak{T}_{\nu(1-(1-\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}, \mathfrak{F}_{\nu(1-(1-\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}], \\ \Big(1-(1-\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) + i\Big(1-(1-\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), -\mid \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}} \mid^{\dot{\mathfrak{A}}} + i(-\mid \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}} \mid^{\dot{\mathfrak{A}}}), \\ \Big((\aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) + i\Big((\aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), -1 + \Big(1+\aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}}\Big)^{\dot{\mathfrak{A}}} + i\Big(-1+(1+\aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}}\Big)^{\dot{\mathfrak{A}}}\Big), \\ \Big(1-(1-R^{\Re}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), \Big(1-(1-R^{\Im}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) \end{array}\right)$$

$$\dot{\mathfrak{A}}_{\text{tn}}\mathfrak{B}_{\aleph_1} = \left(\begin{array}{l} [\mathfrak{T}_{\nu(1-(1-\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}, \mathfrak{F}_{\nu(1-(1-\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}], \\ \Big(1-(1-\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) + i\Big(1-(1-\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), -\mid \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}} \mid^{\dot{\mathfrak{A}}} + i(-\mid \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}} \mid^{\dot{\mathfrak{A}}}), \\ \Big((\aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) + i\Big((\aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), -1 + \Big(1+\aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}}\Big)^{\dot{\mathfrak{A}}} + i\Big(-1+(1+\aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}}\Big)^{\dot{\mathfrak{A}}}\Big), \\ \Big((R^{\Re}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), \Big((R^{\Im}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) \end{array}\right)$$

$$(\mathfrak{B}_{\aleph_1})^{\dot{\mathfrak{A}}_{\text{tcn}}} = \left(\begin{array}{l} [\mathfrak{T}_{\nu(1-(1-\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}, \mathfrak{F}_{\nu(1-(1-\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}], \\ \Big((\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) + i\Big((\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), -1 + \Big(1+\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}}\Big)^{\dot{\mathfrak{A}}} + i\Big(-1+(1+\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}}\Big)^{\dot{\mathfrak{A}}}\Big), \\ \Big(1-(1-\aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) + i\Big(1-(1-\aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), -\mid \aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}} \mid^{\dot{\mathfrak{A}}} + i(-\mid \aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}} \mid^{\dot{\mathfrak{A}}}), \\ \Big(1-(1-R^{\Re}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big), \Big(1-(1-R^{\Im}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\Big) \end{array}\right)$$

$$(\mathfrak{B}_{\aleph_1})^{\dot{\mathfrak{A}}_{\text{tn}}} = \begin{pmatrix} [\mathfrak{T}_{\nu(1-(1-\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}, \mathfrak{F}_{\nu(1-(1-\frac{I_1}{\nu})^{\dot{\mathfrak{A}}})}], \\ \left((\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\right) + i\left((\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\right), -1 + \left(1 + \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}}\right)^{\dot{\mathfrak{A}}} + i\left(-1 + (1 + \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}}\right)^{\dot{\mathfrak{A}}}\right), \\ \left(1 - (1 - \aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\right) + i\left(1 - (1 - \aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\right), - \mid \aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}} \mid^{\dot{\mathfrak{A}}} + i(- \mid \aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}} \mid^{\dot{\mathfrak{A}}}), \\ \left((R^{\Re}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\right), \left((R^{\Im}_{\mathfrak{B}_{\aleph_1}})^{\dot{\mathfrak{A}}}\right) \end{pmatrix}$$

Definition 3.3. For any CBCDVFULNs$\mathfrak{B}_{\aleph_\iota} = \left([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\Re}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\Im}_{\mathfrak{B}_{\aleph_\iota}}\right)$ then the score value $S(\mathfrak{B}_{\aleph_\iota})$ and accuracy value $H(\mathfrak{B}_{\aleph_\iota})$ is

$$S(\mathfrak{B}_{\aleph_\iota}) = [\mathfrak{T}_{(\frac{I_\iota}{\nu})} + \mathfrak{F}_{(\frac{I_\iota}{\nu})}] * \frac{\left(\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + \mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}} + \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}\right)}{4} * \frac{R^{\Re}_{\mathfrak{B}_{\aleph_\iota}} - R^{\Im}_{\mathfrak{B}_{\aleph_\iota}}}{2}$$

$$H(\mathfrak{B}_{\aleph_\iota}) = [\mathfrak{T}_{(\frac{I_\iota}{\nu})} + \mathfrak{F}_{(\frac{I_\iota}{\nu})}] * \frac{\left(\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + \mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}} - \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} - \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}} - \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} - \aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}\right)}{4} * \frac{R^{\Re}_{\mathfrak{B}_{\aleph_\iota}} + R^{\Im}_{\mathfrak{B}_{\aleph_\iota}}}{2}$$

When the radius function is zero, the following model will be taken into consideration:

$$S(\mathfrak{B}_{\aleph_\iota}) = [\mathfrak{T}_{(\frac{I_1}{\nu})} + \mathfrak{F}_{(\frac{I_1}{\nu})}] * \frac{\left(\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + \mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}} + \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}\right)}{4}$$

$$H(\mathfrak{B}_{\aleph_\iota}) = [\mathfrak{T}_{(\frac{I_1}{\nu})} + \mathfrak{F}_{(\frac{I_1}{\nu})}] * \frac{\left(\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + \mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}} - \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} - \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + \aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}} - \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} - \aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}\right)}{4}$$

Definition 3.4. For any two CBCDVFULNs $\mathfrak{B}_{\aleph_\iota} = \left([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\Re}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\Im}_{\mathfrak{B}_{\aleph_\iota}}\right) \iota = 1, 2,$we define

$$\mathfrak{B}_{\aleph_1} \oplus^{\text{tcn}} \mathfrak{B}_{\aleph_2} = \begin{pmatrix} \left[\mathfrak{T}_{\nu\left[1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)\left(\kappa^{1-\frac{I_2}{\nu}}-1\right)}{\kappa-1}\right)\right]}, \mathfrak{F}_{\nu\left[1-\log_\kappa\left(1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)\left(\kappa^{1-\frac{I_2}{\nu}}-1\right)}{\kappa-1}\right)\right]} \right], \\ \left(1 - \log_\kappa\left[1 + \frac{\left(\kappa^{1-\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(1 - \log_\kappa\left[1 + \frac{\left(\kappa^{1-\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(-\log_\kappa\left[1 + \frac{\left(\kappa^{1-\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(-\log_\kappa\left[1 + \frac{\left(\kappa^{1-\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(\log_\kappa\left[1 + \frac{\left(\kappa^{\aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{\aleph^{\Re+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(\log_\kappa\left[1 + \frac{\left(\kappa^{\aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{\aleph^{\Im+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(-1 + \log_\kappa\left[1 + \frac{\left(\kappa^{1+\aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1+\aleph^{\Re-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(-1 + \log_\kappa\left[1 + \frac{\left(\kappa^{1+\aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1+\aleph^{\Im-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(1 - \log_\kappa\left[1 + \frac{\left(\kappa^{1-R^{\Re}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-R^{\Re}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(1 - \log_\kappa\left[1 + \frac{\left(\kappa^{1-R^{\Im}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-R^{\Im}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \end{pmatrix}$$

$$\mathfrak{B}_{\aleph_1} \oplus^{\text{tn}} \mathfrak{B}_{\aleph_2} = \left(\begin{array}{l} \left[\mathfrak{T}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)\left(\kappa^{1-\frac{I_2}{\nu}}-1\right)}{\kappa-1}\right]\right)}, \mathfrak{F}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)\left(\kappa^{1-\frac{I_2}{\nu}}-1\right)}{\kappa-1}\right]\right)} \right], \\ \left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(\log_\kappa\left[1+\frac{\left(\kappa^{\aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{\aleph^{\Re+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(\log_\kappa\left[1+\frac{\left(\kappa^{\aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{\aleph^{\Im+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1+\aleph^{\Re-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1+\aleph^{\Im-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(\log_\kappa\left[1+\frac{\left(\kappa^{R^{\Re}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{R^{\Re}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(\log_\kappa\left[1+\frac{\left(\kappa^{R^{\Im}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{R^{\Im}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \end{array} \right.$$

$$\mathfrak{B}_{\aleph_1} \otimes^{\text{tcn}} \mathfrak{B}_{\aleph_2} = \left(\begin{array}{l} \left[\mathfrak{T}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)\left(\kappa^{1-\frac{I_2}{\nu}}-1\right)}{\kappa-1}\right]\right)}, \mathfrak{F}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)\left(\kappa^{1-\frac{I_2}{\nu}}-1\right)}{\kappa-1}\right]\right)} \right], \\ \left(\log_\kappa\left[1+\frac{\left(\kappa^{\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{\mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(\log_\kappa\left[1+\frac{\left(\kappa^{\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1+\mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1+\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\Re+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\aleph^{\Re+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\Im+}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\aleph^{\Im+}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\Re-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\aleph^{\Re-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\Im-}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-\aleph^{\Im-}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \\ \left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-R^{\Re}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-R^{\Re}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right) + i\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-R^{\Im}_{\mathfrak{B}_{\aleph_1}}}-1\right)\left(\kappa^{1-R^{\Im}_{\mathfrak{B}_{\aleph_2}}}-1\right)}{\kappa-1}\right]\right), \end{array} \right)$$

$$
\mathfrak{B}_{\aleph_1} \otimes^{\mathrm{tn}} \mathfrak{B}_{\aleph_2} = \begin{pmatrix}
\left[\mathfrak{T}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)\left(\kappa^{1-\frac{I_2}{\nu}}-1\right)}{\kappa-1}\right]\right)}, \mathfrak{F}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)\left(\kappa^{1-\frac{I_2}{\nu}}-1\right)}{\kappa-1}\right]\right)} \right], \\
\left(\log_\kappa\left[1+\frac{\left(\kappa^{\mathcal{M}^{\Re+}_{\mathfrak{B}\aleph 1-1}}\right)\left(\kappa^{\mathcal{M}^{\Re+}_{\mathfrak{B}\aleph 2-1}}\right)}{\kappa-1}\right]\right) + i\left(\log_\kappa\left[1+\frac{\left(\kappa^{\mathcal{M}^{\Im+}_{\mathfrak{B}\aleph 1-1}}\right)\left(\kappa^{\mathcal{M}^{\Im+}_{\mathfrak{B}\aleph 2-1}}\right)}{\kappa-1}\right]\right), \\
\left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\mathcal{M}^{\Re-}_{\mathfrak{B}\aleph 1-1}}\right)\left(\kappa^{1+\mathcal{M}^{\Re-}_{\mathfrak{B}\aleph 2-1}}\right)}{\kappa-1}\right]\right) + i\left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\mathcal{M}^{\Im-}_{\mathfrak{B}\aleph 1-1}}\right)\left(\kappa^{1+\mathcal{M}^{\Im-}_{\mathfrak{B}\aleph 2-1}}\right)}{\kappa-1}\right]\right), \\
\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\Re+}_{\mathfrak{B}\aleph 1-1}}\right)\left(\kappa^{1-\aleph^{\Re+}_{\mathfrak{B}\aleph 2-1}}\right)}{\kappa-1}\right]\right) + i\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\Im+}_{\mathfrak{B}\aleph 1-1}}\right)\left(\kappa^{1-\aleph^{\Im+}_{\mathfrak{B}\aleph 2-1}}\right)}{\kappa-1}\right]\right), \\
\left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\Re-}_{\mathfrak{B}\aleph 1-1}}\right)\left(\kappa^{1-\aleph^{\Re-}_{\mathfrak{B}\aleph 2-1}}\right)}{\kappa-1}\right]\right) + i\left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\Im-}_{\mathfrak{B}\aleph 1-1}}\right)\left(\kappa^{1-\aleph^{\Im-}_{\mathfrak{B}\aleph 2-1}}\right)}{\kappa-1}\right]\right), \\
\left(\log_\kappa\left[1+\frac{\left(\kappa^{R^{\Re}_{\mathfrak{B}\aleph 1-1}}\right)\left(\kappa^{R^{\Re}_{\mathfrak{B}\aleph 2-1}}\right)}{\kappa-1}\right]\right) + i\left(\log_\kappa\left[1+\frac{\left(\kappa^{R^{\Im}_{\mathfrak{B}\aleph 1-1}}\right)\left(\kappa^{R^{\Im}_{\mathfrak{B}\aleph 2-1}}\right)}{\kappa-1}\right]\right),
\end{pmatrix}
$$

$$
\dot{\mathfrak{A}}_{\mathrm{tcn}}\mathfrak{B}_{\aleph_1} = \begin{pmatrix}
\left[\mathfrak{T}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right)}, \mathfrak{F}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right)} \right], \\
\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\Re+}_{\mathfrak{B}\aleph 1-1}}\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right) + i\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\Im+}_{\mathfrak{B}\aleph 1-1}}\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right), \\
\left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\Re-}_{\mathfrak{B}\aleph 1-1}}\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right) + i\left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\Im-}_{\mathfrak{B}\aleph 1-1}}\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right), \\
\left(\log_\kappa\left[1+\frac{\left(\kappa^{\aleph^{\Re+}_{\mathfrak{B}\aleph 1-1}}\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right) + i\left(\log_\kappa\left[1+\frac{\left(\kappa^{\aleph^{\Im+}_{\mathfrak{B}\aleph 1-1}}\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right), \\
\left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\aleph^{\Re-}_{\mathfrak{B}\aleph 1-1}}\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}\right]\right) + i\left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\aleph^{\Im-}_{\mathfrak{B}\aleph 1-1}}\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right), \\
\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-R^{\Re}_{\mathfrak{B}\aleph 1-1}}\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right) + i\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-R^{\Im}_{\mathfrak{B}\aleph 1-1}}\right)^{\dot{\mathfrak{A}}_{\mathrm{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\mathrm{tcn}-1}}}\right]\right),
\end{pmatrix} \quad (1)
$$

$$\dot{\mathfrak{A}}_{\text{tn}}\mathfrak{B}_{\aleph_1}=\left(\begin{array}{l}\left[\mathfrak{T}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)},\mathfrak{F}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)}\right],\\ \left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)+i\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right),\\ \left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)+i\left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right),\\ \left(\log_\kappa\left[1+\frac{\left(\kappa^{\aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)+i\left(\log_\kappa\left[1+\frac{\left(\kappa^{\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right),\\ \left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn-1}}}}\right]\right)+i\left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right),\\ \left(\log_\kappa\left[1+\frac{\left(\kappa^{1-R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)+i\left(\log_\kappa\left[1+\frac{\left(\kappa^{1-R^{\mathfrak{I}}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right),\end{array}\right) \tag{2}$$

$$\mathfrak{B}^{\dot{\mathfrak{A}}_{\text{tcn}}}_{\aleph_1}=\left(\begin{array}{l}\left[\mathfrak{T}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)},\mathfrak{F}_{\nu\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)}\right],\\ \left(\log_\kappa\left[1+\frac{\left(\kappa^{\mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)+i\left(\log_\kappa\left[1+\frac{\left(\kappa^{\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right),\\ \left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn-1}}}}\right]\right)+i\left(-1+\log_\kappa\left[1+\frac{\left(\kappa^{1+\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right),\\ \left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)+i\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right),\\ \left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)+i\left(-\log_\kappa\left[1+\frac{\left(\kappa^{1-\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right),\\ \left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)+i\left(1-\log_\kappa\left[1+\frac{\left(\kappa^{1-R^{\mathfrak{I}}_{\mathfrak{B}_{\aleph 1-1}}}\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right),\end{array}\right) \tag{3}$$

$$
\mathfrak{B}_{\aleph_1}^{\dot{\mathfrak{A}}_{\text{tn}}} = \begin{pmatrix}
\left[\mathfrak{T}\,\nu\left(1-\log_{\kappa}\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right), \mathfrak{F}\,\nu\left(1-\log_{\kappa}\left[1+\frac{\left(\kappa^{1-\frac{I_1}{\nu}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}-1}}\right]\right)\right], \\
\left(\log_{\kappa}\left[1+\frac{\left(\kappa^{\mathcal{M}_{\mathfrak{B}_{\aleph 1}}^{\mathfrak{R}+}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}-1}}}\right]\right)+i\left(\log_{\kappa}\left[1+\frac{\left(\kappa^{\mathcal{M}_{\mathfrak{B}_{\aleph 1}}^{\mathfrak{I}+}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}-1}}}\right]\right), \\
\left(-1+\log_{\kappa}\left[1+\frac{\left(\kappa^{1+\mathcal{M}_{\mathfrak{B}_{\aleph 1}}^{\mathfrak{R}-}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}-1}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}}}}\right]\right)+i\left(-1+\log_{\kappa}\left[1+\frac{\left(\kappa^{1+\mathcal{M}_{\mathfrak{B}_{\aleph 1}}^{\mathfrak{I}-}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}-1}}}\right]\right), \\
\left(1-\log_{\kappa}\left[1+\frac{\left(\kappa^{1-\aleph_{\mathfrak{B}_{\aleph 1}}^{\mathfrak{R}+}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}-1}}}\right]\right)+i\left(1-\log_{\kappa}\left[1+\frac{\left(\kappa^{1-\aleph_{\mathfrak{B}_{\aleph 1}}^{\mathfrak{I}+}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}-1}}}\right]\right), \\
\left(-\log_{\kappa}\left[1+\frac{\left(\kappa^{1-\aleph_{\mathfrak{B}_{\aleph 1}}^{\mathfrak{R}-}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}-1}}}\right]\right)+i\left(-\log_{\kappa}\left[1+\frac{\left(\kappa^{1-\aleph_{\mathfrak{B}_{\aleph 1}}^{\mathfrak{I}-}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}-1}}}\right]\right), \\
\left(\log_{\kappa}\left[1+\frac{\left(\kappa^{1-R_{\mathfrak{B}_{\aleph 1}}^{\mathfrak{R}}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}-1}}}\right]\right)+i\left(\log_{\kappa}\left[1+\frac{\left(\kappa^{1-R_{\mathfrak{B}_{\aleph 1}}^{\mathfrak{I}}}-1\right)^{\dot{\mathfrak{A}}_{\text{tcn}}}}{(\kappa-1)^{\dot{\mathfrak{A}}_{\text{tcn}-1}}}\right]\right),
\end{pmatrix} \tag{4}
$$

4 Frank Power Aggregation operator

The methods of CBCDVFULFPA, CBCDVFULFPWA, CBCDVFULFPG, and CBCDVFULFPWG operators are suggested in this section. Additionally, we simplify several of the operators listed above.

Definition 4.1. For any collection of CBCDVFULNs distance measure is $\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}+} + i\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}+}, \mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}-} + i\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}-}, \aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}+} + i\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}+}, \aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}-} + i\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}-}, R_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}} + iR_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}}\Big)\iota = 1, 2, \ldots, \mathfrak{R}$,

$$CBCDVFULFPA^{\text{tn}}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \oplus_{\iota=1}^{tn\mathfrak{R}} \frac{(1 + \aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1 + \aleph(\mathfrak{B}_\iota))}\mathfrak{B}_\iota$$

$$CBCDVFULFPA^{\text{tcn}}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \oplus_{\iota=1}^{tcn\mathfrak{R}} \frac{(1 + \aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1 + \aleph(\mathfrak{B}_\iota))}\mathfrak{B}_\iota$$

as the $CBCDVFULFPA^{\text{tn}}$ for $\mathcal{FTN}$ and the $CBCDVFULFPA^{\text{tcn}}$ $\mathcal{FTCN}$ with $\aleph(\mathfrak{B}_\iota) = \sum_{i\neq\iota=1}^{\mathfrak{R}} S(\mathfrak{B}_{\mathfrak{i}}, \mathfrak{B}_\iota)$, where $S(\mathfrak{B}_{\mathfrak{i}}, \mathfrak{B}_\iota) = 1 - D(\mathfrak{B}_{\mathfrak{i}}, \mathfrak{B}_\iota)$ with conditions,

(1) $S(\mathfrak{B}_{\mathfrak{i}}, \mathfrak{B}_\iota) = S(\mathfrak{B}_\iota, \mathfrak{B}_{\mathfrak{i}})$

(2) $S(\mathfrak{B}_{\mathfrak{i}}, \mathfrak{B}_\iota) \in [0, 1]$

(3) $S(\mathfrak{B}_{\mathfrak{i}}, \mathfrak{B}_\iota) > S(\mathfrak{B}_{\mathfrak{k}}, \mathfrak{B}_{\mathfrak{l}})$, then $D(\mathfrak{B}_{\mathfrak{i}}, \mathfrak{B}_\iota) < D(\mathfrak{B}_{\mathfrak{k}}, \mathfrak{B}_{\mathfrak{l}})$

then, we have

$$D(\mathfrak{B}_{\mathfrak{i}}, \mathfrak{B}_\iota) = \left| \frac{\mathfrak{T}_{I_i} - \mathfrak{T}_{I_\iota}}{\nu} \right| + \left| \frac{\mathfrak{F}_{I_i} - \mathfrak{F}_{I_\iota}}{\nu} \right|$$

$$* \frac{1}{8}\left(\begin{bmatrix} |\mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_i}} - \mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}| & |\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_i}} - \mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}| & |\mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_i}} - \mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}| & |\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_i}} - \mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}| \\ |\aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_i}} - \aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}| & |\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_i}} - \aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}| & |\aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_i}} - \aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}| & |\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_i}} - \aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}| \end{bmatrix}\right) * \left(\frac{|R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_i}} - R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}}| + |R^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_i}} - R^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}|}{2}\right)$$

Theorem 4.2. *For any CBCDVFULNs, show that CBCDVFULFPA operator is also CBCDVFULN, such as* $CBCDVFULFPA^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) =$

$$\left(\begin{array}{l} \left[\mathfrak{T}_{\nu\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\frac{I_\iota}{\nu}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}, \mathfrak{F}_{\nu\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\frac{I_\iota}{\nu}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}\right], \\ \left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left[\kappa^{1+\aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right]^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{1-\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{1-\mathfrak{R}}\left(\kappa^{1-R^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \end{array}\right) \tag{5}$$

$CBCDVFULFPA^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) =$

$$\left(\begin{array}{l} \left[\mathfrak{T}_{\nu\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\frac{\iota_1}{\nu}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}, \mathfrak{F}_{\nu\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\frac{\iota_1}{\nu}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}\right], \\ \left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-R^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \end{array}\right) \tag{6}$$

Property 4.3. *For any collection of CBCDVFULNs* $\mathfrak{B}_{\aleph_\iota} = \left([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\right) \iota = 1, 2, \dots, \mathfrak{R}$, *we obtain*

(1)When $\mathfrak{B}_{\aleph_\iota} = \mathfrak{B}_{\aleph}, \iota = 1, 2, \dots, \mathfrak{R}$

$$CBCDVFULFPA^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) = \mathfrak{B}_{\aleph}$$

$$CBCDVFULFPA^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) = \mathfrak{B}_{\aleph}$$

(2) When $\mathfrak{B}_{\aleph_\iota} \leqslant \mathfrak{B}'_{\aleph_\iota}$,

$$CBCDVFULFPA^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) \leqslant CBCDVFULFPA^{tcn}(\mathfrak{B}'_1, \mathfrak{B}'_2 \dots \mathfrak{B}'_{\mathfrak{R}})$$

$$CBCDVFULFPA^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) \leqslant CBCDVFULFPA^{tn}(\mathfrak{B}'_1, \mathfrak{B}'_2 \dots \mathfrak{B}'_{\mathfrak{R}})$$

(3) When $\mathfrak{B}^-_{\aleph} = min(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}})$ *and* $\mathfrak{B}^+_{\aleph} = max(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}})$

$$\mathfrak{B}^-_{\aleph} \leqslant CBCDVFULFPA^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) \leqslant \mathfrak{B}^+_{\aleph}$$

$$\mathfrak{B}^-_{\aleph} \leqslant CBCDVFULFPA^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) \leqslant \mathfrak{B}^+_{\aleph}$$

Definition 4.4. For any collection of CBCDVFULNs $\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\Big)\iota = 1, 2, \dots, \mathfrak{R}$, we define

$$CBCDVFULFPWA^{\text{tn}}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) = \oplus^{tn\mathfrak{R}}_{\iota=1} \frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}} \dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))} \mathfrak{B}_\iota$$

$$CBCDVFULFPWA^{\text{tcn}}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) = \oplus^{tcn\mathfrak{R}}_{\iota=1} \frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}} \dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))} \mathfrak{B}_\iota$$

as the $CBCDVFULFPWA^{\text{tn}}$ for $\mathcal{FTN}$ and the $CBCDVFULFPWA^{\text{tcn}}$ $\mathcal{FTCN}$.

Theorem 4.5. *For any CBCDVFULNs,show that CBCDVFULFPWA operator is also CBCDVFULN, such as* $CBCDVFULFPWA^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) =$

$$\left(\begin{array}{l}
\left[\mathfrak{T}_{\nu\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\frac{\iota_1}{\nu}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}, \mathfrak{F}_{\nu\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\frac{\iota_1}{\nu}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}\right], \\
\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\
\left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\
\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\
\left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\
\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-R^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right),
\end{array}\right)$$

$CBCDVFULFPWA^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) =$

$$\left(\begin{array}{l} \left[\mathfrak{T}_{\nu\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\frac{I_\iota}{\nu}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}, \mathfrak{F}_{\nu\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\frac{I_\iota}{\nu}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}\right], \\ \left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{R^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \end{array}\right)$$

Property 4.6. *For any collection of CBCDVFULNs*$\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\Big)\iota = 1, 2, \ldots, \mathfrak{R}$,*we obtain*

(1)When $\mathfrak{B}_{\aleph_\iota} = \mathfrak{B}_\aleph, \iota = 1, 2, \ldots, \mathfrak{R}$

$$CBCDVFULFPWA^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \mathfrak{B}_\aleph$$

$$CBCDVFULFPWA^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \mathfrak{B}_\aleph$$

(2) When $\mathfrak{B}_{\aleph_\iota} \leqslant \mathfrak{B}'_{\aleph_\iota}$,

$$CBCDVFULFPWA^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) \leqslant CBCDVFULFPWA^{tcn}(\mathfrak{B}'_1, \mathfrak{B}'_2 \ldots \mathfrak{B}'_{\mathfrak{R}})$$

$$CBCDVFULFPWA^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) \leqslant CBCDVFULFPWA^{tn}(\mathfrak{B}'_1, \mathfrak{B}'_2 \ldots \mathfrak{B}'_{\mathfrak{R}})$$

(3) When $\mathfrak{B}^-_\aleph = min(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}})$ *and* $\mathfrak{B}^+_\aleph = max(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}})$

$$\mathfrak{B}^-_\aleph \leqslant CBCDVFULFPWA^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) \leqslant \mathfrak{B}^+_\aleph$$

$$\mathfrak{B}^-_\aleph \leqslant CBCDVFULFPWA^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) \leqslant \mathfrak{B}^+_\aleph$$

Definition 4.7. For any collection of CBCDVFULNs $\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\Big)\iota = 1, 2, \ldots, \mathfrak{R}$, we define

$$CBCDVFULFPG^{\mathrm{tn}}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \otimes^{tn\mathfrak{R}}_{\iota=1}(\mathfrak{B}_\iota)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}$$

$$CBCDVFULFPG^{\mathrm{tcn}}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \otimes^{tcn\mathfrak{R}}_{\iota=1}(\mathfrak{B}_\iota)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}$$

as the $CBCDVFULFPG^{\mathrm{tn}}$ for $\mathcal{FTN}$ and the $CBCDVFULFPG^{\mathrm{tcn}}$ $\mathcal{FTCN}$.

Theorem 4.8. *For any CBCDVFULNs,show that CBCDVFULFPG operator is also CBCDVFULN, such as* $CBCDVFULFPG^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) =$

$$\left(\begin{array}{l} \left[\mathfrak{T}_{\nu\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\frac{I_\iota}{\nu}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}, \mathfrak{F}_{\nu\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\frac{I_\iota}{\nu}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}\right], \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{1-\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{1-\mathfrak{R}}\left(\kappa^{1-R^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \end{array}\right) \tag{7}$$

$CBCDVFULFPG^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) =$

$$\left(\begin{array}{l} \left[\mathfrak{T}_{\nu\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\frac{I_\iota}{\nu}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}, \mathfrak{F}_{\nu\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\frac{I_\iota}{\nu}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}\right], \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)+i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-R^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}}-1\right)^{\frac{(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \end{array}\right) \tag{8}$$

Property 4.9. *For any collection of CBCDVFULNs* $\mathfrak{B}_{\aleph_\iota} = \left([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\mathfrak{R}-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\mathfrak{I}-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\mathfrak{R}}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\mathfrak{I}}_{\mathfrak{B}_{\aleph_\iota}}\right)\iota = 1, 2, \ldots, \mathfrak{R}$,*we obtain*

(1)When $\mathfrak{B}_{\aleph_\iota} = \mathfrak{B}_{\aleph}, \iota = 1, 2, \ldots, \mathfrak{R}$

$$CBCDVFULFPG^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \mathfrak{B}_{\aleph}$$

$$CBCDVFULFPG^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \mathfrak{B}_{\aleph}$$

(2) When $\mathfrak{B}_{\aleph_\iota} \leqslant \mathfrak{B}'_{\aleph_\iota}$,

$$CBCDVFULFPG^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) \leqslant CBCDVFULFPG^{tcn}(\mathfrak{B}'_1, \mathfrak{B}'_2 \ldots \mathfrak{B}'_{\mathfrak{R}})$$

$$CBCDVFULFPG^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) \leqslant CBCDVFULFPG^{tn}(\mathfrak{B}'_1, \mathfrak{B}'_2 \ldots \mathfrak{B}'_{\mathfrak{R}})$$

(3) When $\mathfrak{B}_{\aleph}^{-} = min(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}})$ *and* $\mathfrak{B}_{\aleph}^{+} = max(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}})$

$$\mathfrak{B}_{\aleph}^{-} \leqslant CBCDVFULFPG^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) \leqslant \mathfrak{B}_{\aleph}^{+}$$

$$\mathfrak{B}_{\aleph}^{-} \leqslant CBCDVFULFPG^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) \leqslant \mathfrak{B}_{\aleph}^{+}$$

Definition 4.10. For any collection of CBCDVFULNs $\mathfrak{B}_{\aleph_\iota} = \left([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}+} + i\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}+}, \mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}-} + i\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}-}, \aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}+} + i\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}+}, \aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}-} + i\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}-}, R_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}} + iR_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}}\right)\iota = 1, 2, \dots, \mathfrak{R}$, we define

$$CBCDVFULFPWG^{\text{tn}}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) = \otimes_{\iota=1}^{tn\mathfrak{R}} \mathfrak{B}_\iota^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}} \dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}$$

$$CBCDVFULFPWG^{\text{tcn}}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) = \otimes_{\iota=1}^{tcn\mathfrak{R}} \mathfrak{B}_\iota^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}} \dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}$$

as the $CBCDVFULFPWG^{\text{tn}}$ for $\mathcal{FTN}$ and the $CBCDVFULFPWG^{\text{tcn}}$ $\mathcal{FTCN}$.

Theorem 4.11. *For any CBCDVFULNs,show that CBCDVFULFPWG operator is also CBCDVFULN, such as*
$CBCDVFULFPWG^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) =$

$$\left(\begin{array}{l} \left[\mathfrak{T}_{\nu\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\frac{I_\iota}{\nu}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}, \mathfrak{F}_{\nu\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\frac{I_\iota}{\nu}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)} \right], \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}+}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}+}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}-}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}-}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}+}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}+}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}-}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}-}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-R_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{1-\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{1-\mathfrak{R}}\left(\kappa^{1-R_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \end{array} \right.$$

$CBCDVFULFPWG^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \dots \mathfrak{B}_{\mathfrak{R}}) =$

$$\left(\begin{array}{l} \left[\mathfrak{T}_{\nu\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\frac{I_\iota}{\nu}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)}, \mathfrak{F}_{\nu\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\frac{I_\iota}{\nu}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right)} \right], \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}+}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}+}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}-}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(-1+\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1+\mathcal{M}_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}-}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}+}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(1-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{1-\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}+}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}-}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(-\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-\aleph_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}-}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \\ \left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-R_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{R}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right) + i\left(\log_\kappa\left[1+\prod_{\iota=1}^{\mathfrak{R}}\left(\kappa^{-R_{\mathfrak{B}_{\aleph_\iota}}^{\mathfrak{I}}}-1\right)^{\frac{\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}{\sum_{\iota=1}^{\mathfrak{R}}\dot{\mathfrak{A}}(1+\aleph(\mathfrak{B}_\iota))}}\right]\right), \end{array} \right.$$

Property 4.12. *For any collection of CBCDVFULNs* $\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\Re}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\Im}_{\mathfrak{B}_{\aleph_\iota}}\Big) \iota = 1, 2, \ldots, \mathfrak{R}$,*we obtain*

(1)When $\mathfrak{B}_{\aleph_\iota} = \mathfrak{B}_{\aleph}, \iota = 1, 2, \ldots, \mathfrak{R}$

$$CBCDVFULFPWG^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \mathfrak{B}_{\aleph}$$

$$CBCDVFULFPWG^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) = \mathfrak{B}_{\aleph}$$

(2) When $\mathfrak{B}_{\aleph_\iota} \leqslant \mathfrak{B}'_{\aleph_\iota}$,

$$CBCDVFULFPWG^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) \leqslant CBCDVFULFPWG^{tcn}(\mathfrak{B}'_1, \mathfrak{B}'_2 \ldots \mathfrak{B}'_{\mathfrak{R}})$$

$$CBCDVFULFPWA^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) \leqslant CBCDVFULFPWG^{tn}(\mathfrak{B}'_1, \mathfrak{B}'_2 \ldots \mathfrak{B}'_{\mathfrak{R}})$$

(3) When $\mathfrak{B}^-_{\aleph} = min(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}})$ *and* $\mathfrak{B}^+_{\aleph} = max(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}})$

$$\mathfrak{B}^-_{\aleph} \leqslant CBCDVFULFPWG^{tcn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) \leqslant \mathfrak{B}^+_{\aleph}$$

$$\mathfrak{B}^-_{\aleph} \leqslant CBCDVFULFPWG^{tn}(\mathfrak{B}_1, \mathfrak{B}_2 \ldots \mathfrak{B}_{\mathfrak{R}}) \leqslant \mathfrak{B}^+_{\aleph}$$

5 MABAC Approach Using the Suggested Operators

The evaluation of the MABAC model using the suggested aggregation operators. Below is a list of the MABAC model's primary steps:

Step 1:Arrange the CBCDVFUL values to compute the matrix, such as

$$DM = [\mathfrak{B}_{i\times \mathrm{F}}]_{\Omega\times n} = \begin{bmatrix} \mathfrak{B}_{11} & \mathfrak{B}_{12} & \ldots & \mathfrak{B}_{1n} \\ \mathfrak{B}_{21} & \mathfrak{B}_{22} & \ldots & \mathfrak{B}_{2n} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ \mathfrak{B}_{\Omega 1} & \mathfrak{B}_{\Omega 2} & \ldots & \mathfrak{B}_{\Omega n} \end{bmatrix}$$

$\mathfrak{B}_{\aleph_\iota} = \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\Re}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\Im}_{\mathfrak{B}_{\aleph_\iota}}\Big) \iota = 1, 2, \ldots, \mathfrak{R}$, is the CBCDVFULNs.

Step 2: We assess normalized data following collection, however when it comes to cost-type data, such as

$$b\mathfrak{B}' = \begin{bmatrix} \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\Re}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\Im}_{\mathfrak{B}_{\aleph_\iota}}\Big) & benefit \\ \Big([\mathfrak{T}_{I_\iota}, \mathfrak{F}_{I_\iota}], \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}}, \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} + i\mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}}, R^{\Re}_{\mathfrak{B}_{\aleph_\iota}} + iR^{\Im}_{\mathfrak{B}_{\aleph_\iota}}\Big) & cost \end{bmatrix}$$

When it comes to benefit forms of data, we neglect normalization.

Step 3: After any necessary normalization, we use the suggested Frank operational rules to extract the weighted decision data as a matrix, using equation(1) (or) (2) (or) (3) (or) (4)

Step 4: Additionally, we use the CBCDVFULFPA and CBCDVFULFPG operator to identify the aggregated data in order to take into account the weighted matrix, (5) and (7)

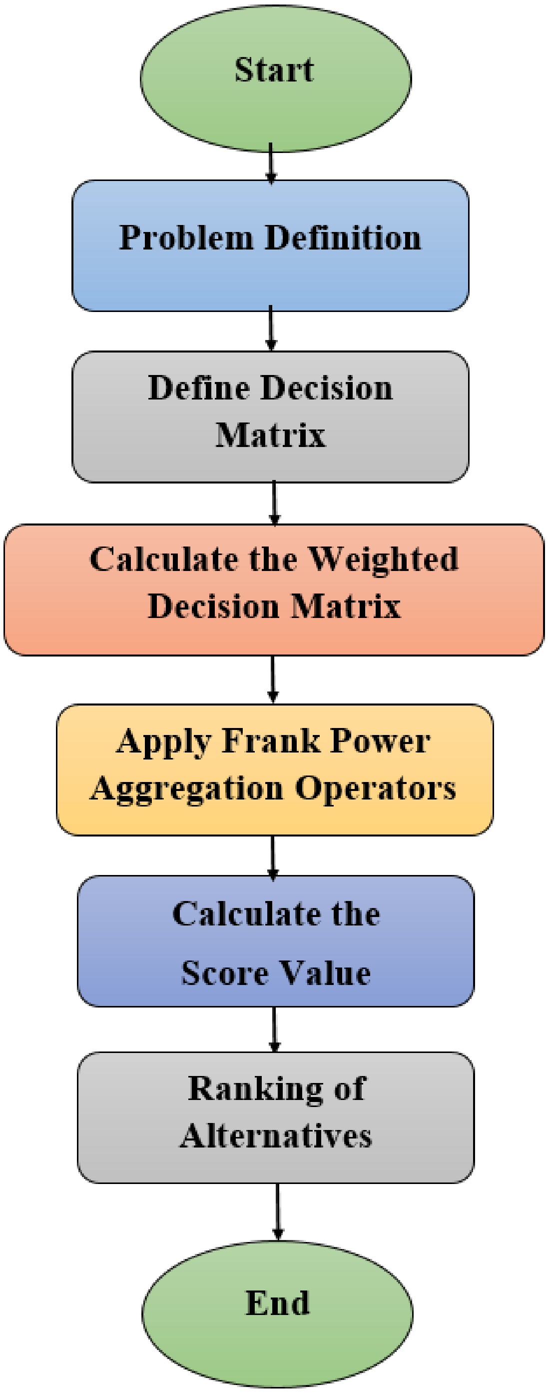

Figure 1: Flow of CBCDVFULSs Group Message Ranking System.

Step 5: We determine the distance measures in order to merge the values of the two matrices mentioned above, including $\mathfrak{B}_{i\mathrm{F}} =$

$$\begin{bmatrix} D(\mathfrak{B}_{\mathrm{i}}, \mathfrak{B}_{\mathrm{F}}) & if \mathfrak{B}_{\mathrm{i}} > \mathfrak{B}_{\mathrm{F}} \\ 0 & if \mathfrak{B}_{\mathrm{i}} = \mathfrak{B}_{\mathrm{F}} \\ -D(\mathfrak{B}_{\mathrm{i}}, \mathfrak{B}_{\mathrm{F}}) & if \mathfrak{B}_{\mathrm{i}} < \mathfrak{B}_{\mathrm{F}} \end{bmatrix}$$

Observe how the main concept of distance measures is explained below, such as

$$\begin{aligned} D(\mathfrak{B}_{\mathrm{i}}, \mathfrak{B}_{\iota}) = & \mid \frac{\mathfrak{T}_{I_i} - \mathfrak{T}_{I_\iota}}{\nu} \mid + \mid \frac{\mathfrak{F}_{I_i} - \mathfrak{F}_{I_\iota}}{\nu} \mid \\ & * \frac{1}{8} \left(\begin{bmatrix} \mid \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_i}} - \mathcal{M}^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} \mid & \mid \mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_i}} - \mathcal{M}^{\Im+}_{\mathfrak{B}_{\aleph_\iota}} \mid & \mid \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_i}} - \mathcal{M}^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} \mid & \mid \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_i}} - \mathcal{M}^{\Im-}_{\mathfrak{B}_{\aleph_\iota}} \mid \\ \mid \aleph^{\Re+}_{\mathfrak{B}_{\aleph_i}} - \aleph^{\Re+}_{\mathfrak{B}_{\aleph_\iota}} \mid & \mid \aleph^{\Im+}_{\mathfrak{B}_{\aleph_i}} - \aleph^{\Im+}_{\mathfrak{B}_{\aleph_\iota}} \mid & \mid \aleph^{\Re-}_{\mathfrak{B}_{\aleph_i}} - \aleph^{\Re-}_{\mathfrak{B}_{\aleph_\iota}} \mid & \mid \aleph^{\Im-}_{\mathfrak{B}_{\aleph_i}} - \aleph^{\Im-}_{\mathfrak{B}_{\aleph_\iota}} \mid \end{bmatrix} \right) \\ & * \left(\frac{\mid R^{\Re}_{\mathfrak{B}_{\aleph_i}} - R^{\Re}_{\mathfrak{B}_{\aleph_\iota}} \mid + \mid R^{\Im}_{\mathfrak{B}_{\aleph_i}} - R^{\Im}_{\mathfrak{B}_{\aleph_\iota}} \mid}{2} \right) \end{aligned}$$

Step 6: Consequently, calculate the appraisal values,

$$\mathcal{S}_i = \frac{1}{m} \sum_{\iota=1}^{m} D(\mathfrak{B}_i, \mathfrak{B}_\iota)$$

Step 7: Lastly, we rank the values based on their appraisal values to determine which of the group of selections is the best choice. In order to increase the value of the developed theory, Finally, we analyze the group message prioritization system for WhatsApp to show the stability and effectiveness of the proposed theory.

6 Smart Group Message Prioritization System For WhatsApp

This section examines the use of the Frank power aggregation operator in combination with the suggested CBCDVFULSs architecture for intelligent message prioritization in WhatsApp group chats. Users of contemporary messaging apps like WhatsApp are usually overloaded with a large number of group messages that differ in terms of their urgency, relevance, and linguistic clarity. These communications are difficult and time-consuming to manually filter since they frequently originate from a variety of senders, include casual and multilingual phrases, and have varying tones. This problem is addressed by the CBCDVFULSs and Frank aggregation strategy, which provides a reliable way to prioritize messages in such ambiguous and contradictory circumstances. This application's goal is to assist users in rapidly identifying the most crucial and contextually appropriate messages within group conversations. We take into consideration five typical messages, designated in order to replicate this. $\mathfrak{L}_1$ is an urgent task notification, $\mathfrak{L}_2$ is an broad group announcement, $\mathfrak{L}_3$ is an emotional personal update, $\mathfrak{L}_4$ is a shared document or link, and $\mathfrak{L}_5$ is a informal meeting reminder. These messages reflect common categories seen in group conversations. Five specific criteria are used to analyze each message: Message urgency $\mathfrak{L}_A^1$ indicates how urgent or important the message's content is. Content type $\mathfrak{L}_A^2$ is the distinguishes between many media forms, including voice notes, documents, photos, and text. User preference relevance $\mathfrak{L}_A^3$ indicates the degree to which the message fits the user's unique communication preferences or interests. Sender importance $\mathfrak{L}_A^4$ evaluates the sender's importance from the viewpoint of the user. Tone polarity $\mathfrak{L}_A^5$ captures the message's subjective or emotional content, including its neutral, negative, or positive tones. Additionally, we have a few weight vectors (0.2,0.15,0.25,0.15,0.25). Non-linear and interactive aggregation that respects the weight of each criterion and the type of uncertainty inherent in the assessments is made possible by this operator. The algorithm ranks the messages according to contextual importance by calculating a priority score for each one after processing these variables. Lastly, we seek to simplify the aforementioned issues in light of the information presented above. To do this, we attempt to analyze some fictitious data using the suggested operators. Additionally, these values will be allocated to every quality in every option.

	$\mathfrak{L}_A^1$	$\mathfrak{L}_A^2$	$\mathfrak{L}_A^3$	$\mathfrak{L}_A^4$	$\mathfrak{L}_A^5$
$\mathfrak{L}_1$	$[[\mathfrak{T}_2, \mathfrak{T}_3]$	$[[\mathfrak{T}_{2.1}, \mathfrak{T}_{3.1}]$	$[[\mathfrak{T}_{2.2}, \mathfrak{T}_{3.2}]$	$[[\mathfrak{T}_{2.3}, \mathfrak{T}_{3.3}]$	$[[\mathfrak{T}_{2.4}, \mathfrak{T}_{3.4}]$
	$.3 + \mathfrak{i}.2$	$.31 + \mathfrak{i}.21$	$.32 + \mathfrak{i}.22$	$.33 + \mathfrak{i}.23$	$.34 + \mathfrak{i}.24$
	$-.2 + \mathfrak{i}(-.5)$	$-.21 + \mathfrak{i}(-.51)$	$-.22 + \mathfrak{i}(-.52)$	$-.23 + \mathfrak{i}(-.53)$	$-.24 + \mathfrak{i}(-.54)$
	$.6 + \mathfrak{i}.4$	$.61 + \mathfrak{i}.41$	$.62 + \mathfrak{i}.42$	$.63 + \mathfrak{i}.43$	$.64 + \mathfrak{i}.44$
	$-.5 + \mathfrak{i}(-.7)$	$-.51 + \mathfrak{i}(-.71)$	$-.52 + \mathfrak{i}(-.72)$	$-.53 + \mathfrak{i}(-.73)$	$-.54 + \mathfrak{i}(-.74)$
	$.2 + \mathfrak{i}.3]$	$.21 + \mathfrak{i}.31]$	$.22 + \mathfrak{i}.32]$	$.23 + \mathfrak{i}.33]$	$.24 + \mathfrak{i}.34]$
$\mathfrak{L}_2$	$[[\mathfrak{T}_3, \mathfrak{T}_4]$	$[[\mathfrak{T}_{3.1}, \mathfrak{T}_{4.1}]$	$[[\mathfrak{T}_{3.2}, \mathfrak{T}_{4.2}]$	$[[\mathfrak{T}_{3.3}, \mathfrak{T}_{4.3}]$	$[[\mathfrak{T}_{3.4}, \mathfrak{T}_{4.4}]$
	$.5 + \mathfrak{i}.3$	$.51 + \mathfrak{i}.31$	$.52 + \mathfrak{i}.32$	$.53 + \mathfrak{i}.33$	$.54 + \mathfrak{i}.34$
	$-.3 + \mathfrak{i}(-.6)$	$-.31 + \mathfrak{i}(-.61)$	$-.32 + \mathfrak{i}(-.62)$	$-.33 + \mathfrak{i}(-.63)$	$-.34 + \mathfrak{i}(-.64)$
	$.4 + \mathfrak{i}.7$	$.41 + \mathfrak{i}.71$	$.42 + \mathfrak{i}.72$	$.43 + \mathfrak{i}.73$	$.44 + \mathfrak{i}.74$
	$-.6 + \mathfrak{i}(-.3)$	$-.61 + \mathfrak{i}(-.31)$	$-.62 + \mathfrak{i}(-.32)$	$-.63 + \mathfrak{i}(-.33)$	$-.64 + \mathfrak{i}(-.34)$
	$.4 + \mathfrak{i}.5]$	$.41 + \mathfrak{i}.51]$	$.42 + \mathfrak{i}.52]$	$.43 + \mathfrak{i}.53]$	$.44 + \mathfrak{i}.54]$
$\mathfrak{L}_3$	$[[\mathfrak{T}_1, \mathfrak{T}_5]$	$[[\mathfrak{T}_{1.1}, \mathfrak{T}_{5.1}]$	$[[\mathfrak{T}_{1.2}, \mathfrak{T}_{5.2}]$	$[[\mathfrak{T}_{1.3}, \mathfrak{T}_{5.3}]$	$[[\mathfrak{T}_{1.4}, \mathfrak{T}_{5.4}]$
	$.7 + \mathfrak{i}.8$	$.71 + \mathfrak{i}.81$	$.72 + \mathfrak{i}.82$	$.73 + \mathfrak{i}.83$	$.74 + \mathfrak{i}.84$
	$-.6 + \mathfrak{i}(-.8)$	$-.61 + \mathfrak{i}(-.81)$	$-.62 + \mathfrak{i}(-.82)$	$-.63 + \mathfrak{i}(-.83)$	$-.64 + \mathfrak{i}(-.84)$
	$.7 + \mathfrak{i}.9$	$.71 + \mathfrak{i}.91$	$.72 + \mathfrak{i}.92$	$.73 + \mathfrak{i}.93$	$.74 + \mathfrak{i}.94$
	$-.3 + \mathfrak{i}(-.4)$	$-.31 + \mathfrak{i}(-.41)$	$-.32 + \mathfrak{i}(-.42)$	$-.33 + \mathfrak{i}(-.43)$	$-.34 + \mathfrak{i}(-.44)$
	$.5 + \mathfrak{i}.8]$	$.51 + \mathfrak{i}.81]$	$.52 + \mathfrak{i}.82]$	$.53 + \mathfrak{i}.83]$	$.54 + \mathfrak{i}.84]$
$\mathfrak{L}_4$	$[[\mathfrak{T}_1, \mathfrak{T}_4]$	$[[\mathfrak{T}_{1.1}, \mathfrak{T}_{4.1}]$	$[[\mathfrak{T}_{1.2}, \mathfrak{T}_{4.2}]$	$[[\mathfrak{T}_{1.3}, \mathfrak{T}_{4.3}]$	$[[\mathfrak{T}_{1.4}, \mathfrak{T}_{4.4}]$
	$.3 + \mathfrak{i}.5$	$.31 + \mathfrak{i}.51$	$.32 + \mathfrak{i}.52$	$.33 + \mathfrak{i}.53$	$.34 + \mathfrak{i}.54$
	$-.2 + \mathfrak{i}(-.4)$	$-.21 + \mathfrak{i}(-.41)$	$-.22 + \mathfrak{i}(-.42)$	$-.23 + \mathfrak{i}(-.43)$	$-.24 + \mathfrak{i}(-.44)$
	$.5 + \mathfrak{i}.1$	$.51 + \mathfrak{i}.11$	$.52 + \mathfrak{i}.12$	$.53 + \mathfrak{i}.13$	$.54 + \mathfrak{i}.14$
	$-.6 + \mathfrak{i}(-.3)$	$-.61 + \mathfrak{i}(-.31)$	$-.62 + \mathfrak{i}(-.32)$	$-.63 + \mathfrak{i}(-.33)$	$-.64 + \mathfrak{i}(-.34)$
	$.4 + \mathfrak{i}.5]$	$.41 + \mathfrak{i}.51]$	$.42 + \mathfrak{i}.52]$	$.43 + \mathfrak{i}.53]$	$.44 + \mathfrak{i}.54]$
$\mathfrak{L}_5$	$[[\mathfrak{T}_2, \mathfrak{T}_5]$	$[[\mathfrak{T}_{2.1}, \mathfrak{T}_{5.1}]$	$[[\mathfrak{T}_{5.2}, \mathfrak{T}_{3.2}]$	$[[\mathfrak{T}_{2.3}, \mathfrak{T}_{5.3}]$	$[[\mathfrak{T}_{2.4}, \mathfrak{T}_{5.4}]$
	$.2 + \mathfrak{i}.6$	$.21 + \mathfrak{i}.61$	$.22 + \mathfrak{i}.62$	$.23 + \mathfrak{i}.63$	$.24 + \mathfrak{i}.64$
	$-.3 + \mathfrak{i}(-.7)$	$-.31 + \mathfrak{i}(-.71)$	$-.32 + \mathfrak{i}(-.72)$	$-.33 + \mathfrak{i}(-.73)$	$-.34 + \mathfrak{i}(-.74)$
	$.2 + \mathfrak{i}.3]$	$.21 + \mathfrak{i}.31]$	$.22 + \mathfrak{i}.32]$	$.23 + \mathfrak{i}.33]$	$.24 + \mathfrak{i}.34]$
	$-.3 + \mathfrak{i}(-.2)$	$-.31 + \mathfrak{i}(-.21)$	$-.32 + \mathfrak{i}(-.22)$	$-.33 + \mathfrak{i}(-.23)$	$-.34 + \mathfrak{i}(-.24)$
	$.5 + \mathfrak{i}.3]$	$.51 + \mathfrak{i}.31]$	$.52 + \mathfrak{i}.32]$	$.53 + \mathfrak{i}.33]$	$.54 + \mathfrak{i}.34]$

Table 1: CBCDVFUL decision matrix

	$\mathfrak{L}_A^1$	$\mathfrak{L}_A^2$	$\mathfrak{L}_A^3$	$\mathfrak{L}_A^4$	$\mathfrak{L}_A^5$
$\mathfrak{L}_1$	$[[\mathfrak{T}_{.1972}, \mathfrak{T}_{.2937}]$	$[[\mathfrak{T}_{.1557}, \mathfrak{T}_{.2287}]$	$[[\mathfrak{T}_{.2697}, \mathfrak{T}_{.3889}]$	$[[\mathfrak{T}_{.1704}, \mathfrak{T}_{.2432}]$	$[[\mathfrak{T}_{.2937}, \mathfrak{T}_{.4124}]$
	$.0663 + \mathfrak{i}.0425$	$.0519 + \mathfrak{i}.0337$	$.0885 + \mathfrak{i}.0587$	$.0557 + \mathfrak{i}.0372$	$.0948 + \mathfrak{i}.0644$
	$-.7510 + \mathfrak{i}(-.8784)$	$-.8133 + \mathfrak{i}(-.9101)$	$-.8180 + \mathfrak{i}(-.9124)$	$-.8225 + \mathfrak{i}(-.9148)$	$-.8268 + \mathfrak{i}(-.9170)$
	$.9076 + \mathfrak{i}.8447$	$.9322 + \mathfrak{i}.8846$	$.9342 + \mathfrak{i}.8873$	$.9363 + \mathfrak{i}.8900$	$.9383 + \mathfrak{i}.8927$
	$-.1215 + \mathfrak{i}(-.1958)$	$-.0946 + \mathfrak{i}(-.1534)$	$-.1579 + \mathfrak{i}(-.2512)$	$-.0995 + \mathfrak{i}(-.1609)$	$-.1658 + \mathfrak{i}(-.2630)$
	$.0425 + \mathfrak{i}.0663]$	$.0337 + \mathfrak{i}.0519]$	$.0587 + \mathfrak{i}.0885]$	$.0372 + \mathfrak{i}.0557]$	$.0644 + \mathfrak{i}.0948]$
$\mathfrak{L}_2$	$[[\mathfrak{T}_{.2937}, \mathfrak{T}_{.3889}]$	$[[\mathfrak{T}_{.2287}, \mathfrak{T}_{.3009}]$	$[[\mathfrak{T}_{.3889}, \mathfrak{T}_{.5059}]$	$[[\mathfrak{T}_{.2432}, \mathfrak{T}_{.3152}]$	$[[\mathfrak{T}_{.4124}, \mathfrak{T}_{.5290}]$
	$.1215 + \mathfrak{i}.0663$	$.0946 + \mathfrak{i}.0519$	$.1579 + \mathfrak{i}.0885$	$.0995 + \mathfrak{i}.0557$	$.1658 + \mathfrak{i}.0948$
	$-.8041 + \mathfrak{i}(-.9076)$	$-.8537 + \mathfrak{i}(-.9332)$	$-.8571 + \mathfrak{i}(-.9342)$	$-.8604 + \mathfrak{i}(-.9363)$	$-.8637 + \mathfrak{i}(-.9383)$
	$.8447 + \mathfrak{i}.9336$	$.8846 + \mathfrak{i}.9518$	$.8873 + \mathfrak{i}.9536$	$.8900 + \mathfrak{i}.9555$	$.8927 + \mathfrak{i}.9573$
	$-.1552 + \mathfrak{i}(-.0663)$	$-.1210 + \mathfrak{i}(-.0519)$	$-.2001 + \mathfrak{i}(-.0885)$	$-.1269 + \mathfrak{i}(-.0557)$	$-.2095 + \mathfrak{i}(-.0948)$
	$.0923 + \mathfrak{i}.1215]$	$.0719 + \mathfrak{i}.0946]$	$.1212 + \mathfrak{i}.1579]$	$.0762 + \mathfrak{i}.0995]$	$.1282 + \mathfrak{i}.1658]$
$\mathfrak{L}_3$	$[[\mathfrak{T}_{.0993}, \mathfrak{T}_{.4826}]$	$[[\mathfrak{T}_{.0820}, \mathfrak{T}_{.3723}]$	$[[\mathfrak{T}_{.1484}, \mathfrak{T}_{.6207}]$	$[[\mathfrak{T}_{.0968}, \mathfrak{T}_{.3865}]$	$[[\mathfrak{T}_{.1728}, \mathfrak{T}_{.6434}]$
	$.1985 + \mathfrak{i}.2489$	$.1534 + \mathfrak{i}.1966$	$.2512 + \mathfrak{i}.3185$	$.1609 + \mathfrak{i}.2074$	$.2630 + \mathfrak{i}.3353$
	$-.9076 + \mathfrak{i}(-.9574)$	$-.9322 + \mathfrak{i}(-.9696)$	$-.9342 + \mathfrak{i}(-.9713)$	$-.9555 + \mathfrak{i}(-.9730)$	$-.9383 + \mathfrak{i}(-.9747)$
	$.9336 + \mathfrak{i}.9794$	$.9518 + \mathfrak{i}.9861$	$.9536 + \mathfrak{i}.9877$	$.9363 + \mathfrak{i}.9892$	$.9573 + \mathfrak{i}.9908$
	$-.0663 + \mathfrak{i}(-.0923)$	$-.0519 + \mathfrak{i}(-.0719)$	$-.0885 + \mathfrak{i}(-.1212)$	$-.0557 + \mathfrak{i}(-.0762)$	$-.0948 + \mathfrak{i}(-.1282)$
	$.1215 + \mathfrak{i}.2489]$	$.0946 + \mathfrak{i}.1966]$	$.1579 + \mathfrak{i}.3185]$	$.0995 + \mathfrak{i}.2074]$	$.1658 + \mathfrak{i}.3353]$
$\mathfrak{L}_4$	$[[\mathfrak{T}_{.0993}, \mathfrak{T}_{.3889}]$	$[[\mathfrak{T}_{.0820}, \mathfrak{T}_{.3009}]$	$[[\mathfrak{T}_{.1484}, \mathfrak{T}_{.5059}]$	$[[\mathfrak{T}_{.0968}, \mathfrak{T}_{.3152}]$	$[[\mathfrak{T}_{.1728}, \mathfrak{T}_{.5290}]$
	$.0663 + \mathfrak{i}.1215$	$.0519 + \mathfrak{i}.0946$	$.0885 + \mathfrak{i}.1579$	$.0557 + \mathfrak{i}.0995$	$.0946 + \mathfrak{i}.1658$
	$-.7510 + \mathfrak{i}(-.8447)$	$-.8133 + \mathfrak{i}(-.8846)$	$-.8180 + \mathfrak{i}(-.8873)$	$-.8225 + \mathfrak{i}(-.8900)$	$-.8268 + \mathfrak{i}(-.8927)$
	$.8784 + \mathfrak{i}.6694$	$.9101 + \mathfrak{i}.7515$	$.9124 + \mathfrak{i}.7595$	$.9148 + \mathfrak{i}.7670$	$.9170 + \mathfrak{i}.7739$
	$-.1552 + \mathfrak{i}(-.0663)$	$-.1210 + \mathfrak{i}(-.0519)$	$-.2001 + \mathfrak{i}(-.0885)$	$-.1269 + \mathfrak{i}(-.0557)$	$-.2095 + \mathfrak{i}(-.0948)$
	$.0923 + \mathfrak{i}.1215]$	$.0719 + \mathfrak{i}.0946]$	$.1212 + \mathfrak{i}.1579]$	$.0762 + \mathfrak{i}.0995]$	$.1282 + \mathfrak{i}.1658]$
$\mathfrak{L}_5$	$[[\mathfrak{T}_{.1972}, \mathfrak{T}_{.4826}]$	$[[\mathfrak{T}_{.1557}, \mathfrak{T}_{.3723}]$	$[[\mathfrak{T}_{.2697}, \mathfrak{T}_{.6207}]$	$[[\mathfrak{T}_{.1704}, \mathfrak{T}_{.3865}]$	$[[\mathfrak{T}_{.2937}, \mathfrak{T}_{.6434}]$
	$.0425 + \mathfrak{i}.1552$	$.0337 + \mathfrak{i}.1210$	$.0587 + \mathfrak{i}.2001$	$.0372 + \mathfrak{i}.1269$	$.0644 + \mathfrak{i}.2095$
	$-.8041 + \mathfrak{i}(-.9336)$	$-.8537 + \mathfrak{i}(-.9518)$	$-.8571 + \mathfrak{i}(-.9536)$	$-.8604 + \mathfrak{i}(-.9555)$	$-.8637 + \mathfrak{i}(-.9573)$
	$.7510 + \mathfrak{i}.8041$	$.8133 + \mathfrak{i}.8537$	$.8180 + \mathfrak{i}.8571$	$.8225 + \mathfrak{i}.8604$	$.8268 + \mathfrak{i}.8637$
	$-.0663 + \mathfrak{i}(-.0425)$	$-.0519 + \mathfrak{i}(-.0337)$	$-.0885 + \mathfrak{i}(-.0587)$	$-.0557 + \mathfrak{i}(-.0372)$	$-.0946 + \mathfrak{i}(-.0644)$
	$.1215 + \mathfrak{i}.0663]$	$.0946 + \mathfrak{i}.0519]$	$.1579 + \mathfrak{i}.0885]$	$.0995 + \mathfrak{i}.0557]$	$.1658 + \mathfrak{i}.0948]$

Table 2: CBCDVFUL weighted matrix

	CBCDVFULPA operator	CBCDVFULPG operator
$\mathfrak{L}_1$	$[[\mathfrak{T}_{.2173}, \mathfrak{T}_{.3133}]$	$[[\mathfrak{T}_{.2089}, \mathfrak{T}_{.3012}]$
	$.0714 + \mathfrak{i}.0473$	$.0686 + \mathfrak{i}.0454$
	$-.8063 + \mathfrak{i}(-.9065)$	$-.8399 + \mathfrak{i}(-.9443)$
	$.9297 + \mathfrak{i}.8798$	$.9684 + \mathfrak{i}.165$
	$-.1278 + \mathfrak{i}(-.2048)$	$-.1229 + \mathfrak{i}(-.1969)$
	$.0473 + \mathfrak{i}.0714]$	$.0454 + \mathfrak{i}.0686]$
$\mathfrak{L}_2$	$[[\mathfrak{T}_{.3127}, \mathfrak{T}_{.4079}]$	$[[\mathfrak{T}_{.3012}, \mathfrak{T}_{.3922}]$
	$.1274 + \mathfrak{i}.0709$	$.1221 + \mathfrak{i}.0682$
	$-.8478 + \mathfrak{i}(-.9292)$	$-.8831 + \mathfrak{i}(-.9679)$
	$.8792 + \mathfrak{i}.9503$	$.9172 + \mathfrak{i}.9899$
	$-.1625 + \mathfrak{i}(-.0709)$	$-.1562 + \mathfrak{i}(-.0682)$
	$.0979 + \mathfrak{i}.1274]$	$.0941 + \mathfrak{i}.1221]$
$\mathfrak{L}_3$	$[[\mathfrak{T}_{.1198}, \mathfrak{T}_{.5011}]$	$[[\mathfrak{T}_{.1151}, \mathfrak{T}_{.4818}]$
	$.2048 + \mathfrak{i}.2613$	$.2033 + \mathfrak{i}.2609$
	$-.9286 + \mathfrak{i}(-.9692)$	$-.9664 + \mathfrak{i}(-.9319)$
	$.9499 + \mathfrak{i}.9866$	$.9886 + \mathfrak{i}.9486$
	$-.0707 + \mathfrak{i}(-.0979)$	$-.0678 + \mathfrak{i}(-.0941)$
	$.1269 + \mathfrak{i}.0972]$	$.1220 + \mathfrak{i}.0934]$
$\mathfrak{L}_4$	$[[\mathfrak{T}_{.0119}, \mathfrak{T}_{.4079}]$	$[[\mathfrak{T}_{.0103}, \mathfrak{T}_{.4583}]$
	$.0709 + \mathfrak{i}.1269$	$.0680 + \mathfrak{i}.1216$
	$-.8058 + \mathfrak{i}(-.8786)$	$-.8387 + \mathfrak{i}(-.9167)$
	$.9052 + \mathfrak{i}.7442$	$.9436 + \mathfrak{i}.7752$
	$-.1263 + \mathfrak{i}(-.0703)$	$-.1211 + \mathfrak{i}(-.0635)$
	$.0963 + \mathfrak{i}.1211]$	$.0935 + \mathfrak{i}.1220]$
$\mathfrak{L}_5$	$[[\mathfrak{T}_{.2173}, \mathfrak{T}_{.5011}]$	$[[\mathfrak{T}_{.2035}, \mathfrak{T}_{.6055}]$
	$.0417 + \mathfrak{i}.1625$	$.0519 + \mathfrak{i}.0337$
	$-.8055 + \mathfrak{i}(-.9500)$	$-.8378 + \mathfrak{i}(-.9890)$
	$.8045 + \mathfrak{i}.8465$	$.8350 + \mathfrak{i}.8822$
	$-.0701 + \mathfrak{i}(-.8045)$	$-.0606 + \mathfrak{i}(-.8357)$
	$.1265 + \mathfrak{i}.0701]$	$.1198 + \mathfrak{i}.0606]$

Table 3: Aggregated matrix

	CBCDVFULPA operator	CBCDVFULPG operator
$\mathfrak{L}_1$	.01370,.01823,.01246,.01696,.01341	.0337,.0027,.00311,.00254,.0334
$\mathfrak{L}_2$	.01300,.02041,.02879,.01946,.01369	.03300,.00541, .00379,.00440,.03869
$\mathfrak{L}_3$	.02775,.03648,.02224,.003358,.02184	.04775,.02148,.0026,.01164,.00316
$\mathfrak{L}_4$	.01509,.01780,.02509,.01632,.01141	.03509,.0028,.00009,.00132,.03641
$\mathfrak{L}_5$	.03187,.03974,.03049,.04182,.03185	.05187,.0247,.00549,.02682,.05685

Table 4: Distance measures

	CBCDVFULPA operator	CBCDVFULPG operator
$\mathfrak{L}_1$	.01495	.01966
$\mathfrak{L}_2$	.01907	.01707
$\mathfrak{L}_3$	.02233	.02301
$\mathfrak{L}_4$	.01714	.014742
$\mathfrak{L}_5$	.0351	.03314

Table 5: Appraisal value

	Ranking	Best optimal
CBCDVFULPA operator	$\mathfrak{L}_5 \geqslant \mathfrak{L}_3 \geqslant \mathfrak{L}_2 \geqslant \mathfrak{L}_4 \geqslant \mathfrak{L}_1$	$\mathfrak{L}_5$
CBCDVFULPG operator	$\mathfrak{L}_5 \geqslant \mathfrak{L}_3 \geqslant \mathfrak{L}_1 \geqslant \mathfrak{L}_2 \geqslant \mathfrak{L}_4$	$\mathfrak{L}_5$

Table 6: Ranking CBCDVFULSs

Method	Score value	Ranking
Bi et al.[27]	$\times$	Not possible
Mahmood et al.[26]	$\times$	Not possible
Jana et al.[25]	$\times$	Not possible
Hu et al.[24]	$\times$	Not possible
CBCDVFULPA	0.01495,0.01907, 0.02233, 0.01714,0.0351	$\mathfrak{L}_5 \geqslant \mathfrak{L}_3 \geqslant \mathfrak{L}_2 \geqslant \mathfrak{L}_4 \geqslant \mathfrak{L}_1$
CBCDVFULPG	0.01966, 0.01707, 0.02301,0.14742, 0.03314	$\mathfrak{L}_5 \geqslant \mathfrak{L}_3 \geqslant \mathfrak{L}_1 \geqslant \mathfrak{L}_2 \geqslant \mathfrak{L}_4$

Table 7: Comparative Analysis

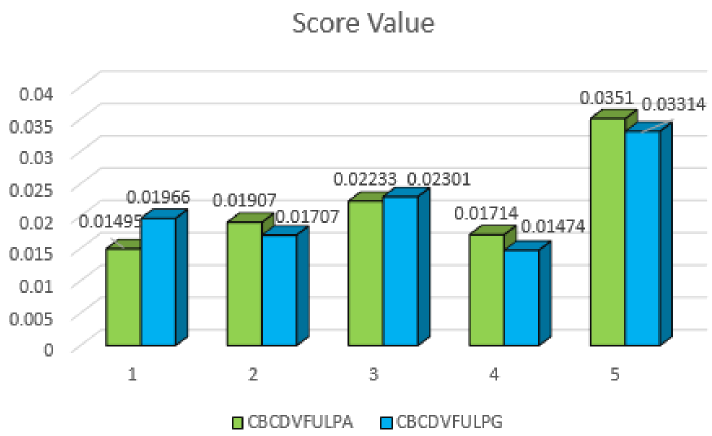

Figure 2: Final ranking using score values.

The algorithm identified Message $\mathfrak{L}_5$, which contains an informal reminder from an important contact, as having the highest priority score based on simulated input data. This finding suggests that, in spite of the message's informal tone, the user found it more meaningful in the context because of its relevance and sender importance. This method makes sure that crucial messages are cleverly recognized and emphasized, which improves the user experience. Such AI-driven fuzzy systems offer scalable and customized communication management solutions as messaging environments become more complex.

6.1 Comparative Analysis

The usefulness and resilience of the suggested CBCDVFULS based MABAC method were evaluated by comparing it against a number of other decision-making techniques. Although these techniques are good at managing simple ambiguity, they frequently fail to capture complicated, circular, and bipolar linguistic information that is present in actual group communication situations. The suggested approach offers a more realistic and expressive modeling framework by combining complex-valued uncertainty, bipolarity, and circularity. Additionally, flexible information fusion is made possible by the employment of Frank power aggregation operators, improving the precision and interpretability of the decision results. The suggested approach captured both the positive and negative language included in the group assessments and produced more consistent and nuanced rankings in the context of WhatsApp group message priority. The comparison findings clearly show that the suggested method offers a more human-aligned decision-making framework and a notable improvement in managing multi-dimensional uncertainty.

7 Conclusion

CBCDVFULSs together with Frank power aggregation operators inside the MABAC approach were used in this study to propose an advanced decision-making framework. The method works well for ranking messages in group communication platforms because it can handle complicated, bipolar, and linguistically unclear material. We showed that the suggested approach provides a reliable, consistent, and interpretable rating mechanism by using the WhatsApp group message prioritization scenario. This approach captures the complex decisions made in real-world conversational dynamics. Future studies can improve the suggested framework by adding sentiment analysis from in the moment group discussions and dynamic language feedback. As user preferences and interaction patterns change, this would enable adaptive message priority. Furthermore, the CBCDVFULS based model's weight assignment procedure may be further automated by using machine learning approaches. The models generalizability across many contexts may also be confirmed by extending the application to other group communication platforms, such as Microsoft Teams or Slack.

References

[1] Zadeh, L.A. Fuzzy sets. *Information and Control* **1965**, *8*(3), 338–353.

[2] Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems* **1986**, *20*(1), 87–96.

[3] Abid, M.N., Yang, M.S., Karamti, H., Ullah, K., Pamucar, D. Similarity measures based on T-spherical fuzzy information with applications to pattern recognition and decision making. *Symmetry* **2022**, *14*(2), 410.

[4] Kasimoglu, F., Bayeg, S., Pinar, A., Utku, D.H. A trapezoidal intuitionistic fuzzy optimization approach for crashing a budget-constrained project. *Ain Shams Engineering Journal* **2024**, *15*(4), 102590.

[5] Khan, M.J., Kumam, P., Liu, P., Kumam, W., Ashraf, S. A novel approach to generalized intuitionistic fuzzy soft sets and its application in decision support system. *Mathematics* **2019**, *7*(8), 742.

[6] Ruspini, E.H., Bezdek, J.C., Keller, J.M. Fuzzy clustering: a historical perspective. *IEEE Computational Intelligence Magazine* **2019**, *14*(1), 45–55.

[7] Zheng, J., Dong, M. Interval-valued intuitionistic fuzzy multi-attribute decision-making based on entropy and bidirectional projection. *International Journal of Computational Intelligence Systems* **2025**, *18*, 39.

[8] Ramot, D., Milo, R., Friedman, M., Kandel, A. Complex fuzzy sets. *IEEE Transactions on Fuzzy Systems* **2002**, *10*(2), 171–186.

[9] Alkouri, A.M.D.J.S., Salleh, A.R. Complex intuitionistic fuzzy sets. In *AIP Conference Proceedings* **2012**, *1482*(1), 464–470. American Institute of Physics.

[10] Ali, J., Naeem, M., Al-Kenani, A.N. Complex T-spherical fuzzy Frank aggregation operators and their application to decision making. *IEEE Access* **2023**, *11*, 88971–89023.

[11] Azeem, M., Ali, J. Complex Fermatean fuzzy partitioned Maclaurin symmetric mean operators and their application to hostel site selection. *Opsearch* **2024**.

[12] Zhang, W.R. Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multiagent decision analysis. In *NAFIPS/IFIS/NASA'94, Proceedings of the First International Joint Conference of the North American Fuzzy Information Processing Society Biannual Conference, The Industrial Fuzzy Control and Intelligence*, IEEE, **1994**, pp. 305–309.

[13] Zhang, W.R. From equilibrium-based business intelligence to information conservational quantum-fuzzy cryptography—A cellular transformation of bipolar fuzzy sets to quantum intelligence machinery. *IEEE Transactions on Fuzzy Systems* **2018**, *26*, 656–669.

[14] Sharma, S.K. Enhancing stock portfolio selection with trapezoidal bipolar fuzzy VIKOR technique with Boruta-GA hybrid optimization model: a multicriteria decision-making approach. *International Journal of Computational Intelligence Systems* **2025**, *18*, 17.

[15] Riaz, M., Habib, A., Saqlain, M., Yang, M.S. Cubic bipolar fuzzy VIKOR method using new distance and entropy measures and Einstein averaging aggregation operators with application to renewable energy. *International Journal of Fuzzy Systems* **2023**, *25*(2), 510–543.

[16] Mahmood, T., de Rehman, U. A novel approach towards bipolar complex fuzzy sets and their applications in generalized similarity measures. *International Journal of Intelligent Systems* **2022**, *37*(1), 535–567.

[17] Khan, M.J., Kumam, W., Alreshidi, N.A. Divergence measures for circular intuitionistic fuzzy sets and their applications. *Engineering Applications of Artificial Intelligence* **2022**, *116*, 105455.

[18] Atanassov, K.T. Circular intuitionistic fuzzy sets. *Journal of Intelligent & Fuzzy Systems* **2020**, *39*(5), 5981–5986.

[19] Chen, T.Y. A circular intuitionistic fuzzy evaluation method based on distances from the average solution to support multiple criteria intelligent decisions. *Engineering Applications of Artificial Intelligence* **2023**, *117*, 105499.

[20] Xu, Z. Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment. *Information Sciences* **2004**, *168*(1–4), 171–184.

[21] Gao, H., Wu, J., Wei, C., Wei, G. MADM method with interval-valued bipolar uncertain linguistic information for evaluating the computer network security. *IEEE Access* **2019**, *7*, 151506–151524.

[22] Mahmood, T., Rehman, U.U., Ali, Z., Aslam, M., Chinram, R. Decision-making approach based on bipolar complex fuzzy uncertain linguistic aggregation operators. *Mathematics* **2024**, *12*(5), 789.

[23] Lan, J., Wu, J., Guo, Y., Wei, C., Wei, G., Gao, H. CODAS methods for multiple attribute group decision making with interval-valued bipolar uncertain linguistic information and their application to risk assessment of Chinese enterprises' overseas mergers and acquisitions. *Economic Research—Ekonomska Istraživanja* **2021**, *34*(1), 3166–3182.

[24] Hu, B., Bi, L., Dai, S. Complex fuzzy power aggregation operators. *Mathematical Problems in Engineering* **2019**, 1–7.

[25] Jana, C., Pal, M., Wang, J.Q. Bipolar fuzzy Dombi prioritized aggregation operators in multiple attribute decision making. *Soft Computing* **2020**, *24*, 3631–3646.

[26] Mahmood, T., ur Rehman, U., Ali, Z., Aslam, M., Chinram, R. Identification and classification of aggregation operators using bipolar complex fuzzy settings and their application in decision support systems. *Mathematics* **2022**, *10*(10), 1726.

[27] Bi, L., Dai, S., Hu, B., Li, S. Complex fuzzy arithmetic aggregation operators. *Journal of Intelligent & Fuzzy Systems* **2019**, *36*(3), 2765–2771.

[28] Ali, Z., Yang, M.S. Analysis of renewable energies based on circular bipolar complex intuitionistic fuzzy linguistic information with Frank power aggregation operators and MABAC model. *Soft Computing* **2025**.

[29] Frank, M.J. On the simultaneous associativity of $(F(x, y))$ and $(x + y - F(x, y))$. *Aequationes Mathematicae* **1978**, *18*(1–2), 266–267.

[30] Frank, M.J. On the simultaneous associativity of $(F(x, y))$ and $(x + y - F(x, y))$. *Aequationes Mathematicae* **1979**, *19*, 194–226.

[31] Alhassan, A. R., Abubakar, M. A., and Alhassan, A. M. "A novel approach for decision-making using fuzzy logic and multi-criteria optimization. *Journal of Applied Mathematics and Computation* **2023**, *5*, 123-134.

[32] Kaviyarasu, M.; Angel, J.; Alqahtani, M. Geometric Accumulation Operators of Dombi Weighted Trapezoidal-Valued Fermatean Fuzzy Numbers with Multi-Attribute Group Decision Making. *Symmetry*, **2025**, 17, 1114. https://doi.org/10.3390/sym17071114