Modeling Financial Uncertainty Using Neutrosophic Ram Awadh Distribution: An Application to Future Economic Growth

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Abstract

Ram Awadh (RA) distribution is flexible to handle skewedness and heavy tailed observations, which are frequent in financial risk management. With flexible structure, it has potential to be a reliable model in financial data modeling and decision-making process in the scenarios of indeterminacy. The new one parameter lifetime distribution is proposed and called as the neutrosophic RA distribution (RAD_N) in this article. We obtain the raw and central moments of it and investigate some important statistical properties such as the coefficient of variation, skewness, kurtosis and index of dispersion. Moreover, some reliability properties such as the hazard rate function mean residual life function, and stochastic orderings of the distribution are considered. The method of maximum likelihood estimation (MLE) is utilized for parameter estimation. A comprehensive simulation study is carried out to evaluate the behavior of the distribution and its statistical properties. Finally, a real-world dataset of economic sector is utilized to illustrate its practical importance.

Keywords: Skewed distribution; Financial model; Neutrosophic probability; Estimation

1. Introduction

Statistical distributions are crucial in financial data modeling, where they form the basis of sampling, forecasting, storage and handling of uncertainty in real world economic and financial systems [1]. Financial market data typically reflects features like skewness, heavy tails, volatility clustering and sudden regime shifts, aspects that defy conventional modeling schemes [2]. Through suitable choice of probability distributions, analysts can provide a succinct description of the asymptotic behavior of variables such as asset returns, interest rates, exchange rates and factor risks [3]. These distributions can then be used to calculate risk measures, including value at risk (VaR), expected shortfall (ES), and other tail-risk metrics that are important for trading strategies and regulatory compliance [4]. For instance, the normal distribution, mathematically congenial as it may be, does not account for the large shocks we see every now and then in real financial exchanges [5]. The statistical distributions are extensively used for time series modeling and fitting of volatility models like GARCH and their extensions, as well as the stochastic simulation of future asset prices in financial econometrics [6]. It is essential to have true distributional assumptions in order to rely on inferential, hypothesis testing, and parameter estimation methods [7]. In addition, in option pricing and derivative valuations, distributions determine the probability of future price movement, and such probability determines the option pricing formulas from models like the Black-Scholes (which assumes the nature of returns to be in lognormal) family of models [8]. In addition, distributions also serve as a central element in portfolio optimization, which describe the joint behavior of the returns on assets and help to determine allocation strategies for investments subject to risk constraints [9]. For credit risk modeling, the probability of the default and loss given default are modelled using adequate distributions in order to price the exchange of default linked debt instruments and to control the financial stability of institutions [10].Moreover, given the rise of big data and machine learning in finance, it is crucial for model selection, evaluation, and

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interpretability to know the underlying distribution of features. The distribution-based methods are used with increasing frequency in stress testing, fraud detection, algorithmic trading, and economic forecasting. In such scenarios, by capturing outliers, volatility, or structure breaks, distributions would help to make decisions under uncertainty [11-12]. The development of hybrid and generalized distributions, possibly including fuzzy logic, neutrosophic or entropy-related traits, has additionally improved the ability towards modeling ambiguous and imprecise financial data [13]. Such methods offer more powerful means for studying market patterns in uncertain, incomplete or conflicting knowledge disorders. The key point of the previous discussion is that statistical distributions are not just mathematical tools, but they also play a key role in representing the richness and complex nature of financial systems, in enhancing the efficacy of prediction, in risk management and in supporting rational economic and financial decisions.

Neutrosophic distributions on other hand are an effective generalization of classical probability distributions, which describe uncertainty more flexibly and realistically by considering truth, indeterminacy, and falsehood [14-16]. In financial data modeling, where information is usually vague, uncertain, imprecise, contradictory, and incomplete or with lack of information, the neutrosophic distributions provide a considerable advantage [17]. They permit the analyst to capture not only randomness but also vagueness and ambiguity, which often occur in economic systems driven by human behavior, political outcomes, and market psychology [18]. Standard statistical models, providing poor forecasts and biased estimates of uncertainty, can miss these subtleties. Invoking neutrosophic logic in distribution theory can allow researchers to be more accurate in dealing with complex phenomena comprising irregular changes of the market, uncertain income from investments and ambiguous credit ranking. As a result, neutrosophic distributions are highly applicable in such fields as risk management, portfolio optimization and economic decision in a circumstance of uncertainty [19]. Their capacity for generalizing classical models and to accommodate multiple sources of uncertainty makes them a key instrument in contemporary financial analytics, as they represent an extension of the descriptive and predictive strength of financial models [20].

The RA distribution is a new model with applications to lifetime and reliability data with economic applications [21]. It has a mathematical form that can describe skewed and heavy-tailed data (such as the ones we experience in economic indicators, like GDP growth, inflation rates or investment returns). Better fits real world financial data beyond conventional symmetric distributions [22]. Use the RA distribution and the improved distribution to help better understand the underlying economic dynamics and assess related risk [23]. For the prediction of future economic development, this distribution can well capture irregular trajectories and abrupt jumps and, therefore, provides better estimate accuracy and reliability. With its flexibility, the proposed model is very useful in various applications in which economic data is affected by various uncertain and nonlinear factors. Consequently, the RA distribution is a useful tool in the development of predictive modelling and economic planning.

To the best of the authors' knowledge, the neutrosophic structure of RA distribution has not been previously presented in the literature. As a new and robust framework for addressing the intrinsic uncertainties in the modeling of future economic growth and the analysis of financial data, the proposed model has its important theoretical value and practical significance. It incorporates neutrosophic logic to present a broader description of indeterminacy, vagueness and incomplete information in financial environments.

The rest of the study is organized as follows. Section 2 presents the classical orientation of the proposed model. Section 3 introduces the neutrosophic structure of RA distribution and its neutrosophic characteristics. Section 4 estimation method contains estimation method. Section 5 illustrates the real application of the proposed method. Findings of the work are concluded in Section 6.

2. Classical Ram Awadh Distribution

A new one-parameter lifetime distribution, characterized by the parameter β is defined through its probability density function (PDF).

$$f(t;\beta) = \frac{\beta^6}{\beta^6+120}(\beta + t^5)e^{-\beta t}\,; t > 0, \beta > 0. \tag{1}$$

The PDF in Eq (1) is known as the Ram Awadh Distribution. It is a combination of two well-known distributions known as an exponential distribution and a gamma distribution. Shape of the PDF for different values of scale parameter β is shown in Figure 1.

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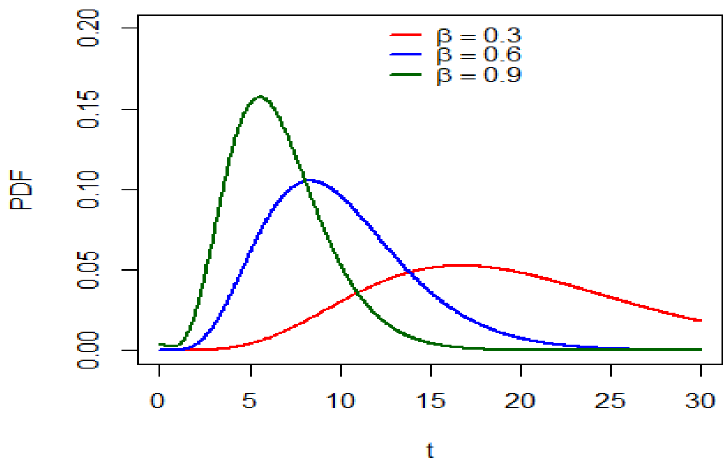

Figure 1. PDF curves of crisp RA distribution with different shape parameter values

The PDF of the RA distribution for three values of its parameters is illustrated in Figure 1. The horizontal axis is crisp random variable, and the vertical axis represents the probability density values. The three curves are red plot for 0.3, blue plot for 0.6 and green plot that corresponds to parameter value being 0.9 respectively. The top legend also indicates the value of each parameter corresponding to the curve. This plot shows how changing the scale parameter changes the shape and spread of the distribution. Next associate function is cumulative distribution function (CDF). The CDF is quite useful, because it gives you the probability that a random variable is less than or equal to some particular point. This gives us a sense of how the data is distributed in general, some random variables and their probabilities. CDF plays a crucial role as well in statistical inference, simulation, hypothesis testing, and decision-making, in applied sciences. The CDF of RA distribution is given by:

$$\mathcal{F}(t;\,\beta) = 1 - \left[1 + \frac{\beta t(\beta^4 t^4 + 5\beta^3 t^3 + 20\beta^2 t^2 + 60\beta t + 120)}{\beta^6 + 120}\right] e^{-\beta t} \;\; ; t > 0, \beta > 0 \qquad (2)$$

The CDF plot of the RA distribution is shown in Figure 2.

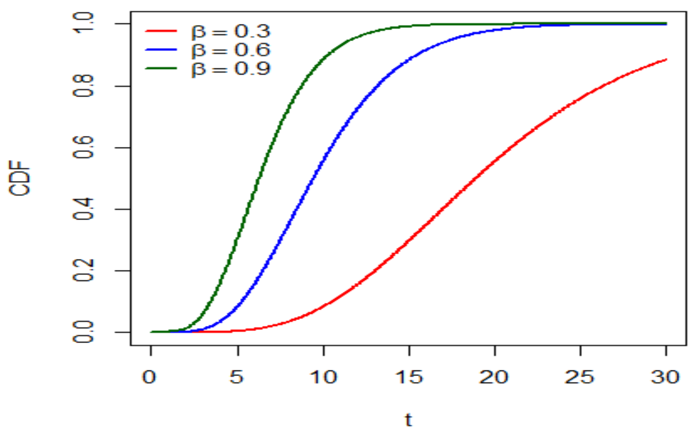

Figure 2. CDF curves of crisp RA distribution with different shape parameter values

Figure 2 represents CDF curves for the RA distribution with some specific values of the scale parameter values. Figure 2 has the crisp random variable on the horizontal axis and shown the cumulative probability which was referred from 0 to 1 in vertical axes. The red, blue and green curves are for the parameter values 0.3, 0.6 and 0.9 respectively. Every curve also starts at 0, grows linearly and approaches 1. The legend labels the curves according to their parameter values indicating a different rate at which the cumulative probability goes up with different settings of the distribution

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The mean and variance of the RA distribution can easily be derived as:

$$\text{Mean}=E(T) = \int_0^{\infty} tf(t,\beta)dt = \frac{\beta^6+720}{(\beta^6+120)\beta} \quad (3)$$

$$\text{Variance}=\frac{86400+3840\beta^6+\beta^{12}}{(\beta^6+120)^2\beta^2} \quad (4)$$

Similarly higher moments about origin can be established to find the expressions from the rth moment expression of the distribution. The (central moments) are the most important expectation values and can describe the distribution well. The average, or center-value, showing the predominant characteristic of the values, is given by this first moment. The second moment describes the spreading or variance (or, alternately, the degree to which the values differ from their means). The third moment accounts for the skewness, that is, how much the distribution tilts left or right. The fourth moment measures the peak or the flatness (referred to as kurtosis), which reflects the form of the distribution with regard to that of the normal curve. As a pair, these moments provide a complete description of the behavior of the distribution and are used heavily in statistics, data analysis, and decision theory.

The primary aim of this paper is to introduce a new lifetime distribution that offers greater flexibility compared to existing one-parameter distributions proposed in the literature. The proposed neutrosophic form of the RA distribution is described in the next section. The proposed RAD_N describe fully with its characteristics in the following distribution

3. Neutrosophic Ram Awadh Model

This section presents the mathematical principles underlying the proposed model and organizes them in a logical manner. A random variable t_N is said to follow a (RAD_N) with parameter β_n if its probability density function (pdf) is defined as follows:

$$f_n(t;\beta_n) = \frac{\beta_n}{\beta_n+120}(\beta_n + t^5)e^{-\beta_n t}; t > 0, \beta_n > 0. \quad (5)$$

The RAD_N is an extension of the original RA model for which the parameter is represented as an interval rather than crisp. This helps ensure that the distribution can better capture uncertainty, indeterminacy, and imprecision which are often the case in real-world data. It's a probability density function because it summarizes the likelihood of observing a value drawn from the random variable, but instead of being based on a specific value of the parameter, it's covering a certain range of parameters within the interval. This makes the neutrosophic variant more flexible and adequate for representing situations in which there is incomplete or vague information, which provides a more general picture of probability behavior under uncertainty. The basic plot of neutrosophic PDF is shown in Figure 3.

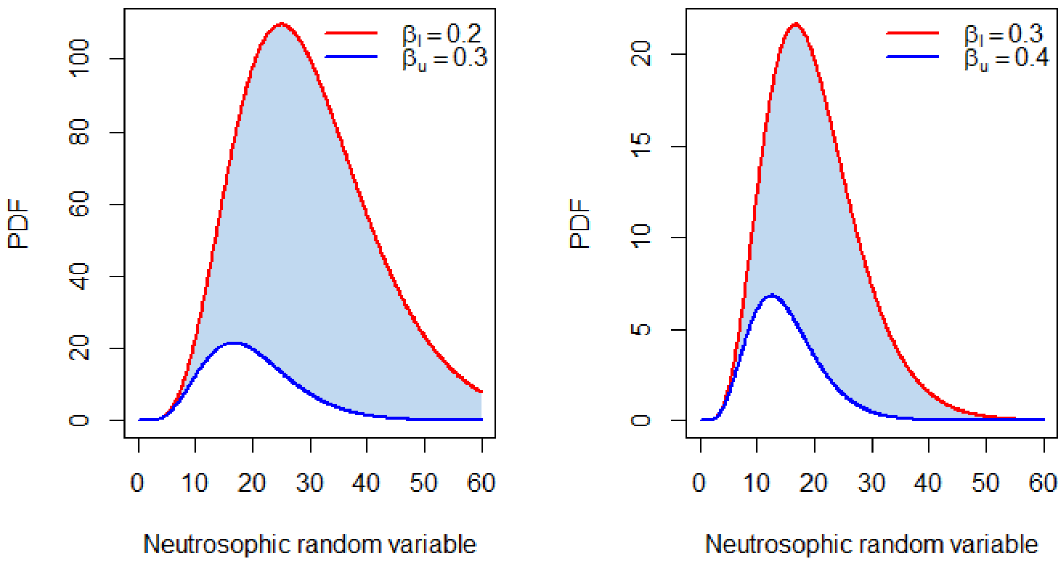

Figure 3. Neutrosophic PDF curves of RAD_N

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The subplots in Figure 3 illustrate the RAD_N for two ranges of its parameter respectively. The lower bound and upper bound curves are depicted in red and blue, respectively in each panel; while the shaded region denotes the neutrosophic area, which describes the uncertainty between these bounds. The neutrosophic random variable is given on the horizontal axis and the probability density on the vertical axis. It is apparent from the plots how the distribution varies as the parameter varies within a given range that shows the significance of using neutrosophic modeling to describe data in which imprecision or vagueness is involved.

The other function associated with PDF is CDF, which can be represented under neutrosophic logic as:

$$\mathcal{F}_n(t;\beta_n) = 1 - \left[1 + \frac{\beta_n t(\beta_n^4 t^4 + 5\beta_n^3 t^3 + 20\beta_n^2 t^2 + 60\beta_n t + 120)}{\beta_n^6 + 120}\right] e^{-\beta_n t} \;; t > 0, \beta_n > 0 \qquad (6)$$

The neutrosophic CDF of proposed distribution generalizes the classical form by assuming that the parameter can assume interval values as opposed to constant. This extension allows the distribution to model uncertainty, indeterminacy, and vagueness found in real data. The CDF is the cumulative probability that the neutrosophic random variable has values up to a given value. The neutrosophic CDF has a range rather than a single curve, when by introducing interval parameters, which represents the spread of the uncertainty in the system. This makes it especially applicable in cases where data or parameter values cannot be known exactly and instead are characterized by bounds. The shape of neutrosophic CDF is shown in Figure 4.

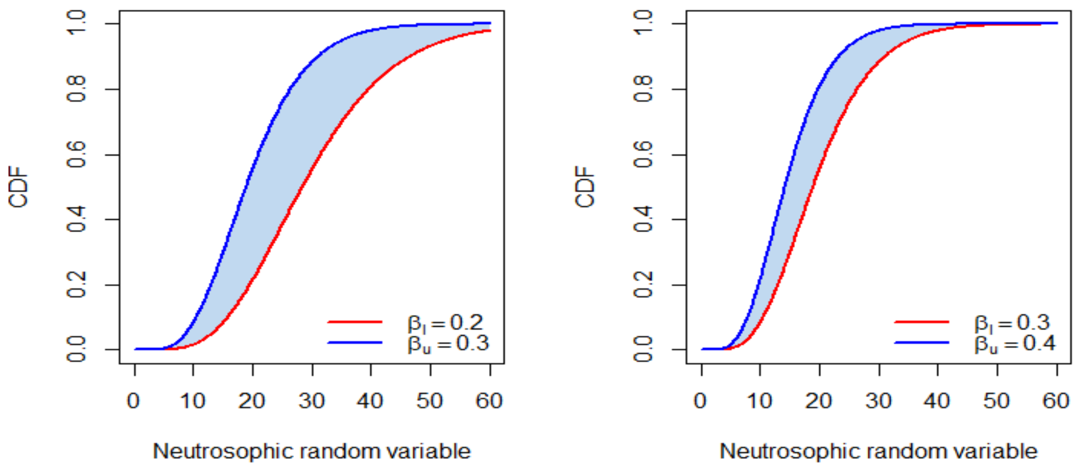

Figure 4. Neutrosophic CDF curves of RAD_N

Figure 4 represents the Neutrosophic cumulative distribution function of RA distribution for two ranges of parameters. Each panel depicts the accumulated probability curves for the lower and upper values of the parameter, in red and blue, and their shaded area in between is the neutrosophic region of uncertainty. Moreover, the cumulative probability is plotted along the vertical axis that values from zero to one. The plots show more clearly imprecision and indeterminacy in the data: we do not know the parameter, but we know a range for the cumulative behaviors. Now if we write the general expression for jth moment, it becomes easy to derive its higher moment's coefficients.

The j^{th} moment concerning the origins of the (RAD_N) may be retrieved as:

$$\mu_{jn} = E(T^j) = \int_0^\infty t^j f(t,\beta)dt = \frac{j![\beta_n^6 + (j+1)(j+2)(j+3)(j+4)(j+5)]}{\beta_n^j(\beta_n^6+120)} \;; j = 1,2,3, \ldots \qquad (7)$$

Thus, the first four moments concerning the formation of the (RAD_N) are given by

$$\acute{\mu}_{1n} = \frac{\beta_n^6 + 720}{\beta_n(\beta_n^6+120)}$$

$$\acute{\mu}_{2n} = \frac{2(\beta_n^6 + 2520)}{\beta_n^2(\beta_n^6+120)}$$

$$\acute{\mu}_{3n} = \frac{6(\beta_n^6 + 6720)}{\beta_n^3(\beta_n^6+120)}$$

$$\acute{\mu}_{4n} = \frac{24(\beta_n^6 + 15120)}{\beta_n^4(\beta_n^6+120)}$$

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Now using the following relationships, we can find moments about mean which are also called central moments.

$$\mu_{1n} = 0$$

$$\mu_{2n} = \mu'_{2n} - (\mu'_{1n})^2$$

$$\mu_{3n} = \mu'_{3n} - 3\mu'_{2n}\mu'_{1n} + 2(\mu'_{1n})^3$$

$$\mu_{4n} = \mu'_{4n} - 4\mu'_{3n}\mu'_{1n} + 6\mu'_{2n}(\mu'_{1n})^2 - 3(\mu'_{1n})^4$$

The central moments of the (RAD_N) are calculated as follows:

$$\mu_{2n} = \frac{(\beta_n^{12}+3840\beta_n^6+86400)}{\beta_n^2(\beta_n^6+120)^2}$$

$$\mu_{3n} = \frac{2(\beta_n^{18}+12960\beta_n^{12}-172800\beta_n^6+1036800)}{\beta_n^3(\beta_n^6+120)^3}$$

$$\mu_{4n} = \frac{9(\beta_n^{24}+25280\beta_n^{18}+2054400\beta_n^{12}+271872000\beta_n^6+3317760000)}{\beta_n^4(\beta_n^6+120)^4}$$

Now the coefficient of variation (CV) can be expressed as:

$$CV = \frac{\sigma}{\mu_{1n}} = \frac{\sqrt{(\beta_n^{12}+3840\beta_n^6+86400)}}{(\beta_n^6+720)} \tag{8}$$

The CV is a statistic, which can be defined as the ratio of the standard deviation to the mean of the set of data. In a neutrosophic environment when data may be fuzzy, indeterminate or interval-valued, the CV may provide a powerful tool to investigate the consistency or the extent of spread of the data taking into consideration the indeterminacy. It enables users to compare variation in datasets or random variables by normalizing the measure of uncertainty between them, even if the absolute scales of the two are different. This measure can formalize the intrinsic uncertainty and the imprecision by using the neutrosophic approach, and it is an effective tool for solving decision-making, risk assessment and statistical applications when precise information is not easily available.

In neutrosophic form, skewness and kutosis coefficients using above moments can be expressed as follows:

$$Skewness = \frac{\mu_{3n}}{\mu_{2n}} = \frac{2(\beta_n^{18}+\beta_n^{12}12960-172800\beta_n^6+1036800)}{(\beta_n^{12}+3840\beta_n^6+86400)^{3/2}} \tag{9}$$

$$Kurtosis = \frac{\mu_{4n}}{\mu_{2n}} = \frac{9(\beta_n^{24}+25280\beta_n^{18}+2054400\beta_n^{12}+271872000\beta_n^6+3317760000)}{(\beta_n^{12}+3840\beta_n^6+86400)^4} \tag{10}$$

Skewness and kurtosis are statistical properties that reflect some characteristics of the distribution shape other than the mean and the variance. Skewness measures the amount of asymmetry of the distribution of data and reveals whether the values are concentrated on one side of mean more than other is, which in turn may act as a hint to detect a biasness or a trend in data. Kurtosis is a measure of the sharpness or flatness of the distribution indicating whether data have heavy tails or center more around the mean compared to the normal distribution. In combination, these coefficients offer superior detailed insights into data properties and are useful for improved modeling and risk assessment and decision making, for instance, when deviations from the normal could affect significantly the conclusions.

The proposed distribution also has important implications in reliability theory. The reliability hazard rate function of the proposed model can be expressed as:

$$h_n(t) = \frac{\beta_n^6(\beta_n+t^5)}{(\beta_n^5t^5+5\beta_n^4t^4+20\beta_n^3t^3+60\beta_n^2t^2+120\beta_nt+\beta_n^6+120)} \tag{11}$$

Mean residual function of the proposed model can be expressed as:

$$MR_n(t) = \frac{(\beta_n^5t^5+10\beta_n^4t^4+60\beta_n^3t^3++240\beta_n^2t^2+600\beta_nt+\beta_n^6+720)}{\beta_n(\beta_n^5t^5+5\beta_n^4t^4+20\beta_n^3t^3++60\beta_n^2t^2+120\beta_nt+\beta_n^6+120)} \tag{12}$$

The hazard rate function and the mean residual life function are two features of the behavior of lifetimes or durations that are opposed to one another. The hazard rate measures the instantaneous risk or probability of failure at a particular time, given that the subject has made it through to that time. On the other hand, the mean residual life function measures the expected time that will occur before the lifespan under questioning, given that life has

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been observed for some time. While the hazard pays focus on the immediate odds of failure, the mean residual life reflects the average remaining lifetime. They both provide a much clearer understanding of reliability, risk, or survival behavior in varied applications. Now assuming different values of neutrosophic parameter of the proposed model, statistical characteristics are shown in Table 1.

Table 1: Statistical characteristics of RAD_N for different values of neutrosophic parameter

β_n	Neutrosophic Mean	Neutrosophic Variance	Neutrosophic Skewness	Neutrosophic Kurtosis
[0.1,0.25]	[24.000,60.000]	[96.001,600.000]	[0.082,0.082]	[3.999,4.000]
[0.2,0.3]	[19.999,30.000]	[66.668,150.000]	[0.082,0.082]	[3.999,4.000]
[0.3,0.45]	[13.333,19.999]	[29.636,66.668]	[0.081,0.082]	[3.999,3.999]
[0.5,0.7]	[8.564,11.999]	[12.285,24.010]	[0.079,0.081]	[3.997,3.999]

The neutrosophic moments of the proposed distribution for different ranges of the parameter are tabulated in Table 1. Table 1 contains, for each interval, the lower and upper bounds of the ranges of the mean, variance, skewness, and kurtosis, thus taking into account the uncertainty in the parameter. Neutrosophic range is used to list the lower and upper bounds for each statistic together such that readers can see both the central tendency, spread and shape properties of a distribution for imprecise or interval-based parameter settings. This gives the user an overall picture of how the variability and uncertainty of the parameter influences the behavior of the distribution.

4. Estimation Method

The maximum likelihood estimation (MLE) is a popular statistical method for estimating the unknown parameters of a probability distribution by maximizing the likelihood function, which quantifies how effectively the assumed model accounts for the observation. The goal is to pick values of the parameters so that the data we have observed is most likely to be observed. MLE seems to be popular, cause its estimates have few nice properties given regular conditions, they are consistent (converging to the true parameter as sample size goes to infinity), asymptotically unbiased (i.e. it doesn't "systematically overestimate or underestimate anything") and efficient (they reach the lowest possible variance). It is of primary interest because it gives a unified and adaptable approach for parameter estimation in a wide array of contexts including, but not limited to, economics, engineering, biology and machine learning, thus becoming one of the backbones of modern statistical inference. In case of our proposed distribution for the sample $t_1, t_2, \dots t_n$ the likelihood function can be written as:

$$L(\beta) = \prod_{i=1}^{n} \frac{{\beta_n}^6}{\beta^6+120}\left(\beta_n + t_i^5\right)e^{-\beta_n t_i} \tag{13}$$

The log likelihood function of Eq (13) can be expressed as:

$$\mathrm{l}(\beta_n) = \sum_{i=1}^{n}\left[\ln\left(\frac{{\beta_n}^6}{{\beta_n}^6+120}\right) + \ln\left(\beta_n + t_i^5\right) - \beta_n t_i\right] \tag{14}$$

Maximizing Eq (14) with respect to unknown parameter yielded:

$$\sum_{i=1}^{n}\left[\frac{720}{\beta_n(\beta_n^6+120)} + \frac{1}{\beta_n + t_i^5} - t_i\right] = 0 \tag{15}$$

Eq (15) cannot be solved analytically but some numerical programs written in R can be used to find the solution. Now if we generate random samples using the R package “new.dist” with set seed (123), the MLE using R program can be obtained as:

$$\hat{\beta}_n = [1.005, 1.4979] \tag{16}$$

A random data is generated using function of the “new. dist” package with known parameter value $\beta_n = [1, 1.5]$ then 100 samples are generated from RA distribution with lower and upper values of neutrosophic parameter. Thus, the neutrosophic data based on known parameter value provide reliable results as shown in Eq (15).

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5. Real Data Demonstration

In this section, proposed model has been employed on Saudi Arabia's GDP per capita based on purchasing power parity (PPP) in current international dollars. Data is taken from open public available data source [24]. Data supply annual estimates of the country's economic output per person, accounting for differences in price levels among countries. The dataset sheds light on the living standards and welfare of people in Saudi Arabia for almost two decades. It shows how local buying power in the nation has changed over the years in line with both domestic conditions and the international economy. It should be noted that this information was last released on May 30, 2024 and is just one in a global suite of PPP statistics that allow cross-country comparisons of income and living standards to be made. Even if the dataset on Saudi Arabia's the GDP per capita (PPP, current international $) from 2005 to 2024 appears to be crisp and exact in terms of numbers, it is likely to be compound with various dimensions of uncertainty in the real world. These uncertainties arise from differences in data collection and estimation methodologies, exchange rates adjustments and purchasing power parity estimates obtained from organizations such as the World Bank, OECD, Eurostat and IMF. Furthermore, national account revisions, economic shocks, policy adjustments, and global external elements such as oil price changes or geopolitics, all that add to the uncertainty that stays behind the numbers published. This means that, although the data looks precise, the interpretation of the numbers must reflect the fuzziness and the potential mismatch, between the actual measurements and the complex economic realities that these numbers reflect. To represent the inherent uncertainties in Saudi Arabia's GDP per capita data, we extend each crisp value into an interval. This is achieved by generating small random variations from uniform distribution that act as possible deviations from the reported figures. For each value in the dataset, an upper bound is obtained by slightly increasing it with a random component, (i.e., random generated samples are added in actual variable while a lower bound is obtained by subtracting sample random samples from actual data. The result is a neutrosophic representation of the data, where each observation is expressed as an interval instead of a single point, reflecting the uncertainty and imprecision present in real-world economic measurements. Data generated by this method is shown in Table 2.

:**Table 2** Neutrosophic GDP data for year 2005-2024

[53547.89, 53549.07]	[54637.95, 54639.43]	[55029.44, 55030.49]	[57235.15, 57237.03]
[54786.4, 54787.99]	[58828.66, 58829.42]	[64124.69, 64125.76]	[65032.72, 65034.21]
[62202.33, 62203.18]	[62577.61, 62578.22]	[53929.87, 53931.79]	[49936.61, 49937.97]
[53119.78, 53121.15]	[59377.23, 59378.6]	[59560.15, 59560.28]	[47517.59, 47518.97]
[62690.25, 62691.26]	[71968.03, 71968.29]	[71564.47, 71565.12]	[71242.48, 71243.66]

Now structure of data from Table 2 shows that classical model RA distribution cannot be used to describe interval values data. However, the proposed that is equipped with neutrosophic logic can be used to find neutrosophic parametric value. Based on MLE program written in R, the estimated parameter of the proposed model is given by:

$\hat{\beta}_{\mathrm{n}} =$ [8.229806e-05, 8.229806e-05]

The output of neutrosophic parameter shows the crips value in this case because of minor uncertainties assumed in the processing data. Thus, the neutrosophic model is a more general and flexible form which can reduce to existing model under zero or minor imprecision in the study parameters.

6. Conclusion

In this paper, we have proposed a new one-parameter neutrosophic RA distribution, which serve as a generalized approach for handling skewed and heavy-tailed data, which is highly prevalent in financial economic growth model under imprecise environment. Raw and central moments have been obtained, and several important statistical measures including coefficient variation, skewness, kurtosis and index of dispersion have been analyzed. Moreover, reliability notions, such as hazard rate function and mean residual life function were studied and it was shown that the distribution was sound for reliability analysis. Parameter estimation has been achieved with the MLE approach, which was evaluated by means of extensive simulation studies. The practical applicability of the suggested distribution was further evidenced with an economic real dataset of Saudi Arabia's GDP, establishing the distribution as a promising model for real data studies. In general, the RAD_N provides a powerful extension of the class of lifetime distributions and interesting perspectives for both theoretical and applied side in vague and ambiguous settings in economic data modeling.

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