Neutrosophic Extension of the Transmuted Lindley Distribution: Theory and Properties

Afrah Al Bossly[1,*]
[1]Department of Mathematics, College of Science and Humanities in Al-Kharj, Prince Sattam Bin Abdulaziz University, Al-Kharj, 11942, Saudi Arabia
Email: a.basli@psau.edu.sa

Abstract

In this paper, we introduce a new extension of the transmuted Lindley distribution (TLD) by utilizing neutrosophic logic to handle uncertainties that are often found in real life data. As classical probability models are not flexible enough for dealing with vague, imprecise, ambiguous, and incomplete information, neutrosophic theory is more general as it handles indeterminacy part associated with data. The proposed neutrosophic transmuted Lindey distribution (NTLD) combines indeterminacy concept, yielding a powerful statistical distribution, which is suitable for modeling both randomness and indeterminacy. Major functions such as probability density function (PDF), cumulative function (CDF), reliability function (RF) and hazard rate function (HRF) are established in this framework. Graphical analysis and simulated data are used to illustrate the performance of the model. Moreover, important moments such as mean, variance, skewness, and kurtosis are computed for different values of the neutrosophic parameters. The proposed distribution provides a generalized approach to model complex and uncertain data in reliability engineering, survival analysis, and decision-making. A real electricity consumption data from energy sector is utilized to show the proposed model applicability.

Keywords: Neutrosophic logic; Lindley model; Neutrosophic probability; Estimation

1. Introduction

Probability distributions are basic tools of statistical and data analysis used to mathematically define uncertainty and the variability of random processes [1]. They permit practitioners to simulate and study a wide variety of processes that occur in practice in fields including engineering, medicine, economics, and social sciences [2]. Providing a characterization of the likelihood of specific outcomes, probability distributions allow the development of predictive models, statistical inference and decision making under uncertainty [3]. In addition, they are building blocks for hypothesis test and parameter estimation and have been used widely in simulation studies, so they are critical for understanding the patterns in data and practical interpretation of empirical data [4-5]. Several classical probability distributions such as the normal, exponential, Poisson, and Gamma find applications in practice to model a wide variety of real-world phenomena, since they have well-known theoretical properties and are easy to apply [6]. Nevertheless, in reality, empirical data can behave in complex ways that is not entirely consistent with the prediction of classical models. To avoid these limitations, various generalizations of classical distributions have been considered during the last decades. These are generalized models that are meant to be more flexible and versatile than the other models in data modeling by adding parameters or structures [7]. Therefore, in recent times, there are numerous modified, transmuted, and hybrid probability distributions that increase the flexibility to model a variety of uncertain and diverse data effectively developed in reliability analysis, finance and environmental sciences in the literature [8].

In recent years, the transformed extension of probability distributions has attracted a great deal of interest in introducing other shape parameters that increase the flexibility of classical models [9]. Utilizing a transmutation scheme, statisticians are given a means to construct new families of distributions that preserve the essential features of a base distribution but are better able to fit actual data patterns. As transmuted distributions possess this extra

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degree of freedom, these can more closely follow the skewness, tail behavior as well as other anomalous behavior of real data thus proving to be highly effective in real life applications [10]. As such as reliability engineering, survival analysis, risk assessment, economics, etc., have received particular attention from the researchers interested in transmuted models, which are required to cope with such very complex, non-symmetric and fuzzy structures of data. Therefore, the transmuted extensions of distributions are effective means for more realistic modeling, better data fitting, and sound inferential procedures. The Lindley distribution, introduced in Bayesian statistics as one parameter probability density function that has been established as a useful tool in reliability, medical, and health problems as well as in modeling lifetime data and survival data [11]. The Lindley model, however, lacks flexibility once real-life complex phenomena are to be modeled. To address this deficiency, the TLD has been proposed [12]. This extension introduces an extra parameter that enables more flexibility when shaping this distribution (for example, skewness and kurtosis). TLD has better modeling flexibility, which allows it to be more flexible and customized for lifetime, medical and reliability applications [13]. However, many real-world variables are not measured in exact based information. Fuzzy and neutrosophic sciences are applied to solve the inherent vagueness, ambiguity and uncertainty associated with real world data [14]. In classical probability-based methods, it is difficult to effectively handle imprecise, inconsistent and incomplete information [15-18]. Fuzzy logic can be used to model such ambiguity when partial membership is allowed, and this becomes applicable when binary logic caters to only one of the situations in problems [19-20]. It is generalized by the neutrosophic theory that uses the values truth, indeterminacy, and falsity to characterize more aspects of uncertainty. It renders neutrosophic models particularly useful for complex systems as found in medical diagnosis, decision-making, engineering, social sciences, etc. Including different levels of uncertainty, these methods provide more flexible, realistic models when we wish to analyze data that cannot exclusively be modelled using classical statistical methods [21].

In line with the wide range of applications and the increasing relevance in the use of neutrosophic logic in dealing with indeterminacy, inconsistency, and incompleteness of information, we further extend its application to the TLD in this work. By formulating the background of transmuted mode, and neutrosophic logic into the classical transmuted Lindley modeling approach, we propose the neutrosophic transmuted Lindly (NTL) distribution as a superior model for uncertainty inference in real data problems. The derived distribution considers the inherent fuzziness and lack of clarity evident in most application occurrence scenarios, especially in fields such as reliability analysis, uncertainty in decision-making, and risk assessment. The developed approach offers superior flexibility to the classical distribution by incorporation of the neutrosophic parameters for a highly detailed representation of fuzzy phenomenon.

2. Transmuted Lindley Model

The classical structure of TLD is explained in this section. Prior to definition of Lindley distribution, it is essential to define first the transmuted distribution. A random variable is said to follow transmuted distribution if it is developed by using the transformation:

$$G(z) = (1+\delta)F(z) - \delta F^2(z), \quad |\delta| \le 1 \tag{1}$$

Where $F(z)$ is CDF of the original distribution. It is important to note that at zero value of δ distribution converts to conventional model. Moreover, the resulting transmuted distribution introduces an additional parameter δ and transmuted form is a flexible generalization technique used to modify existing probability distributions to better handle the behavior of real-world scenario. Transmuted forms retain the basic shape of the original distribution and provide additional flexibility to better meet different applications. These models have been extensively used in such areas as reliability analysis, survival studies, risk modeling, finance, etc., where the standard distributions might not be flexible enough to accommodate for observed data features.

Now the Lindley model with parameter β is defined as:

$$g(z,\beta) = \frac{\beta^2}{\beta+1}(1+z)e^{-\beta z}, \quad z>0,\ \beta>0 \tag{2}$$

The Lindley distribution is a continuous probability model widely employed to model lifetime or waiting time data. It is unimodal with one parameter, and is especially useful for positively skewed data. The form of distribution enables it to capture behaviors that are not well described by simpler models such as exponential distribution. The shape of distribution for different values of shape parameter β can be seen in Figure 1.

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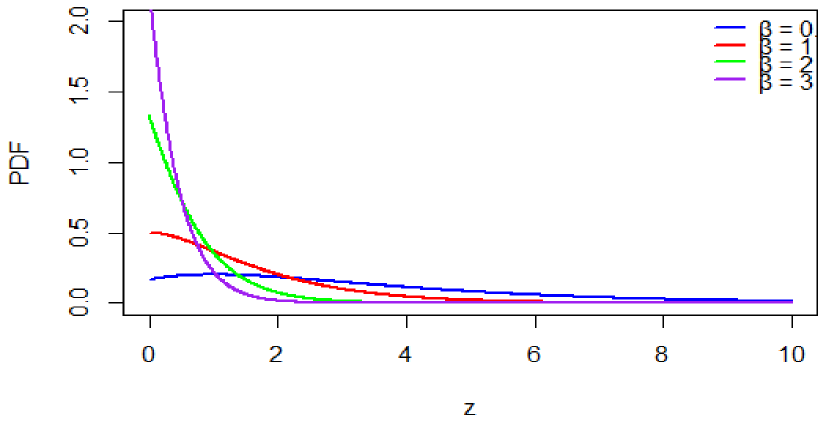

Figure 1. PDF curves of Lindely model for different values of shape parameter

Figure 1 shows the PDF of the Lindley distribution, which is increasing in the beginning, it has a modal value and after that it gradually decreases, which makes it appropriate for modeling problems in which the probability of an event to happen increases for a short period and then decreases. This flexibility has resulted in the Lindley distribution to be useful in reliability engineering, survival analysis and risk analysis. The cumulative distribution function (CDF) of the Lindley distribution can be defined as:

$$G(z) = 1 - \frac{\beta+1+\beta z}{\beta+1} e^{-\beta z}, \quad z > 0, \beta > 0 \tag{3}$$

The CDF again depends on shape parameter β . The CDF curves for different values of shape parameter values can is depicted in Figure 2.

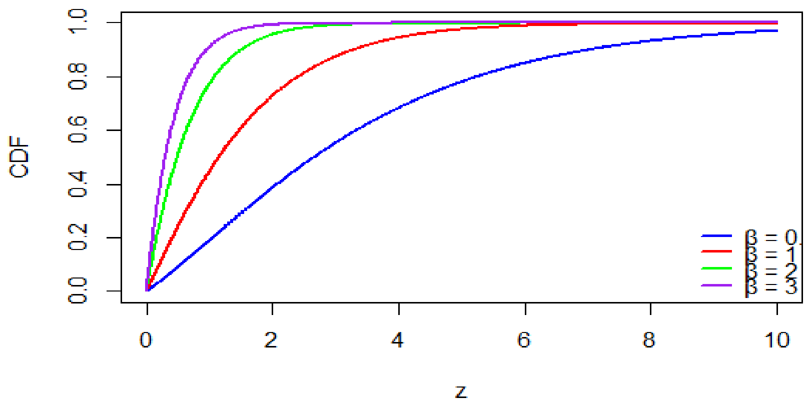

Figure 2. CDF curves of Lindely model for different values of shape parameter

Figure 2 shows the CDF of the Lindley distribution for different beta $\beta = 0.5,\ 1, 2, 3$. Indeed, the curves demonstrate distinctive rates of probability mass accumulation across β values. For lower β, the distribution accumulates slower, which means that it has a heavier tail and is more variable. On the other hand, higher β values result in steeper curves, indicating faster accumulation and smaller variability. This phenomenon is an example of how robust the Lindley distribution is in tailoring different types of skewed and non-monotonic data according to the selection of the β parameter.

Now the CDF of the transmuted model can be written as:

$$TG(z) = \left(1 - \frac{\beta+1+\beta z}{\beta+1} e^{-\beta z}\right)\left(1 + \delta \frac{\beta+1+\beta z}{\beta+1} e^{-\beta z}\right) \tag{4}$$

Based on Eq (3), the PDF of the transmuted Lindley model can be derived as :

$$Tg(z) = \frac{\beta^2}{\beta+1}(1+z)e^{-\beta z}\left(1 - \delta + 2\delta \frac{\beta+1+\beta z}{\beta+1} e^{-\beta z}\right) \tag{5}$$

The structures of PDF and CDF curves of the TLD can be seen in Figure3.

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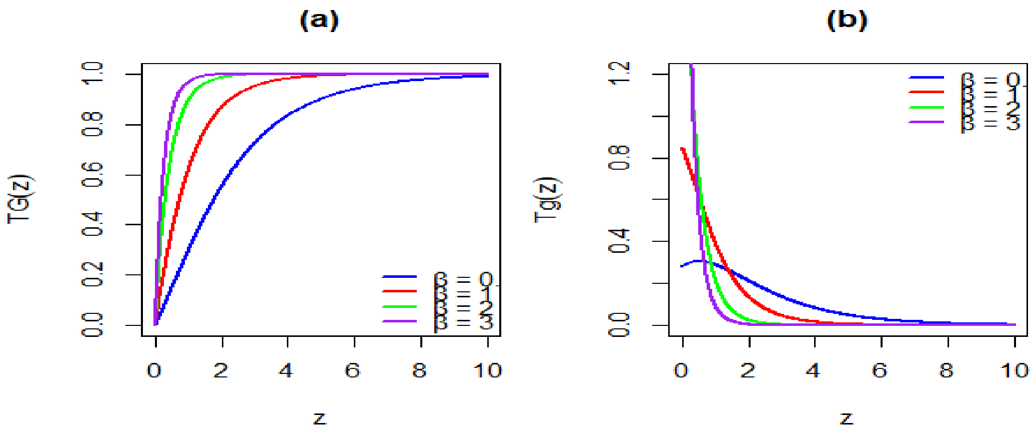

Figure 3. Transmuted Lindley distribution at transmuted parameter $\delta = 0.7$ (a) CDF (b) PDF

Figure 3 illustrates some of the possible shapes of PDF and CDF of the transmuted Lindley distribution. Figure 3 shows the TLD is an enhanced statistical model to handle more complex data due to transmuted parameters.

The mean and variance of the TLD can be established from its rth moment that can be found as:

$$E(Z^r) = \frac{r!}{\beta^r(\beta+1)}\left[(1-\delta)(\beta+r+1) + \delta \cdot \frac{\beta}{2^{r-1}(\beta+1)}(2\beta+3\beta+r)\right] \quad (6)$$

Thus, mean and variance can be established by using:

$$E(Z) = \frac{1}{\beta(\beta+1)}\left[(1-\delta)(\beta+2) + 2\delta \cdot \frac{\beta}{\beta+1}(\beta+2)\right] \quad (7)$$

$$E(Z^2) = \frac{2}{\beta^2(\beta+1)}\left[(1-\delta)(\beta+3) + \delta \cdot \frac{\beta}{\beta+1}(2\beta+5)\right] \quad (8)$$

Now utilizing Eq (6) and Eq (7) variance can be established as

$$var(z) = E(Z^2) - [E(Z)]^2 \quad (9)$$

The other significant key property is the inverse CDF, however the equation given below does not have closed form solution.

$$TG(z) = U \quad (10)$$

The Eq (9) is nonlinear and not analytically invertible, so there is no closed-form inverse CDF. So some numerical approximation methods using R package can be used to generate random samples from the TLD. Using the R program, 30 random samples from the TLD were generated with a fixed seed set to 123, as presented in Table 1.

Table 1: Random data generation from the transmuted Lindely model

Simulated samples from conventional model				
0.1685	0.7776	0.2609	1.0884	1.4539
0.0233	0.3725	1.1332	0.3977	0.3025
1.6312	0.2995	0.5631	0.4219	0.0542
1.1714	0.1405	0.0215	0.1973	1.6021
1.1190	0.5876	0.5084	2.7622	0.5301
0.6142	0.3896	0.4476	0.1696	0.0793
1.7171	1.1848	0.5841	0.7954	0.0125
0.3222	0.7099	0.1213	0.1902	0.1310

Table 1 indicates some random values which are generated from the conventional model with parameter setting $\beta = 2$ and $\delta = 0.5$. Many other random data can be found with variety of parameters and seed settings.

As the transmuted Lindley model has significant applications in reliability theory. Therefore, it is important to study some key functions commonly used in this domain. The reliability function of the proposed model can be written as:

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$$\Psi(z) = \frac{\beta+1+\beta z}{\beta+1} e^{-\beta z}\left[(1-\delta) + \delta \cdot \frac{\beta+1+\beta z}{\beta+1} e^{-\beta z}\right] \tag{11}$$

The reliability function is a basic concept in reliability engineering and survival analysis. It signifies the likelihood that a system, component or process will meet a reliability objective by performing the required function without failure for a specified measure of time. This role is fundamental in developing and testing the durability and performance of products in all fields including manufacturing, health care, aerospace, and electronics. Through the reliability function, engineers and designers are able to estimate the life of a product, optimize its maintenance scheduling, reduce costs and enhance safety. It is also a main part when it comes to the comparison of different designs or models under which their competitiveness is determined in terms of survival probability over time. In other words, the reliability function makes it possible to have better planning, evaluation of risk, and quality control in the theoretical and applied situations. Additional related concept is hazard rate function, which can be written as:

$$\eta(z) = \frac{\beta^2(1+z)}{\beta+1+\beta z} \cdot \frac{1-\delta+2\delta \cdot \frac{\beta+1+\beta z}{\beta+1} e^{-\beta z}}{\delta-1-\delta \cdot \frac{\beta+1+\beta z}{\beta+1} e^{-\beta z}} \tag{12}$$

The hazard rate function (or failure rate function) is an important subject in reliability theory and survival analysis. It indicates the instantaneous rate of failure at any time t, given the item or system survived up to time t. Essentially it tells us how likely something is to fail in the next instant, if so, far it has not yet failed? This role enables researchers, practitioners, and analysts to grasp and model the behavior of system components over time, particularly in systems where failure is an issue. The other properties can be established based on expression given in Eq (4).

3. Neutrosophic Transmuted Lindley Model

In this section, we present the neutrosophic extension of the classical TLD using neutrosophic logic. The classical PDF is constructed on the base of accurate and full information. It is believed that any random event is associated to outcomes that can be characterized by exact probabilities whose sum is equal to one. 1 This form fits when uncertainty is purely random, and the data is unambiguous, complete, and well defined. Neutrosophic Probability Distributions, on the contrary, can deal with the more general types of uncertainty where vague, incomplete, inconsistent and indeterminate information is concerned. In comparison with the traditional models, neutrosophic distributions consider not just true information, but also provide a treatment for what is indeterminate or unknown.

The NTLD can be expressed as:

$$g_N(z) = \frac{\beta_N^2}{\beta_N+1}(1+z)e^{-\beta_N z}\left(1-\delta+2\delta\frac{\beta_N+1+\beta_N z}{\beta_N+1} e^{-\beta_N z}\right) \tag{13}$$

Note that $\beta_N = [\beta_l, \beta_u]$ is assumed to be in interval form and no accurate information is available for it whereas the other transmuted parameter δ is a crip value between 0 and 1. The neutrosophic structure of PDF for different values of shape parameter is given in Figure 4.

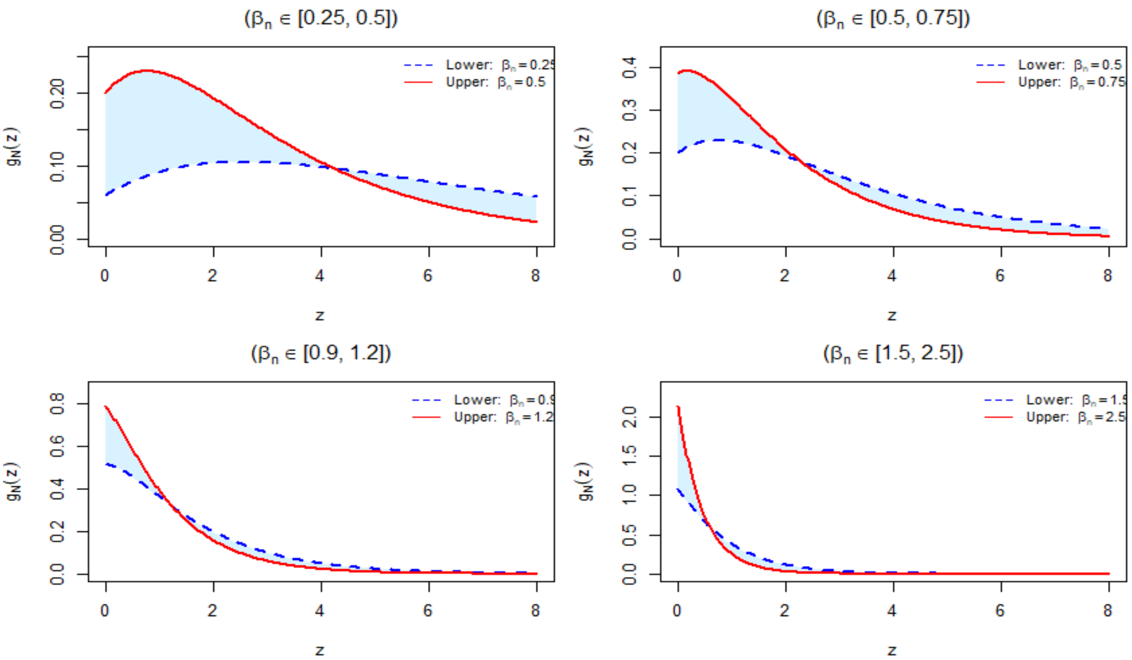

Figure 4. Neutrosophic PDF curves of the proposed model

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Figure 4 shows the panel plots of the neutrosophic PDF of the proposed model for different intervals of the neutrosophic parameter β_N with a fixed value of δ. Each panel corresponds to a specific interval of β_N , namely [0.25,0.5], [0.5, 0.75], [0.9,1.2] and [1.5, 2.5]. The shaded region between the two curves represents the neutrosophic uncertainty zone of the neutrosophic PDF. This visualization indicates uncertainty and variability in the distribution shape under the neutrosophic environment.

Associated function with PDF is CDF, which is under neutrosophic framework, can be written as:

$$G_N(z) = \left(1 - \frac{\beta_n+1+\beta_n z}{\beta_n+1} e^{-\beta_n z}\right)\left(1 + \delta \frac{\beta_n+1+\beta_n z}{\beta_n+1} e^{-\beta_n z}\right) \quad (14)$$

The CDF of the TLD under neutrosophic form is the probability that a random variable is less than or equal to a particular value with uncertainties in model parameters. In this setting, the traditional parameter is encoded by a neutrosophic and quantified with a range or vagueness, which is usual in realistic data. This altered CDF includes randomness but also uncertainty such as cause from incomplete, inconsistent, or imprecise information. The neutrosophic logic allows for flexibility and adaptability of the model, thus making it applicable in problems with data deviation from exact patterns (structures) or models of processes with vague or uncertain characteristics. The graphical structure of CDF curves is shown in Figure 5.

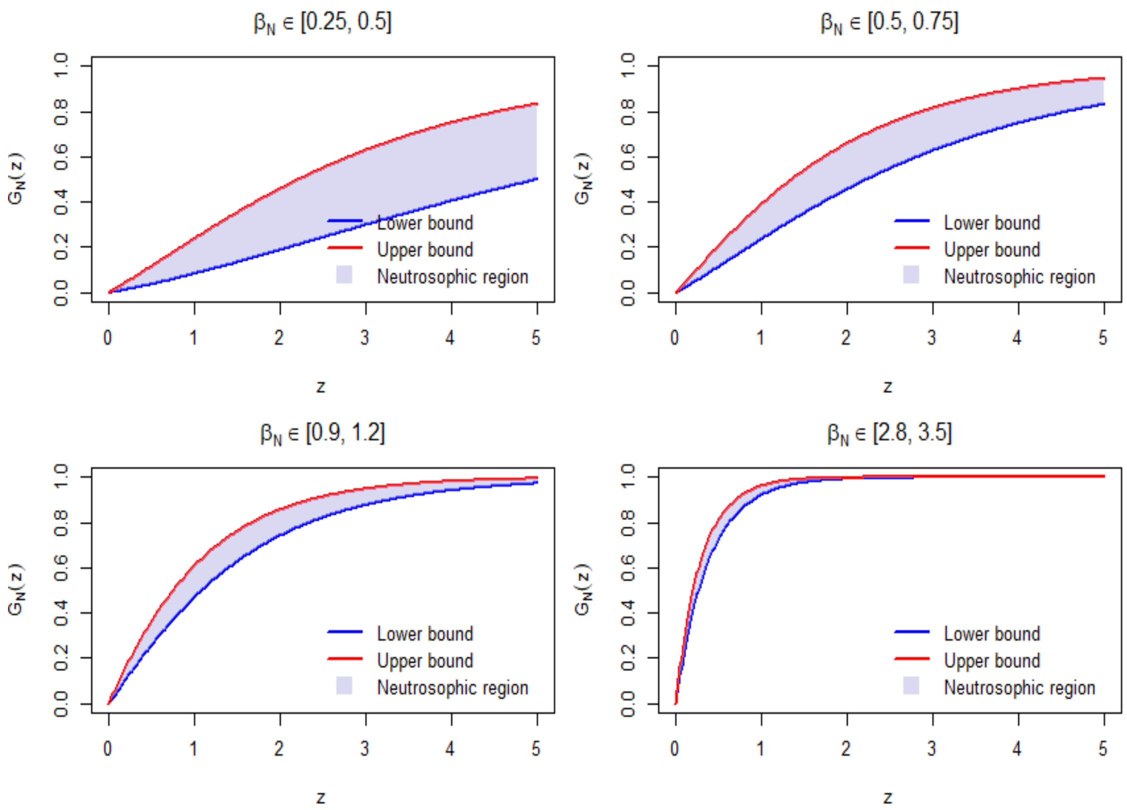

Figure 5. Neutrosophic CDF curves of the proposed model

Figure 5 illustrates the cumulative performance of the NTLD for various intervals of the shape parameter when the transmutation phenomenon is held fixed. The plot is broken down in four panels, each of them displaying the evolution of the distribution when the parameter takes value in a given range. The shaded region between lower and upper curves in each panel represents the uncertainty or indeterminacy stemming from the imprecision in the parameter values. With a shape parameter, increasing the curves become steeper, and the probability accumulates faster for smaller values of the variable. It is visual from a figure that the distribution varies under uncertainty and shows the adaptability of neutrosophic modeling.

The rth moment of the NTLD can be written as:

$$E(Z^r) = \frac{r!}{\beta_N^r(\beta_N+1)}\left[(1-\delta)(\beta_N + r + 1) + \delta \cdot \frac{\beta_N}{2^{r-1}(\beta_N+1)}(2\beta_N + 3\beta_N + r)\right] \quad (15)$$

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Utilizing the following relation we can find mean, variance , skewness and kurtosis of the proposed model:

$$\mu_1 = E(Z)$$

$$\mu_2 = E(Z^2) - \mu_1^2$$

$$\mu_3 = E[(Z-\mu_1)^3] = E(Z^3) - 3\mu_1 E(Z^2) + 2\mu_1^3$$

$$\mu_4 = E[(Z-\mu_1)^4] = E(Z^4) - 4\mu_1 E(Z^3) + 6\mu_1^2 E(Z^2) - 3\mu_1^4 \quad (16)$$

If we assume four different values β_N at fixed value of transmuted parameter δ equal to 0.2, statistical characteristics of the proposed are given in Table 2.

Table 2: Statistical characteristics of neutrosophic transmuted Lindley distribution

β_N	Mean	Variance	Skewness	Kurtosis
[0.25, 0.5]	[2.977, 6.048]	[6.866, 31.645]	[1.472, 1.689]	[6.139, 7.092]
[0.5, 0.75]	[1.986, 2.977]	[2.65, 6.862]	[1.689, 1.953]	[7.092, 8.216]
[1.5, 2]	[0.777, 1.018]	[0.195, 0.445]	[3.153, 4.442]	[12.760, 17.139]
[3, 4.5]	[0.365, 0.533]	[0.010, 0.053]	[9.166, 33.08]	[31.984, 106.85]

Table 2 provides a summary of some essential characteristics of the proposed model for range of different neutrosophic parameter values. For each interval, Table 4 presents interval values for the mean, variance, skewness, and kurtosis. These intervals provide a flexible data description, indicating a range of possibilities instead of a fixed point. We use the mean to understand the central tendency of the data within each range whereas the variance reveals how much the data points scatter. Skewness reflects the symmetrical form of the data, while kurtosis provides an idea of sharpness of the distribution's peak. Noticeable changes can be observed in statistical characteristics as the values of β_N indicating how the behavior of the data shifts across different conditions.

Now neutrosophic reliability functions can be written as:

$$\Psi_N(z) = \frac{\beta_N+1+\beta_N z}{\beta_N+1} e^{-\beta_N z}\left[(1-\delta) + \delta \cdot \frac{\beta_N+1+\beta_N z}{\beta_N+1} e^{-\beta_N z}\right] \quad (17)$$

The neutrosophic reliability function is more flexible than the classical concept since it allows uncertainty and imprecision to be taken directly into the model. Classical reliability involves having specific, fixed values for parameters, whereas the neutrosophic approach accepts possibilities in intervallic sense, vague sense and/or indeterminate sense, giving it an enhanced ability to model real life systems when exact information is unavailable or impractical. This is of particular significance in practical situations characterized by imprecise data, expert feedback, or incomplete information, as it leads to a more general and comprehensive picture of system performance throughout time. The graphical display of reliability can be seen in Figure 6

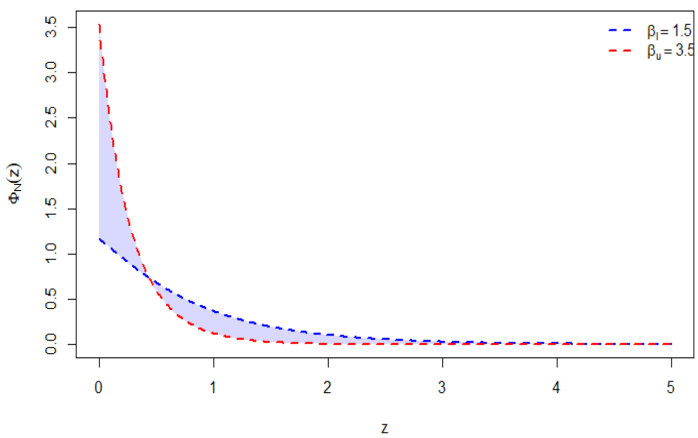

Figure 6. Reliability function of the proposed model

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Figure 6 shows the reliability curve of the proposed model with transmuted parameter equal to 0.3. This complemented part of the CDF curve.

Similarly, neutrosophic hazard function can be obtained as:

$$\eta_N(z) = \frac{\beta_N^2(1+z)}{\beta_N+1+\beta_N z} \cdot \frac{1-\delta+2\delta \cdot \frac{\beta_N+1+\beta_N z}{\beta_N+1} e^{-\beta_N z}}{\delta-1-\delta \cdot \frac{\beta_N+1+\beta_N z}{\beta_N+1} e^{-\beta_N z}} \quad (18)$$

The classical hazard rate function is extended to a neutrosophic hazard rate by including hesitancy and indeterminacy in the parameter. In contrast with the classic model, which assumes that parameters are fixed and fully known, the neutrosophic model permits the parameters such as the rate or the shape to belong to interval numbers. This represents not purely randomness, but instead any type of ambiguity or imprecision which might be present because of limited data, expert disagreements of environmental variability. Thus, the neutrosophic hazard rate can more realistically and flexible reflect the fair wear and tear behavior over time, notably in uncertain or complex system.

4. Real Data Example

In this section, we have utilized our proposed NTLD to energy sector by utilizing electricity consumption data for Saudi Arabia for the time of 2000-2020. Electricity consumption data is an essential tool for understanding energy usage patterns, forecasting demand and proper resourcing. Policymakers, utility companies, and researchers analyze this data to identify peak periods of demand, to the wasteful practices of distributing natural resources more efficiently which causes infrastructure upgrade and shutdowns as a means of keeping the power supply stable. This type of well-timed consumption data also ensures interoperability with wind and solar power generation as an important feature for the balance between supply and demand in newly formatted smart grids. It has essential functions in economic planning, environmental impact assessments and energy-saving policymaking. It is the cornerstone of sustainable development and security of our energy supply. The data in neutrosophic form is available in the source [22]. Neutrosophic structure of this dataset is shown in Figure 7.

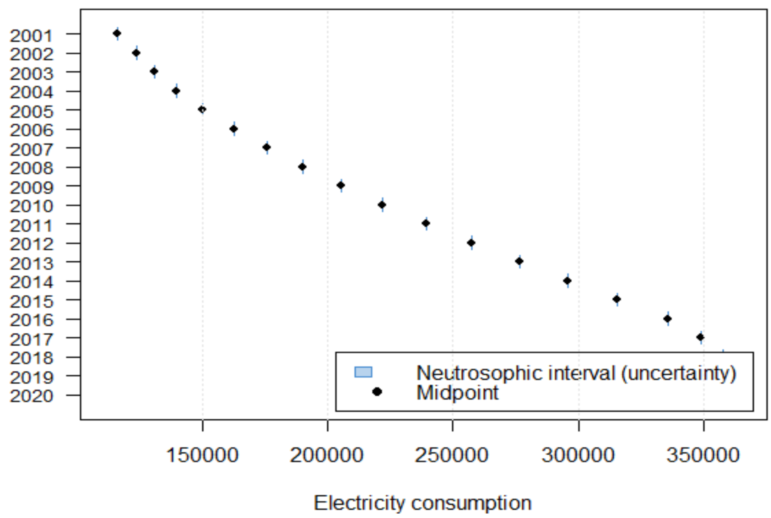

Figure 7. Electricity consumption data for Saudi Arabia for time period 2000-2020

Figure 7 reveals the neutrosophic electricity usage data in Saudi Arabia from 2001 to2020. Every bar shows the full likely value range for that year (shaded region signifies measurement uncertainty or indeterminacy of degrees) Solid points at the midpoint of each year provide the best estimate of consumption for that year. The trend over the two decades is generally clear, with an increase in electricity used that peaked around 2007 followed by a slight decrease more recently. The neutrosophic representation is indeed able to represent both the variability and the uncertainty in the data set, so that users are not only capable of visualizing trends (or patterns) but also corresponding confidence intervals around their reported values. This offers a more comprehensive view than single-point time series, allowing for greater context when making decisions on energy planning and policy. However, the conventional version of a transmuted Lindley model in its crisp form cannot be used for an adequate analysis of Saudi Arabian electricity consumption data illustrated in Figure 7. Crisp models require precise, single-valued observations and are unable to account for the uncertainty reflected in the interval neutrosophic data. Due to this limitation, crisp models neglect the indeterminacy and the potential variability in the measured values, which results in the generation of partial and biased findings. In its turn, the neutrosophic form of the TLD has a more general structure and can work with determinate, indeterminate, and inconsistent information at the same

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time. This ability helps to include in the analysis the uncertainty that exists in interval data and ensure a more relevant examination of general patterns. Now based on neutrosophic data the estimated mean of the proposed model is given by:

$\mu_N = [\, 238125.2\, ,\ 238126.3\,]$

The neutrosophic mean interval shows the region that covers almost all-average electricity consumption about its true location, taking in consideration the uncertainty of the interval-valued data. The lower bound and the upper bound of the expected consumption is the minimum and maximum respectively expected from separately neutrosophic transmuted Lindley model parameters fitted from either lower or upper sides of data values. Taken together, these ranges provide a more complete representation of central tendency in the presence of data uncertainty as compared to traditional single-value estimates, thus allowing for with much more justifiable decisions and interpretation.

5. Conclusion

A neutrosophic version of the TLD has been proposed in this work to accommodate indeterminate, imprecise and vague data. The proposed NTLD is more flexible and can be widely accepted to fit the statistical data. The work provides an analytical explanation and intuitive plot of how this model is more flexible towards uncertainties in practice. Main statistical characteristics such as the PDF, CDF, HF and HRF have been developed. A numerical approach has been established for finding distribution moments. Moments are numerically investigated for different ranges of the neutrosophic parameters. The results show that the NTLD can model a variety of data behaviors and the proposed NTLD can be used as a general framework for representing the complexity of the system, in those areas such as reliability, healthcare survival, and risk assessment, where uncertainty is envisaged as an integral aspect of the process. This paper enriches neutrosophic statistics and provides some new directions for uncertain data modeling. Lastly, we inspect neutrosophic electricity consumption data of energy sector to understand the inner uncertainty and variation more precisely and broadly. This provides more detailed reading compared to conventional methods. Proposed model gives also other information and is a wider scale model of current Model.

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Conflicts of Interest: The authors declare no conflict of interest.

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